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    2. Greatest Common Factor and Factor by Grouping
    3. Factor Quadratic Trinomials with Leading Coefficient 1
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    6. General Strategy for Factoring Polynomials

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7. Rational Expressions
    1. Introduction
    2. Simplify Rational Expressions
    3. Multiply and Divide Rational Expressions
    4. Add and Subtract Rational Expressions with a Common Denominator
    5. Add and Subtract Rational Expressions with Unlike Denominators
    6. Review & Practice Test
8. Square Root Expressions
    1. Introduction
    2. Simplify and Use Square Roots
    3. Simplify Square Root
    4. Review & Practice Test

Introduction class="introduction"

In order to be structurally sound, the foundation of a building must be carefully constructed.

Just like a building needs a firm foundation to support it, your study of algebra needs to have a firm foundation. To ensure this, we begin this book with a review of arithmetic operations with whole numbers, integers, fractions, and decimals, so that you have a solid base that will support your study of algebra.

Introduction to Whole Numbers
By the end of this section, you will be able to:

- Use place value with whole numbers
- Identify multiples and and apply divisibility tests
- Find prime factorizations and least common multiples

As we begin our study of elementary algebra, we need to refresh some of our skills and vocabulary. This chapter will focus on whole numbers, integers, fractions, decimals, and real numbers. We will also begin our use of algebraic notation and vocabulary.

Use Place Value with Whole Numbers

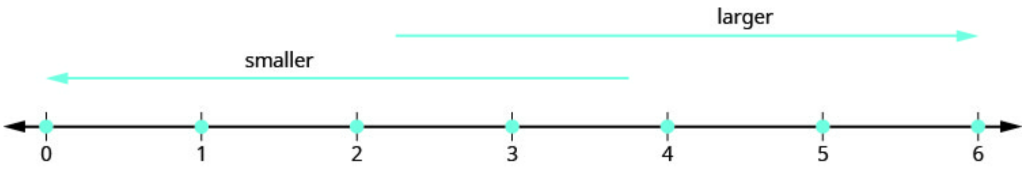
The most basic numbers used in algebra are the numbers we use to count objects in our world: 1, 2, 3, 4, and so on. These are called the **counting numbers**. Counting numbers are also called *natural numbers*. If we add zero to the counting numbers, we get the set of **whole numbers**.

Counting Numbers: 1, 2, 3, …

Whole Numbers: 0, 1, 2, 3, …

The notation "…" is called ellipsis and means "and so on," or that the pattern continues endlessly.

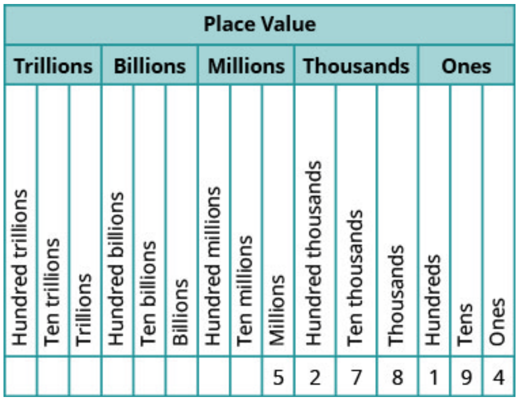
We can visualize counting numbers and whole numbers on a **number line** (see [link]).



The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left. While this number line shows only the whole numbers 0 through 6, the numbers keep going without end.

Note: Doing the Manipulative Mathematics activity "Number Line-Part 1" will help you develop a better understanding of the counting numbers and the whole numbers.

Our number system is called a place value system, because the value of a digit depends on its position in a number. [link] shows the place values. The place values are separated into groups of three, which are called periods. The periods are *ones*, *thousands*, *millions*, *billions*, *trillions*, and so on. In a written number, commas separate the periods.



Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

The number 5,278,194 is shown in the chart. The digit 5 is in the millions place. The digit 2 is in the hundred-thousands place. The digit 7 is in the ten-thousands place. The digit 8 is in the thousands place. The digit 1 is in the hundreds place. The digit 9 is in the tens place. The digit 4 is in the ones place.

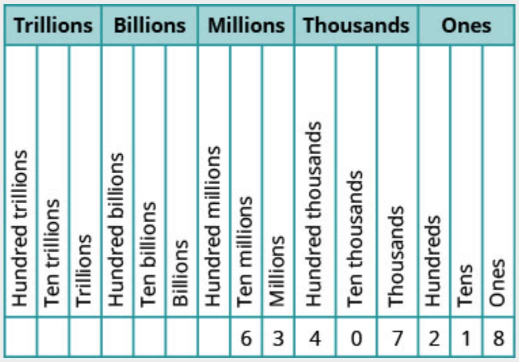
Example:
Exercise:

Problem: **In the number 63,407,218, find the place value of each digit:

ⓐ 7
ⓑ 0
ⓒ 1
ⓓ 6
ⓔ 3

Solution:
Solution

Place the number in the place value chart:



Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

ⓐ The 7 is in the thousands place.
ⓑ The 0 is in the ten thousands place.
ⓒ The 1 is in the tens place.
ⓓ The 6 is in the ten-millions place.
ⓔ The 3 is in the millions place.

Note:
Exercise:

TRY IT :: 1.1

Problem: For the number 27,493,615, find the place value of each digit:

ⓐ 2 ⓑ 1 ⓒ 4 ⓓ 7 ⓔ 5

Solution:

ⓐ ten millions ⓑ tens ⓒ hundred thousands ⓓ millions ⓔ ones

Note:
Exercise:

TRY IT :: 1.2

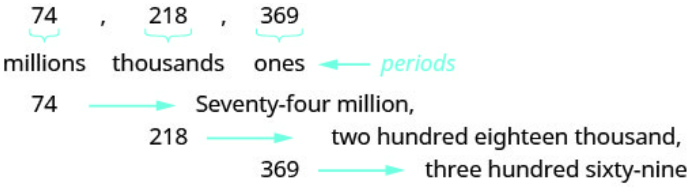
Problem: For the number 519,711,641,328, find the place value of each digit:

ⓐ 9 ⓑ 4 ⓒ 2 ⓓ 6 ⓔ 7

Solution:

ⓐ billions ⓑ ten thousands ⓒ tens ⓓ hundred thousands ⓔ hundred millions

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period, followed by the name of the period, without the *s* at the end. Start at the left, where the periods have the largest value. The ones period is not named. The commas separate the periods, so wherever there is a comma in the number, put a comma between the words (see [link]). The number 74,218,369 is written as seventy-four million, two hundred eighteen thousand, three hundred sixty-nine.



Note:

Name a Whole Number in Words.

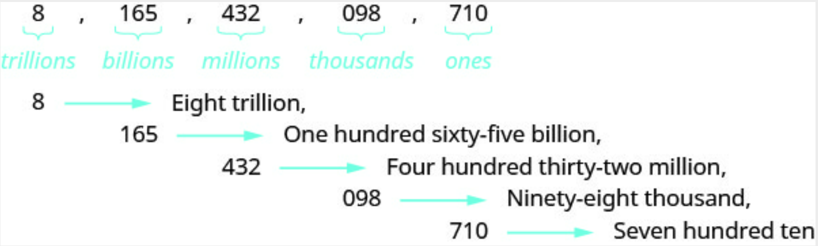
Start at the left and name the number in each period, followed by the period name.
Put commas in the number to separate the periods.
Do not name the ones period.

Example:
Exercise:

Problem: **Name the number 8,165,432,098,710 using words.

Solution:
Solution

Name the number in each period, followed by the period name.



Put the commas in to separate the periods.

So, $8, 165, 432, 098, 710$ is named as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Note:
Exercise:

TRY IT :: 1.3
Problem: Name the number $9, 258, 137, 904, 061$ using words.

Solution:

nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

Note:
Exercise:

TRY IT :: 1.4
Problem: Name the number $17, 864, 325, 619, 004$ using words.

Solution:

seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand four

We are now going to reverse the process by writing the digits from the name of the number. To write the number in digits, we first look for the clue words that indicate the periods. It is helpful to draw three blanks for the needed periods and then fill in the blanks with the numbers, separating the periods with commas.

Note:
Write a Whole Number Using Digits.

Identify the words that indicate periods. (Remember, the ones period is never named.)
Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Name the number in each period and place the digits in the correct place value position.

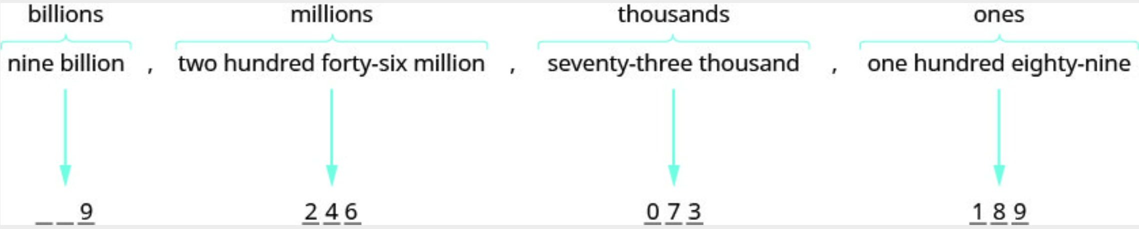
Example:
Exercise:

Problem:

**Write *nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine* as a whole number using digits.

Solution:
Solution

Identify the words that indicate periods.
Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Then write the digits in each period.



The number is 9,246,073,189.

Note:
Exercise:

Problem:

TRY IT :: 1.5
Write the number two billion, four hundred sixty-six million, seven hundred fourteen thousand, fifty-one as a whole number using digits.

Solution:

2,466,714,051

Note:
Exercise:

Problem:

TRY IT :: 1.6
Write the number eleven billion, nine hundred twenty-one million, eight hundred thirty thousand, one hundred six as a whole number using digits.

Solution:

11,921,830,106

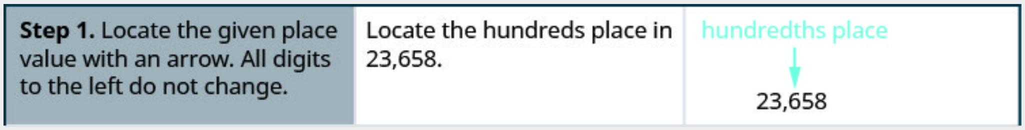
In 2013, the U.S. Census Bureau estimated the population of the state of New York as 19,651,127. We could say the population of New York was approximately 20 million. In many cases, you don't need the exact value; an approximate number is good enough.

The process of approximating a number is called rounding. Numbers are rounded to a specific place value, depending on how much accuracy is needed. Saying that the population of New York is approximately 20 million means that we rounded to the millions place.

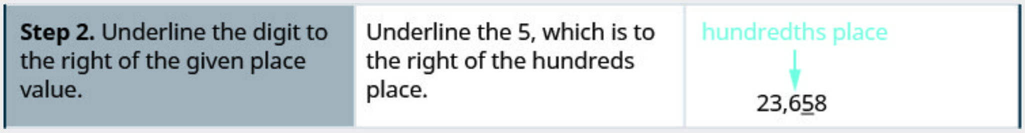
Example:
How to Round Whole Numbers
Exercise:

Problem: **Round 23,658 to the nearest hundred.

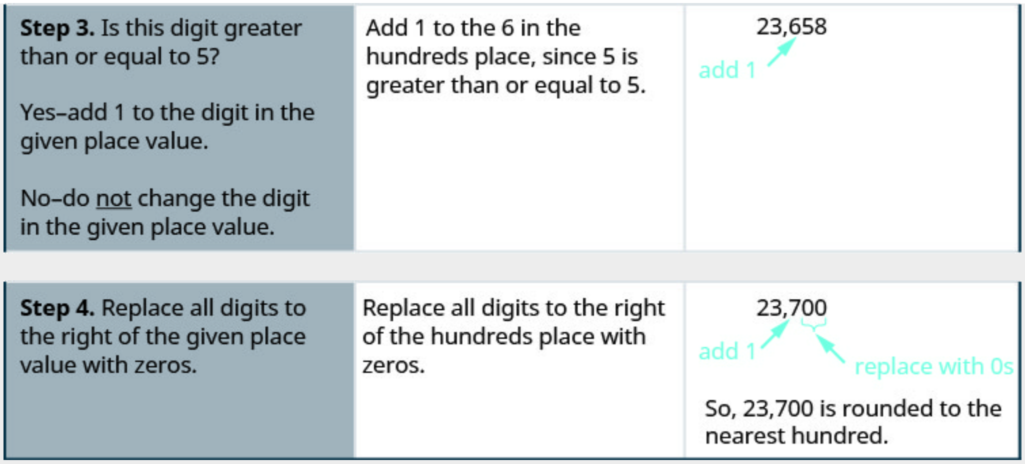
Solution:
Solution



Step 1. Locate the given place value with an arrow. All digits to the left do not change.	Locate the hundreds place in 23,658.	hundredths place ↓ 23,658



Step 2. Underline the digit to the right of the given place value.	Underline the 5, which is to the right of the hundreds place.	hundredths place ↓ $23{,}6\underline{5}8$



Step 3. Is this digit greater than or equal to 5? Yes–add 1 to the digit in the given place value. No–do not change the digit in the given place value.	Add 1 to the 6 in the hundreds place, since 5 is greater than or equal to 5.	23,658 add 1
Step 4. Replace all digits to the right of the given place value with zeros.	Replace all digits to the right of the hundreds place with zeros.	23,700 add 1 replace with 0s So, 23,700 is rounded to the nearest hundred.

Note:
Exercise:

TRY IT :: 1.7

Problem: Round to the nearest hundred: 17,852.

Solution:

17,900

Note:
Exercise:

TRY IT :: 1.8

Problem: Round to the nearest hundred: 468,751.

Solution:

468,800

Note:
Round Whole Numbers.

Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
Underline the digit to the right of the given place value.
Is this digit greater than or equal to 5?

- Yes–add 1 to the digit in the given place value.
- No–do not change the digit in the given place value.

Replace all digits to the right of the given place value with zeros.

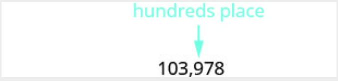
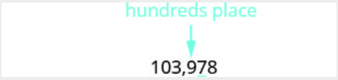
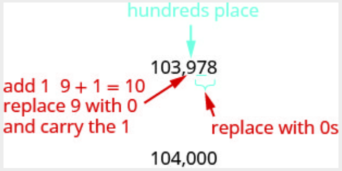
Example:
Exercise:

Problem: **Round 103,978 to the nearest:

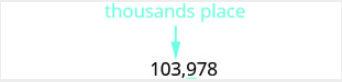
ⓐ hundred
ⓑ thousand
ⓒ ten thousand

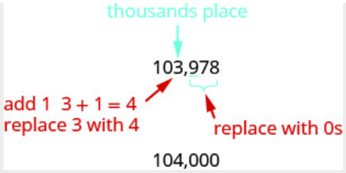
Solution:
Solution

ⓐ

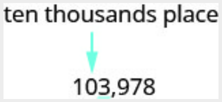
Locate the hundreds place in 103,978.	
Underline the digit to the right of the hundreds place.	
Since 7 is greater than or equal to 5, add 1 to the 9. Replace all digits to the right of the hundreds place with zeros.	 hundreds place 103,978 add 1 9 + 1 = 10 replace 9 with 0 and carry the 1 replace with 0s 104,000
	So, 104,000 is 103,978 rounded to the nearest hundred.

ⓑ

Locate the thousands place and underline the digit to the right of the thousands place.	

Since 9 is greater than or equal to 5, add 1 to the 3. Replace all digits to the right of the hundreds place with zeros.	 thousands place 103,978 add 1 3 + 1 = 4 replace 3 with 4 replace with 0s 104,000
	So, 104,000 is 103,978 rounded to the nearest thousand.

ⓒ

Locate the ten thousands place and underline the digit to the right of the ten thousands place.	
Since 3 is less than 5, we leave the 0 as is, and then replace the digits to the right with zeros.	100,000
	So, 100,000 is 103,978 rounded to the nearest ten thousand.

Note:
Exercise:

TRY IT :: 1.9
Problem: Round 206,981 to the nearest: ⓐ hundred ⓑ thousand ⓒ ten thousand.

Solution:

ⓐ 207,000 ⓑ 207,000 ⓒ 210,000

Note:
Exercise:

TRY IT :: 1.10
Problem: Round 784,951 to the nearest: ⓐ hundred ⓑ thousand ⓒ ten thousand.

Solution:

ⓐ 785,000 ⓑ 785,000 ⓒ 780,000

Identify Multiples and Apply Divisibility Tests

The numbers 2, 4, 6, 8, 10, and 12 are called **multiples** of 2. A multiple of 2 can be written as the product of a counting number and 2.

2,	4,	6,	8,	10,	12, …
2 • 1,	2 • 2,	2 • 3,	2 • 4,	2 • 5,	2 • 6

Similarly, a multiple of 3 would be the product of a counting number and 3.

3,	6,	9,	12,	15,	18, …
3 • 1,	3 • 2,	3 • 3,	3 • 4,	3 • 5,	3 • 6

We could find the multiples of any number by continuing this process.

Note: Doing the Manipulative Mathematics activity “Multiples” will help you develop a better understanding of multiples.

[link] shows the multiples of 2 through 9 for the first 12 counting numbers.

Counting Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Multiples of 2	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84

Counting Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108
Multiples of 10	10	20	30	40	50	60	70	80	90	100	110	120

Note:
Multiple of a Number
A number is a **multiple** of *n* if it is the product of a counting number and *n*.

Another way to say that 15 is a multiple of 3 is to say that 15 is **divisible** by 3. That means that when we divide 3 into 15, we get a counting number. In fact, $15 \div 3$ is 5, so 15 is $5 \cdot 3$.

Note:
Divisible by a Number
If a number *m* is a multiple of *n*, then *m* is **divisible** by *n*.

Look at the multiples of 5 in [link]. They all end in 5 or 0. Numbers with last digit of 5 or 0 are divisible by 5. Looking for other patterns in [link] that shows multiples of the numbers 2 through 9, we can discover the following divisibility tests:

Note:
Divisibility Tests
A number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8.
- 3 if the sum of the digits is divisible by 3.
- 5 if the last digit is 5 or 0.
- 6 if it is divisible by both 2 and 3.
- 10 if it ends with 0.

Example:
Exercise:

Problem: **Is 5,625 divisible by 2? By 3? By 5? By 6? By 10?

Solution:
Solution

Is 5,625 divisible by 2?	
Does it end in 0,2,4,6, or 8?	No. 5,625 is not divisible by 2.
Is 5,625 divisible by 3?	
What is the sum of the digits?	$5+6+2+5=18$
Is the sum divisible by 3?	Yes. 5,625 is divisble by 3.
Is 5,625 divisible by 5 or 10?	
What is the last digit? It is 5.	5,625 is divisble by 5 but not by 10.
Is 5,625 divisible by 6?	
Is it divisible by both 2 or 3?	No, 5,625 is not divisible by 2, so 5,625 is not divisible by 6.

Note:
Exercise:

TRY IT :: 1.11
Problem: Determine whether 4,962 is divisible by 2, by 3, by 5, by 6, and by 10.

Solution:

by 2, 3, and 6

Note:
Exercise:

TRY IT :: 1.12
Problem: Determine whether 3,765 is divisible by 2, by 3, by 5, by 6, and by 10.

Solution:

by 3 and 5

Find Prime Factorizations and Least Common Multiples

In mathematics, there are often several ways to talk about the same ideas. So far, we've seen that if *m* is a multiple of *n*, we can say that *m* is divisible by *n*. For example, since 72 is a multiple of 8, we say 72 is divisible by 8. Since 72 is a multiple of 9, we say 72 is divisible by 9. We can express this still another way.

Since $8 \cdot 9 = 72$, we say that 8 and 9 are **factors** of 72. When we write $72 = 8 \cdot 9$, we say we have factored 72.

$$\underbrace{8 \cdot 9}_{\text{factors}} = \underbrace{72}_{\text{product}}$$

Other ways to factor 72 are $1 \cdot 72, 2 \cdot 36, 3 \cdot 24, 4 \cdot 18, \text{ and } 6 \cdot 12$. Seventy-two has many factors: 1, 2, 3, 4, 6, 8, 9, 12, 18, 36, and 72.

Note:
Factors
If $a \cdot b = m$, then *a* and *b* are **factors** of *m*.

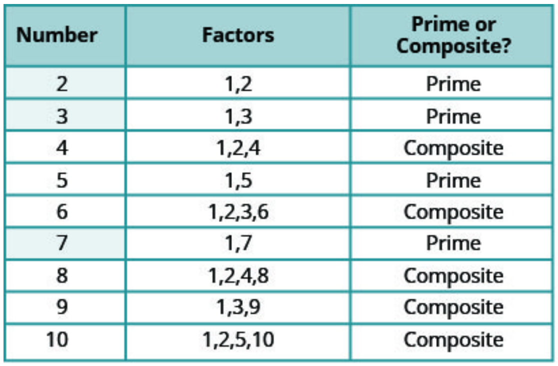
Some numbers, like 72, have many factors. Other numbers have only two factors.

Note: Doing the Manipulative Mathematics activity "Model Multiplication and Factoring" will help you develop a better understanding of multiplication and factoring.

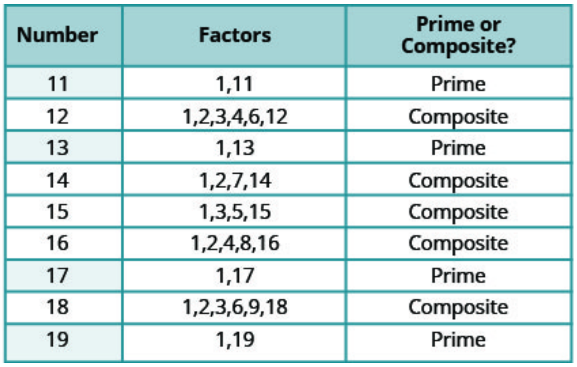
Note:
Prime Number and Composite Number
A **prime number** is a counting number greater than 1, whose only factors are 1 and itself.
A **composite number** is a counting number that is not prime. A composite number has factors other than 1 and itself.

Note: Doing the Manipulative Mathematics activity "Prime Numbers" will help you develop a better understanding of prime numbers.

The counting numbers from 2 to 19 are listed in [link], with their factors. Make sure to agree with the "prime" or "composite" label for each!



Number	Factors	Prime or Composite?
2	1,2	Prime
3	1,3	Prime
4	1,2,4	Composite
5	1,5	Prime
6	1,2,3,6	Composite
7	1,7	Prime
8	1,2,4,8	Composite
9	1,3,9	Composite
10	1,2,5,10	Composite



Number	Factors	Prime or Composite?
11	1,11	Prime
12	1,2,3,4,6,12	Composite
13	1,13	Prime
14	1,2,7,14	Composite
15	1,3,5,15	Composite
16	1,2,4,8,16	Composite
17	1,17	Prime
18	1,2,3,6,9,18	Composite
19	1,19	Prime

The **prime numbers** less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. Notice that the only even prime number is 2.

A composite number can be written as a unique product of primes. This is called the **prime factorization** of the number. Finding the prime factorization of a composite number will be useful later in this course.

Note:
Prime Factorization
The **prime factorization** of a number is the product of prime numbers that equals the number.

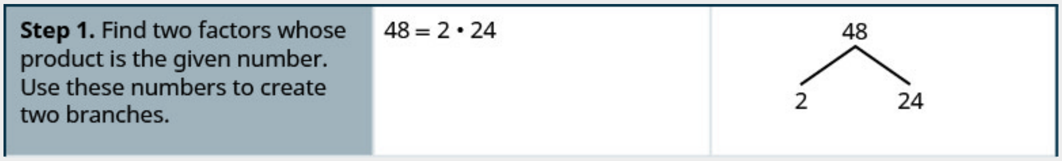
To find the prime factorization of a composite number, find any two factors of the number and use them to create two branches. If a factor is prime, that branch is complete. Circle that prime!

If the factor is not prime, find two factors of the number and continue the process. Once all the branches have circled primes at the end, the factorization is complete. The composite number can now be written as a product of prime numbers.

Example:
How to Find the Prime Factorization of a Composite Number
Exercise:

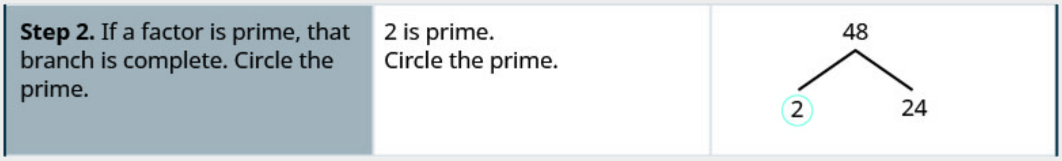
Problem: **Find the prime factorization of 48.

Solution:
Solution



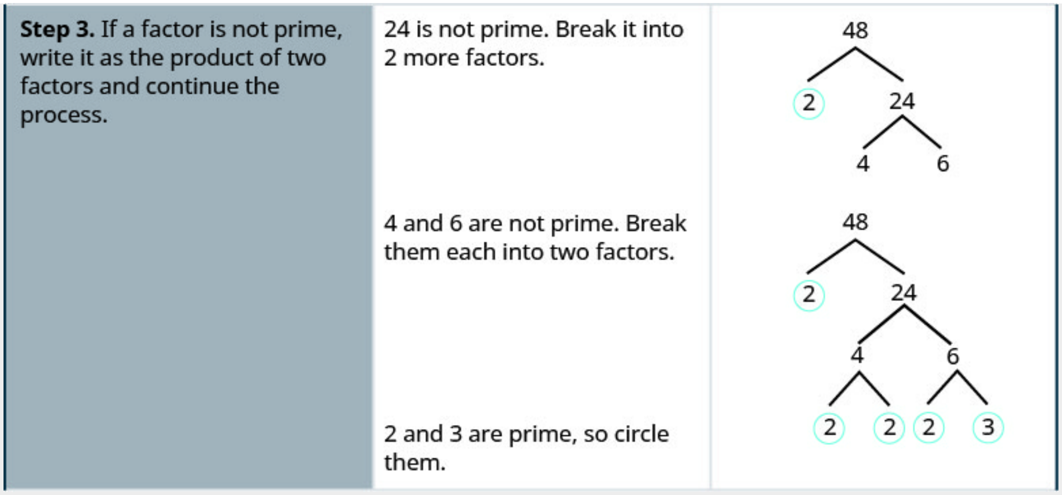
Step 1. Find two factors whose product is the given number. Use these numbers to create two branches.

$48 = 2 \cdot 24$

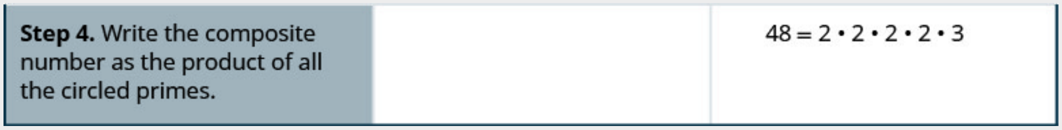


Step 2. If a factor is prime, that branch is complete. Circle the prime.

2 is prime.
Circle the prime.



Step 3. If a factor is not prime, write it as the product of two factors and continue the process.	24 is not prime. Break it into 2 more factors. 4 and 6 are not prime. Break them each into two factors. 2 and 3 are prime, so circle them.	



Step 4. Write the composite number as the product of all the circled primes.		48 = 2 • 2 • 2 • 2 • 3

We say $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ is the prime factorization of 48. We generally write the primes in ascending order. Be sure to multiply the factors to verify your answer!

If we first factored 48 in a different way, for example as $6 \cdot 8$, the result would still be the same. Finish the prime factorization and verify this for yourself.

Note:
Exercise:

TRY IT :: 1.13
Problem: Find the prime factorization of 80.

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

Note:
Exercise:

TRY IT :: 1.14
Problem: Find the prime factorization of 60.

Solution:

$2 \cdot 2 \cdot 3 \cdot 5$

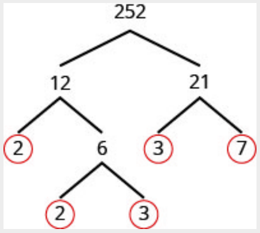
Note:
Find the Prime Factorization of a Composite Number.

Find two factors whose product is the given number, and use these numbers to create two branches.
If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
If a factor is not prime, write it as the product of two factors and continue the process.
Write the composite number as the product of all the circled primes.

Example:
Exercise:

Problem: **Find the prime factorization of 252.

Solution:
Solution

Step 1. Find two factors whose product is 252. 12 and 21 are not prime. Break 12 and 21 into two more factors. Continue until all primes are factored.	
Step 2. Write 252 as the product of all the circled primes.	$252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$

Note:
Exercise:

TRY IT :: 1.15
Problem: Find the prime factorization of 126.

Solution:

$2 \cdot 3 \cdot 3 \cdot 7$

Note:
Exercise:

TRY IT :: 1.16

Problem: Find the prime factorization of 294.

Solution:

$2 \cdot 3 \cdot 7 \cdot 7$

One of the reasons we look at multiples and primes is to use these techniques to find the **least common multiple** of two numbers. This will be useful when we add and subtract fractions with different denominators. Two methods are used most often to find the least common multiple and we will look at both of them.

The first method is the Listing Multiples Method. To find the least common multiple of 12 and 18, we list the first few multiples of 12 and 18:

12: 12, 24, **36**, 48, 60, **72**, 84, 96, **108**...

18: 18, **36**, 54, **72**, 90, **108**...

Common Multiples: 36, 72, 108...

Least Common Multiple: 36

Notice that some numbers appear in both lists. They are the **common multiples** of 12 and 18.

We see that the first few common multiples of 12 and 18 are 36, 72, and 108. Since 36 is the smallest of the common multiples, we call it the *least common multiple.* We often use the abbreviation LCM.

Note:
Least Common Multiple
The **least common multiple** (LCM) of two numbers is the smallest number that is a multiple of both numbers.

The procedure box lists the steps to take to find the LCM using the prime factors method we used above for 12 and 18.

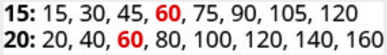
Note:
Find the Least Common Multiple by Listing Multiples.

List several multiples of each number.
Look for the smallest number that appears on both lists.
This number is the LCM.

Example:
Exercise:

Problem: **Find the least common multiple of 15 and 20 by listing multiples.

Solution:
Solution

Make lists of the first few multiples of 15 and of 20, and use them to find the least common multiple.	 **15:** 15, 30, 45, 60, 75, 90, 105, 120 **20:** 20, 40, 60, 80, 100, 120, 140, 160
Look for the smallest number that appears in both lists.	The first number to appear on both lists is 60, so 60 is the least common multiple of 15 and 20.

Notice that 120 is in both lists, too. It is a common multiple, but it is not the *least* common multiple.

Note:
Exercise:

TRY IT :: 1.17
Problem: Find the least common multiple by listing multiples: 9 and 12.

Solution:

36

Note:
Exercise:

TRY IT :: 1.18
Problem: Find the least common multiple by listing multiples: 18 and 24.

Solution:

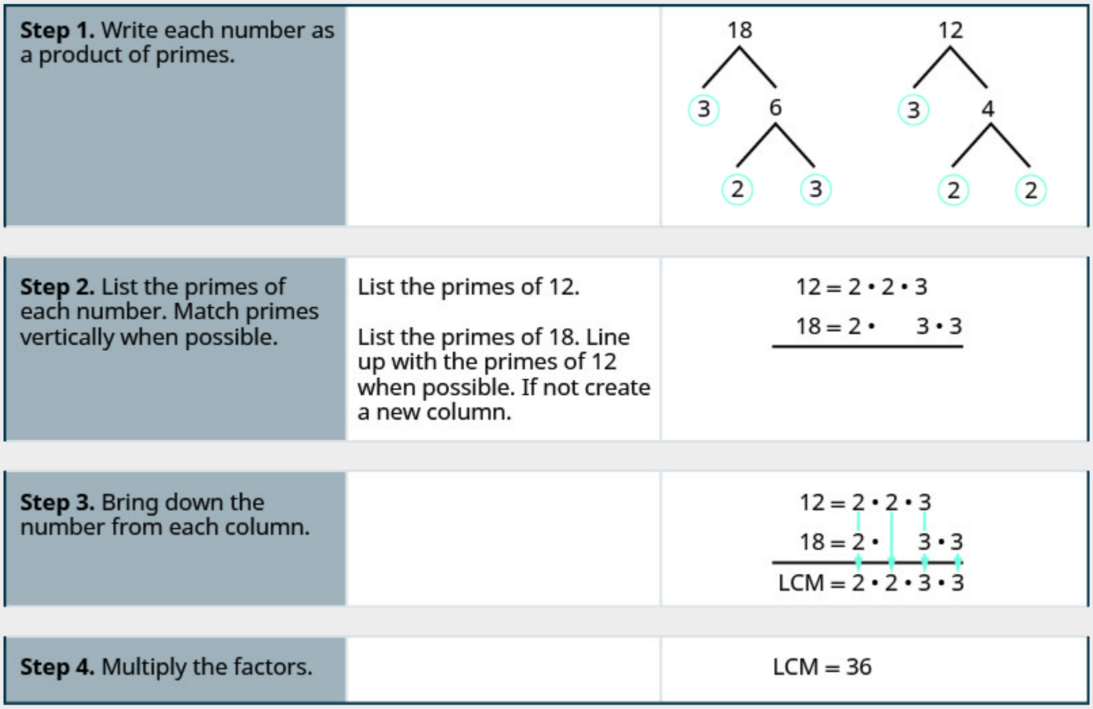
72

Our second method to find the least common multiple of two numbers is to use The Prime Factors Method. Let's find the LCM of 12 and 18 again, this time using their prime factors.

Example:
How to Find the Least Common Multiple Using the Prime Factors Method
Exercise:

Problem: **Find the Least Common Multiple (LCM) of 12 and 18 using the prime factors method.

Solution:
Solution



Step 1. Write each number as a product of primes.		18 3 6 2 3 12 3 4 2 2
Step 2. List the primes of each number. Match primes vertically when possible.	List the primes of 12. List the primes of 18. Line up with the primes of 12 when possible. If not create a new column.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$
Step 3. Bring down the number from each column.		$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3$
Step 4. Multiply the factors.		$\text{LCM} = 36$

Notice that the prime factors of 12 $(2 \cdot 2 \cdot 3)$ and the prime factors of 18 $(2 \cdot 3 \cdot 3)$ are included in the LCM $(2 \cdot 2 \cdot 3 \cdot 3)$. So 36 is the least common multiple of 12 and 18.

By matching up the common primes, each common prime factor is used only once. This way you are sure that 36 is the *least* common multiple.

Note:
Exercise:

TRY IT :: 1.19
Problem: Find the LCM using the prime factors method: 9 and 12.

Solution:

36

Note:
Exercise:

TRY IT :: 1.20

Problem: Find the LCM using the prime factors method: 18 and 24.

Solution:

72

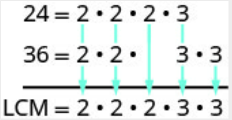
Note:
Find the Least Common Multiple Using the Prime Factors Method.

Write each number as a product of primes.
List the primes of each number. Match primes vertically when possible.
Bring down the columns.
Multiply the factors.

Example:
Exercise:

Problem: **Find the Least Common Multiple (LCM) of 24 and 36 using the prime factors method.

Solution:
Solution

Find the primes of 24 and 36. Match primes vertically when possible. Bring down all columns.	 24 = 2 • 2 • 2 • 3 36 = 2 • 2 • 3 • 3 LCM = 2 • 2 • 2 • 3 • 3
Multiply the factors.	$\text{LCM} = 72$
	The LCM of 24 and 36 is 72.

Note:
Exercise:

TRY IT :: 1.21

Problem: Find the LCM using the prime factors method: 21 and 28.

Solution:

84

Note:
Exercise:

TRY IT :: 1.22

Problem: Find the LCM using the prime factors method: 24 and 32.

Solution:

96

Note:
Access this online resource for additional instruction and practice with using whole numbers. You will need to enable Java in your web browser to use the application.

- Sieve of Eratosthenes

Key Concepts

- **Place Value** as in [link].
- **Name a Whole Number in Words**

  Start at the left and name the number in each period, followed by the period name.
  Put commas in the number to separate the periods.
  Do not name the ones period.

- **Write a Whole Number Using Digits**

  Identify the words that indicate periods. (Remember the ones period is never named.)
  Draw 3 blanks to indicate the number of places needed in each period. Separate the periods by commas.
  Name the number in each period and place the digits in the correct place value position.

- **Round Whole Numbers**

  Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
  Underline the digit to the right of the given place value.
  Is this digit greater than or equal to 5?
    - Yes—add 1 to the digit in the given place value.
    - No—do not change the digit in the given place value.

  Replace all digits to the right of the given place value with zeros.

- **Divisibility Tests:** A number is divisible by:
  - 2 if the last digit is 0, 2, 4, 6, or 8.

  - 3 if the sum of the digits is divisible by 3.
  - 5 if the last digit is 5 or 0.
  - 6 if it is divisible by both 2 and 3.
  - 10 if it ends with 0.
- **Find the Prime Factorization of a Composite Number**

  Find two factors whose product is the given number, and use these numbers to create two branches.
  If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
  If a factor is not prime, write it as the product of two factors and continue the process.
  Write the composite number as the product of all the circled primes.
- **Find the Least Common Multiple by Listing Multiples**

  List several multiples of each number.
  Look for the smallest number that appears on both lists.
  This number is the LCM.
- **Find the Least Common Multiple Using the Prime Factors Method**

  Write each number as a product of primes.
  List the primes of each number. Match primes vertically when possible.
  Bring down the columns.
  Multiply the factors.

Practice Makes Perfect

Use Place Value with Whole Numbers

In the following exercises, find the place value of each digit in the given numbers.

Exercise:

51,493
ⓐ 1
ⓑ 4
ⓒ 9
ⓓ 5
Problem: ⓔ 3

Solution:

ⓐ thousands ⓑ hundreds ⓒ tens ⓓ ten thousands ⓔ ones

Exercise:

87,210
ⓐ 2
ⓑ 8
ⓒ 0
ⓓ 7
Problem: ⓔ 1

Exercise:

164,285
ⓐ 5
ⓑ 6
ⓒ 1
ⓓ 8
Problem: ⓔ 2

Solution:

ⓐ ones ⓑ ten thousands ⓒ hundred thousands ⓓ tens ⓔ hundreds

Exercise:

395,076
ⓐ 5
ⓑ 3
ⓒ 7
ⓓ 0
Problem: ⓔ 9

Exercise:

93,285,170
ⓐ 9
ⓑ 8
ⓒ 7
ⓓ 5
Problem: ⓔ 3

Solution:

ⓐ ten millions ⓑ ten thousands ⓒ tens ⓓ thousands ⓔ millions

Exercise:

36,084,215
ⓐ 8
ⓑ 6
ⓒ 5
ⓓ 4
Problem: ⓔ 3

Exercise:

7,284,915,860,132
ⓐ 7
ⓑ 4
ⓒ 5
ⓓ 3
Problem: ⓔ 0

Solution:

ⓐ trillions ⓑ billions ⓒ millions ⓓ tens ⓔ thousands

Exercise:

2,850,361,159,433
ⓐ 9
ⓑ 8
ⓒ 6
ⓓ 4
Problem: ⓔ 2

In the following exercises, name each number using words.

Exercise:

Problem: 1,078

Solution:

one thousand, seventy-eight

Exercise:

Problem: 5,902

Exercise:

Problem: 364,510

Solution:

three hundred sixty-four thousand, five hundred ten

Exercise:

Problem: 146,023

Exercise:

Problem: 5,846,103

Solution:

five million, eight hundred forty-six thousand, one hundred three

Exercise:

Problem: 1,458,398

Exercise:

Problem: 37,889,005

Solution:

thirty-seven million, eight hundred eighty-nine thousand, five

Exercise:

Problem: 62,008,465

In the following exercises, write each number as a whole number using digits.

Exercise:

Problem: four hundred twelve

Solution:

412

Exercise:

Problem: two hundred fifty-three

Exercise:

Problem: thirty-five thousand, nine hundred seventy-five

Solution:

35,975

Exercise:

Problem: sixty-one thousand, four hundred fifteen

Exercise:

Problem: eleven million, forty-four thousand, one hundred sixty-seven

---

Solution:

11,044,167

Exercise:

Problem: eighteen million, one hundred two thousand, seven hundred eighty-three

Exercise:

Problem: three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

---

Solution:

3,226,512,017

Exercise:

Problem: eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

In the following, round to the indicated place value.

Exercise:

Problem: Round to the nearest ten.

ⓐ 386 ⓑ 2,931

Solution:

ⓐ 390 ⓑ 2,930

Exercise:

Problem: Round to the nearest ten.

ⓐ 792 ⓑ 5,647

Exercise:

Problem: Round to the nearest hundred.

ⓐ 13,748 ⓑ 391,794

Solution:

ⓐ 13,700 ⓑ 391,800

Exercise:

Problem: Round to the nearest hundred.

ⓐ 28,166 ⓑ 481,628

Exercise:

Problem: Round to the nearest ten.

ⓐ 1,492 ⓑ 1,497

Solution:

ⓐ 1,490 ⓑ 1,500

Exercise:

Problem: Round to the nearest ten.

ⓐ 2,791 ⓑ 2,795

Exercise:

Problem: Round to the nearest hundred.

ⓐ 63,994 ⓑ 63,940

Solution:

ⓐ 64,000 ⓑ 63,900

Exercise:

Problem: Round to the nearest hundred.

ⓐ 49,584 ⓑ 49,548

In the following exercises, round each number to the nearest ⓐ hundred, ⓑ thousand, ⓒ ten thousand.

Exercise:

Problem: 392,546

Solution:

ⓐ 392,500 ⓑ 393,000 ⓒ 390,000

Exercise:

Problem: 619,348

Exercise:

Problem: 2,586,991

Solution:

ⓐ 2,587,000 ⓑ 2,587,000 ⓒ 2,590,000

Exercise:

Problem: 4,287,965

Identify Multiples and Factors

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, 3, 5, 6, and 10.

Exercise:

Problem: 84

Solution:

divisible by 2, 3, and 6

Exercise:

Problem: 9,696

Exercise:

Problem: 75

Solution:

divisible by 3 and 5

Exercise:

Problem: 78

Exercise:

Problem: 900

Solution:

divisible by 2, 3, 5, 6, and 10

Exercise:

Problem: 800

Exercise:

Problem: 986

Solution:

divisible by 2

Exercise:

Problem: 942

Exercise:

Problem: 350

Solution:

divisible by 2, 5, and 10

Exercise:

Problem: 550

Exercise:

Problem: 22,335

Solution:

divisible by 3 and 5

Exercise:

Problem: 39,075

Find Prime Factorizations and Least Common Multiples

In the following exercises, find the prime factorization.

Exercise:

Problem: 86

Solution:

$2 \cdot 43$

Exercise:

Problem: 78

Exercise:

Problem: 132

Solution:

$2 \cdot 2 \cdot 3 \cdot 11$

Exercise:

Problem: 455

Exercise:

Problem: 693

Solution:

$3 \cdot 3 \cdot 7 \cdot 11$

Exercise:

Problem: 400

Exercise:

Problem: 432

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

Exercise:

Problem: 627

Exercise:

Problem: 2,160

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$

Exercise:

Problem: 2,520

In the following exercises, find the least common multiple of the each pair of numbers using the multiples method.

Exercise:

Problem: 8, 12

Solution:

24

Exercise:

Problem: 4, 3

Exercise:

Problem: 12, 16

Solution:

48

Exercise:

Problem: 30, 40

Exercise:

Problem: 20, 30

Solution:

60

Exercise:

Problem: 44, 55

In the following exercises, find the least common multiple of each pair of numbers using the prime factors method.

Exercise:

Problem: 8, 12

Solution:

24

Exercise:

Problem: 12, 16

Exercise:

Problem: 28, 40

Solution:

280

Exercise:

Problem: 84, 90

Exercise:

Problem: 55, 88

Solution:

440

Exercise:

Problem: 60, 72

Everyday Math

Exercise:

Problem:

Writing a Check Jorge bought a car for $24,493. He paid for the car with a check. Write the purchase price in words.

Solution:

twenty-four thousand, four hundred ninety-three dollars

Exercise:

Problem:

Writing a Check Marissa’s kitchen remodeling cost $18,549. She wrote a check to the contractor. Write the amount paid in words.

Exercise:

Problem:

Buying a Car Jorge bought a car for $24,493. Round the price to the nearest ⓐ ten ⓑ hundred ⓒ thousand; and ⓓ ten-thousand.

Solution:

ⓐ $24,490 ⓑ $24,500 ⓒ $24,000 ⓓ $20,000

Exercise:

Problem:

Remodeling a Kitchen Marissa’s kitchen remodeling cost $18,549, Round the cost to the nearest ⓐ ten ⓑ hundred ⓒ thousand and ⓓ ten-thousand.

Exercise:

Problem:

Population The population of China was 1,339,724,852 on November 1, 2010. Round the population to the nearest ⓐ billion ⓑ hundred-million; and ⓒ million.

Solution:

ⓐ 1,000,000,000 ⓑ 1,300,000,000 ⓒ 1,340,000,000

Exercise:

Problem:

Astronomy The average distance between Earth and the sun is 149,597,888 kilometers. Round the distance to the nearest ⓐ hundred-million ⓑ ten-million; and ⓒ million.

Exercise:

Problem:

Grocery Shopping Hot dogs are sold in packages of 10, but hot dog buns come in packs of eight. What is the smallest number that makes the hot dogs and buns come out even?

Solution:

40

Exercise:

Problem:

Grocery Shopping Paper plates are sold in packages of 12 and party cups come in packs of eight. What is the smallest number that makes the plates and cups come out even?

Writing Exercises

Exercise:

Problem: Give an everyday example where it helps to round numbers.

Exercise:

Problem: If a number is divisible by 2 and by 3 why is it also divisible by 6?

Exercise:

Problem: What is the difference between prime numbers and composite numbers?

Solution:

Answers may vary.

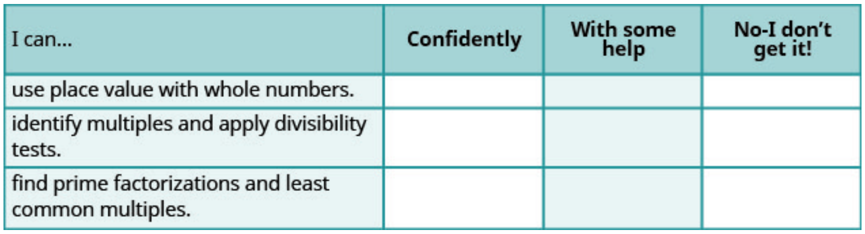
Exercise:

Problem:

Explain in your own words how to find the prime factorization of a composite number, using any method you prefer.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
use place value with whole numbers.			
identify multiples and apply divisibility tests.			
find prime factorizations and least common multiples.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

composite number

A composite number is a counting number that is not prime. A composite number has factors other than 1 and itself.

counting numbers

The counting numbers are the numbers 1, 2, 3, …

divisible by a number

If a number m is a multiple of n, then m is divisible by n. (If 6 is a multiple of 3, then 6 is divisible by 3.)

factors

If $a \cdot b = m$, then a and b are factors of m. Since $3 \cdot 4 = 12$, then 3 and 4 are factors of 12.

least common multiple

The least common multiple of two numbers is the smallest number that is a multiple of both numbers.

multiple of a number

A number is a multiple of n if it is the product of a counting number and n.

number line

A number line is used to visualize numbers. The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left.

origin

The origin is the point labeled 0 on a number line.

prime factorization

The prime factorization of a number is the product of prime numbers that equals the number.

prime number	A prime number is a counting number greater than 1, whose only factors are 1 and itself.
whole numbers	The whole numbers are the numbers 0, 1, 2, 3, ...

Use the Language of Algebra
By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Simplify expressions using the order of operations
- Evaluate an expression
- Identify and combine like terms
- Translate an English phrase to an algebraic expression

Use Variables and Algebraic Symbols

Suppose this year Greg is 20 years old and Alex is 23. You know that Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right? In the language of algebra, we say that Greg's age and Alex's age are **variables** and the 3 is a **constant**. The ages change ("vary") but the 3 years between them always stays the same ("constant"). Since Greg's age and Alex's age will always differ by 3 years, 3 is the *constant*.

In algebra, we use letters of the alphabet to represent variables. So if we call Greg's age g, then we could use $g+3$ to represent Alex's age. See [link].

Greg's age	**Alex's age**
12	15
20	23
35	38
$\boldsymbol{g}$	$\boldsymbol{g}+3$

The letters used to represent these changing ages are called *variables*. The letters most commonly used for variables are x, y, a, b, and c.

Note:
Variable
A **variable** is a letter that represents a number whose value may change.

Note:
Constant
A **constant** is a number whose value always stays the same.

To write algebraically, we need some operation symbols as well as numbers and variables. There are several types of symbols we will be using.

There are four basic arithmetic operations: addition, subtraction, multiplication, and division. We'll list the symbols used to indicate these operations below ([link]). You'll probably recognize some of them.

Operation	**Notation**	**Say:**	**The result is…**
Addition	$a+b$	*a* plus *b*	the sum of *a* and *b*
Subtraction	$a-b$	*a* minus *b*	the difference of *a* and *b*
Multiplication	$a\cdot b, ab, (a)\,(b),$ $(a)b, a\,(b)$	*a* times *b*	the product of *a* and *b*
Division	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$	*a* divided by *b*	the quotient of *a* and *b*, *a* is called the dividend, and *b* is called the divisor

We perform these operations on two numbers. When translating from symbolic form to English, or from English to symbolic form, pay attention to the words "of" and "and."

- The *difference of* 9 and 2 means subtract 9 and 2, in other words, 9 minus 2, which we write symbolically as $9-2$.
- The *product of* 4 and 8 means multiply 4 and 8, in other words 4 times 8, which we write symbolically as $4\cdot8$.

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does 3*xy* mean $3\ \times\ y$ ('three times *y*') or $3\cdot x\cdot y$ (three times *x* times *y*)? To make it clear, use · or parentheses for multiplication.

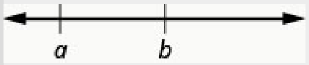
When two quantities have the same value, we say they are equal and connect them with an **equal sign**.

Note:
Equality Symbol
$a = b$ is read “a is equal to b”
The symbol “$=$” is called the **equal sign**.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols “<” and “>.”

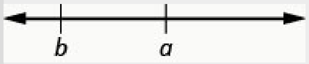
Note:
Inequality
Equation:

$a < b$ is read “a is less than b”
a is to the left of b on the number line



Equation:

$a > b$ is read “a is greater than b”
a is to the right of b on the number line



The expressions $a < b$ or $a > b$ can be read from left to right or right to left, though in English we usually read from left to right ([link]). In general, $a < b$ is equivalent to $b > a$. For example $7 < 11$ is equivalent to $11 > 7$. And $a > b$ is equivalent to $b < a$. For example $17 > 4$ is equivalent to $4 < 17$.

Inequality Symbols	**Words**
$a \neq b$	*a* is *not equal to b*
$a < b$	*a* is *less than b*
$a \leq b$	*a* is *less than or equal to b*
$a > b$	*a* is *greater than b*
$a \geq b$	*a* is *greater than or equal to b*

Example:
Exercise:

Problem: **Translate from algebra into English:

ⓐ $17 \leq 26$ ⓑ $8 \neq 17 - 3$ ⓒ $12 > 27 \div 3$ ⓓ $y + 7 < 19$

Solution:
Solution

ⓐ $17 \leq 26$
17 is less than or equal to 26

ⓑ $8 \neq 17 - 3$
8 is not equal to 17 minus 3

ⓒ $12 > 27 \div 3$
12 is greater than 27 divided by 3

ⓓ $y + 7 < 19$
y plus 7 is less than 19

Note:
Exercise:

TRY IT :: 1.23
Problem: Translate from algebra into English:

ⓐ $14 \leq 27$ ⓑ $19 - 2 \neq 8$ ⓒ $12 > 4 \div 2$ ⓓ $x - 7 < 1$

Solution:

ⓐ 14 is less than or equal to 27 ⓑ 19 minus 2 is not equal to 8 ⓒ 12 is greater than 4 divided by 2 ⓓ *x* minus 7 is less than 1

Note:
Exercise:

TRY IT :: 1.24

Problem: Translate from algebra into English:

ⓐ $19 \geq 15$ ⓑ $7 = 12 - 5$ ⓒ $15 \div 3 < 8$ ⓓ $y + 3 > 6$

Solution:

ⓐ 19 is greater than or equal to 15 ⓑ 7 is equal to 12 minus 5 ⓒ 15 divided by 3 is less than 8 ⓓ *y* plus 3 is greater than 6

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in English. They help to make clear which expressions are to be kept together and separate from other expressions. We will introduce three types now.

Note:
Grouping Symbols
Equation:

$$\begin{array}{ll}\text{Parentheses} & () \\ \text{Brackets} & [] \\ \text{Braces} & \{\}\end{array}$$

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.
Equation:

$$8(14 - 8) \qquad 21 - 3[2 + 4(9 - 8)] \qquad 24 \div \{13 - 2[1(6 - 5) + 4]\}$$

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have *expressions* and *equations*.

Note:
Expression
An **expression** is a number, a variable, or a combination of numbers and variables using operation symbols.

An **expression** is like an English phrase. Here are some examples of expressions:

Expression	Words	English Phrase
$3+5$	3 plus 5	the sum of three and five
$n-1$	*n* minus one	the difference of *n* and one
$6\cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	*x* divided by *y*	the quotient of *x* and *y*

Notice that the English phrases do not form a complete sentence because the phrase does not have a verb.

An **equation** is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb.

Note:
Equation
An **equation** is two expressions connected by an equal sign.

Here are some examples of equations.

Equation	**English Sentence**
$3+5=8$	The sum of three and five is equal to eight.
$n-1=14$	*n* minus one equals fourteen.
$6\cdot 7=42$	The product of six and seven is equal to forty-two.
$x=53$	*x* is equal to fifty-three.
$y+9=2y-3$	*y* plus nine is equal to two *y* minus three.

Example:
Exercise:

Problem: **Determine if each is an expression or an equation:

ⓐ $2(x+3)=10$ ⓑ $4(y-1)+1$ ⓒ $x\div 25$ ⓓ $y+8=40$

Solution:
Solution

ⓐ $2(x+3)=10$	This is an *equation*—two expressions are connected with an equal sign.
ⓑ $4(y-1)+1$	This is an *expression*—no equal sign.
ⓒ $x\div 25$	This is an *expression*—no equal sign.
ⓓ $y+8=40$	This is an *equation*—two expressions are connected with an equal sign.

Note:
Exercise:

Problem:

TRY IT :: 1.25
Determine if each is an expression or an equation: ⓐ $3(x-7)=27$ ⓑ $5(4y-2)-7$.

Solution:

ⓐ equation ⓑ expression

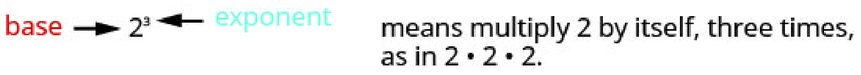
Note:
Exercise:

TRY IT :: 1.26
Problem: Determine if each is an expression or an equation: ⓐ $y^3 \div 14$ ⓑ $4x-6=22$.

Solution:

ⓐ expression ⓑ equation

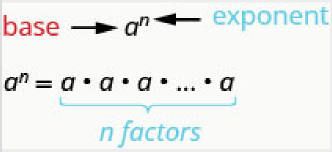
Suppose we need to multiply 2 nine times. We could write this as $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$. This is tedious and it can be hard to keep track of all those 2s, so we use exponents. We write $2\cdot2\cdot2$ as 2^3 and $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$ as 2^9. In expressions such as 2^3, the 2 is called the *base* and the 3 is called the *exponent*. The exponent tells us how many times we need to multiply the base.



We read 2^3 as “two to the third power” or “two cubed.”

We say 2^3 is in *exponential notation* and $2\cdot2\cdot2$ is in *expanded notation*.

Note:
Exponential Notation
a^n means multiply a by itself, n times.



The expression a^n is read a to the n^{th} power.

While we read a^n as “a to the n^{th} power,” we usually read:

- a^2 “a squared”
- a^3 “a cubed”

We’ll see later why a^2 and a^3 have special names.

[link] shows how we read some expressions with exponents.

Expression	**In Words**
7^2	7 to the second power or 7 squared
5^3	5 to the third power or 5 cubed
9^4	9 to the fourth power
12^5	12 to the fifth power

Example:
Exercise:

Problem: **Simplify: 3^4.

Solution:
Solution

	3^4
Expand the expression.	$3 \cdot 3 \cdot 3 \cdot 3$
Multiply left to right.	$9 \cdot 3 \cdot 3$

Multiply.	$27 \cdot 3$
Multiply.	81

Note:
Exercise:

TRY IT :: 1.27
Problem: Simplify: ⓐ 5^3 ⓑ 1^7.

Solution:

ⓐ 125 ⓑ 1

Note:
Exercise:

TRY IT :: 1.28
Problem: Simplify: ⓐ 7^2 ⓑ 0^5.

Solution:

ⓐ 49 ⓑ 0

Simplify Expressions Using the Order of Operations

To **simplify an expression** means to do all the math possible. For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:

Equation:

$$\begin{gathered} 4 \cdot 2 + 1 \\ 8 + 1 \\ 9 \end{gathered}$$

By not using an equal sign when you simplify an expression, you may avoid confusing expressions with equations.

Note:
Simplify an Expression
To **simplify an expression**, do all operations in the expression.

We’ve introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values. For example, consider the expression:

Equation:

$$4+3\cdot 7$$

If you simplify this expression, what do you get?

Some students say 49,

Equation:

$$\begin{array}{lc} & 4+3\cdot 7 \\ \text{Since } 4+3 \text{ gives } 7. & 7\cdot 7 \\ \text{And } 7\cdot 7 \text{ is } 49. & 49 \end{array}$$

Others say 25,

Equation:

$$\begin{array}{lc} & 4+3\cdot 7 \\ \text{Since } 3\cdot 7 \text{ is } 21. & 4+21 \\ \text{And } 21+4 \text{ makes } 25. & 25 \end{array}$$

Imagine the confusion in our banking system if every problem had several different correct answers!

The same expression should give the same result. So mathematicians early on established some guidelines that are called the Order of Operations.

Note:
Perform the Order of Operations.

Parentheses and Other

Grouping Symbols	◦ Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
Exponents	◦ Simplify all expressions with exponents.
Multiplication and Division	◦ Perform all multiplication and division in order from left to right. These operations have equal priority.
Addition **S**ubtraction and	◦ Perform all addition and subtraction in order from left to right. These operations have equal priority.

Note: Doing the Manipulative Mathematics activity “Game of 24” give you practice using the order of operations.

Students often ask, “How will I remember the order?” Here is a way to help you remember: Take the first letter of each key word and substitute the silly phrase: “Please Excuse My Dear Aunt Sally.”

Equation:

Parentheses	**P**lease
Exponents	**E**xcuse
Multiplication **D**ivision	**M**y **D**ear
Addition **S**ubtraction	**A**unt **S**ally

It’s good that “**M**y **D**ear” goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, “**A**unt **S**ally” goes together and so reminds us that **a**ddition and **s**ubtraction also have equal priority and we do them in order from left to right.

Let’s try an example.

Example:
Exercise:

Problem: **Simplify: ⓐ $4+3\cdot 7$ ⓑ $(4+3)\cdot 7$.

Solution:
Solution

ⓐ

	$4 + 3 \cdot 7$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply first.	$4 + 3 \cdot 7$
Add.	$4 + 21$
	25

ⓑ

	$(4 + 3) \cdot 7$
Are there any **p**arentheses? Yes.	$(4 + 3) \cdot 7$
Simplify inside the parentheses.	$(7)7$

Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply.	49

Note:
Exercise:

TRY IT :: 1.29
Problem: Simplify: ⓐ $12-5\cdot 2$ ⓑ $(12-5)\cdot 2$.

Solution:

ⓐ 2 ⓑ 14

Note:
Exercise:

TRY IT :: 1.30
Problem: Simplify: ⓐ $8+3\cdot 9$ ⓑ $(8+3)\cdot 9$.

Solution:

ⓐ 35 ⓑ 99

Example:
Exercise:

Problem: **Simplify: $18\div 6+4\left(5-2\right)$.

Solution:
Solution

Parentheses? Yes, subtract first.	$18 \div 6 + 4\left(5 - 2\right)$ $18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	$18 \div 6 + 4(3)$
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

Note:
Exercise:

TRY IT :: 1.31

Problem: Simplify: $30 \div 5 + 10\left(3 - 2\right)$.

Solution:

16

Note:
Exercise:

TRY IT :: 1.32

Problem: Simplify: $70 \div 10 + 4\left(6 - 2\right)$.

Solution:

23

When there are multiple grouping symbols, we simplify the innermost parentheses first and work outward.

Example:
Exercise:

Problem: **Simplify: $5+2^3+3\left[6-3\left(4-2\right)\right]$.

Solution:
Solution

	$5+2^3+3[6-3(4-2)]$
Are there any parentheses (or other grouping symbol)? Yes.	
Focus on the parentheses that are inside the brackets.	$5+2^3+3[6-3(4-2)]$
Subtract.	$5+2^3+3[6-3(2)]$
Continue inside the brackets and multiply.	$5+2^3+3[6-6]$
Continue inside the brackets and subtract.	$5+2^3+3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	$5+2^3+3[0]$

Simplify exponents.	$5+8+3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5+8+0$
Is there any addition or subtraction? Yes.	
Add.	$13+0$
Add.	13

Note:
Exercise:

TRY IT :: 1.33

Problem: Simplify: $9+5^3-[4(9+3)]$.

Solution:

86

Note:
Exercise:

TRY IT :: 1.34

Problem: Simplify: $7^2-2[4(5+1)]$.

Solution:

1

Evaluate an Expression

In the last few examples, we simplified expressions using the order of operations. Now we'll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

Note:
Evaluate an Expression
To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Example:
Exercise:

Problem: **Evaluate $7x - 4$, when ⓐ $x = 5$ ⓑ $x = 1$.

Solution:
Solution

ⓐ

when $x = 5$	$7x - 4$
	$7(5) - 4$
Multiply.	$35 - 4$
Subtract.	31

ⓑ

when $x = 1$	$7x - 4$
	$7(1) - 4$
Multiply.	$7 - 4$
Subtract.	3

Note:
Exercise:

TRY IT :: 1.35

Problem: Evaluate $8x - 3$, when ⓐ $x = 2$ ⓑ $x = 1$.

Solution:

ⓐ 13 ⓑ 5

Note:
Exercise:

TRY IT ;; 1.36

Problem: Evaluate $4y - 4$, when ⓐ $y = 3$ ⓑ $y = 5$.

Solution:

ⓐ 8 ⓑ 16

Example:
Exercise:

Problem: **Evaluate $x = 4$, when ⓐ x^2 ⓑ 3^x.

Solution:
Solution

ⓐ

	x^2
Replace *x* with 4.	4^2
Use definition of exponent.	$4 \cdot 4$
Simplify.	16

ⓑ

	3^x
Replace *x* with 4.	3^x
Use definition of exponent.	$3 \cdot 3 \cdot 3 \cdot 3$
Simplify.	81

Note:
Exercise:

TRY IT :: 1.37

Problem: Evaluate $x = 3$, when ⓐ x^2 ⓑ 4^x.

Solution:

ⓐ 9 ⓑ 64

Note:
Exercise:

TRY IT :: 1.38

Problem: Evaluate $x = 6$, when ⓐ x^3 ⓑ 2^x.

Solution:

ⓐ 216 ⓑ 64

Example:
Exercise:

Problem: **Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Solution:
Solution

	$2x^2 + 3x + 8$
Substitute $x = 4$.	$2x^2 + 3x + 8$
Follow the order of operations.	$2(16) + 3(4) + 8$
	$32 + 12 + 8$
	52

Note:
Exercise:

TRY IT :: 1.39

Problem: Evaluate $3x^2+4x+1$ when $x=3$.

Solution:

40

Note:
Exercise:

TRY IT :: 1.40

Problem: Evaluate $6x^2-4x-7$ when $x=2$.

Solution:

9

Identify and Combine Like Terms

Algebraic expressions are made up of terms. A **term** is a constant, or the product of a constant and one or more variables.

Note:
Term
A **term** is a constant, or the product of a constant and one or more variables.

Examples of terms are $7, y, 5x^2, 9a, \text{and } b^5$.

The constant that multiplies the variable is called the **coefficient**.

Note:
Coefficient
The **coefficient** of a term is the constant that multiplies the variable in a term.

Think of the coefficient as the number in front of the variable. The coefficient of the term $3x$ is 3. When we write x, the coefficient is 1, since $x = 1 \cdot x$.

Example:
Exercise:

Problem: **Identify the coefficient of each term: ⓐ $14y$ ⓑ $15x^2$ ⓒ a.

Solution:
Solution

ⓐ The coefficient of $14y$ is 14.

ⓑ The coefficient of $15x^2$ is 15.

ⓒ The coefficient of a is 1 since $a = 1\ a$.

Note:
Exercise:

TRY IT :: 1.41
Problem: Identify the coefficient of each term: ⓐ $17x$ ⓑ $41b^2$ ⓒ z.

Solution:

ⓐ 17 ⓑ 41 ⓒ 1

Note:
Exercise:

TRY IT :: 1.42
Problem: Identify the coefficient of each term: ⓐ $9p$ ⓑ $13a^3$ ⓒ y^3.

Solution:

ⓐ 9 ⓑ 13 ⓒ 1

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?

Equation:

$$5x \qquad 7 \qquad n^2 \qquad 4 \qquad 3x \qquad 9n^2$$

The 7 and the 4 are both constant terms.

The *5x* and the 3*x* are both terms with *x*.

The n^2 and the $9n^2$ are both terms with n^2.

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- 5*x* and 3*x* are like terms.
- x^2 and $9x^2$ are like terms.

Note:

Like Terms

Terms that are either constants or have the same variables raised to the same powers are called **like terms**.

Example:

Exercise:

Problem: **Identify the like terms: y^3, $7x^2$, 14, 23, $4y^3$, 9*x*, $5x^2$.

Solution:

Solution

y^3 and $4y^3$ are like terms because both have y^3; the variable and the exponent match.

$7x^2$ and $5x^2$ are like terms because both have x^2; the variable and the exponent match.

14 and 23 are like terms because both are constants.

There is no other term like 9*x*.

Note:
Exercise:

TRY IT :: 1.43
Problem: Identify the like terms: 9, $2x^3$, y^2, $8x^3$, 15, $9y$, $11y^2$.

Solution:

9 and 15, y^2 and $11y^2$, $2x^3$ and $8x^3$

Note:
Exercise:

TRY IT :: 1.44
Problem: Identify the like terms: $4x^3$, $8x^2$, 19, $3x^2$, 24, $6x^3$.

Solution:

19 and 24, $8x^2$ and $3x^2$, $4x^3$ and $6x^3$

Adding or subtracting terms forms an expression. In the expression $2x^2 + 3x + 8$, from [link], the three terms are $2x^2$, $3x$, and 8.

Example:
Exercise:

Problem: **Identify the terms in each expression.

ⓐ $9x^2 + 7x + 12$
ⓑ $8x + 3y$

Solution:
Solution

ⓐ The terms of $9x^2 + 7x + 12$ are $9x^2$, $7x$, and 12.

ⓑ The terms of $8x + 3y$ are $8x$ and $3y$.

Note:
Exercise:

TRY IT :: 1.45

Problem: Identify the terms in the expression $4x^2 + 5x + 17$.

Solution:

$4x^2, 5x, 17$

Note:
Exercise:

TRY IT :: 1.46

Problem: Identify the terms in the expression $5x + 2y$.

Solution:

5*x*, 2*y*

If there are like terms in an expression, you can simplify the expression by combining the like terms. What do you think $4x + 7x + x$ would simplify to? If you thought 12*x*, you would be right!

Equation:

$$\begin{array}{c} 4x + 7x + x \\ x + x + x + x \quad + x + x + x + x + x + x + x \quad + x \\ 12x \end{array}$$

Add the coefficients and keep the same variable. It doesn’t matter what *x* is—if you have 4 of something and add 7 more of the same thing and then add 1 more, the result is 12 of them. For example, 4 oranges plus 7 oranges plus 1 orange is 12 oranges. We will discuss the mathematical properties behind this later.

**Simplify: $4x + 7x + x$.

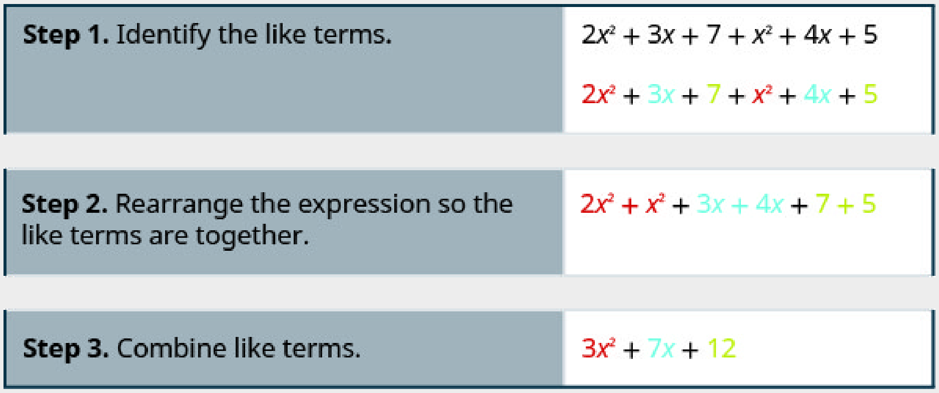
Add the coefficients. 12*x*

Example:
How To Combine Like Terms

Exercise:

Problem: Simplify: $2x^2 + 3x + 7 + x^2 + 4x + 5$.

Solution:
Solution



Step 1. Identify the like terms.	$2x^2 + 3x + 7 + x^2 + 4x + 5$ $2x^2 + 3x + 7 + x^2 + 4x + 5$
Step 2. Rearrange the expression so the like terms are together.	$2x^2 + x^2 + 3x + 4x + 7 + 5$
Step 3. Combine like terms.	$3x^2 + 7x + 12$

Note:
Exercise:

TRY IT :: 1.47
Problem: Simplify: $3x^2 + 7x + 9 + 7x^2 + 9x + 8$.

Solution:

$10x^2 + 16x + 17$

Note:
Exercise:

TRY IT :: 1.48
Problem: Simplify: $4y^2 + 5y + 2 + 8y^2 + 4y + 5$.

Solution:

$12y^2 + 9y + 7$

Note:
Combine Like Terms.

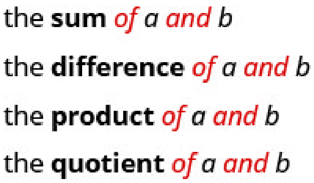
Identify like terms.
Rearrange the expression so like terms are together.
Add or subtract the coefficients and keep the same variable for each group of like terms.

Translate an English Phrase to an Algebraic Expression

In the last section, we listed many operation symbols that are used in algebra, then we translated expressions and equations into English phrases and sentences. Now we'll reverse the process. We'll translate English phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. [link] summarizes them.

Operation	**Phrase**	**Expression**
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a+b$
Subtraction	a minus b the difference of a and b a decreased by b b less than a b subtracted from a	$a-b$
Multiplication	a times b the product of a and b twice a	$a\cdot b, ab, a(b), (a)(b)$ $2a$
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$

Look closely at these phrases using the four operations:


the **sum** *of* *a* *and* *b*

the **difference** *of* *a* *and* *b*

the **product** *of* *a* *and* *b*

the **quotient** *of* *a* *and* *b*

Each phrase tells us to operate on two numbers. Look for the words *of* and *and* to find the numbers.

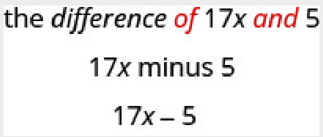
Example:
Exercise:

Problem:

**Translate each English phrase into an algebraic expression: ⓐ the difference of $17x$ and 5 ⓑ the quotient of $10x^2$ and 7.

Solution:
Solution

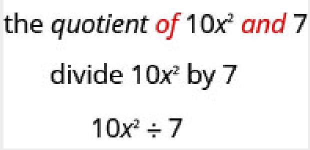
ⓐ The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.


the *difference* *of* $17x$ *and* 5

$17x$ minus 5

$17x - 5$

ⓑ The key word is “quotient,” which tells us the operation is division.


the *quotient* *of* $10x^2$ *and* 7

divide $10x^2$ by 7

$10x^2 \div 7$

This can also be written $10x^2/7$ or $\frac{10x^2}{7}$.

Note:
Exercise:

Problem:

TRY IT :: 1.49
Translate the English phrase into an algebraic expression: ⓐ the difference of $14x^2$ and 13 ⓑ the quotient of 12*x* and 2.

Solution:

ⓐ $14x^2 - 13$ ⓑ $12x \div 2$

Note:
Exercise:

Problem:

TRY IT :: 1.50
Translate the English phrase into an algebraic expression: ⓐ the sum of $17y^2$ and 19 ⓑ the product of 7 and y.

Solution:

ⓐ $17y^2 + 19$ ⓑ 7*y*

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight "more than" means 8 added to your present age. How old were you seven years ago? This is 7 years less than your age now. You subtract 7 from your present age. Seven "less than" means 7 subtracted from your present age.

Example:
Exercise:

Problem:

**Translate the English phrase into an algebraic expression: ⓐ Seventeen more than *y* ⓑ Nine less than $9x^2$.

Solution:
Solution

ⓐ The key words are *more than.* They tell us the operation is addition. *More than* means "added to."
Equation:

$$\text{Seventeen more than } y$$
$$\text{Seventeen added to } y$$
$$y + 17$$

ⓑ The key words are *less than*. They tell us to subtract. *Less than* means "subtracted from."

Equation:

$$\text{Nine less than } 9x^2$$
$$\text{Nine subtracted from } 9x^2$$
$$9x^2 - 9$$

Note:
Exercise:

Problem:

TRY IT :: 1.51

Translate the English phrase into an algebraic expression: ⓐ Eleven more than x ⓑ Fourteen less than $11a$.

Solution:

ⓐ $x + 11$ ⓑ $11a - 14$

Note:
Exercise:

Problem:

TRY IT :: 1.52

Translate the English phrase into an algebraic expression: ⓐ 13 more than z ⓑ 18 less than $8x$.

Solution:

ⓐ $z + 13$ ⓑ $8x - 18$

Example:
Exercise:

Problem:

**Translate the English phrase into an algebraic expression: ⓐ five times the sum of *m* and *n* ⓑ the sum of five times *m* and *n*.

Solution:
Solution

There are two operation words—*times* tells us to multiply and *sum* tells us to add.

ⓐ Because we are multiplying 5 times the sum we need parentheses around the sum of *m* and *n*, $(m+n)$. This forces us to determine the sum first. (Remember the order of operations.)
Equation:

$$\begin{array}{c}\text{five times the sum of } m \text{ and } n \\ 5\left(m+n\right)\end{array}$$

ⓑ To take a sum, we look for the words "of" and "and" to see what is being added. Here we are taking the sum *of* five times *m* and *n*.
Equation:

$$\begin{array}{c}\text{the sum of five times } m \text{ and } n \\ 5m+n\end{array}$$

Note:
Exercise:

Problem:

TRY IT :: 1.53
Translate the English phrase into an algebraic expression: ⓐ four times the sum of *p* and *q* ⓑ the sum of four times *p* and *q*.

Solution:

ⓐ $4\left(p+q\right)$ ⓑ $4p+q$

Note:

Exercise:

Problem:

TRY IT :: 1.54
Translate the English phrase into an algebraic expression: ⓐ the difference of two times *x* and 8, ⓑ two times the difference of *x* and 8.

Solution:

ⓐ $2x - 8$ ⓑ $2\left(x - 8\right)$

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression. We'll see how to do this in the next two examples.

Example:
Exercise:

Problem:

**The length of a rectangle is 6 less than the width. Let *w* represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution:
Solution

Write a phrase about the length of the rectangle.	6 less than the width
Substitute w for "the width."	6 less than w
Rewrite "less than" as "subtracted from."	6 subtracted from w
Translate the phrase into algebra.	$w - 6$

Note:

Exercise:

Problem:

TRY IT :: 1.55
The length of a rectangle is 7 less than the width. Let *w* represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution:

$w - 7$

Note:
Exercise:

Problem:

TRY IT :: 1.56
The width of a rectangle is 6 less than the length. Let *l* represent the length of the rectangle. Write an expression for the width of the rectangle.

Solution:

$l - 6$

Example:
Exercise:

Problem:

**June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let *q* represent the number of quarters. Write an expression for the number of dimes.

Solution:
Solution

Write a phrase about the number of dimes.	three less than four times the number of quarters

Substitute q for the number of quarters.	3 less than 4 times q
Translate "4 times q."	3 less than $4q$
Translate the phase into algebra.	$4q-3$

Note:
Exercise:

Problem:

TRY IT :: 1.57
Geoffrey has dimes and quarters in his pocket. The number of dimes is eight less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution:

$4q-8$

Note:
Exercise:

Problem:

TRY IT :: 1.58
Lauren has dimes and nickels in her purse. The number of dimes is three more than seven times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Solution:

$7n+3$

Key Concepts

Notation	**The result is…**

$a+b$	the sum of a and b
$a-b$	the difference of a and b
$a \cdot b, ab, (a)\,(b)\ \ (a)b, a\,(b)$	the product of a and b
$a \div b, a/b, \frac{a}{b}, b\overline{)a}$	the quotient of a and b

- **Inequality**

$a < b$ is read "a is less than b"	a is to the left of b on the number line
$a > b$ is read "a is greater than b"	a is to the right of b on the number line

Inequality Symbols	**Words**
$a \neq b$	a is **not equal to** b
$a < b$	a is **less than** b
$a \leq b$	a is **less than or equal to** b
$a > b$	a is **greater than** b
$a \geq b$	a is **greater than or equal to** b

- **Grouping Symbols**
  - Parentheses ( )
  - Brackets [ ]
  - Braces { }

- **Exponential Notation**
  - a^n means multiply a by itself, n times. The expression a^n is read a to the n^{th} power.

- **Order of Operations:** When simplifying mathematical expressions perform the operations in the following order:

  Parentheses and other Grouping Symbols: Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
  Exponents: Simplify all expressions with exponents.
  Multiplication and Division: Perform all multiplication and division in order from left to right. These operations have equal priority.
  Addition and Subtraction: Perform all addition and subtraction in order from left to right. These operations have equal priority.

- **Combine Like Terms**

  Identify like terms.
  Rearrange the expression so like terms are together.
  Add or subtract the coefficients and keep the same variable for each group of like terms.

Practice Makes Perfect

Use Variables and Algebraic Symbols

In the following exercises, translate from algebra to English.

Exercise:

Problem: $16 - 9$

Solution:

16 minus 9, the difference of sixteen and nine

Exercise:

Problem: $3 \cdot 9$

Exercise:

Problem: $28 \div 4$

Solution:

28 divided by 4, the quotient of twenty-eight and four

Exercise:

Problem: $x + 11$

Exercise:

Problem: $(2)(7)$

Solution:

2 times 7, the product of two and seven

Exercise:

Problem: $(4)(8)$

Exercise:

Problem: $14 < 21$

Solution:

fourteen is less than twenty-one

Exercise:

Problem: $17 < 35$

Exercise:

Problem: $36 \geq 19$

Solution:

thirty-six is greater than or equal to nineteen

Exercise:

Problem: $6n = 36$

Exercise:

Problem: $y - 1 > 6$

Solution:

y minus 1 is greater than 6, the difference of *y* and one is greater than six

Exercise:

Problem: $y - 4 > 8$

Exercise:

Problem: $2 \leq 18 \div 6$

Solution:

2 is less than or equal to 18 divided by 6; 2 is less than or equal to the quotient of eighteen and six

Exercise:

Problem: $a \neq 1 \cdot 12$

In the following exercises, determine if each is an expression or an equation.

Exercise:

Problem: $9 \cdot 6 = 54$

Solution:

equation

Exercise:

Problem: $7 \cdot 9 = 63$

Exercise:

Problem: $5 \cdot 4 + 3$

Solution:

expression

Exercise:

Problem: $x + 7$

Exercise:

Problem: $x + 9$

Solution:

expression

Exercise:

Problem: $y - 5 = 25$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify each expression.

Exercise:

Problem: 5^3

Solution:

125

Exercise:

Problem: 8^3

Exercise:

Problem: 2^8

Solution:

256

Exercise:

Problem: 10^5

In the following exercises, simplify using the order of operations.

Exercise:

Problem: ⓐ $3+8\cdot 5$ ⓑ $(3+8)\cdot 5$

Solution:

ⓐ 43 ⓑ 55

Exercise:

Problem: ⓐ $2+6\cdot 3$ ⓑ $(2+6)\cdot 3$

Exercise:

Problem: $2^3-12\div(9-5)$

Solution:

5

Exercise:

Problem: $3^2-18\div(11-5)$

Exercise:

Problem: $3\cdot 8+5\cdot 2$

Solution:

34

Exercise:

Problem: $4\cdot 7+3\cdot 5$

Exercise:

Problem: $2+8(6+1)$

Solution:

58

Exercise:

Problem: $4+6(3+6)$

Exercise:

Problem: $4\cdot 12/8$

Solution:

6

Exercise:

Problem: $2\cdot 36/6$

Exercise:

Problem: $(6+10)\div(2+2)$

Solution:

4

Exercise:

Problem: $(9+12)\div(3+4)$

Exercise:

Problem: $20\div 4+6\cdot 5$

Solution:

35

Exercise:

Problem: $33\div 3+8\cdot 2$

Exercise:

Problem: 3^2+7^2

Solution:

58

Exercise:

Problem: $(3+7)^2$

Exercise:

Problem: $3(1+9\cdot 6)-4^2$

Solution:

149

Exercise:

Problem: $5(2+8\cdot 4)-7^2$

Exercise:

Problem: $2\left[1+3\left(10-2\right)\right]$

Solution:

50

Exercise:

Problem: $5\left[2+4\left(3-2\right)\right]$

Evaluate an Expression

In the following exercises, evaluate the following expressions.

Exercise:

Problem: $7x+8$ when $x=2$

Solution:

22

Exercise:

Problem: $8x-6$ when $x=7$

Exercise:

Problem: x^2 when $x=12$

Solution:

144

Exercise:

Problem: x^3 when $x = 5$

Exercise:

Problem: x^5 when $x = 2$

Solution:

32

Exercise:

Problem: 4^x when $x = 2$

Exercise:

Problem: $x^2 + 3x - 7$ when $x = 4$

Solution:

21

Exercise:

Problem: $6x + 3y - 9$ when $x = 6, y = 9$

Exercise:

Problem: $(x - y)^2$ when $x = 10, y = 7$

Solution:

9

Exercise:

Problem: $(x + y)^2$ when $x = 6, y = 9$

Exercise:

Problem: $a^2 + b^2$ when $a = 3, b = 8$

Solution:

73

Exercise:

Problem: $r^2 - s^2$ when $r = 12, s = 5$

Exercise:

Problem: $2l + 2w$ when $l = 15, w = 12$

Solution:

54

Exercise:

Problem: $2l + 2w$ when $l = 18, w = 14$

Simplify Expressions by Combining Like Terms

In the following exercises, identify the coefficient of each term.

Exercise:

Problem: $8a$

Solution:

8

Exercise:

Problem: $13m$

Exercise:

Problem: $5r^2$

Solution:

5

Exercise:

Problem: $6x^3$

In the following exercises, identify the like terms.

Exercise:

Problem: $x^3, 8x, 14, 8y, 5, 8x^3$

Solution:

x^3 and $8x^3$, 14 and 5

Exercise:

Problem: $6z, 3w^2, 1, 6z^2, 4z, w^2$

Exercise:

Problem: $9a, a^2, 16, 16b^2, 4, 9b^2$

Solution:

16 and 4, $16b^2$ and $9b^2$

Exercise:

Problem: $3, 25r^2, 10s, 10r, 4r^2, 3s$

In the following exercises, identify the terms in each expression.

Exercise:

Problem: $15x^2 + 6x + 2$

Solution:

$15x^2, 6x, 2$

Exercise:

Problem: $11x^2 + 8x + 5$

Exercise:

Problem: $10y^3 + y + 2$

Solution:

$10y^3, y, 2$

Exercise:

Problem: $9y^3 + y + 5$

In the following exercises, simplify the following expressions by combining like terms.
Exercise:

Problem: $10x + 3x$

Solution:

$13x$

Exercise:

Problem: $15x + 4x$

Exercise:

Problem: $4c + 2c + c$

Solution:

$7c$

Exercise:

Problem: $6y + 4y + y$

Exercise:

Problem: $7u + 2 + 3u + 1$

Solution:

$10u + 3$

Exercise:

Problem: $8d + 6 + 2d + 5$

Exercise:

Problem: $10a + 7 + 5a - 2 + 7a - 4$

Solution:

$22a + 1$

Exercise:

Problem: $7c + 4 + 6c - 3 + 9c - 1$

Exercise:

Problem: $3x^2 + 12x + 11 + 14x^2 + 8x + 5$

Solution:

$17x^2 + 20x + 16$

Exercise:

Problem: $5b^2 + 9b + 10 + 2b^2 + 3b - 4$

Translate an English Phrase to an Algebraic Expression

In the following exercises, translate the phrases into algebraic expressions.

Exercise:

Problem: the difference of 14 and 9

Solution:

$14 - 9$

Exercise:

Problem: the difference of 19 and 8

Exercise:

Problem: the product of 9 and 7

Solution:

$9 \cdot 7$

Exercise:

Problem: the product of 8 and 7

Exercise:

Problem: the quotient of 36 and 9

Solution:

$36 \div 9$

Exercise:

Problem: the quotient of 42 and 7

Exercise:

Problem: the sum of *8x* and 3*x*

Solution:

$8x + 3x$

Exercise:

Problem: the sum of *13x* and 3*x*

Exercise:

Problem: the quotient of *y* and 3

Solution:

$\frac{y}{3}$

Exercise:

Problem: the quotient of *y* and 8

Exercise:

Problem: eight times the difference of *y* and nine

Solution:

$8\left(y - 9\right)$

Exercise:

Problem: seven times the difference of *y* and one

Exercise:

Problem:

Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let *c* represent the number of classical CDs. Write an expression for the number of rock CDs.

Solution:

$c + 3$

Exercise:

Problem:

The number of girls in a second-grade class is 4 less than the number of boys. Let b represent the number of boys. Write an expression for the number of girls.

Exercise:

Problem:

Greg has nickels and pennies in his pocket. The number of pennies is seven less than twice the number of nickels. Let n represent the number of nickels. Write an expression for the number of pennies.

Solution:

$2n-7$

Exercise:

Problem:

Jeannette has $5 and $10 bills in her wallet. The number of fives is three more than six times the number of tens. Let t represent the number of tens. Write an expression for the number of fives.

Everyday Math

Exercise:

Problem:

Car insurance Justin's car insurance has a $750 deductible per incident. This means that he pays $750 and his insurance company will pay all costs beyond $750. If Justin files a claim for $2,100.

ⓐ how much will he pay?
ⓑ how much will his insurance company pay?

Solution:

ⓐ $750 ⓑ $1,350

Exercise:

Problem:

Home insurance Armando's home insurance has a $2,500 deductible per incident. This means that he pays $2,500 and the insurance company will pay all costs beyond $2,500. If Armando files a claim for $19,400.

ⓐ how much will he pay?
ⓑ how much will the insurance company pay?

Writing Exercises

Exercise:

Problem: Explain the difference between an expression and an equation.

Solution:

Answers may vary

Exercise:

Problem: Why is it important to use the order of operations to simplify an expression?

Exercise:

Problem:

Explain how you identify the like terms in the expression $8a^2 + 4a + 9 - a^2 - 1$.

Solution:

Answers may vary

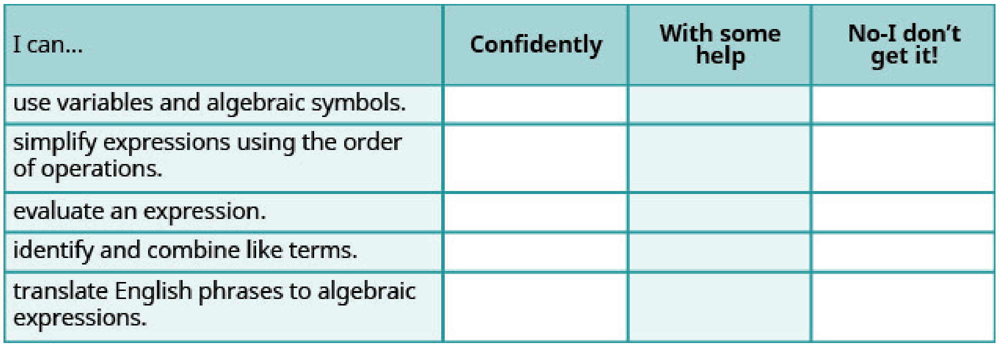
Exercise:

Problem:

Explain the difference between the phrases “4 times the sum of *x* and *y*” and “the sum of 4 times *x* and *y*.”

Self Check

ⓐ Use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
use variables and algebraic symbols.			
simplify expressions using the order of operations.			
evaluate an expression.			
identify and combine like terms.			
translate English phrases to algebraic expressions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

coefficient

The coefficient of a term is the constant that multiplies the variable in a term.

constant

A constant is a number whose value always stays the same.

equality symbol

The symbol “$=$” is called the equal sign. We read $a=b$ as “a is equal to b.”

equation

An equation is two expressions connected by an equal sign.

evaluate an expression

To evaluate an expression means to find the value of the expression when the variable is replaced by a given number.

expression

An expression is a number, a variable, or a combination of numbers and variables using operation symbols.

like terms

Terms that are either constants or have the same variables raised to the same powers are called like terms.

simplify an expression

To simplify an expression, do all operations in the expression.

term

A term is a constant or the product of a constant and one or more variables.

variable

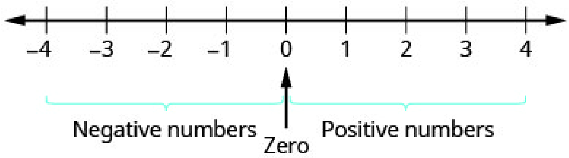
A variable is a letter that represents a number whose value may change.

Add and Subtract Integers
By the end of this section, you will be able to:

- Use negatives and opposites
- Simplify: expressions with absolute value
- Add integers
- Subtract integers

Use Negatives and Opposites

Our work so far has only included the counting numbers and the whole numbers. But if you have ever experienced a temperature below zero or accidentally overdrawn your checking account, you are already familiar with negative numbers. **Negative numbers** are numbers less than 0. The negative numbers are to the left of zero on the number line. See [link].

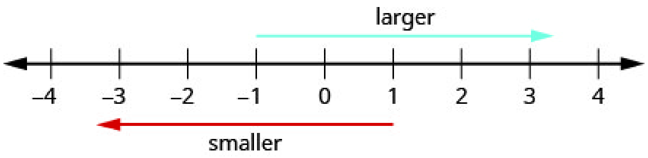


The number line shows the location of positive and negative numbers.

The arrows on the ends of the number line indicate that the numbers keep going forever. There is no biggest positive number, and there is no smallest negative number.

Is zero a positive or a negative number? Numbers larger than zero are positive, and numbers smaller than zero are negative. Zero is neither positive nor negative.

Consider how numbers are ordered on the number line. Going from left to right, the numbers increase in value. Going from right to left, the numbers decrease in value. See [link].



The numbers on a number line increase in value going from left to right and decrease in value going from right to left.

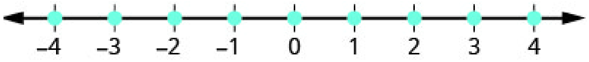
Note: Doing the Manipulative Mathematics activity “Number Line-part 2” will help you develop a better understanding of integers.

Remember that we use the notation:

a < *b* (read “a is less than b”) when a is to the left of b on the number line.

a > *b* (read “*a* is greater than *b*”) when *a* is to the right of *b* on the number line.

Now we need to extend the number line which showed the whole numbers to include negative numbers, too. The numbers marked by points in [link] are called the integers. The integers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3\ldots$



All the marked numbers are called *integers*.

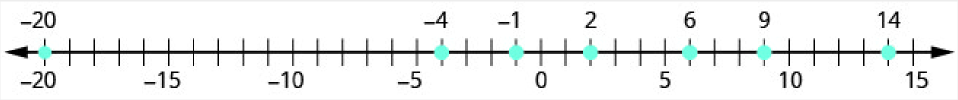
Example:
Exercise:

Problem:

**Order each of the following pairs of numbers, using < or >: ⓐ 14 ___ 6 ⓑ -1 ___ 9 ⓒ -1 ___ -4 ⓓ 2 ___ -20.

Solution:
Solution

It may be helpful to refer to the number line shown.



ⓐ 14 is to the right of 6 on the number line.	14 ___ 6 $14 > 6$
ⓑ −1 is to the left of 9 on the number line.	-1 ___ 9 $-1 < 9$

ⓒ −1 is to the right of −4 on the number line.	-1 ___ -4 $-1 > -4$
ⓓ 2 is to the right of −20 on the number line.	2 ___ -20 $2 > -20$

Note:
Exercise:

Problem:

TRY IT :: 1.59
Order each of the following pairs of numbers, using $<$ or $>$: ⓐ 15 ___ 7 ⓑ -2 ___ 5 ⓒ -3 ___ -7
ⓓ 5 ___ $-17.$

Solution:

ⓐ > ⓑ < ⓒ > ⓓ >

Note:
Exercise:

Problem:

TRY IT :: 1.60
Order each of the following pairs of numbers, using $<$ or $>$: ⓐ 8 ___ 13 ⓑ 3 ___ -4 ⓒ -5 ___ -2
ⓓ 9 ___ $-21.$

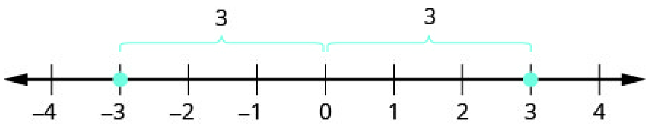
Solution:

ⓐ < ⓑ > ⓒ < ⓓ >

You may have noticed that, on the number line, the negative numbers are a mirror image of the positive numbers, with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, they are called **opposites**. The opposite of 2 is -2, and the opposite of -2 is 2.

Note:
Opposite
The **opposite** of a number is the number that is the same distance from zero on the number line but on the opposite side of zero.

[link] illustrates the definition.



The opposite of 3 is -3.

Sometimes in algebra the same symbol has different meanings. Just like some words in English, the specific meaning becomes clear by looking at how it is used. You have seen the symbol “−” used in three different ways.

Equation:

$10-4$	Between two numbers, it indicates the operation of *subtraction*. We read $10-4$ as “10 minus 4.”
-8	In front of a number, it indicates a *negative* number. We read -8 as “negative eight.”
$-x$	In front of a variable, it indicates the *opposite*. We read $-x$ as “the opposite of x.”
$-(-2)$	Here there are two “ − ” signs. The one in the parentheses tells us the number is negative 2. The one outside the parentheses tells us to take the *opposite* of -2. We read $-(-2)$ as “the opposite of negative two.”

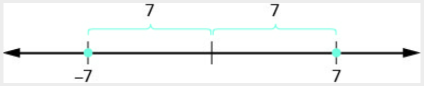
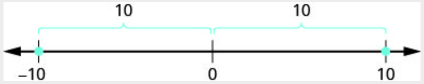
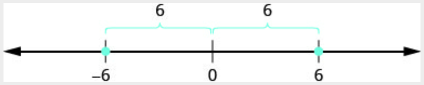
$10-4$	Between two numbers, it indicates the operation of *subtraction*. We read $10-4$ as "10 minus 4."
-8	In front of a number, it indicates a *negative* number. We read −8 as "negative eight."
$-x$	In front of a variable, it indicates the *opposite*. We read $-x$ as "the opposite of x."
$-(-2)$	Here there are two "−" signs. The one in the parentheses tells us the number is negative 2. The one outside the parentheses tells us to take the *opposite* of −2. We read $-(-2)$ as "the opposite of negative two."

Note:
Opposite Notation
$-a$ means the opposite of the number a.
The notation $-a$ is read as “the opposite of a.”

Example:
Exercise:

Problem: **Find: ⓐ the opposite of 7 ⓑ the opposite of -10 ⓒ $-(-6)$.

Solution:
Solution

ⓐ −7 is the same distance from 0 as 7, but on the opposite side of 0.	 The opposite of 7 is −7.
ⓑ 10 is the same distance from 0 as −10, but on the opposite side of 0.	 The opposite of −10 is 10.
ⓒ −(−6)	 −(−6), read as "the opposite of (-6)", is 6.

Note:
Exercise:

TRY IT :: 1.61
Problem: Find: ⓐ the opposite of 4 ⓑ the opposite of -3 ⓒ $-(-1)$.

Solution:

ⓐ -4 ⓑ 3 ⓒ 1

Note:
Exercise:

TRY IT :: 1.62
Problem: Find: ⓐ the opposite of 8 ⓑ the opposite of -5 ⓒ $-(-5)$.

Solution:

ⓐ -8 ⓑ 5 ⓒ 5

Our work with opposites gives us a way to define the integers.The whole numbers and their opposites are called the **integers**. The integers are the numbers $\ldots-3,-2,-1,0,1,2,3\ldots$

Note:
Integers
The whole numbers and their opposites are called the **integers**.
The integers are the numbers
Equation:

$$\ldots-3,-2,-1,0,1,2,3\ldots$$

When evaluating the opposite of a variable, we must be very careful. Without knowing whether the variable represents a positive or negative number, we don't know whether $-x$ is positive or negative. We can see this in [link].

Example:
Exercise:

Problem: **Evaluate ⓐ $-x$, when $x=8$ ⓑ $-x$, when $x=-8$.

Solution:
Solution

ⓐ

To evaluate when $x = 8$ means to substitute 8 for x.	
	$-x$
Substitute 8 for x.	$-(8)$
Write the opposite of 8.	-8

ⓑ

To evaluate when $x = -8$ means to substitute -8 for $-x$.	
	$-x$
Substitute -8 for x.	$-(-8)$
Write the opposite of -8.	8

Note:
Exercise:

TRY IT :: 1.63

Problem: Evaluate $-n$, when ⓐ $n = 4$ ⓑ $n = -4$.

Solution:

ⓐ -4 ⓑ 4

Note:
Exercise:

TRY IT :: 1.64
Problem: Evaluate $-m$, when ⓐ $m = 11$ ⓑ $m = -11$.

Solution:

ⓐ -11 ⓑ 11

Simplify: Expressions with Absolute Value

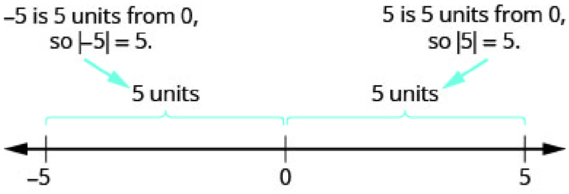
We saw that numbers such as 2 and -2 are opposites because they are the same distance from 0 on the number line. They are both two units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number.

Note:
Absolute Value
The **absolute value** of a number is its distance from 0 on the number line.
The absolute value of a number *n* is written as $|n|$.

For example,

- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.

[link] illustrates this idea.



The integers 5 and are 5 units away from 0.

The absolute value of a number is never negative (because distance cannot be negative). The only number with absolute value equal to zero is the number zero itself, because the distance from 0 to 0 on the number line is zero units.

Note:
Property of Absolute Value
$|n| \geq 0$ for all numbers
Absolute values are always greater than or equal to zero!

Mathematicians say it more precisely, “absolute values are always non-negative.” Non-negative means greater than or equal to zero.

Example:
Exercise:

Problem: **Simplify: ⓐ $|3|$ ⓑ $|-44|$ ⓒ $|0|$.

Solution:
Solution

The absolute value of a number is the distance between the number and zero. Distance is never negative, so the absolute value is never negative.

ⓐ $|3|$
3

ⓑ $|-44|$
44

ⓒ $|0|$
0

Note:
Exercise:

TRY IT :: 1.65
Problem: Simplify: ⓐ $|4|$ ⓑ $|-28|$ ⓒ $|0|$.

Solution:

ⓐ 4 ⓑ 28 ⓒ 0

Note:
Exercise:

TRY IT :: 1.66

Problem: Simplify: ⓐ $|-13|$ ⓑ $|47|$ ⓒ $|0|$.

Solution:

ⓐ 13 ⓑ 47 ⓒ 0

In the next example, we'll order expressions with absolute values. Remember, positive numbers are always greater than negative numbers!

Example:
Exercise:

Problem: **Fill in $<, >,$ or $=$ for each of the following pairs of numbers:

ⓐ $|-5|$ ___ $-|-5|$ ⓑ 8 ___ $-|-8|$ ⓒ -9 ___ $-|-9|$ ⓓ $-(-16)$ ___ $-|-16|$

Solution:
Solution

ⓐ Simplify. Order.	$\begin{array}{ccc} \lvert-5\rvert & \underline{\quad} & -\lvert-5\rvert \\ 5 & \underline{\quad} & -5 \\ 5 & > & -5 \\ \lvert-5\rvert & > & -\lvert-5\rvert \end{array}$
ⓑ Simplify. Order.	$\begin{array}{ccc} 8 & \underline{\quad} & -\lvert-8\rvert \\ 8 & \underline{\quad} & -8 \\ 8 & > & -8 \\ 8 & > & -\lvert-8\rvert \end{array}$
ⓒ Simplify. Order.	$\begin{array}{ccc} 9 & \underline{\quad} & -\lvert-9\rvert \\ -9 & \underline{\quad} & -9 \\ -9 & = & -9 \\ -9 & = & -\lvert-9\rvert \end{array}$
ⓓ Simplify. Order.	

	$\begin{array}{ccc} -(-16) & \_\_\_ & -\lvert -16 \rvert \\ 16 & \_\_\_ & -16 \\ 16 & > & -16 \\ -(-16) & > & -\lvert -16 \rvert \end{array}$

Note:
Exercise:

Problem:

TRY IT :: 1.67
Fill in <, >, or = for each of the following pairs of numbers: ⓐ $|-9|$ ___ $-|-9|$ ⓑ 2 ___ $-|-2|$
ⓒ -8 ___ $|-8|$
ⓓ $-(-9)$ ___ $-|-9|$.

Solution:

ⓐ > ⓑ > ⓒ < ⓓ >

Note:
Exercise:

Problem:

TRY IT :: 1.68
Fill in <, >, or = for each of the following pairs of numbers: ⓐ 7 ___ $-|-7|$ ⓑ
$-(-10)$ ___ $-|-10|$
ⓒ $|-4|$ ___ $-|-4|$ ⓓ -1 ___ $|-1|$.

Solution:

ⓐ > ⓑ > ⓒ > ⓓ <

We now add absolute value bars to our list of grouping symbols. When we use the order of operations, first we simplify inside the absolute value bars as much as possible, then we take the absolute value of the resulting number.

Note:
Grouping Symbols
Equation:

Parentheses	$()$	Braces	$\{\}$
Brackets	$[]$	Absolute value	$\|\;\|$

In the next example, we simplify the expressions inside absolute value bars first, just like we do with parentheses.

Example:
Exercise:

Problem: **Simplify: $24 - |19 - 3\,(6 - 2)|$.

Solution:
Solution

	$24 - \|19 - 3\,(6 - 2)\|$
Work inside parentheses first: subtract 2 from 6.	$24 - \|19 - 3\,(4)\|$
Multiply 3(4).	$24 - \|19 - 12\|$
Subtract inside the absolute value bars.	$24 - \|7\|$
Take the absolute value.	$24 - 7$
Subtract.	17

Note:
Exercise:

TRY IT :: 1.69
Problem: Simplify: $19 - |11 - 4\,(3 - 1)|$.

Solution:

16

Note:
Exercise:

TRY IT :: 1.70

Problem: Simplify: $9 - |8 - 4\,(7 - 5)|$.

Solution:

9

Example:
Exercise:

Problem:

**Evaluate: ⓐ $|x|$ when $x = -35$ ⓑ $|-y|$ when $y = -20$ ⓒ $-|u|$ when $u = 12$ ⓓ $-|p|$ when $p = -14$.

Solution:
Solution

ⓐ $|x|$ when $x = -35$

	$\lvert x\rvert$
Substitute −35 for *x*.	$\lvert -35\rvert$
Take the absolute value.	35

ⓑ $|-y|$ when $y = -20$

	$\lvert -y\rvert$
Substitute −20 for *y*.	$\lvert -(-20)\rvert$

Simplify.	$\lvert 20 \rvert$
Take the absolute value.	20

ⓒ $-\lvert u \rvert$ when $u = 12$

	$-\lvert u \rvert$
Substitute 12 for *u*.	$-\lvert 12 \rvert$
Take the absolute value.	-12

ⓓ $-\lvert p \rvert$ when $p = -14$

	$-\lvert p \rvert$
Substitute -14 for *p*.	$-\lvert -14 \rvert$
Take the absolute value.	-14

Note:
Exercise:

Problem:

TRY IT :: 1.71
Evaluate: ⓐ $\lvert x \rvert$ when $x = -17$ ⓑ $\lvert -y \rvert$ when $y = -39$ ⓒ $-\lvert m \rvert$ when $m = 22$ ⓓ $-\lvert p \rvert$ when $p = -11$.

Solution:

ⓐ 17 ⓑ 39 ⓒ -22 ⓓ -11

Note:
Exercise:

Problem:

TRY IT :: 1.72
Evaluate: ⓐ $|y|$ when $y=-23$ ⓑ $|-y|$ when $y=-21$ ⓒ $-|n|$ when $n=37$ ⓓ $-|q|$ when $q=-49$.

Solution:

ⓐ 23 ⓑ 21 ⓒ -37 ⓓ -49

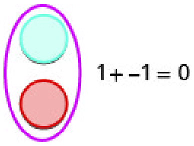
Add Integers

Most students are comfortable with the addition and subtraction facts for positive numbers. But doing addition or subtraction with both positive and negative numbers may be more challenging.

Note: Doing the Manipulative Mathematics activity “Addition of Signed Numbers” will help you develop a better understanding of adding integers.”

We will use two color counters to model addition and subtraction of negatives so that you can visualize the procedures instead of memorizing the rules.

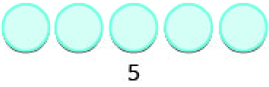
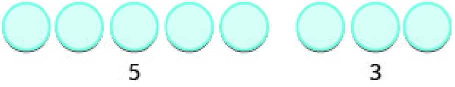
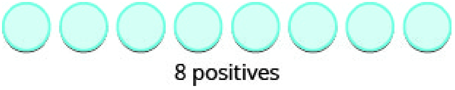
We let one color (blue) represent positive. The other color (red) will represent the negatives. If we have one positive counter and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero.



We will use the counters to show how to add the four addition facts using the numbers $5, -5$ and $3, -3$.
Equation:

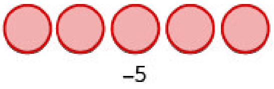
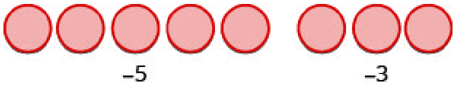
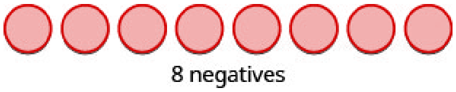
$$5+3 \qquad -5+(-3) \qquad -5+3 \qquad 5+(-3)$$

To add $5+3$, we realize that $5+3$ means the sum of 5 and 3.

We start with 5 positives.	
And then we add 3 positives.	
We now have 8 positives. The sum of 5 and 3 is 8.	

Now we will add $-5 + (-3)$. Watch for similarities to the last example $5 + 3 = 8$.

To add $-5 + (-3)$, we realize this means the sum of -5 and -3.

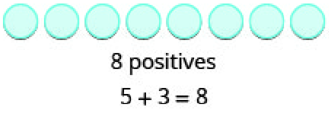
We start with 5 negatives.	
And then we add 3 negatives.	
We now have 8 negatives. The sum of −5 and −3 is −8.	

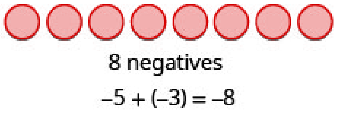
In what ways were these first two examples similar?

- The first example adds 5 positives and 3 positives—both positives.
- The second example adds 5 negatives and 3 negatives—both negatives.

In each case we got 8—either 8 positives or 8 negatives.

When the signs were the same, the counters were all the same color, and so we added them.



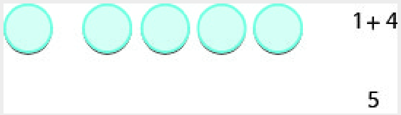


Example:
Exercise:

Problem: **Add: ⓐ $1 + 4$ ⓑ $-1 + (-4)$.

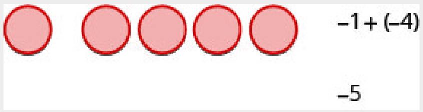
Solution:
Solution

ⓐ



1 positive plus 4 positives is 5 positives.

ⓑ



1 negative plus 4 negatives is 5 negatives.

Note:
Exercise:

TRY IT :: 1.73

Problem: Add: ⓐ $2+4$ ⓑ $-2+(-4)$.

Solution:

ⓐ 6 ⓑ -6

Note:

Exercise:

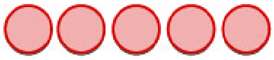
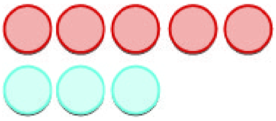
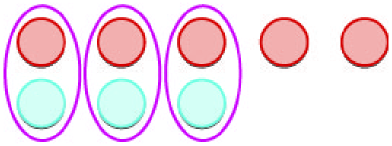
TRY IT :: 1.74

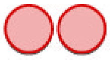
Problem: Add: ⓐ $2+5$ ⓑ $-2+(-5)$.

Solution:

ⓐ 7 ⓑ -7

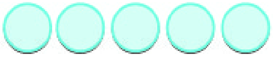
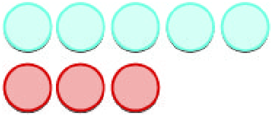
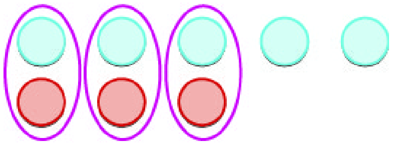
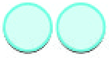
So what happens when the signs are different? Let’s add $-5+3$. We realize this means the sum of -5 and 3. When the counters were the same color, we put them in a row. When the counters are a different color, we line them up under each other.

	−5 + 3 means the sum of −5 and 3.
We start with 5 negatives.	
And then we add 3 positives.	
We remove any neutral pairs.	

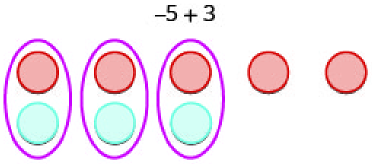
We have 2 negatives left.	 2 negatives
The sum of −5 and 3 is −2.	−5 + 3 = −2

Notice that there were more negatives than positives, so the result was negative.

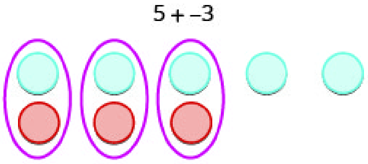
Let’s now add the last combination, $5+(-3)$.

	5 + (−3) means the sum of 5 and −3.
We start with 5 positives.	
And then we add 3 negatives.	
We remove any neutral pairs.	
We have 2 positives left.	 2 positives
The sum of 5 and −3 is 2.	5 + (−3) = 2

When we use counters to model addition of positive and negative integers, it is easy to see whether there are more positive or more negative counters. So we know whether the sum will be positive or negative.



More negatives – the sum is negative.



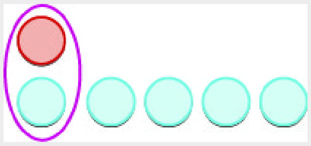
More positives – the sum is positive.

Example:
Exercise:

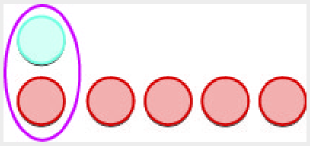
Problem: **Add: ⓐ $-1+5$ ⓑ $1+(-5)$.

Solution:
Solution

ⓐ

	−1 + 5
	
There are more positives, so the sum is positive.	4

ⓑ

	1 + (−5)
	
There are more negatives, so the sum is negative.	−4

Note:

Exercise:

TRY IT :: 1.75

Problem: Add: ⓐ $-2+4$ ⓑ $2+(-4)$.

Solution:

ⓐ 2 ⓑ -2

Note:
Exercise:

TRY IT :: 1.76

Problem: Add: ⓐ $-2+5$ ⓑ $2+(-5)$.

Solution:

ⓐ 3 ⓑ -3

Now that we have added small positive and negative integers with a model, we can visualize the model in our minds to simplify problems with any numbers.

When you need to add numbers such as $37+(-53)$, you really don’t want to have to count out 37 blue counters and 53 red counters. With the model in your mind, can you visualize what you would do to solve the problem?

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more red (negative) counters than blue (positive) counters, the sum would be *negative*. How many more red counters would there be? Because $53-37=16$, there are 16 more red counters.

Therefore, the sum of $37+(-53)$ is -16.
Equation:

$$37+(-53)=-16$$

Let’s try another one. We’ll add $-74+(-27)$. Again, imagine 74 red counters and 27 more red counters, so we’d have 101 red counters. This means the sum is -101.
Equation:

$$-74+(-27)=-101$$

Let’s look again at the results of adding the different combinations of 5, -5 and 3, -3.

Note:

Addition of Positive and Negative Integers
Equation:

$$\begin{array}{cc} 5+3 & -5+(-3) \\ 8 & -8 \\ \text{both positive, sum positive} & \text{both negative, sum negative} \end{array}$$

When the signs are the same, the counters would be all the same color, so add them.
Equation:

$$\begin{array}{cc} -5+3 & 5+(-3) \\ -2 & 2 \\ \text{different signs, more negatives, sum negative} & \text{different signs, more positives, sum positive} \end{array}$$

When the signs are different, some of the counters would make neutral pairs, so subtract to see how many are left.

Visualize the model as you simplify the expressions in the following examples.

Example:
Exercise:

Problem: **Simplify: ⓐ $19+(-47)$ ⓑ $-14+(-36)$.

Solution:
Solution

ⓐ Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

$$\begin{array}{ll} & 19+(-47) \\ \text{Add.} & -28 \end{array}$$

ⓑ Since the signs are the same, we add. The answer will be negative because there are only negatives.

$$\begin{array}{ll} & -14+(-36) \\ \text{Add.} & -50 \end{array}$$

Note:
Exercise:

TRY IT :: 1.77
Problem: Simplify: ⓐ $-31+(-19)$ ⓑ $15+(-32)$.

Solution:

ⓐ -50 ⓑ -17

Note:
Exercise:

TRY IT :: 1.78
Problem: Simplify: ⓐ $-42+(-28)$ ⓑ $25+(-61)$.

Solution:

ⓐ -70 ⓑ -36

The techniques used up to now extend to more complicated problems, like the ones we've seen before. Remember to follow the order of operations!

Example:
Exercise:

Problem: **Simplify: $-5+3(-2+7)$.

Solution:
Solution

	$-5+3(-2+7)$
Simplify inside the parentheses.	$-5+3(5)$
Multiply.	$-5+15$
Add left to right.	10

Note:
Exercise:

TRY IT :: 1.79

Problem: Simplify: $-2+5(-4+7)$.

Solution:

13

Note:
Exercise:

TRY IT :: 1.80

Problem: Simplify: $-4+2(-3+5)$.

Solution:

0

Subtract Integers

Note: Doing the Manipulative Mathematics activity “Subtraction of Signed Numbers” will help you develop a better understanding of subtracting integers.

We will continue to use counters to model the subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative numbers.

Perhaps when you were younger, you read “$5-3$” as “5 take away 3.” When you use counters, you can think of subtraction the same way!

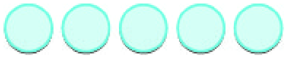
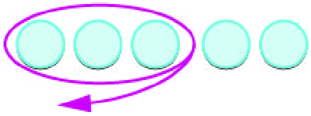
We will model the four subtraction facts using the numbers 5 and 3.

Equation:

$$5-3 \qquad -5-(-3) \qquad -5-3 \qquad 5-(-3)$$

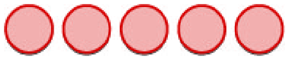
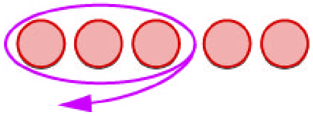
To subtract $5-3$, we restate the problem as “5 take away 3.”

We start with 5 positives.	

	
We ‘take away’ 3 positives.	
We have 2 positives left.	
The difference of 5 and 3 is 2.	2

Now we will subtract $-5-(-3)$. Watch for similarities to the last example $5-3=2$.

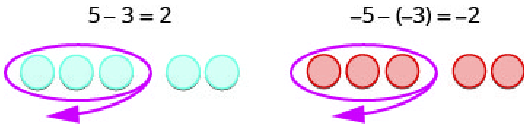
To subtract $-5-(-3)$, we restate this as “–5 take away –3”

We start with 5 negatives.	
We ‘take away’ 3 negatives.	
We have 2 negatives left.	
The difference of −5 and −3 is −2.	−2

Notice that these two examples are much alike: The first example, we subtract 3 positives from 5 positives and end up with 2 positives.

In the second example, we subtract 3 negatives from 5 negatives and end up with 2 negatives.

Each example used counters of only one color, and the “take away” model of subtraction was easy to apply.



Example:
Exercise:

Problem: **Subtract: ⓐ $7 - 5$ ⓑ $-7 - (-5)$.

Solution:
Solution

ⓐ Take 5 positive from 7 positives and get 2 positives.	$7 - 5$ 2
ⓑ Take 5 negatives from 7 negatives and get 2 negatives.	$-7 - (-5)$ -2

Note:
Exercise:

TRY IT :: 1.81
Problem: Subtract: ⓐ $6 - 4$ ⓑ $-6 - (-4)$.

Solution:

ⓐ 2 ⓑ -2

Note:
Exercise:

TRY IT :: 1.82
Problem: Subtract: ⓐ $7 - 4$ ⓑ $-7 - (-4)$.

Solution:

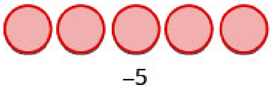
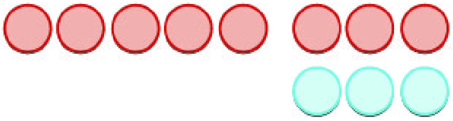
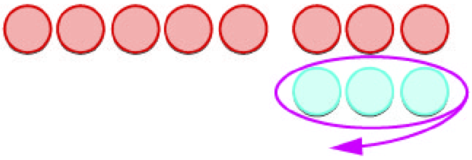
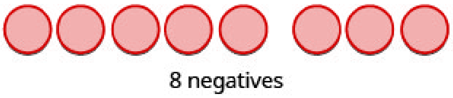
ⓐ 3 ⓑ -3

What happens when we have to subtract one positive and one negative number? We'll need to use both white and red counters as well as some neutral pairs. Adding a neutral pair does not change the value. It is like changing quarters to nickels—the value is the same, but it looks different.

- To subtract $-5-3$, we restate it as -5 take away 3.

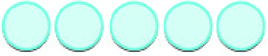
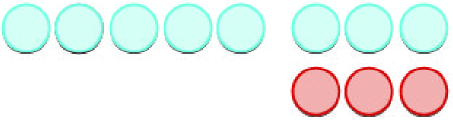
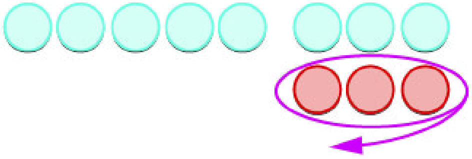
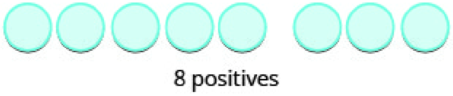
We start with 5 negatives. We need to take away 3 positives, but we do not have any positives to take away.

Remember, a neutral pair has value zero. If we add 0 to 5 its value is still 5. We add neutral pairs to the 5 negatives until we get 3 positives to take away.

	−5 − 3 means −5 take away 3.
We start with 5 negatives.	
We now add the neutrals needed to get 3 positives.	
We remove the 3 positives.	
We are left with 8 negatives.	
The difference of −5 and 3 is −8.	−5 − 3 = −8

And now, the fourth case, $5-(-3)$. We start with 5 positives. We need to take away 3 negatives, but there are no negatives to take away. So we add neutral pairs until we have 3 negatives to take away.

	5 − (−3) means 5 take away −3.

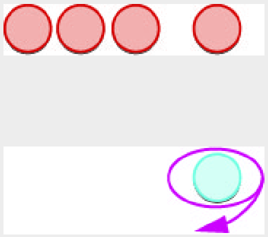
We start with 5 positives.	
We now add the needed neutrals pairs.	
We remove the 3 negatives.	
We are left with 8 positives.	
The difference of 5 and −3 is 8.	5 − (−3) = 8

Example:
Exercise:

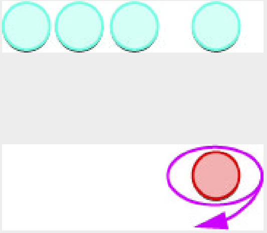
Problem: **Subtract: ⓐ −3 − 1 ⓑ 3 − (−1).

Solution:
Solution

ⓐ

Take 1 positive from the one added neutral pair.		−3 − 1 −4

ⓑ

Take 1 negative from the one added neutral pair.		$3 - (-1)$ 4

Note:
Exercise:

TRY IT :: 1.83
Problem: Subtract: ⓐ $-6 - 4$ ⓑ $6 - (-4)$.

Solution:

ⓐ -10 ⓑ 10

Note:
Exercise:

TRY IT :: 1.84
Problem: Subtract: ⓐ $-7 - 4$ ⓑ $7 - (-4)$.

Solution:

ⓐ -11 ⓑ 11

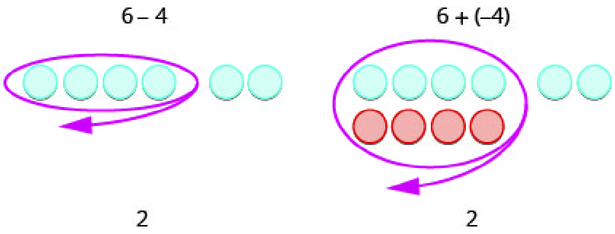
Have you noticed that *subtraction of signed numbers can be done by adding the opposite*? In [link], $-3 - 1$ is the same as $-3 + (-1)$ and $3 - (-1)$ is the same as $3 + 1$. You will often see this idea, the **subtraction property**, written as follows:

Note:
Subtraction Property
Equation:

$$a - b = a + (-b)$$

Subtracting a number is the same as adding its opposite.

Look at these two examples.



Equation:

$$6 - 4 \text{ gives the same answer as } 6 + (-4).$$

Of course, when you have a subtraction problem that has only positive numbers, like $6 - 4$, you just do the subtraction. You already knew how to subtract $6 - 4$ long ago. But *knowing* that $6 - 4$ gives the same answer as $6 + (-4)$ helps when you are subtracting negative numbers. Make sure that you understand how $6 - 4$ and $6 + (-4)$ give the same results!

Example:
Exercise:

Problem: **Simplify: ⓐ $13 - 8$ and $13 + (-8)$ ⓑ $-17 - 9$ and $-17 + (-9)$.

Solution:
Solution

ⓐ Subtract.	$13 - 8$ 5	$13 + (-8)$ 5
ⓑ Subtract.	$-17 - 9$ -26	$-17 + (-9)$ -26

Note:
Exercise:

TRY IT :: 1.85

Problem: Simplify: ⓐ $21 - 13$ and $21 + (-13)$ ⓑ $-11 - 7$ and $-11 + (-7)$.

Solution:

ⓐ 8 ⓑ -18

Note:
Exercise:

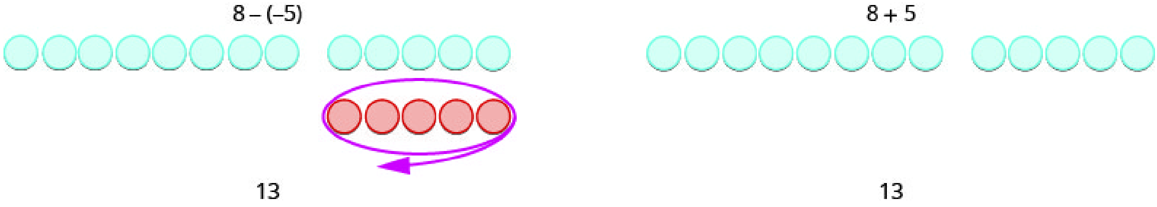
TRY IT :: 1.86

Problem: Simplify: ⓐ $15 - 7$ and $15 + (-7)$ ⓑ $-14 - 8$ and $-14 + (-8)$.

Solution:

ⓐ 8 ⓑ -22

Look at what happens when we subtract a negative.



Equation:

$$8 - (-5) \text{ gives the same answer as } 8 + 5$$

Subtracting a negative number is like adding a positive!

You will often see this written as $a - (-b) = a + b$.

Does that work for other numbers, too? Let’s do the following example and see.

Example:
Exercise:

Problem: **Simplify: ⓐ $9 - (-15)$ and $9 + 15$ ⓑ $-7 - (-4)$ and $-7 + 4$.

Solution:
Solution

ⓐ Subtract.	$9 - (-15)$ 24	$9 + 15$ 24
ⓑ Subtract.	$-7 - (-4)$ -3	$-7 + 4$ -3

Note:
Exercise:

TRY IT :: 1.87
Problem: Simplify: ⓐ $6 - (-13)$ and $6 + 13$ ⓑ $-5 - (-1)$ and $-5 + 1$.

Solution:

ⓐ 19 ⓑ -4

Note:
Exercise:

TRY IT :: 1.88
Problem: Simplify: ⓐ $4 - (-19)$ and $4 + 19$ ⓑ $-4 - (-7)$ and $-4 + 7$.

Solution:

ⓐ 23 ⓑ 3

Let’s look again at the results of subtracting the different combinations of $5, -5$ and $3, -3$.

Note:
Subtraction of Integers
Equation:

$$\begin{array}{cc} 5-3 & -5-(-3) \\ 2 & -2 \\ \text{5 positives take away 3 positives} & \text{5 negatives take away 3 negatives} \\ \text{2 positives} & \text{2 negatives} \end{array}$$

When there would be enough counters of the color to take away, subtract.

Equation:

$$\begin{array}{cc} -5-3 & 5-(-3) \\ -8 & 8 \\ \text{5 negatives, want to take away 3 positives} & \text{5 positives, want to take away 3 negatives} \\ \text{need neutral pairs} & \text{need neutral pairs} \end{array}$$

When there would be not enough counters of the color to take away, add.

What happens when there are more than three integers? We just use the order of operations as usual.

Example:
Exercise:

Problem: **Simplify: $7-(-4-3)-9$.

Solution:
Solution

	$7-(-4-3)-9$
Simplify inside the parentheses first.	$7-(-7)-9$
Subtract left to right.	$14-9$
Subtract.	5

Note:
Exercise:

TRY IT :: 1.89

Problem: Simplify: $8 - (-3 - 1) - 9$.

Solution:

3

Note:

Exercise:

TRY IT :: 1.90

Problem: Simplify: $12 - (-9 - 6) - 14$.

Solution:

13

Note:

Access these online resources for additional instruction and practice with adding and subtracting integers. You will need to enable Java in your web browser to use the applications.

- Add Colored Chip
- Subtract Colored Chip

Key Concepts

- **Addition of Positive and Negative Integers**

$5 + 3$ 8 both positive, sum positive	$-5 + (-3)$ -8 both negative, sum negative
$-5 + 3$ -2 different signs, more negatives sum negative	$5 + (-3)$ 2 different signs, more positives sum positive

- **Property of Absolute Value**: $|n| \geq 0$ for all numbers. Absolute values are always greater than or equal to zero!

- **Subtraction of Integers**

$5 - 3$	$-5 - (-3)$
2	-2
5 positives	5 negatives
take away 3 positives	take away 3 negatives
2 positives	2 negatives
$-5 - 3$	$5 - (-3)$
-8	8
5 negatives, want to subtract 3 positives	5 positives, want to subtract 3 negatives
need neutral pairs	need neutral pairs

- **Subtraction Property:** Subtracting a number is the same as adding its opposite.

Practice Makes Perfect

Use Negatives and Opposites of Integers

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise:

ⓐ 9 ___ 4
ⓑ -3 ___ 6
ⓒ -8 ___ -2
Problem: ⓓ 1 ___ -10

Solution:

ⓐ > ⓑ < ⓒ < ⓓ >

Exercise:

ⓐ -7 ___ 3
ⓑ -10 ___ -5
ⓒ 2 ___ -6
Problem: ⓓ 8 ___ 9

In the following exercises, find the opposite of each number.

Exercise:

ⓐ 2
Problem: ⓑ -6

Solution:

ⓐ -2 ⓑ 6

Exercise:

ⓐ 9
Problem: ⓑ -4

In the following exercises, simplify.

Exercise:

Problem: $-(-4)$

Solution:

4

Exercise:

Problem: $-(-8)$

Exercise:

Problem: $-(-15)$

Solution:

15

Exercise:

Problem: $-(-11)$

In the following exercises, evaluate.

Exercise:

$-c$ when
ⓐ $c=12$
Problem: ⓑ $c=-12$

Solution:

ⓐ -12 ⓑ 12

Exercise:

$-d$ when
ⓐ $d=21$
Problem: ⓑ $d=-21$

Simplify Expressions with Absolute Value

In the following exercises, simplify.

Exercise:

ⓐ $|-32|$
ⓑ $|0|$
Problem: ⓒ $|16|$

Solution:

ⓐ 32 ⓑ 0 ⓒ 16

Exercise:

ⓐ $|0|$
ⓑ $|-40|$
Problem: ⓒ $|22|$

In the following exercises, fill in <, >, or $=$ for each of the following pairs of numbers.

Exercise:

ⓐ -6 ___ $|-6|$
Problem: ⓑ $-|-3|$ ___ -3

Solution:

ⓐ $<$ ⓑ $=$

Exercise:

ⓐ $|-5|$ ___ $-|-5|$
Problem: ⓑ 9 ___ $-|-9|$

In the following exercises, simplify.

Exercise:

Problem: $-(-5)$ and $-|-5|$

Solution:

$5, -5$

Exercise:

Problem: $-|-9|$ and $-(-9)$

Exercise:

Problem: $8\left|-7\right|$

Solution:

56

Exercise:

Problem: $5\left|-5\right|$

Exercise:

Problem: $\left|15-7\right|-\left|14-6\right|$

Solution:

0

Exercise:

Problem: $\left|17-8\right|-\left|13-4\right|$

Exercise:

Problem: $18-\left|2\left(8-3\right)\right|$

Solution:

8

Exercise:

Problem: $18-\left|3\left(8-5\right)\right|$

In the following exercises, evaluate.
Exercise:

Problem: ⓐ $-\left|p\right|$ when $p=19$
ⓑ $-\left|q\right|$ when $q=-33$

Solution:

ⓐ -19 ⓑ -33

Exercise:

Problem: ⓐ $-\left|a\right|$ when $a=60$
ⓑ $-\left|b\right|$ when $b=-12$

Add Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-21 + (-59)$

Solution:

-80

Exercise:

Problem: $-35 + (-47)$

Exercise:

Problem: $48 + (-16)$

Solution:

32

Exercise:

Problem: $34 + (-19)$

Exercise:

Problem: $-14 + (-12) + 4$

Solution:

-22

Exercise:

Problem: $-17 + (-18) + 6$

Exercise:

Problem: $135 + (-110) + 83$

Solution:

108

Exercise:

Problem: $-38 + 27 + (-8) + 12$

Exercise:

Problem: $19+2\left(-3+8\right)$

Solution:

29

Exercise:

Problem: $24+3\left(-5+9\right)$

Subtract Integers

In the following exercises, simplify.

Exercise:

Problem: $8-2$

Solution:

6

Exercise:

Problem: $-6-\left(-4\right)$

Exercise:

Problem: $-5-4$

Solution:

-9

Exercise:

Problem: $-7-2$

Exercise:

Problem: $8-\left(-4\right)$

Solution:

12

Exercise:

Problem: $7-\left(-3\right)$

Exercise:

ⓐ $44 - 28$
Problem: ⓑ $44 + (-28)$

Solution:

ⓐ 16 ⓑ 16

Exercise:

ⓐ $35 - 16$
Problem: ⓑ $35 + (-16)$

Exercise:

ⓐ $27 - (-18)$
Problem: ⓑ $27 + 18$

Solution:

ⓐ 45 ⓑ 45

Exercise:

ⓐ $46 - (-37)$
Problem: ⓑ $46 + 37$

In the following exercises, simplify each expression.

Exercise:

Problem: $15 - (-12)$

Solution:

27

Exercise:

Problem: $14 - (-11)$

Exercise:

Problem: $48 - 87$

Solution:

-39

Exercise:

Problem: $45 - 69$

Exercise:

Problem: $-17 - 42$

Solution:

-59

Exercise:

Problem: $-19 - 46$

Exercise:

Problem: $-103 - (-52)$

Solution:

-51

Exercise:

Problem: $-105 - (-68)$

Exercise:

Problem: $-45 - (-54)$

Solution:

9

Exercise:

Problem: $-58 - (-67)$

Exercise:

Problem: $8 - 3 - 7$

Solution:

-2

Exercise:

Problem: $9 - 6 - 5$

Exercise:

Problem: $-5-4+7$

Solution:

-2

Exercise:

Problem: $-3-8+4$

Exercise:

Problem: $-14-(-27)+9$

Solution:

22

Exercise:

Problem: $64+(-17)-9$

Exercise:

Problem: $(2-7)-(3-8)$

Solution:

0

Exercise:

Problem: $(1-8)-(2-9)$

Exercise:

Problem: $-(6-8)-(2-4)$

Solution:

4

Exercise:

Problem: $-(4-5)-(7-8)$

Exercise:

Problem: $25-[10-(3-12)]$

Solution:

6

Exercise:

Problem: $32 - [5 - (15 - 20)]$

Exercise:

Problem: $6.3 - 4.3 - 7.2$

Solution:

-8

Exercise:

Problem: $5.7 - 8.2 - 4.9$

Exercise:

Problem: $5^2 - 6^2$

Solution:

-11

Exercise:

Problem: $6^2 - 7^2$

Everyday Math

Exercise:

Problem:

Elevation The highest elevation in the United States is Mount McKinley, Alaska, at 20,320 feet above sea level. The lowest elevation is Death Valley, California, at 282 feet below sea level.

Use integers to write the elevation of:

ⓐ Mount McKinley.
ⓑ Death Valley.

Solution:

ⓐ 20,329 ⓑ -282

Exercise:

Problem:

Extreme temperatures The highest recorded temperature on Earth was 58° Celsius, recorded in the Sahara Desert in 1922. The lowest recorded temperature was 90° below 0° Celsius, recorded in Antarctica in 1983.

Use integers to write the:

ⓐ highest recorded temperature.
ⓑ lowest recorded temperature.

Exercise:

Problem:

State budgets In June, 2011, the state of Pennsylvania estimated it would have a budget surplus of $540 million. That same month, Texas estimated it would have a budget deficit of $27 billion.

Use integers to write the budget of:

ⓐ Pennsylvania.
ⓑ Texas.

Solution:

ⓐ $540 million ⓑ $-\$27$ billion

Exercise:

Problem:

College enrollments Across the United States, community college enrollment grew by 1,400,000 students from Fall 2007 to Fall 2010. In California, community college enrollment declined by 110,171 students from Fall 2009 to Fall 2010.

Use integers to write the change in enrollment:

ⓐ in the U.S. from Fall 2007 to Fall 2010.
ⓑ in California from Fall 2009 to Fall 2010.

Exercise:

Problem:

Stock Market The week of September 15, 2008 was one of the most volatile weeks ever for the US stock market. The closing numbers of the Dow Jones Industrial Average each day were:

Monday	-504

Tuesday	$+142$
Wednesday	-449
Thursday	$+410$
Friday	$+369$

What was the overall change for the week? Was it positive or negative?

Solution:

-32

Exercise:

Problem:

Stock Market During the week of June 22, 2009, the closing numbers of the Dow Jones Industrial Average each day were:

Monday	-201
Tuesday	-16
Wednesday	-23
Thursday	$+172$
Friday	-34

What was the overall change for the week? Was it positive or negative?

Writing Exercises

Exercise:

Problem: Give an example of a negative number from your life experience.

Solution:

Answers may vary

Exercise:

Problem: What are the three uses of the “ $-$ ” sign in algebra? Explain how they differ.

Exercise:

Problem: Explain why the sum of -8 and 2 is negative, but the sum of 8 and -2 is positive.

Solution:

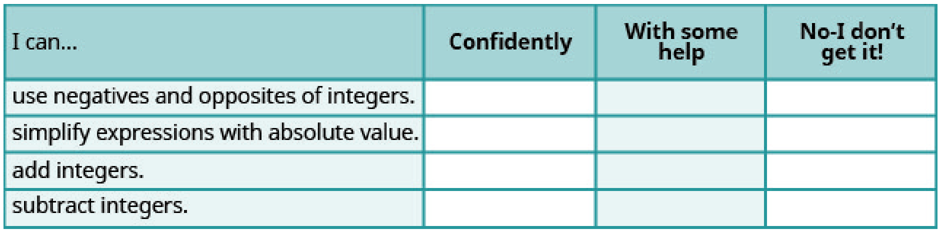
Answers may vary

Exercise:

Problem: Give an example from your life experience of adding two negative numbers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
use negatives and opposites of integers.			
simplify expressions with absolute value.			
add integers.			
subtract integers.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

absolute value

The absolute value of a number is its distance from 0 on the number line. The absolute value of a number n is written as $|n|$.

integers

The whole numbers and their opposites are called the integers: ...−3, −2, −1, 0, 1, 2, 3...

opposite

The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of zero: $-a$ means the opposite of the number. The notation $-a$ is read “the opposite of a.”

Multiply and Divide Integers

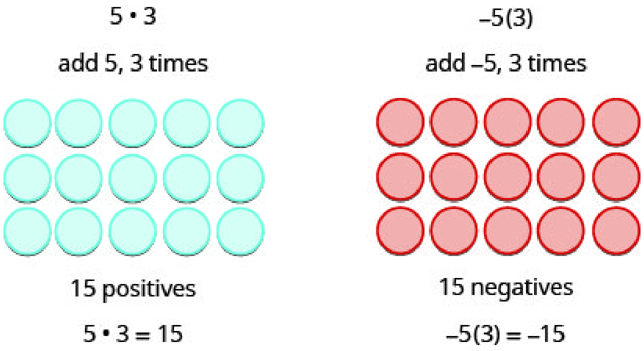
By the end of this section, you will be able to:

- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate English phrases to algebraic expressions
- Use integers in applications

Multiply Integers

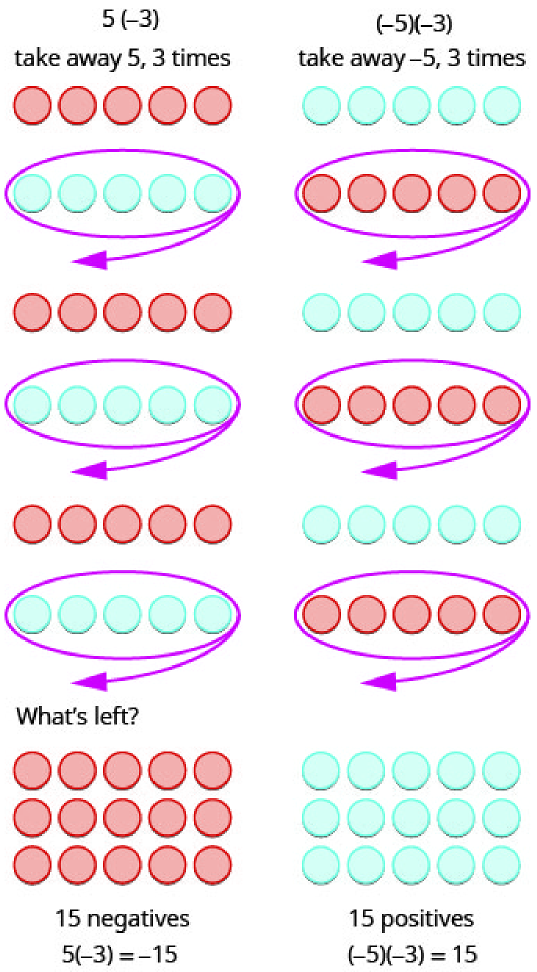
Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show multiplication of integers. Let’s look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction. Here, we will use the model just to help us discover the pattern.

We remember that $a \cdot b$ means add a, b times. Here, we are using the model just to help us discover the pattern.



The next two examples are more interesting.

What does it mean to multiply 5 by -3? It means subtract 5, 3 times. Looking at subtraction as “taking away,” it means to take away 5, 3 times. But there is nothing to take away, so we start by adding neutral pairs on the workspace. Then we take away 5 three times.



In summary:
Equation:

$$\begin{array}{rclcrcl} 5 \cdot 3 & = & 15 & \quad & -5\left(3\right) & = & -15 \\ 5\left(-3\right) & = & -15 & & \left(-5\right)\left(-3\right) & = & 15 \end{array}$$

Notice that for multiplication of two signed numbers, when the:

- signs are the *same*, the product is *positive*.
- signs are *different*, the product is *negative*.

We'll put this all together in the chart below.

Note:
Multiplication of Signed Numbers
For multiplication of two signed numbers:

Same signs	Product	Example
Two positives Two negatives	Positive Positive	$7 \cdot 4 = 28$ $-8(-6) = 48$

Different signs	Product	Example
Positive · negative Negative · positive	Negative Negative	$7(-9) = -63$ $-5 \cdot 10 = -50$

Example:
Exercise:

Problem: **Multiply: ⓐ $-9 \cdot 3$ ⓑ $-2(-5)$ ⓒ $4(-8)$ ⓓ $7 \cdot 6$.

Solution:
Solution

ⓐ Multiply, noting that the signs are different so the product is negative.	$-9 \cdot 3$ -27
ⓑ Multiply, noting that the signs are the same so the product is positive.	$-2(-5)$ 10
ⓒ Multiply, with different signs.	$4(-8)$ -32
ⓓ Multiply, with same signs.	$7 \cdot 6$ 42

Note:
Exercise:

TRY IT :: 1.91

Problem: Multiply: ⓐ $-6 \cdot 8$ ⓑ $-4\,(-7)$ ⓒ $9\,(-7)$ ⓓ $5 \cdot 12$.

Solution:

ⓐ -48 ⓑ 28 ⓒ -63 ⓓ 60

Note:
Exercise:

TRY IT :: 1.92

Problem: Multiply: ⓐ $-8 \cdot 7$ ⓑ $-6\,(-9)$ ⓒ $7\,(-4)$ ⓓ $3 \cdot 13$.

Solution:

ⓐ -56 ⓑ 54 ⓒ -28 ⓓ 39

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1? Let's multiply a positive number and then a negative number by -1 to see what we get.

Equation:

$$\begin{array}{lcc} & -1 \cdot 4 & -1(-3) \\ \text{Multiply.} & -4 & 3 \\ & -4 \text{ is the opposite of } 4. & 3 \text{ is the opposite of } -3. \end{array}$$

Each time we multiply a number by -1, we get its opposite!

Note:
Multiplication by -1
Equation:

$$-1a = -a$$

Multiplying a number by -1 gives its opposite.

Example:
Exercise:

Problem: **Multiply: ⓐ $-1 \cdot 7$ ⓑ $-1\,(-11)$.

Solution:
Solution

ⓐ Multiply, noting that the signs are different so the product is negative.	$-1 \cdot 7$ -7 -7 is the opposite of 7.
ⓑ Multiply, noting that the signs are the same so the product is positive.	$-1\,(-11)$ 11 11 is the opposite of -11.

Note:
Exercise:

TRY IT :: 1.93
Problem: Multiply: ⓐ $-1 \cdot 9$ ⓑ $-1 \cdot (-17)$.

Solution:

ⓐ -9 ⓑ 17

Note:
Exercise:

TRY IT :: 1.94
Problem: Multiply: ⓐ $-1 \cdot 8$ ⓑ $-1 \cdot (-16)$.

Solution:

ⓐ -8 ⓑ 16

Divide Integers

What about division? Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $15 \cdot 3 = 5$. In words, this expression says that 15 can be divided into three groups of five each because adding five three times gives 15. Look at some examples of multiplying integers, to figure out the rules for dividing integers.

Equation:

$$\begin{array}{rclcrcl} 5 \cdot 3 & = & 15 \text{ so } 15 \div 3 & = & 5 & \qquad -5\left(3\right) & = & -15 \text{ so } -15 \div 3 & = & -5 \\ \left(-5\right)\left(-3\right) & = & 15 \text{ so } 15 \div \left(-3\right) & = & -5 & \qquad 5\left(-3\right) & = & -15 \text{ so } -15 \div \left(-3\right) & = & 5 \end{array}$$

Division follows the same rules as multiplication!

For division of two signed numbers, when the:

- signs are the *same*, the quotient is *positive*.
- signs are *different*, the quotient is *negative*.

And remember that we can always check the answer of a division problem by multiplying.

Note:
Multiplication and Division of Signed Numbers
For multiplication and division of two signed numbers:

- If the signs are the same, the result is positive.
- If the signs are different, the result is negative.

Same signs	**Result**
Two positives Two negatives	Positive Positive
If the signs are the same, the result is positive.	

Different signs	**Result**
Positive and negative Negative and positive	Negative Negative
If the signs are different, the result is negative.	

Example:
Exercise:

Problem: **Divide: ⓐ $-27 \div 3$ ⓑ $-100 \div (-4)$.

Solution:
Solution

ⓐ Divide. With different signs, the quotient is negative.	$-27 \div 3$ -9
ⓑ Divide. With signs that are the same, the quotient is positive.	$-100 \div (-4)$ 25

Note:
Exercise:

TRY IT :: 1.95
Problem: Divide: ⓐ $-42 \div 6$ ⓑ $-117 \div (-3)$.

Solution:

ⓐ -7 ⓑ 39

Note:

Exercise:

TRY IT :: 1.96

Problem: Divide: ⓐ $-63 \div 7$ ⓑ $-115 \div (-5)$.

Solution:

ⓐ -9 ⓑ 23

Simplify Expressions with Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember My Dear Aunt Sally?

Let’s try some examples. We’ll simplify expressions that use all four operations with integers—addition, subtraction, multiplication, and division. Remember to follow the order of operations.

Example:
Exercise:

Problem: **Simplify: $7(-2)+4(-7)-6$.

Solution:
Solution

	$7(-2)+4(-7)-6$
Multiply first.	$-14+(-28)-6$
Add.	$-42-6$
Subtract.	-48

Note:

Exercise:

TRY IT :: 1.97

Problem: Simplify: $8\left(-3\right)+5\left(-7\right)-4$.

Solution:

-63

Note:
Exercise:

TRY IT :: 1.98

Problem: Simplify: $9\left(-3\right)+7\left(-8\right)-1$.

Solution:

-84

Example:
Exercise:

Problem: **Simplify: ⓐ $\left(-2\right)^{4}$ ⓑ -2^{4}.

Solution:
Solution

ⓐ Write in expanded form. Multiply. Multiply. Multiply.	$\left(-2\right)^{4}$ $\left(-2\right)\left(-2\right)\left(-2\right)\left(-2\right)$ $4\left(-2\right)\left(-2\right)$ $-8\left(-2\right)$ 16
ⓑ Write in expanded form. We are asked to find the opposite of 2^{4}.	

Multiply. Multiply. Multiply.	-2^4 $-(2\cdot 2\cdot 2\cdot 2)$ $-(4\cdot 2\cdot 2)$ $-(8\cdot 2)$ -16

Notice the difference in parts ⓐ and ⓑ. In part ⓐ, the exponent means to raise what is in the parentheses, the (-2) to the 4th power. In part ⓑ, the exponent means to raise just the 2 to the 4th power and then take the opposite.

Note:
Exercise:

TRY IT :: 1.99

Problem: Simplify: ⓐ $(-3)^4$ ⓑ -3^4.

Solution:

ⓐ 81 ⓑ −81

Note:
Exercise:

TRY IT :: 1.100

Problem: Simplify: ⓐ $(-7)^2$ ⓑ -7^2.

Solution:

ⓐ 49 ⓑ −49

The next example reminds us to simplify inside parentheses first.

Example:
Exercise:

Problem: **Simplify: $12-3(9-12)$.

Solution:
Solution

	$12 - 3\,(9 - 12)$
Subtract in parentheses first.	$12 - 3\,(-3)$
Multiply.	$12 - (-9)$
Subtract.	21

Note:
Exercise:

TRY IT :: 1.101
Problem: Simplify: $17 - 4\,(8 - 11)$.

Solution:

29

Note:
Exercise:

TRY IT :: 1.102
Problem: Simplify: $16 - 6\,(7 - 13)$.

Solution:

52

Example:
Exercise:

Problem: **Simplify: $8\left(-9\right) \div \left(-2\right)^3$.

Solution:
Solution

	$8\left(-9\right) \div \left(-2\right)^3$
Exponents first.	$8\left(-9\right) \div \left(-8\right)$
Multiply.	$-72 \div \left(-8\right)$
Divide.	9

Note:
Exercise:

TRY IT :: 1.103
Problem: Simplify: $12\left(-9\right) \div \left(-3\right)^3$.

Solution:

4

Note:
Exercise:

TRY IT :: 1.104
Problem: Simplify: $18\left(-4\right) \div \left(-2\right)^3$.

Solution:

9

Example:
Exercise:

Problem: **Simplify: $-30 \div 2 + (-3)(-7)$.

Solution:
Solution

	$-30 \div 2 + (-3)(-7)$
Multiply and divide left to right, so divide first.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add.	6

Note:
Exercise:

TRY IT :: 1.105
Problem: Simplify: $-27 \div 3 + (-5)(-6)$.

Solution:

21

Note:
Exercise:

TRY IT :: 1.106
Problem: Simplify: $-32 \div 4 + (-2)(-7)$.

Solution:

6

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers.

Example:
Exercise:

Problem: **When $n = -5$, evaluate: ⓐ $n + 1$ ⓑ $-n + 1$.

Solution:
Solution

ⓐ

	$n + 1$
Substitute -5 for n.	$-5 + 1$
Simplify.	-4

ⓑ

	$-n + 1$
Substitute -5 for n.	$-(-5) + 1$
Simplify.	$5 + 1$

Add.	6

Note:
Exercise:

TRY IT :: 1.107

Problem: When $n = -8$, evaluate ⓐ $n + 2$ ⓑ $-n + 2$.

Solution:

ⓐ -6 ⓑ 10

Note:
Exercise:

TRY IT :: 1.108

Problem: When $y = -9$, evaluate ⓐ $y + 8$ ⓑ $-y + 8$.

Solution:

ⓐ -1 ⓑ 17

Example:
Exercise:

Problem: **Evaluate $(x + y)^2$ when $x = -18$ and $y = 24$.

Solution:
Solution

	$(x + y)^2$

Substitute −18 for *x* and 24 for *y*.	56.3
Add inside parenthesis.	$(6)^2$
Simplify.	36

Note:
Exercise:

TRY IT :: 1.109

Problem: Evaluate $(x+y)^2$ when $x=-15$ and $y=29$.

Solution:

196

Note:
Exercise:

TRY IT :: 1.110

Problem: Evaluate $(x+y)^3$ when $x=-8$ and $y=10$.

Solution:

8

Example:
Exercise:

Problem: **Evaluate $20-z$ when ⓐ $z=12$ and ⓑ $z=-12$.

Solution:
Solution

ⓐ

	20 – *z*
Substitute 12 for *z*.	20 – 12
Subtract.	8

ⓑ

	20 – *z*
Substitute –12 for *z*.	20 – (–12)
Subtract.	32

Note:
Exercise:

TRY IT :: 1.111

Problem: Evaluate: $17 - k$ when ⓐ $k = 19$ and ⓑ $k = -19$.

Solution:

ⓐ -2 ⓑ 36

Note:
Exercise:

TRY IT :: 1.112

Problem: Evaluate: $-5-b$ when ⓐ $b=14$ and ⓑ $b=-14$.

Solution:

ⓐ -19 ⓑ 9

Example:
Exercise:

Problem: **Evaluate: $2x^2+3x+8$ when $x=4$.

Solution:
Solution

Substitute 4 for x. Use parentheses to show multiplication.

	$2x^2+3x+8$
Substitute.	$2(4)^2+3(4)+8$
Evaluate exponents.	$2(16)+3(4)+8$
Multiply.	$32+12+8$
Add.	52

Note:
Exercise:

TRY IT :: 1.1113

Problem: Evaluate: $3x^2 - 2x + 6$ when $x = -3$.

Solution:

39

Note:
Exercise:

TRY IT :: 1.114

Problem: Evaluate: $4x^2 - x - 5$ when $x = -2$.

Solution:

13

Translate Phrases to Expressions with Integers

Our earlier work translating English to algebra also applies to phrases that include both positive and negative numbers.

Example:
Exercise:

Problem: **Translate and simplify: the sum of 8 and -12, increased by 3.

Solution:
Solution

	the **sum** of 8 and -12, increased by 3.
Translate.	$[8 + (-12)] + 3$

Simplify. Be careful not to confuse the brackets with an absolute value sign.	$(-4)+3$
Add.	-1

Note:
Exercise:

TRY IT :: 1.115
Problem: Translate and simplify the sum of 9 and -16, increased by 4.

Solution:

$(9+(-16))+4\text{; simplifies to }-3$

Note:
Exercise:

TRY IT :: 1.116
Problem: Translate and simplify the sum of -8 and -12, increased by 7.

Solution:

$(-8+(-12))+7\text{; simplifies to }-13$

When we first introduced the operation symbols, we saw that the expression may be read in several ways. They are listed in the chart below.

$a-b$

$a-b$
a minus b the difference of a and b b subtracted from a b less than a

Be careful to get *a* and *b* in the right order!

Example:
Exercise:

Problem:

**Translate and then simplify ⓐ the difference of 13 and -21 ⓑ subtract 24 from -19.

Solution:
Solution

ⓐ Translate. Simplify.	$\text{the } \mathbf{difference}\ of\ 13\ and\ -21$ $13-(-21)$ 34
ⓑ Translate. Remember, "subtract b from a means $a-b$. Simplify.	$\mathbf{subtract}\ 24\ \mathbf{from}\ -19$ $-19-24$ 43

Note:
Exercise:

TRY IT :: 1.117

Problem: Translate and simplify ⓐ the difference of 14 and -23 ⓑ subtract 21 from -17.

Solution:

ⓐ $14-(-23) ; 37$ ⓑ $-17-21 ; -38$

Note:
Exercise:

TRY IT :: 1.118

Problem: Translate and simplify ⓐ the difference of 11 and -19 ⓑ subtract 18 from -11.

Solution:

ⓐ $11-(-19) ; 30$ ⓑ $-11-18 ; -29$

Once again, our prior work translating English to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the key word for multiplication is “product” and for division is “quotient.”

Example:
Exercise:

Problem:

**Translate to an algebraic expression and simplify if possible: the product of -2 and 14.

Solution:
Solution

	the product $of -2\ and\ 14$
Translate.	$(-2)(14)$
Simplify.	-28

Note:

Exercise:

Problem:

TRY IT :: 1.119
Translate to an algebraic expression and simplify if possible: the product of -5 and 12.

Solution:

$-5\,(12); -60$

Note:
Exercise:

Problem:

TRY IT :: 1.120
Translate to an algebraic expression and simplify if possible: the product of 8 and -13.

Solution:

$-8\,(13); -104$

Example:
Exercise:

Problem:

**Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7.

Solution:
Solution

	the quotient $of\ -56\ and\ -7$
Translate.	$-56 \div (-7)$
Simplify.	8

Note:
Exercise:

Problem:

TRY IT :: 1.121
Translate to an algebraic expression and simplify if possible: the quotient of -63 and -9.

Solution:

$-63 \div (-9); 7$

Note:
Exercise:

Problem:

TRY IT :: 1.122
Translate to an algebraic expression and simplify if possible: the quotient of -72 and -9.

Solution:

$-72 \div (-9); 8$

Use Integers in Applications

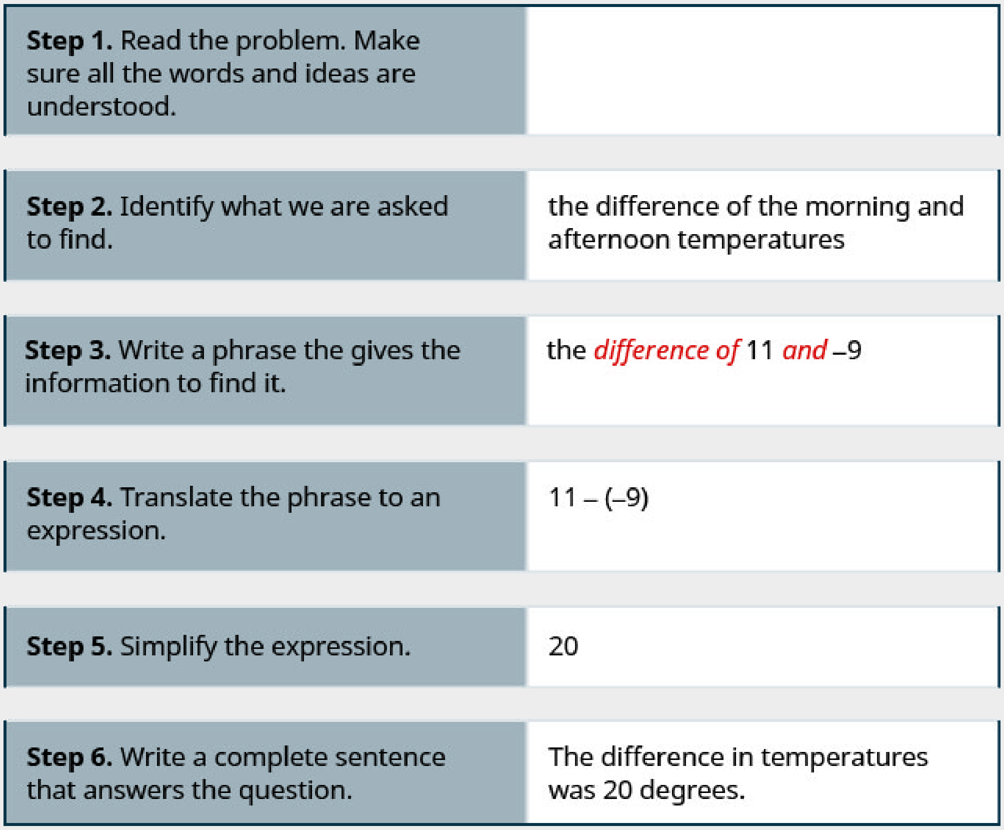
We'll outline a plan to solve applications. It's hard to find something if we don't know what we're looking for or what to call it! So when we solve an application, we first need to determine what the problem is asking us to find. Then we'll write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

Example:
How to Apply a Strategy to Solve Applications with Integers
Exercise:

Problem:

**The temperature in Urbana, Illinois one morning was 11 degrees. By mid-afternoon, the temperature had dropped to -9 degrees. What was the difference of the morning and afternoon temperatures?

Solution:
Solution



Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the difference of the morning and afternoon temperatures
Step 3. Write a phrase the gives the information to find it.	the *difference of* 11 *and* –9
Step 4. Translate the phrase to an expression.	11 – (–9)
Step 5. Simplify the expression.	20
Step 6. Write a complete sentence that answers the question.	The difference in temperatures was 20 degrees.

Note:
Exercise:

Problem:

TRY IT :: 1.123
The temperature in Anchorage, Alaska one morning was 15 degrees. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference in the morning and afternoon temperatures?

Solution:

The difference in temperatures was 45 degrees.

Note:

Exercise:

Problem:

TRY IT :: 1.124
The temperature in Denver was -6 degrees at lunchtime. By sunset the temperature had dropped to -15 degrees. What was the difference in the lunchtime and sunset temperatures?

Solution:

The difference in temperatures was 9 degrees.

Note:
Apply a Strategy to Solve Applications with Integers.

Read the problem. Make sure all the words and ideas are understood
Identify what we are asked to find.
Write a phrase that gives the information to find it.
Translate the phrase to an expression.
Simplify the expression.
Answer the question with a complete sentence.

Example:
Exercise:

Problem:

**The Mustangs football team received three penalties in the third quarter. Each penalty gave them a loss of fifteen yards. What is the number of yards lost?

Solution:
Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the number of yards lost

Step 3. Write a phrase that gives the information to find it.	three times a 15-yard penalty
Step 4. Translate the phrase to an expression.	$3\left(-15\right)$
Step 5. Simplify the expression.	-45
Step 6. Answer the question with a complete sentence.	The team lost 45 yards.

Note:
Exercise:

Problem:

TRY IT :: 1.125
The Bears played poorly and had seven penalties in the game. Each penalty resulted in a loss of 15 yards. What is the number of yards lost due to penalties?

Solution:

The Bears lost 105 yards.

Note:
Exercise:

Problem:

TRY IT :: 1.126
Bill uses the ATM on campus because it is convenient. However, each time he uses it he is charged a $2 fee. Last month he used the ATM eight times. How much was his total fee for using the ATM?

Solution:

A $16 fee was deducted from his checking account.

Key Concepts

- **Multiplication and Division of Two Signed Numbers**

    - Same signs—Product is positive
    - Different signs—Product is negative

- **Strategy for Applications**

  Identify what you are asked to find.
  Write a phrase that gives the information to find it.
  Translate the phrase to an expression.
  Simplify the expression.
  Answer the question with a complete sentence.

Practice Makes Perfect

Multiply Integers

In the following exercises, multiply.

Exercise:

Problem: $-4 \cdot 8$

Solution:

-32

Exercise:

Problem: $-3 \cdot 9$

Exercise:

Problem: $9\left(-7\right)$

Solution:

-63

Exercise:

Problem: $13\left(-5\right)$

Exercise:

Problem: -1.6

Solution:

-6

Exercise:

Problem: -1.3

Exercise:

Problem: $-1\left(-14\right)$

Solution:

14

Exercise:

Problem: $-1\left(-19\right)$

Divide Integers

In the following exercises, divide.

Exercise:

Problem: $-24 \div 6$

Solution:

-4

Exercise:

Problem: $35 \div \left(-7\right)$

Exercise:

Problem: $-52 \div \left(-4\right)$

Solution:

13

Exercise:

Problem: $-84 \div \left(-6\right)$

Exercise:

Problem: $-180 \div 15$

Solution:

-12

Exercise:

Problem: $-192 \div 12$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $5\left(-6\right)+7\left(-2\right)-3$

Solution:

-47

Exercise:

Problem: $8\left(-4\right)+5\left(-4\right)-6$

Exercise:

Problem: ${\left(-2\right)}^{6}$

Solution:

64

Exercise:

Problem: ${\left(-3\right)}^{5}$

Exercise:

Problem: $-{4}^{2}$

Solution:

-16

Exercise:

Problem: $-{6}^{2}$

Exercise:

Problem: $-3\left(-5\right)\left(6\right)$

Solution:

90

Exercise:

Problem: $-4\left(-6\right)\left(3\right)$

Exercise:

Problem: $\left(8-11\right)\left(9-12\right)$

Solution:

9

Exercise:

Problem: $\left(6-11\right)\left(8-13\right)$

Exercise:

Problem: $26-3\left(2-7\right)$

Solution:

41

Exercise:

Problem: $23-2\left(4-6\right)$

Exercise:

Problem: $65\div\left(-5\right)+\left(-28\right)\div\left(-7\right)$

Solution:

-9

Exercise:

Problem: $52\div\left(-4\right)+\left(-32\right)\div\left(-8\right)$

Exercise:

Problem: $9-2\left[3-8\left(-2\right)\right]$

Solution:

-29

Exercise:

Problem: $11-3\left[7-4\left(-2\right)\right]$

Exercise:

Problem: $\left(-3\right)^{2}-24\div\left(8-2\right)$

Solution:

5

Exercise:

Problem: $\left(-4\right)^{2}-32\div\left(12-4\right)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $y+\left(-14\right)$ when
ⓐ $y=-33$
ⓑ $y=30$

Solution:

ⓐ -47 ⓑ 16

Exercise:

Problem: $x+\left(-21\right)$ when
ⓐ $x=-27$
ⓑ $x=44$

Exercise:

Problem:
ⓐ $a+3$ when $a=-7$
ⓑ $-a+3$ when $a=-7$

Solution:

ⓐ -4 ⓑ 10

Exercise:

Problem: ⓐ $d+(-9)$ when $d=-8$
ⓑ $-d+(-9)$ when $d=-8$

Exercise:

Problem: $m+n$ when $m=-15, n=7$

Solution:

-8

Exercise:

Problem: $p+q$ when $p=-9, q=17$

Exercise:

Problem: $r+s$ when $r=-9, s=-7$

Solution:

-16

Exercise:

Problem: $t+u$ when $t=-6, u=-5$

Exercise:

Problem: $(x+y)^2$ when $x=-3, y=14$

Solution:

121

Exercise:

Problem: $(y+z)^2$ when $y=-3, z=15$

Exercise:

$-2x+17$ when
ⓐ $x=8$
Problem: ⓑ $x=-8$

Solution:

ⓐ 1 ⓑ 33

Exercise:

$-5y+14$ when
ⓐ $y=9$
Problem: ⓑ $y=-9$

Exercise:

$10-3m$ when
ⓐ $m=5$
Problem: ⓑ $m=-5$

Solution:

ⓐ -5 ⓑ 25

Exercise:

$18-4n$ when
ⓐ $n=3$
Problem: ⓑ $n=-3$

Exercise:

$2w^2-3w+7$ when
Problem: $w=-2$

Solution:

21

Exercise:

Problem: $3u^2-4u+5$ when $u=-3$

Exercise:

$9a-2b-8$ when
Problem: $a=-6$ and $b=-3$

Solution:

-56

Exercise:

Problem: $7m-4n-2$ when $m=-4$ and $n=-9$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise:

Problem: the sum of 3 and -15, increased by 7

Solution:

$(3+(-15))+7;-5$

Exercise:

Problem: the sum of -8 and -9, increased by 23

Exercise:

Problem: the difference of 10 and -18

Solution:

$10-(-18);28$

Exercise:

Problem: subtract 11 from -25

Exercise:

Problem: the difference of -5 and -30

Solution:

$-5-(-30);25$

Exercise:

Problem: subtract -6 from -13

Exercise:

Problem: the product of -3 and 15

Solution:

$-3 \cdot 15; -45$

Exercise:

Problem: the product of -4 and 16

Exercise:

Problem: the quotient of -60 and -20

Solution:

$-60 \div (-20); 3$

Exercise:

Problem: the quotient of -40 and -20

Exercise:

Problem: the quotient of -6 and the sum of a and b

Solution:

$\frac{-6}{a+b}$

Exercise:

Problem: the quotient of -7 and the sum of m and n

Exercise:

Problem: the product of -10 and the difference of p and q

Solution:

$-10\left(p-q\right)$

Exercise:

Problem: the product of -13 and the difference of c and d

Use Integers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature On January 15, the high temperature in Anaheim, California, was 84°. That same day, the high temperature in Embarrass, Minnesota was -12°. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

Solution:

96°

Exercise:

Problem:

Temperature On January 21, the high temperature in Palm Springs, California, was 89°, and the high temperature in Whitefield, New Hampshire was -31°. What was the difference between the temperature in Palm Springs and the temperature in Whitefield?

Exercise:

Problem:

Football At the first down, the Chargers had the ball on their 25 yard line. On the next three downs, they lost 6 yards, gained 10 yards, and lost 8 yards. What was the yard line at the end of the fourth down?

Solution:

21

Exercise:

Problem:

Football At the first down, the Steelers had the ball on their 30 yard line. On the next three downs, they gained 9 yards, lost 14 yards, and lost 2 yards. What was the yard line at the end of the fourth down?

Exercise:

Problem:

Checking Account Mayra has \$124 in her checking account. She writes a check for \$152. What is the new balance in her checking account?

Solution:

$-\$28$

Exercise:

Problem:

Checking Account Selina has $165 in her checking account. She writes a check for $207. What is the new balance in her checking account?

Exercise:

Problem:

Checking Account Diontre has a balance of $-\$38$ in his checking account. He deposits $225 to the account. What is the new balance?

Solution:

$187

Exercise:

Problem:

Checking Account Reymonte has a balance of $-\$49$ in his checking account. He deposits $281 to the account. What is the new balance?

Everyday Math

Exercise:

Problem:

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped $12 per share. What was the total effect on Javier's portfolio?

Solution:

$-\$3600$

Exercise:

Problem:

Weight loss In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Writing Exercises

Exercise:

Problem: In your own words, state the rules for multiplying integers.

Solution:

Answers may vary

Exercise:

Problem: In your own words, state the rules for dividing integers.

Exercise:

Problem: Why is $-2^4 \neq (-2)^4$?

Solution:

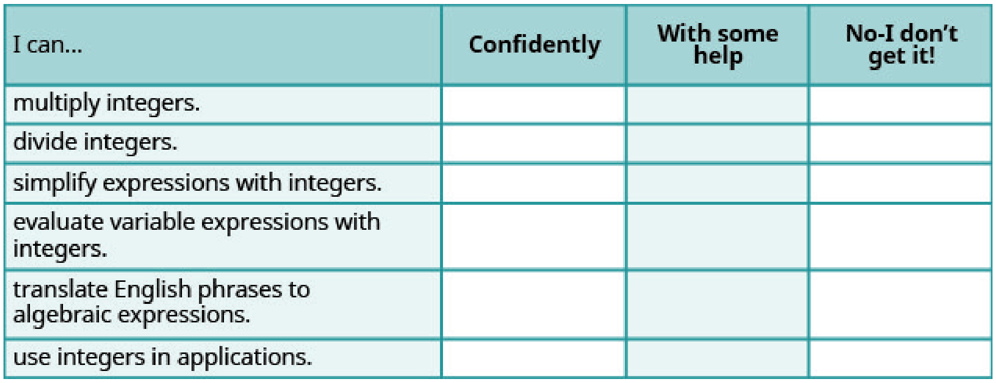
Answers may vary

Exercise:

Problem: Why is $-4^3 = (-4)^3$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
multiply integers.			
divide integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate English phrases to algebraic expressions.			
use integers in applications.			

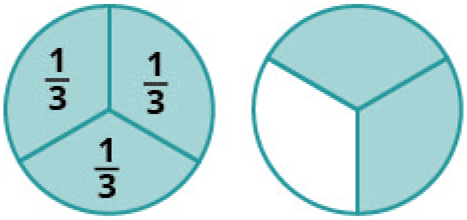
ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Visualize Fractions
By the end of this section, you will be able to:

- Find equivalent fractions
- Simplify fractions
- Multiply fractions
- Divide fractions
- Simplify expressions written with a fraction bar
- Translate phrases to expressions with fractions

Find Equivalent Fractions

Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts. See [link]. The fraction $\frac{2}{3}$ represents two of three equal parts. In the fraction $\frac{2}{3}$, the 2 is called the **numerator** and the 3 is called the **denominator**.



The circle on the left has been divided into 3 equal parts. Each part is $\frac{1}{3}$ of the 3 equal parts. In the circle on the right, $\frac{2}{3}$ of the circle is shaded (2 of the 3 equal parts).

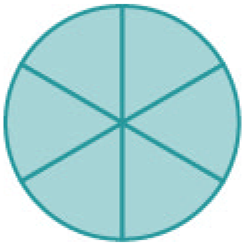
Note: Doing the Manipulative Mathematics activity “Model Fractions” will help you develop a better understanding of fractions, their numerators and denominators.

Note:
Fraction
A **fraction** is written $\frac{a}{b}$, where $b \neq 0$ and

- a is the **numerator** and b is the **denominator**.

A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

If a whole pie has been cut into 6 pieces and we eat all 6 pieces, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie.



So $\frac{6}{6} = 1$. This leads us to the property of one that tells us that any number, except zero, divided by itself is 1.

Note:
Property of One
Equation:

$$\frac{a}{a} = 1 \qquad (a \neq 0)$$

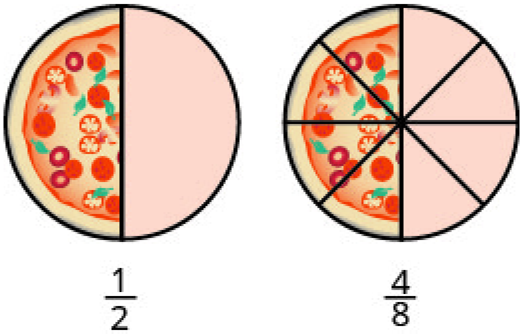
Any number, except zero, divided by itself is one.

Note: Doing the Manipulative Mathematics activity “Fractions Equivalent to One” will help you develop a better understanding of fractions that are equivalent to one.

If a pie was cut in 6 pieces and we ate all 6, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie. If the pie was cut into 8 pieces and we ate all 8, we ate $\frac{8}{8}$ pieces, or one whole pie. We ate the same amount—one whole pie.

The fractions $\frac{6}{6}$ and $\frac{8}{8}$ have the same value, 1, and so they are called equivalent fractions. **Equivalent fractions** are fractions that have the same value.

Let’s think of pizzas this time. [link] shows two images: a single pizza on the left, cut into two equal pieces, and a second pizza of the same size, cut into eight pieces on the right. This is a way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. In other words, they are equivalent fractions.

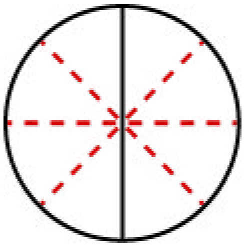


Since the same amount is of each pizza is shaded, we see that $\frac{1}{2}$ is equivalent to

$\frac{4}{8}$. They are equivalent fractions.

Note:
Equivalent Fractions
Equivalent fractions are fractions that have the same value.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could we take a pizza that is cut into 2 pieces and cut it into 8 pieces? We could cut each of the 2 larger pieces into 4 smaller pieces! The whole pizza would then be cut into 8 pieces instead of just 2. Mathematically, what we've described could be written like this as $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$. See [link].



Cutting each half of the pizza into 4 pieces, gives us pizza cut into 8 pieces: $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$.

This model leads to the following property:

Note:
Equivalent Fractions Property
If a, b, c are numbers where $b \neq 0, c \neq 0$, then
Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

If we had cut the pizza differently, we could get

$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4}$$

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{so} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

So, we say $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Note: Doing the Manipulative Mathematics activity “Equivalent Fractions” will help you develop a better understanding of what it means when two fractions are equivalent.

Example:
Exercise:

Problem: **Find three fractions equivalent to $\frac{2}{5}$.

Solution:
Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number. We can choose any number, except for zero. Let's multiply them by 2, 3, and then 5.

$$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10} \qquad \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15} \qquad \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Note:
Exercise:

TRY IT :: 1.127

Problem: Find three fractions equivalent to $\frac{3}{5}$.

Solution:

$\frac{6}{10}$, $\frac{9}{15}$, $\frac{12}{20}$; answers may vary

Note:
Exercise:

TRY IT :: 1.128

Problem: Find three fractions equivalent to $\frac{4}{5}$.

Solution:

$\frac{8}{10}, \frac{12}{15}, \frac{16}{20}$; answers may vary

Simplify Fractions

A fraction is considered **simplified** if there are no common factors, other than 1, in its numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

Note:
Simplified Fraction
A fraction is considered **simplified** if there are no common factors in its numerator and denominator.

The phrase *reduce a fraction* means to simplify the fraction. We simplify, or reduce, a fraction by removing the common factors of the numerator and denominator. A fraction is not simplified until all common factors have been removed. If an expression has fractions, it is not completely simplified until the fractions are simplified.

In [link], we used the equivalent fractions property to find equivalent fractions. Now we’ll use the equivalent fractions property in reverse to simplify fractions. We can rewrite the property to show both forms together.

Note:
Equivalent Fractions Property
If a, b, c are numbers where $b \neq 0, c \neq 0,$
Equation:

$$\text{then} \quad \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

Example:
Exercise:

Problem: **Simplify: $-\frac{32}{56}$.

Solution:
Solution

	$-\frac{32}{56}$
Rewrite the numerator and denominator showing the common factors.	$-\frac{4 \cdot 8}{7 \cdot 8}$
Simplify using the equivalent fractions property.	$-\frac{4}{7}$

Notice that the fraction $-\frac{4}{7}$ is simplified because there are no more common factors.

Note:
Exercise:

TRY IT :: 1.129

Problem: Simplify: $-\frac{42}{54}$.

Solution:

$-\frac{7}{9}$

Note:
Exercise:

TRY IT :: 1.130

Problem: Simplify: $-\frac{45}{81}$.

Solution:

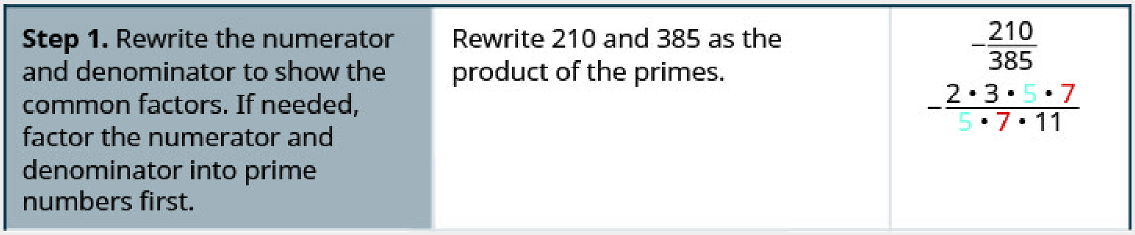
$-\frac{5}{9}$

Sometimes it may not be easy to find common factors of the numerator and denominator. When this happens, a good idea is to factor the numerator and the denominator into prime numbers. Then divide out the common factors using the equivalent fractions property.

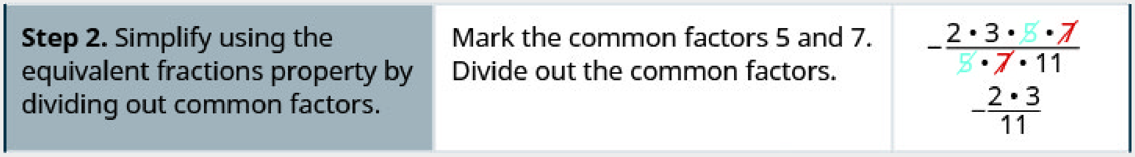
Example:
How to Simplify a Fraction
Exercise:

Problem: **Simplify: $-\frac{210}{385}$.

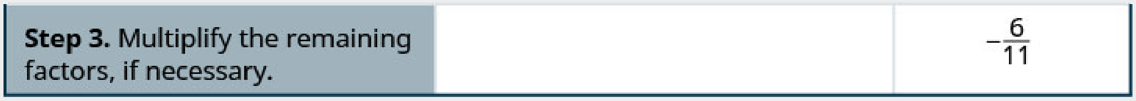
Solution:
Solution



Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.	Rewrite 210 and 385 as the product of the primes.	$-\frac{210}{385}$ $-\frac{2\cdot3\cdot5\cdot7}{5\cdot7\cdot11}$



Step 2. Simplify using the equivalent fractions property by dividing out common factors.	Mark the common factors 5 and 7. Divide out the common factors.	$-\frac{2\cdot3\cdot\cancel{5}\cdot\cancel{7}}{\cancel{5}\cdot\cancel{7}\cdot11}$ $-\frac{2\cdot3}{11}$



Step 3. Multiplify the remaining factors, if necessary.		$-\frac{6}{11}$

Note:
Exercise:

TRY IT :: 1.131

Problem: Simplify: $-\frac{69}{120}$.

Solution:

$-\frac{23}{40}$

Note:
Exercise:

TRY IT :: 1.132

Problem: Simplify: $-\frac{120}{192}$.

Solution:

$-\frac{5}{8}$

We now summarize the steps you should follow to simplify fractions.

Note:

Simplify a Fraction.

Rewrite the numerator and denominator to show the common factors.

If needed, factor the numerator and denominator into prime numbers first.

Simplify using the equivalent fractions property by dividing out common factors.

Multiply any remaining factors, if needed.

Example:

Exercise:

Problem: **Simplify: $\frac{5x}{5y}$.

Solution:

Solution

	$\frac{5x}{5y}$
Rewrite showing the common factors, then divide out the common factors.	 $\frac{5x}{5y}$ Rewrite showing the common factors, then divide out the common factors. $\frac{\cancel{5}\cdot x}{\cancel{5}\cdot y}$ Simplify. $\frac{x}{y}$
Simplify.	$\frac{x}{y}$

Note:
Exercise:

TRY IT :: 1.133

Problem: Simplify: $\frac{7x}{7y}$.

Solution:

$\frac{x}{y}$

Note:
Exercise:

TRY IT :: 1.134

Problem: Simplify: $\frac{3a}{3b}$.

Solution:

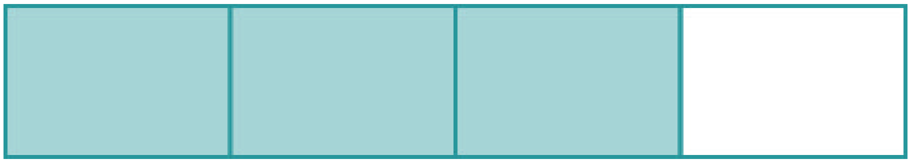
$\frac{a}{b}$

Multiply Fractions

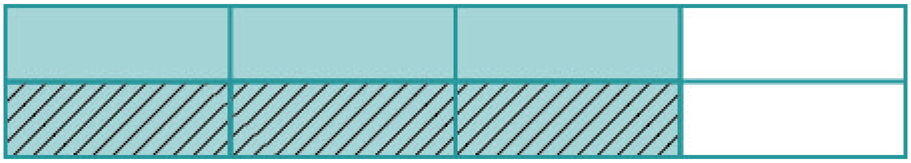
Many people find multiplying and dividing fractions easier than adding and subtracting fractions. So we will start with fraction multiplication.

Note: Doing the Manipulative Mathematics activity "Model Fraction Multiplication" will help you develop a better understanding of multiplying fractions.

We'll use a model to show you how to multiply two fractions and to help you remember the procedure. Let's start with $\frac{3}{4}$.



Now we'll take $\frac{1}{2}$ of $\frac{3}{4}$.



Notice that now, the whole is divided into 8 equal parts. So $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

To multiply fractions, we multiply the numerators and multiply the denominators.

Note:

Fraction Multiplication

If a, b, c and d are numbers where $b \neq 0$ and $d \neq 0$, then

Equation:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To multiply fractions, multiply the numerators and multiply the denominators.

When multiplying fractions, the properties of positive and negative numbers still apply, of course. It is a good idea to determine the sign of the product as the first step. In [link], we will multiply negative and a positive, so the product will be negative.

Example:

Exercise:

Problem: **Multiply: $-\frac{11}{12} \cdot \frac{5}{7}$.

Solution:

Solution

The first step is to find the sign of the product. Since the signs are the different, the product is negative.

	$-\frac{11}{12} \cdot \frac{5}{7}$
Determine the sign of the product; multiply.	$-\frac{11 \cdot 5}{12 \cdot 7}$

Are there any common factors in the numerator and the demoninator? No.	$-\frac{55}{84}$

Note:
Exercise:

TRY IT :: 1.135

Problem: Multiply: $-\frac{10}{28} \cdot \frac{8}{15}$.

Solution:

$-\frac{4}{21}$

Note:
Exercise:

TRY IT :: 1.136

Problem: Multiply: $-\frac{9}{20} \cdot \frac{5}{12}$.

Solution:

$-\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, *a*, can be written as $\frac{a}{1}$. So, for example, $3 = \frac{3}{1}$.

Example:
Exercise:

Problem: **Multiply: $-\frac{12}{5}(-20x)$.

Solution:
Solution

Determine the sign of the product. The signs are the same, so the product is positive.

	$-\frac{12}{5}(-20x)$
Write $20x$ as a fraction.	$\frac{12}{5}\left(\frac{20x}{1}\right)$
Multiply.	
Rewrite 20 to show the common factor 5 and divide it out.	$\frac{12 \cdot 4 \cdot \cancel{5}x}{\cancel{5} \cdot 1}$
Simplify.	$48x$

Note:
Exercise:

TRY IT :: 1.137

Problem: Multiply: $\frac{11}{3}(-9a)$.

Solution:

$-33a$

Note:
Exercise:

TRY IT :: 1.138

Problem: Multiply: $\frac{13}{7}(-14b)$.

Solution:

$-26b$

Divide Fractions

Now that we know how to multiply fractions, we are almost ready to divide. Before we can do that, that we need some vocabulary.

The **reciprocal** of a fraction is found by inverting the fraction, placing the numerator in the denominator and the denominator in the numerator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Notice that $\frac{2}{3} \cdot \frac{3}{2} = 1$. A number and its reciprocal multiply to 1.

To get a product of positive 1 when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

The reciprocal of $-\frac{10}{7}$ is $-\frac{7}{10}$, since $-\frac{10}{7}\left(-\frac{7}{10}\right) = 1$.

Note:
Reciprocal
The **reciprocal** of $\frac{a}{b}$ is $\frac{b}{a}$.
A number and its reciprocal multiply to one $\frac{a}{b} \cdot \frac{b}{a} = 1$.

Note: Doing the Manipulative Mathematics activity “Model Fraction Division” will help you develop a better understanding of dividing fractions.

To divide fractions, we multiply the first fraction by the reciprocal of the second.

Note:
Fraction Division
If a, b, c and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then
Equation:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide fractions, we multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don’t divide by zero!

Example:
Exercise:

Problem: **Divide: $-\frac{2}{3} \div \frac{n}{5}$.

Solution:
Solution

	$-\frac{2}{3} \div \frac{n}{5}$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{2}{3} \cdot \frac{5}{n}$
Multiply.	$-\frac{10}{3n}$

Note:
Exercise:

TRY IT :: 1.139
Problem: Divide: $-\frac{3}{5} \div \frac{p}{7}$.

Solution:

$-\frac{21}{5p}$

Note:
Exercise:

TRY IT :: 1.140

Problem: Divide: $-\frac{5}{8} \div \frac{q}{3}$.

Solution:

$-\frac{15}{8q}$

Example:
Exercise:

Problem: **Find the quotient: $-\frac{7}{18} \div \left(-\frac{14}{27}\right)$.

Solution:
Solution

	$-\frac{7}{18} \div \left(-\frac{14}{27}\right)$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{7}{18} \cdot -\frac{27}{14}$
Determine the sign of the product, and then multiply..	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{\cancel{7} \cdot \cancel{9} \cdot 3}{\cancel{9} \cdot 2 \cdot \cancel{7} \cdot 2}$

Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

Note:
Exercise:

TRY IT :: 1.141

Problem: Find the quotient: $-\frac{7}{27} \div \left(-\frac{35}{36}\right)$.

Solution:

$\frac{4}{15}$

Note:
Exercise:

TRY IT :: 1.142

Problem: Find the quotient: $-\frac{5}{14} \div \left(-\frac{15}{28}\right)$.

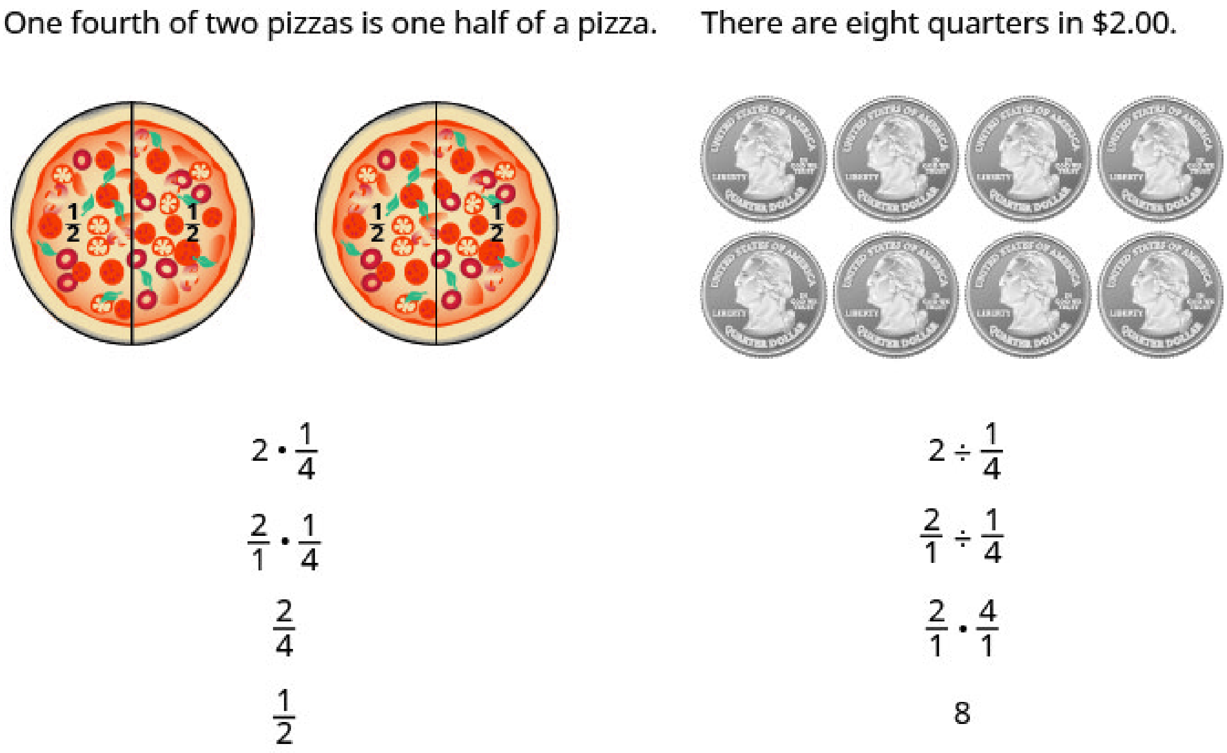
Solution:

$\frac{2}{3}$

There are several ways to remember which steps to take to multiply or divide fractions. One way is to repeat the call outs to yourself. If you do this each time you do an exercise, you will have the steps memorized.

- “To multiply fractions, multiply the numerators and multiply the denominators.”
- “To divide fractions, multiply the first fraction by the reciprocal of the second.”

Another way is to keep two examples in mind:



One fourth of two pizzas is one half of a pizza.

$$2 \cdot \frac{1}{4}$$

$$\frac{2}{1} \cdot \frac{1}{4}$$

$$\frac{2}{4}$$

$$\frac{1}{2}$$

There are eight quarters in $2.00.

$$2 \div \frac{1}{4}$$

$$\frac{2}{1} \div \frac{1}{4}$$

$$\frac{2}{1} \cdot \frac{4}{1}$$

$$8$$

The numerators or denominators of some fractions contain fractions themselves. A fraction in which the numerator or the denominator is a fraction is called a **complex fraction**.

Note:
Complex Fraction
A **complex fraction** is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:
Equation:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}}$$

To simplify a complex fraction, we remember that the fraction bar means division. For example, the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ means $\frac{3}{4} \div \frac{5}{8}$.

Example:
Exercise:

Problem: **Simplify: $\frac{\frac{3}{4}}{\frac{5}{8}}$.

Solution:
Solution

	$\frac{\frac{3}{4}}{\frac{5}{8}}$
Rewrite as division.	$\frac{3}{4} \div \frac{5}{8}$
Multiply the first fraction by the reciprocal of the second.	$\frac{3}{4} \cdot \frac{8}{5}$
Multiply.	$\frac{3 \cdot 8}{4 \cdot 5}$

Look for common factors.	$\frac{3\cdot\cancel{4}\cdot 2}{\cancel{4}\cdot 5}$
Divide out common factors and simplify.	$\frac{6}{5}$

Note:
Exercise:

TRY IT :: 1.143

Problem: Simplify: $\frac{\frac{2}{3}}{\frac{5}{6}}$.

Solution:

$\frac{4}{5}$

Note:
Exercise:

TRY IT :: 1.144

Problem: Simplify: $\frac{\frac{3}{7}}{\frac{6}{11}}$.

Solution:

$\frac{11}{14}$

Example:

Exercise:

Problem: **Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$.

Solution:
Solution

	$\frac{\frac{x}{2}}{\frac{xy}{6}}$
Rewrite as division.	$\frac{x}{2} \div \frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2} \cdot \frac{6}{xy}$
Multiply.	$\frac{x \cdot 6}{2 \cdot xy}$
Look for common factors.	$\frac{\cancel{x} \cdot 3 \cdot \cancel{2}}{\cancel{2} \cdot \cancel{x} \cdot y}$
Divide out common factors and simplify.	$\frac{3}{y}$

Note:
Exercise:

TRY IT :: 1.145

Problem: Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$.

Solution:

$\frac{3}{4b}$

Note:
Exercise:

TRY IT :: 1.146

Problem: Simplify: $\frac{\frac{p}{2}}{\frac{pq}{8}}$.

Solution:

$\frac{4}{q}$

Simplify Expressions with a Fraction Bar

The line that separates the numerator from the denominator in a fraction is called a fraction bar. A fraction bar acts as grouping symbol. The order of operations then tells us to simplify the numerator and then the denominator. Then we divide.

To simplify the expression $\frac{5-3}{7+1}$, we first simplify the numerator and the denominator separately. Then we divide.
Equation:

$$\frac{5-3}{7+1}$$

Equation:

$$\frac{2}{8}$$

Equation:

$$\frac{1}{4}$$

Note:
Simplify an Expression with a Fraction Bar.

Simplify the expression in the numerator. Simplify the expression in the denominator.
Simplify the fraction.

Example:
Exercise:

Problem: **Simplify: $\frac{4-2(3)}{2^2+2}$.

Solution:
Solution

	$\frac{4-2(3)}{2^2+2}$
Use the order of operations to simpliy the numerator and the denominator.	$\frac{4-6}{4+2}$
Simplify the numerator and the denominator.	$\frac{-2}{6}$
Simplify. A negative divided by a positive is negative.	$-\frac{1}{3}$

Note:
Exercise:

TRY IT :: 1.147

Problem: Simplify: $\frac{6-3(5)}{3^2+3}$.

Solution:

$-\frac{3}{4}$

Note:
Exercise:

TRY IT :: 1.148

Problem: Simplify: $\frac{4-4(6)}{3^2+3}$.

Solution:

$-\frac{5}{3}$

Where does the negative sign go in a fraction? Usually the negative sign is in front of the fraction, but you will sometimes see a fraction with a negative numerator, or sometimes with a negative denominator. Remember that fractions represent division. When the numerator and denominator have different signs, the quotient is negative.

Equation:

$$\begin{array}{ccc} \frac{-1}{3} = -\frac{1}{3} & & \frac{\text{negative}}{\text{positive}} = \text{negative} \\ \frac{1}{-3} = -\frac{1}{3} & & \frac{\text{positive}}{\text{negative}} = \text{negative} \end{array}$$

Note:

Placement of Negative Sign in a Fraction

For any positive numbers *a* and *b*,

Equation:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Example:

Exercise:

Problem: **Simplify: $\frac{4(-3)+6(-2)}{-3(2)-2}$.

Solution:

Solution

	$\frac{4(-3)+6(-2)}{-3(2)-2}$
Multiply.	$\frac{-12+(-12)}{-6-2}$
Simplify.	$\frac{-24}{-8}$
Divide.	3

Note:
Exercise:

TRY IT :: 1.149

Problem: Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$.

Solution:

4

Note:
Exercise:

TRY IT :: 1.150

Problem: Simplify: $\frac{7(-1)+9(-3)}{-5(3)-2}$.

Solution:

2

Translate Phrases to Expressions with Fractions

Now that we have done some work with fractions, we are ready to translate phrases that would result in expressions with fractions.

The English words quotient and ratio are often used to describe fractions. Remember that "quotient" means division. The quotient of a and b is the result we get from dividing a by b, or $\frac{a}{b}$.

Example:
Exercise:

Problem:

**Translate the English phrase into an algebraic expression: the quotient of the difference of m and n, and p.

Solution:
Solution

We are looking for the *quotient of* the difference of m and n, *and* p. This means we want to divide the difference of m and n by p.
Equation:

$$\frac{m-n}{p}$$

Note:
Exercise:

Problem:

TRY IT :: 1.151
Translate the English phrase into an algebraic expression: the quotient of the difference of a and b, and cd.

Solution:

$\frac{a-b}{cd}$

Note:
Exercise:

Problem:

TRY IT :: 1.152
Translate the English phrase into an algebraic expression: the quotient of the sum of p and q, and r

Solution:

$\frac{p+q}{r}$

Key Concepts

- **Equivalent Fractions Property:** If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.
- **Fraction Division:** If a, b, c and d are numbers where $b \neq 0, c \neq 0,$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. To divide fractions, multiply the first fraction by the reciprocal of the second.

- **Fraction Multiplication:** If a, b, c and d are numbers where $b \neq 0,$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. To multiply fractions, multiply the numerators and multiply the denominators.
- **Placement of Negative Sign in a Fraction:** For any positive numbers a and b, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.
- **Property of One:** $\frac{a}{a} = 1$; Any number, except zero, divided by itself is one.
- **Simplify a Fraction**

  Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.
  Simplify using the equivalent fractions property by dividing out common factors.
  Multiply any remaining factors.

- **Simplify an Expression with a Fraction Bar**

  Simplify the expression in the numerator. Simplify the expression in the denominator.
  Simplify the fraction.

Practice Makes Perfect

Find Equivalent Fractions

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise:

Problem: $\frac{3}{8}$

Solution:

$\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$ answers may vary

Exercise:

Problem: $\frac{5}{8}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

$\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$ answers may vary

Exercise:

Problem: $\frac{1}{8}$

Simplify Fractions

In the following exercises, simplify.

Exercise:

Problem: $-\frac{40}{88}$

Solution:

$-\frac{5}{11}$

Exercise:

Problem: $-\frac{63}{99}$

Exercise:

Problem: $-\frac{108}{63}$

Solution:

$-\frac{12}{7}$

Exercise:

Problem: $-\frac{104}{48}$

Exercise:

Problem: $\frac{120}{252}$

Solution:

$\frac{10}{21}$

Exercise:

Problem: $\frac{182}{294}$

Exercise:

Problem: $-\frac{3x}{12y}$

Solution:

$-\frac{x}{4y}$

Exercise:

Problem: $-\frac{4x}{32y}$

Exercise:

Problem: $\frac{14x^2}{21y}$

Solution:

$\frac{2x^2}{3y}$

Exercise:

Problem: $\frac{24a}{32b^2}$

Multiply Fractions

In the following exercises, multiply.

Exercise:

Problem: $\frac{3}{4} \cdot \frac{9}{10}$

Solution:

$\frac{27}{40}$

Exercise:

Problem: $\frac{4}{5} \cdot \frac{2}{7}$

Exercise:

Problem: $-\frac{2}{3}\left(-\frac{3}{8}\right)$

Solution:

$\frac{1}{4}$

Exercise:

Problem: $-\frac{3}{4}\left(-\frac{4}{9}\right)$

Exercise:

Problem: $-\frac{5}{9} \cdot \frac{3}{10}$

Solution:

$-\frac{1}{6}$

Exercise:

Problem: $-\frac{3}{8} \cdot \frac{4}{15}$

Exercise:

Problem: $\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$

Solution:

$-\frac{21}{50}$

Exercise:

Problem: $\left(-\frac{9}{10}\right)\left(\frac{25}{33}\right)$

Exercise:

Problem: $\left(-\frac{63}{84}\right)\left(-\frac{44}{90}\right)$

Solution:

$\frac{11}{30}$

Exercise:

Problem: $\left(-\frac{63}{60}\right)\left(-\frac{40}{88}\right)$

Exercise:

Problem: $4 \cdot \frac{5}{11}$

Solution:

$\frac{20}{11}$

Exercise:

Problem: $5 \cdot \frac{8}{3}$

Exercise:

Problem: $\frac{3}{7} \cdot 21n$

Solution:

$9n$

Exercise:

Problem: $\frac{5}{6} \cdot 30m$

Exercise:

Problem: $-8\left(\frac{17}{4}\right)$

Solution:

-34

Exercise:

Problem: $(-1)\left(-\frac{6}{7}\right)$

Divide Fractions

In the following exercises, divide.

Exercise:

Problem: $\frac{3}{4} \div \frac{2}{3}$

Solution:

$\frac{9}{8}$

Exercise:

Problem: $\frac{4}{5} \div \frac{3}{4}$

Exercise:

Problem: $-\frac{7}{9} \div \left(-\frac{7}{4}\right)$

Solution:

1

Exercise:

Problem: $-\frac{5}{6} \div \left(-\frac{5}{6}\right)$

Exercise:

Problem: $\frac{3}{4} \div \frac{x}{11}$

Solution:

$\frac{33}{4x}$

Exercise:

Problem: $\frac{2}{5} \div \frac{y}{9}$

Exercise:

Problem: $\frac{5}{18} \div \left(-\frac{15}{24}\right)$

Solution:

$-\frac{4}{9}$

Exercise:

Problem: $\frac{7}{18} \div \left(-\frac{14}{27}\right)$

Exercise:

Problem: $\frac{8u}{15} \div \frac{12v}{25}$

Solution:

$\frac{10u}{9v}$

Exercise:

Problem: $\frac{12r}{25} \div \frac{18s}{35}$

Exercise:

Problem: $-5 \div \frac{1}{2}$

Solution:

-10

Exercise:

Problem: $-3 \div \frac{1}{4}$

Exercise:

Problem: $\frac{3}{4} \div (-12)$

Solution:

$-\frac{1}{16}$

Exercise:

Problem: $-15 \div \left(-\frac{5}{3}\right)$

In the following exercises, simplify.

Exercise:

Problem: $\frac{-\frac{8}{21}}{\frac{12}{35}}$

Solution:

$-\frac{10}{9}$

Exercise:

Problem: $\frac{-\frac{9}{16}}{\frac{33}{40}}$

Exercise:

Problem: $\frac{-\frac{4}{5}}{2}$

Solution:

$-\frac{2}{5}$

Exercise:

Problem: $\frac{5}{\frac{3}{10}}$

Exercise:

Problem: $\frac{\frac{m}{3}}{\frac{n}{2}}$

Solution:

$\frac{2m}{3n}$

Exercise:

Problem: $\frac{-\frac{3}{8}}{-\frac{y}{12}}$

Simplify Expressions Written with a Fraction Bar

In the following exercises, simplify.

Exercise:

Problem: $\frac{22+3}{10}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{19-4}{6}$

Exercise:

Problem: $\frac{48}{24-15}$

Solution:

$\frac{16}{3}$

Exercise:

Problem: $\frac{46}{4+4}$

Exercise:

Problem: $\frac{-6+6}{8+4}$

Solution:

0

Exercise:

Problem: $\frac{-6+3}{17-8}$

Exercise:

Problem: $\frac{4\cdot 3}{6\cdot 6}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{6\cdot 6}{9\cdot 2}$

Exercise:

Problem: $\frac{4^2-1}{25}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{7^2+1}{60}$

Exercise:

Problem: $\frac{8 \cdot 3+2 \cdot 9}{14+3}$

Solution:

$2\frac{8}{17}$

Exercise:

Problem: $\frac{9 \cdot 6-4 \cdot 7}{22+3}$

Exercise:

Problem: $\frac{5 \cdot 6-3 \cdot 4}{4 \cdot 5-2 \cdot 3}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{8 \cdot 9-7 \cdot 6}{5 \cdot 6-9 \cdot 2}$

Exercise:

Problem: $\frac{5^2-3^2}{3-5}$

Solution:

$-8>$

Exercise:

Problem: $\frac{6^2-4^2}{4-6}$

Exercise:

Problem: $\frac{7\cdot4-2(8-5)}{9\cdot3-3\cdot5}$

Solution:

$\frac{11}{6}$

Exercise:

Problem: $\frac{9\cdot7-3(12-8)}{8\cdot7-6\cdot6}$

Exercise:

Problem: $\frac{9(8-2)-3(15-7)}{6(7-1)-3(17-9)}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{8(9-2)-4(14-9)}{7(8-3)-3(16-9)}$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

Exercise:

Problem: the quotient of r and the sum of s and 10

Solution:

$\frac{r}{s+10}$

Exercise:

Problem: the quotient of A and the difference of 3 and B

Exercise:

Problem: the quotient of the difference of x and $y,$ and -3

Solution:

$\frac{x-y}{-3}$

Exercise:

Problem: the quotient of the sum of m and $n,$ and $4q$

Everyday Math

Exercise:

Problem:

Baking. A recipe for chocolate chip cookies calls for $\frac{3}{4}$ cup brown sugar. Imelda wants to double the recipe. ⓐ How much brown sugar will Imelda need? Show your calculation. ⓑ Measuring cups usually come in sets of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the cookie recipe.

Solution:

ⓐ $1\frac{1}{2}$ cups ⓑ answers will vary

Exercise:

Problem:

Baking. Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk. ⓐ How much condensed milk will Nina need? Show your calculation. ⓑ Measuring cups usually come in sets of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2},$ and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk needed for 4 pans of fudge.

Exercise:

Problem:

Portions Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?

Solution:

20 bags

Exercise:

Problem:

Portions Kristen has $\frac{3}{4}$ yards of ribbon that she wants to cut into 6 equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Writing Exercises

Exercise:

Problem:

Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.

Solution:

Answers may vary

Exercise:

Problem:

Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Exercise:

Problem: Explain how you find the reciprocal of a fraction.

Solution:

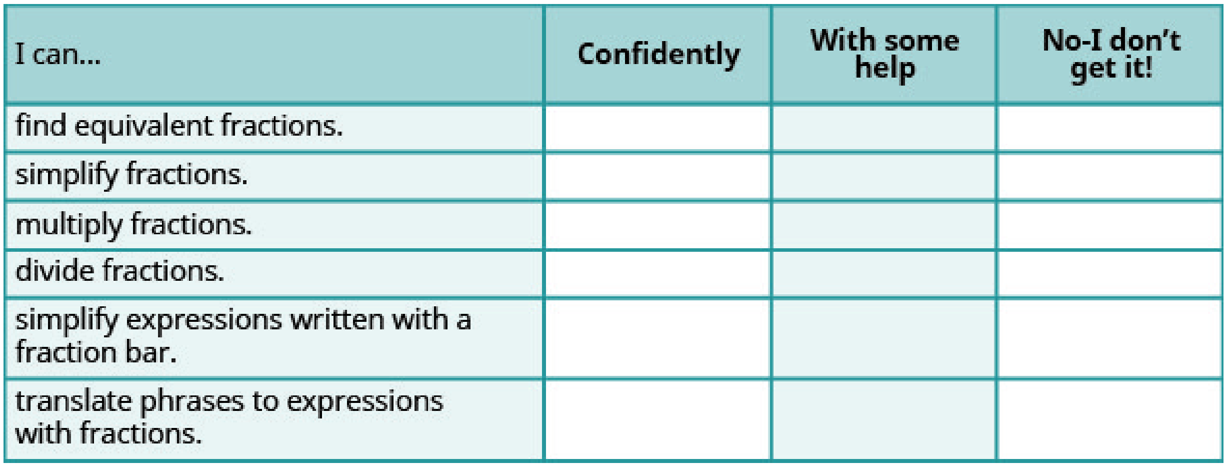
Answers may vary

Exercise:

Problem: Explain how you find the reciprocal of a negative number.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
find equivalent fractions.			
simplify fractions.			
multiply fractions.			
divide fractions.			
simplify expressions written with a fraction bar.			
translate phrases to expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

Glossary

complex fraction

A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

denominator

The denominator is the value on the bottom part of the fraction that indicates the number of equal parts into which the whole has been divided.

equivalent fractions

Equivalent fractions are fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$, where $b \neq 0$ a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

numerator

The numerator is the value on the top part of the fraction that indicates how many parts of the whole are included.

reciprocal

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. A number and its reciprocal multiply to one: $\frac{a}{b} \cdot \frac{b}{a} = 1$.

simplified fraction

A fraction is considered simplified if there are no common factors in its numerator and denominator.

Add and Subtract Fractions
By the end of this section, you will be able to:

- Add or subtract fractions with a common denominator
- Add or subtract fractions with different denominators
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

Add or Subtract Fractions with a Common Denominator

When we multiplied fractions, we just multiplied the numerators and multiplied the denominators right straight across. To add or subtract fractions, they must have a common denominator.

Note:
Fraction Addition and Subtraction
If $a, b,$ and c are numbers where $c \neq 0$, then
Equation:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \text{and} \quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

Note: Doing the Manipulative Mathematics activities “Model Fraction Addition” and “Model Fraction Subtraction” will help you develop a better understanding of adding and subtracting fractions.

Example:
Exercise:

Problem: **Find the sum: $\frac{x}{3} + \frac{2}{3}$.

Solution:
Solution

	$\frac{x}{3} + \frac{2}{3}$
Add the numerators and place the sum over the common denominator.	$\frac{x+2}{3}$

Note:
Exercise:

TRY IT :: 1.153
Problem: Find the sum: $\frac{x}{4} + \frac{3}{4}$.

Solution:

$\frac{x+3}{4}$

Note:
Exercise:

TRY IT :: 1.154

Problem: Find the sum: $\frac{y}{8}+\frac{5}{8}$.

Solution:

$\frac{y+5}{8}$

Example:
Exercise:

Problem: **Find the difference: $-\frac{23}{24}-\frac{13}{24}$.

Solution:
Solution

	$-\frac{23}{24}-\frac{13}{24}$
Subtract the numerators and place the difference over the common denominator.	$\frac{-23-13}{24}$
Simplify.	$\frac{-36}{24}$
Simplify. Remember, $-\frac{a}{b}=\frac{-a}{b}$.	$-\frac{3}{2}$

Note:
Exercise:

TRY IT :: 1.155

Problem: Find the difference: $-\frac{19}{28}-\frac{7}{28}$.

Solution:

$-\frac{13}{14}$

Note:
Exercise:

TRY IT :: 1.156

Problem: Find the difference: $-\frac{27}{32}-\frac{1}{32}$.

Solution:

$-\frac{7}{8}$

Example:
Exercise:

Problem: **Simplify: $-\frac{10}{x}-\frac{4}{x}$.

Solution:
Solution

	$-\frac{10}{x}-\frac{4}{x}$
Subtract the numerators and place the difference over the common denominator.	$\frac{-14}{x}$
Rewrite with the sign in front of the fraction.	$-\frac{14}{x}$

Note:
Exercise:

TRY IT :: 1.157

Problem: Find the difference: $-\frac{9}{x}-\frac{7}{x}$.

Solution:

$-\frac{16}{x}$

Note:
Exercise:

TRY IT :: 1.158

Problem: Find the difference: $-\frac{17}{a}-\frac{5}{a}$.

Solution:

$-\frac{22}{a}$

Now we will do an example that has both addition and subtraction.

Example:
Exercise:

Problem: **Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$.

Solution:
Solution

Add and subtract fractions—do they have a common denominator? Yes.	$\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$
Add and subtract the numerators and place the difference over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify left to right.	$\frac{-2-1}{8}$
Simplify.	$-\frac{3}{8}$

Note:
Exercise:

TRY IT :: 1.159

Problem: Simplify: $\frac{2}{5}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Solution:

-1

Note:
Exercise:

TRY IT :: 1.160

Problem: Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Solution:

$-\frac{2}{3}$

Add or Subtract Fractions with Different Denominators

As we have seen, to add or subtract fractions, their denominators must be the same. The **least common denominator** (LCD) of two fractions is the smallest number that can be used as a common denominator of the fractions. The LCD of the two fractions is the least common multiple (LCM) of their denominators.

Note:
Least Common Denominator
The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

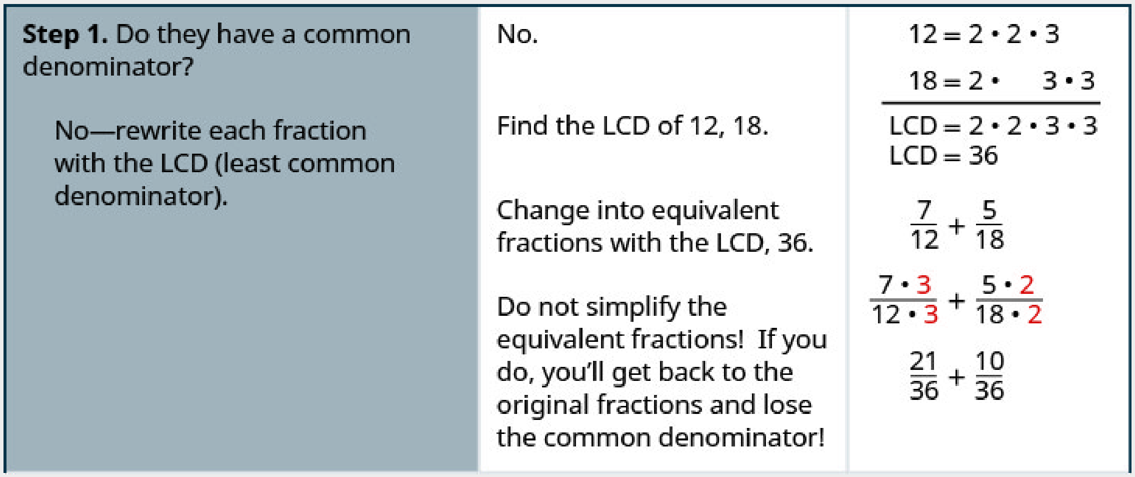
Note: Doing the Manipulative Mathematics activity “Finding the Least Common Denominator” will help you develop a better understanding of the LCD.

After we find the least common denominator of two fractions, we convert the fractions to equivalent fractions with the LCD. Putting these steps together allows us to add and subtract fractions because their denominators will be the same!

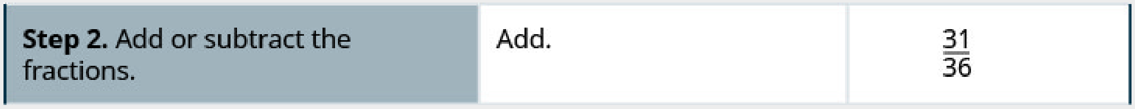
Example:
How to Add or Subtract Fractions
Exercise:

Problem: **Add: $\frac{7}{12}+\frac{5}{18}$.

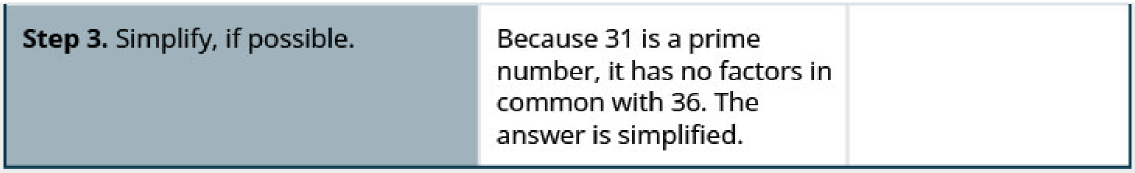
Solution:
Solution



Step 1. Do they have a common denominator?	No.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$
No—rewrite each fraction with the LCD (least common denominator).	Find the LCD of 12, 18.	$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3$ $\text{LCD} = 36$
	Change into equivalent fractions with the LCD, 36.	$\frac{7}{12}+\frac{5}{18}$
	Do not simplify the equivalent fractions! If you do, you’ll get back to the original fractions and lose the common denominator!	$\frac{7 \cdot 3}{12 \cdot 3}+\frac{5 \cdot 2}{18 \cdot 2}$ $\frac{21}{36}+\frac{10}{36}$



Step 2. Add or subtract the fractions.	Add.	$\frac{31}{36}$



Step 3. Simplify, if possible.	Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.	

Note:
Exercise:

TRY IT :: 1.161

Problem: Add: $\frac{7}{12} + \frac{11}{15}$.

Solution:

$\frac{79}{60}$

Note:
Exercise:

TRY IT :: 1.162

Problem: Add: $\frac{13}{15} + \frac{17}{20}$.

Solution:

$\frac{103}{60}$

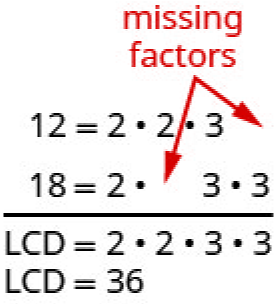
Note:
Add or Subtract Fractions.

Do they have

a common denominator?	○ Yes—go to step 2. ○ No—rewrite each fraction with the LCD (least common denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.
Add or subtract the fractions.	
Simplify, if possible.	

When finding the equivalent fractions needed to create the common denominators, there is a quick way to find the number we need to multiply both the numerator and denominator. This method works if we found the LCD by factoring into primes.

Look at the factors of the LCD and then at each column above those factors. The “missing” factors of each denominator are the numbers we need.



In [link], the LCD, 36, has two factors of 2 and two factors of 3.

The numerator 12 has two factors of 2 but only one of 3—so it is “missing” one 3—we multiply the numerator and denominator by 3.

The numerator 18 is missing one factor of 2—so we multiply the numerator and denominator by 2.

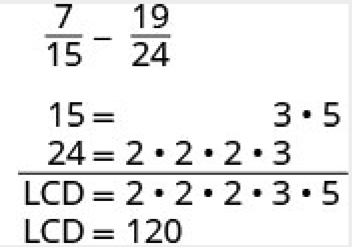
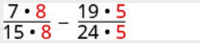
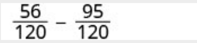
We will apply this method as we subtract the fractions in [link].

Example:
Exercise:

Problem: **Subtract: $\frac{7}{15} - \frac{19}{24}$.

Solution:
Solution

Do the fractions have a common denominator? No, so we need to find the LCD.

Find the LCD.  $\frac{7}{15}-\frac{19}{24}$ $15 = 3 \cdot 5$ $24 = 2 \cdot 2 \cdot 2 \cdot 3$ $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ $\text{LCD} = 120$	
Notice, 15 is “missing” three factors of 2 and 24 is “missing” the 5 from the factors of the LCD. So we multiply 8 in the first fraction and 5 in the second fraction to get the LCD.	
Rewrite as equivalent fractions with the LCD.	 $\frac{7 \cdot 8}{15 \cdot 8}-\frac{19 \cdot 5}{24 \cdot 5}$
Simplify.	 $\frac{56}{120}-\frac{95}{120}$

Subtract.	$-\frac{39}{120}$
Check to see if the answer can be simplified.	$-\frac{13 \cdot 3}{40 \cdot 3}$
Both 39 and 120 have a factor of 3.	
Simplify.	$-\frac{13}{40}$

Do not simplify the equivalent fractions! If you do, you’ll get back to the original fractions and lose the common denominator!

Note:
Exercise:

TRY IT :: 1.163

Problem: Subtract: $\frac{13}{24} - \frac{17}{32}$.

Solution:

$\frac{1}{96}$

Note:
Exercise:

TRY IT :: 1.164

Problem: Subtract: $\frac{21}{32} - \frac{9}{28}$.

Solution:

$\frac{75}{224}$

In the next example, one of the fractions has a variable in its numerator. Notice that we do the same steps as when both numerators are numbers.

Example:
Exercise:

Problem: **Add: $\frac{3}{5}+\frac{x}{8}$.

Solution:
Solution

The fractions have different denominators.

		$\frac{3}{5}+\frac{x}{8}$
Find the LCD. $5 = 5$ $8 = 2 \cdot 2 \cdot 2$ $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 5$ $\text{LCD} = 40$		
Rewrite as equivalent fractions with the		

LCD.		$\frac{3 \cdot 8}{5 \cdot 8}+\frac{x \cdot 5}{8 \cdot 5}$
Simplify.		$\frac{24}{40}+\frac{5x}{40}$
Add.		$\frac{24+5x}{40}$

Remember, we can only add like terms: 24 and 5*x* are not like terms.

Note:
Exercise:

TRY IT :: 1.165

Problem: Add: $\frac{y}{6}+\frac{7}{9}$.

Solution:

$\frac{3y+14}{18}$

Note:
Exercise:

TRY IT :: 1.166

Problem: Add: $\frac{x}{6}+\frac{7}{15}$.

Solution:

$\frac{5x+14}{30}$

We now have all four operations for fractions. [link] summarizes fraction operations.

Fraction Multiplication	**Fraction Division**
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ Multiply the numerators and multiply the denominators	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ Multiply the first fraction by the reciprocal of the second.
Fraction Addition	**Fraction Subtraction**
$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ Add the numerators and place the sum over the common denominator.	$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$ Subtract the numerators and place the difference over the common denominator.
To multiply or divide fractions, an LCD is NOT needed. To add or subtract fractions, an LCD is needed.	

Example:
Exercise:

Problem: **Simplify: ⓐ $\frac{5x}{6}-\frac{3}{10}$ ⓑ $\frac{5x}{6}\cdot\frac{3}{10}$.

Solution:
Solution

First ask, “What is the operation?” Once we identify the operation that will determine whether we need a common denominator. Remember, we need a common denominator to add or subtract, but not to multiply or divide.

ⓐ What is the operation? The operation is subtraction. Do the fractions have a common denominator? No. Rewrite each fraction as an equivalent fraction with the LCD. Subtract the numerators and place the difference over the common denominators. Simplify, if possible. There are no common factors. The fraction is simplified.	$\frac{5x}{6}-\frac{3}{10}$ $\frac{5x\cdot5}{6\cdot5}-\frac{3\cdot3}{10\cdot3}$ $\frac{25x}{30}-\frac{9}{30}$ $\frac{25x-9}{30}$
ⓑ What is the operation? Multiplication. To multiply fractions, multiply the numerators and multiply the denominators. Rewrite, showing common factors. Remove common factors. Simplify.	$\frac{5x}{6}\cdot\frac{3}{10}$ $\frac{5x\cdot3}{6\cdot10}$ $\frac{\cancel{5}x\cdot\cancel{3}}{2\cdot\cancel{3}\cdot2\cdot\cancel{5}}$ $\frac{x}{4}$

Notice we needed an LCD to add $\frac{5x}{6} - \frac{3}{10}$, but not to multiply $\frac{5x}{6} \cdot \frac{3}{10}$.

Note:
Exercise:

TRY IT :: 1.167

Problem: Simplify: ⓐ $\frac{3a}{4} - \frac{8}{9}$ ⓑ $\frac{3a}{4} \cdot \frac{8}{9}$.

Solution:

ⓐ $\frac{27a-32}{36}$ ⓑ $\frac{2a}{3}$

Note:
Exercise:

TRY IT :: 1.168

Problem: Simplify: ⓐ $\frac{4k}{5} - \frac{1}{6}$ ⓑ $\frac{4k}{5} \cdot \frac{1}{6}$.

Solution:

ⓐ $\frac{24k-5}{30}$ ⓑ $\frac{2k}{15}$

Use the Order of Operations to Simplify Complex Fractions

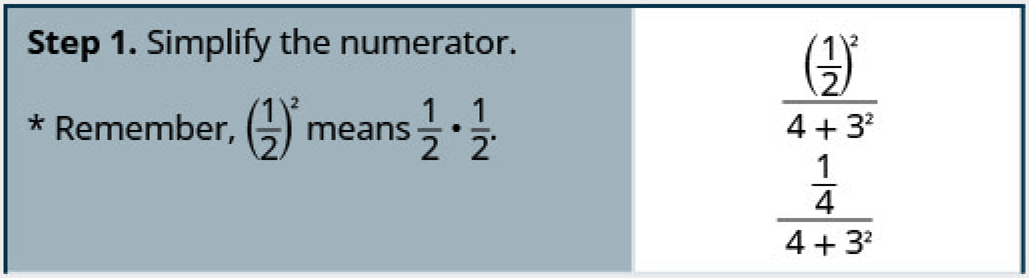
We have seen that a complex fraction is a fraction in which the numerator or denominator contains a fraction. The fraction bar indicates division. We simplified the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ by dividing $\frac{3}{4}$ by $\frac{5}{8}$.

Now we’ll look at complex fractions where the numerator or denominator contains an expression that can be simplified. So we first must completely simplify the numerator and denominator separately using the order of operations. Then we divide the numerator by the denominator.

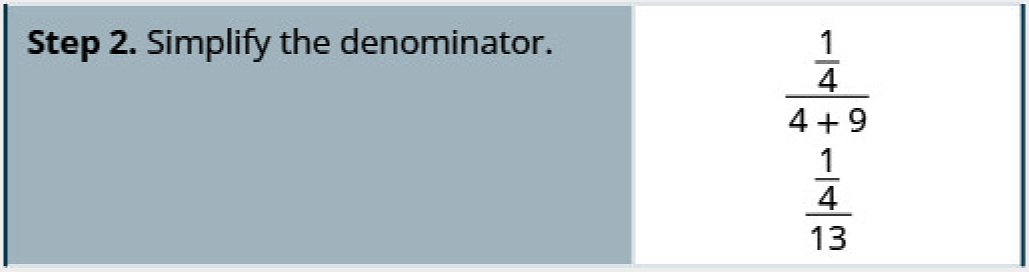
Example:
How to Simplify Complex Fractions
Exercise:

Problem: **Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

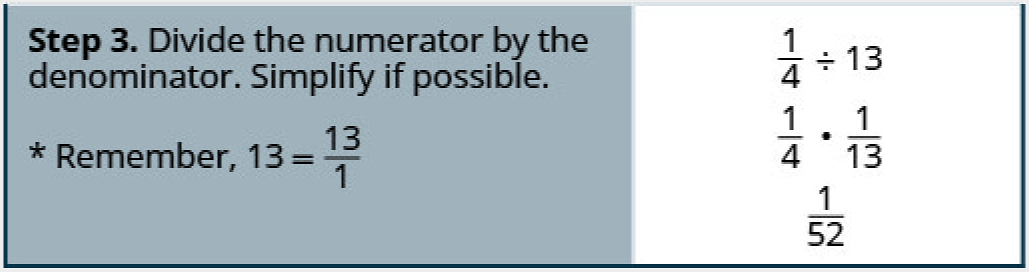
Solution:
Solution



Step 1. Simplify the numerator. * Remember, $\left(\frac{1}{2}\right)^2$ means $\frac{1}{2}\cdot\frac{1}{2}$.	$\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$ $\frac{\frac{1}{4}}{4+3^2}$



Step 2. Simplify the denominator.	$\frac{\frac{1}{4}}{4+9}$ $\frac{\frac{1}{4}}{13}$



Step 3. Divide the numerator by the denominator. Simplify if possible. * Remember, $13=\frac{13}{1}$	$\frac{1}{4}\div 13$ $\frac{1}{4}\cdot\frac{1}{13}$ $\frac{1}{52}$

Note:
Exercise:

TRY IT :: 1.169

Problem: Simplify: $\frac{\left(\frac{1}{3}\right)^2}{2^3+2}$.

Solution:

$\frac{1}{90}$

Note:
Exercise:

TRY IT :: 1.170

Problem: Simplify: $\frac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Solution:

272

Note:
Simplify Complex Fractions.

Simplify the numerator.
Simplify the denominator.
Divide the numerator by the denominator. Simplify if possible.

Example:

Exercise:

Problem: **Simplify: $\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution:
Solution

It may help to put parentheses around the numerator and the denominator.

	$\frac{\left(\frac{1}{2}+\frac{2}{3}\right)}{\left(\frac{3}{4}-\frac{1}{6}\right)}$
Simplify the numerator (LCD = 6) and simplify the denominator (LCD = 12).	$\frac{\left(\frac{3}{6}+\frac{4}{6}\right)}{\left(\frac{9}{12}-\frac{2}{12}\right)}$
Simplify.	$\frac{\left(\frac{7}{6}\right)}{\left(\frac{7}{12}\right)}$
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$
Simplify.	$\frac{7}{6} \cdot \frac{12}{7}$
Divide out common factors.	$\frac{7 \cdot 6 \cdot 2}{6 \cdot 7}$
Simplify.	2

Note:
Exercise:

TRY IT :: 1.171

Problem: Simplify: $\frac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$.

Solution:

2

Note:
Exercise:

TRY IT :: 1.172

Problem: Simplify: $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Solution:

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

Example:
Exercise:

Problem: **Evaluate $x+\frac{1}{3}$ when ⓐ $x=-\frac{1}{3}$ ⓑ $x=-\frac{3}{4}$.

Solution:
Solution

ⓐ To evaluate $x+\frac{1}{3}$ when $x=-\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Simplify.	0

ⓑ To evaluate $x+\frac{1}{3}$ when $x=-\frac{3}{4}$, we substitute $-\frac{3}{4}$ for x in the expression.

	$x+\frac{1}{3}$

Substitute $-\frac{3}{4}$ for x.	$-\frac{3}{4}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot 3}{4\cdot 3}+\frac{1\cdot 4}{3\cdot 4}$
Simplify.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

Note:
Exercise:

TRY IT :: 1.173

Problem: Evaluate $x+\frac{3}{4}$ when ⓐ $x=-\frac{7}{4}$ ⓑ $x=-\frac{5}{4}$.

Solution:

ⓐ -1 ⓑ $-\frac{1}{2}$

Note:
Exercise:

TRY IT :: 1.174

Problem: Evaluate $y+\frac{1}{2}$ when ⓐ $y=\frac{2}{3}$ ⓑ $y=-\frac{3}{4}$.

Solution:

ⓐ $\frac{7}{6}$ ⓑ $-\frac{1}{4}$

Example:
Exercise:

Problem: **Evaluate $-\frac{5}{6}-y$ when $y=-\frac{2}{3}$.

Solution:
Solution

	$-\frac{5}{6}-y$
Substitute $-\frac{2}{3}$ for y.	$-\frac{5}{6}-\left(-\frac{2}{3}\right)$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{5}{6}-\left(-\frac{4}{6}\right)$
Subtract.	$\frac{-5-(-4)}{6}$
Simplify.	$-\frac{1}{6}$

Note:
Exercise:

TRY IT :: 1.175

Problem: Evaluate $-\frac{1}{2}-y$ when $y=-\frac{1}{4}$.

Solution:

$-\frac{1}{4}$

Note:
Exercise:

TRY IT :: 1.176

Problem: Evaluate $-\frac{3}{8}-x$ when $x=-\frac{5}{2}$.

Solution:

$\frac{17}{8}$

Example:
Exercise:

Problem: **Evaluate $2x^2y$ when $x=\frac{1}{4}$ and $y=-\frac{2}{3}$.

Solution:
Solution

Substitute the values into the expression.

	$2x^2y$
Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply. Divide out the common factors. Notice we write 16 as $2 \cdot 2 \cdot 4$ to make it easy to remove common factors.	$-\frac{\cancel{2}\cdot 1\cdot \cancel{2}}{\cancel{2}\cdot \cancel{2}\cdot 4\cdot 3}$
Simplify.	$-\frac{1}{12}$

Note:
Exercise:

TRY IT :: 1.177

Problem: Evaluate $3ab^2$ when $a = -\frac{2}{3}$ and $b = -\frac{1}{2}$.

Solution:

$-\frac{1}{2}$

Note:
Exercise:

TRY IT :: 1.178

Problem: Evaluate $4c^3d$ when $c = -\frac{1}{2}$ and $d = -\frac{4}{3}$.

Solution:

$\frac{2}{3}$

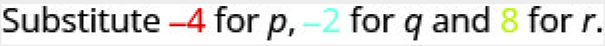
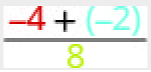
The next example will have only variables, no constants.

Example:
Exercise:

Problem: **Evaluate $\frac{p+q}{r}$ when $p = -4, q = -2,$ and $r = 8$.

Solution:
Solution

To evaluate $\frac{p+q}{r}$ when $p = -4, q = -2,$ and $r = 8$, we substitute the values into the expression.

	$\frac{p+q}{r}$
 Substitute –4 for p, –2 for q and 8 for r.	 $\frac{-4+(-2)}{8}$

Add in the numerator first.	$\frac{-6}{8}$
Simplify.	$-\frac{3}{4}$

Note:
Exercise:

TRY IT :: 1.179

Problem: Evaluate $\frac{a+b}{c}$ when $a=-8, b=-7,$ and $c=6.$

Solution:

$-\frac{5}{2}$

Note:
Exercise:

TRY IT :: 1.180

Problem: Evaluate $\frac{x+y}{z}$ when $x=9, y=-18,$ and $z=-6.$

Solution:

$\frac{3}{2}$

Key Concepts

- **Fraction Addition and Subtraction:** If $a, b,$ and c are numbers where $c \neq 0$, then
  $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.
  To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.
- **Strategy for Adding or Subtracting Fractions**

Do they have a common denominator?	Yes—go to step 2.	No—Rewrite each fraction with the LCD (Least Common Denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.

  Add or subtract the fractions.
  Simplify, if possible. To multiply or divide fractions, an LCD IS NOT needed. To add or subtract fractions, an LCD IS needed.

- **Simplify Complex Fractions**

  Simplify the numerator.
  Simplify the denominator.
  Divide the numerator by the denominator. Simplify if possible.

Practice Makes Perfect

Add and Subtract Fractions with a Common Denominator

In the following exercises, add.

Exercise:

Problem: $\frac{6}{13} + \frac{5}{13}$

Solution:

$\frac{11}{13}$

Exercise:

Problem: $\frac{4}{15} + \frac{7}{15}$

Exercise:

Problem: $\frac{x}{4} + \frac{3}{4}$

Solution:

$\frac{x+3}{4}$

Exercise:

Problem: $\frac{8}{q} + \frac{6}{q}$

Exercise:

Problem: $-\frac{3}{16} + \left(-\frac{7}{16}\right)$

Solution:

$-\frac{5}{8}$

Exercise:

Problem: $-\frac{5}{16} + \left(-\frac{9}{16}\right)$

Exercise:

Problem: $-\frac{8}{17} + \frac{15}{17}$

Solution:

$\frac{7}{17}$

Exercise:

Problem: $-\frac{9}{19}+\frac{17}{19}$

Exercise:

Problem: $\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$

Solution:

$-\frac{16}{13}$

Exercise:

Problem: $\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$

In the following exercises, subtract.

Exercise:

Problem: $\frac{11}{15}-\frac{7}{15}$

Solution:

$\frac{4}{15}$

Exercise:

Problem: $\frac{9}{13}-\frac{4}{13}$

Exercise:

Problem: $\frac{11}{12}-\frac{5}{12}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{7}{12} - \frac{5}{12}$

Exercise:

Problem: $\frac{19}{21} - \frac{4}{21}$

Solution:

$\frac{5}{7}$

Exercise:

Problem: $\frac{17}{21} - \frac{8}{21}$

Exercise:

Problem: $\frac{5y}{8} - \frac{7}{8}$

Solution:

$\frac{5y-7}{8}$

Exercise:

Problem: $\frac{11z}{13} - \frac{8}{13}$

Exercise:

Problem: $-\frac{23}{u} - \frac{15}{u}$

Solution:

$-\frac{38}{u}$

Exercise:

Problem: $-\frac{29}{v} - \frac{26}{v}$

Exercise:

Problem: $-\frac{3}{5} - \left(-\frac{4}{5}\right)$

Solution:

$\frac{1}{5}$

Exercise:

Problem: $-\frac{3}{7} - \left(-\frac{5}{7}\right)$

Exercise:

Problem: $-\frac{7}{9} - \left(-\frac{5}{9}\right)$

Solution:

$-\frac{2}{9}$

Exercise:

Problem: $-\frac{8}{11} - \left(-\frac{5}{11}\right)$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $-\frac{5}{18} \cdot \frac{9}{10}$

Solution:

$-\frac{1}{4}$

Exercise:

Problem: $-\frac{3}{14} \cdot \frac{7}{12}$

Exercise:

Problem: $\frac{n}{5} - \frac{4}{5}$

Solution:

$\frac{n-4}{5}$

Exercise:

Problem: $\frac{6}{11} - \frac{s}{11}$

Exercise:

Problem: $-\frac{7}{24} + \frac{2}{24}$

Solution:

$-\frac{5}{24}$

Exercise:

Problem: $-\frac{5}{18} + \frac{1}{18}$

Exercise:

Problem: $\frac{8}{15} \div \frac{12}{5}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{7}{12} \div \frac{9}{28}$

Add or Subtract Fractions with Different Denominators

In the following exercises, add or subtract.

Exercise:

Problem: $\frac{1}{2} + \frac{1}{7}$

Solution:

$\frac{9}{14}$

Exercise:

Problem: $\frac{1}{3} + \frac{1}{8}$

Exercise:

Problem: $\frac{1}{3} - \left(-\frac{1}{9}\right)$

Solution:

$\frac{4}{9}$

Exercise:

Problem: $\frac{1}{4} - \left(-\frac{1}{8}\right)$

Exercise:

Problem: $\frac{7}{12} + \frac{5}{8}$

Solution:

$\frac{29}{24}$

Exercise:

Problem: $\frac{5}{12}+\frac{3}{8}$

Exercise:

Problem: $\frac{7}{12}-\ \frac{9}{16}$

Solution:

$\frac{1}{48}$

Exercise:

Problem: $\frac{7}{16}-\ \frac{5}{12}$

Exercise:

Problem: $\frac{2}{3}-\ \frac{3}{8}$

Solution:

$\frac{7}{24}$

Exercise:

Problem: $\frac{5}{6}-\ \frac{3}{4}$

Exercise:

Problem: $-\ \frac{11}{30}+\frac{27}{40}$

Solution:

$\frac{37}{120}$

Exercise:

Problem: $-\frac{9}{20}+\frac{17}{30}$

Exercise:

Problem: $-\frac{13}{30}+\frac{25}{42}$

Solution:

$\frac{17}{105}$

Exercise:

Problem: $-\frac{23}{30}+\frac{5}{48}$

Exercise:

Problem: $-\frac{39}{56}-\frac{22}{35}$

Solution:

$-\frac{53}{40}$

Exercise:

Problem: $-\frac{33}{49}-\frac{18}{35}$

Exercise:

Problem: $-\frac{2}{3}-\left(-\frac{3}{4}\right)$

Solution:

$\frac{1}{12}$

Exercise:

Problem: $-\frac{3}{4}-\left(-\frac{4}{5}\right)$

Exercise:

Problem: $1+\frac{7}{8}$

Solution:

$$\frac{15}{8}$$

Exercise:

Problem: $1-\frac{3}{10}$

Exercise:

Problem: $\frac{x}{3}+\frac{1}{4}$

Solution:

$$\frac{4x+3}{12}$$

Exercise:

Problem: $\frac{y}{2}+\frac{2}{3}$

Exercise:

Problem: $\frac{y}{4}-\frac{3}{5}$

Solution:

$$\frac{4y-12}{20}$$

Exercise:

Problem: $\frac{x}{5} - \frac{1}{4}$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: ⓐ $\frac{2}{3} + \frac{1}{6}$ ⓑ $\frac{2}{3} \div \frac{1}{6}$

Solution:

ⓐ $\frac{5}{6}$ ⓑ 4

Exercise:

Problem: ⓐ $-\frac{2}{5} - \frac{1}{8}$ ⓑ $-\frac{2}{5} \cdot \frac{1}{8}$

Exercise:

Problem: ⓐ $\frac{5n}{6} \div \frac{8}{15}$ ⓑ $\frac{5n}{6} - \frac{8}{15}$

Solution:

ⓐ $\frac{25n}{16}$ ⓑ $\frac{25n-16}{30}$

Exercise:

Problem: ⓐ $\frac{3a}{8} \div \frac{7}{12}$ ⓑ $\frac{3a}{8} - \frac{7}{12}$

Exercise:

Problem: $-\frac{3}{8} \div \left(-\frac{3}{10}\right)$

Solution:

$\frac{5}{4}$

Exercise:

Problem: $-\frac{5}{12} \div \left(-\frac{5}{9}\right)$

Exercise:

Problem: $-\frac{3}{8} + \frac{5}{12}$

Solution:

$\frac{1}{24}$

Exercise:

Problem: $-\frac{1}{8} + \frac{7}{12}$

Exercise:

Problem: $\frac{5}{6} - \frac{1}{9}$

Solution:

$\frac{13}{18}$

Exercise:

Problem: $\frac{5}{9} - \frac{1}{6}$

Exercise:

Problem: $-\frac{7}{15} - \frac{y}{4}$

Solution:

$\frac{-28-15y}{60}$

Exercise:

Problem: $-\frac{3}{8} - \frac{x}{11}$

Exercise:

Problem: $\frac{11}{12a} \cdot \frac{9a}{16}$

Solution:

$\frac{33}{64}$

Exercise:

Problem: $\frac{10y}{13} \cdot \frac{8}{15y}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

Exercise:

Problem: $\frac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$

Solution:

54

Exercise:

Problem: $\frac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$

Exercise:

Problem: $\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$

Solution:

$\frac{49}{25}$

Exercise:

Problem: $\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$

Exercise:

Problem: $\frac{2}{\frac{1}{3}+\frac{1}{5}}$

Solution:

$\frac{15}{4}$

Exercise:

Problem: $\frac{5}{\frac{1}{4}+\frac{1}{3}}$

Exercise:

Problem: $\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$

Solution:

$\frac{5}{21}$

Exercise:

Problem: $\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$

Exercise:

Problem: $\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$

Solution:

$\frac{7}{9}$

Exercise:

Problem: $\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$

Exercise:

Problem: $1-\ \frac{3}{5}\div\frac{1}{10}$

Solution:

-5

Exercise:

Problem: $1-\ \frac{5}{6}\div\frac{1}{12}$

Exercise:

Problem: $\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$

Solution:

$\frac{19}{12}$

Exercise:

Problem: $\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$

Exercise:

Problem: $\frac{3}{8} - \frac{1}{6} + \frac{3}{4}$

Solution:

$\frac{23}{24}$

Exercise:

Problem: $\frac{2}{5} + \frac{5}{8} - \frac{3}{4}$

Exercise:

Problem: $12\left(\frac{9}{20} - \frac{4}{15}\right)$

Solution:

$\frac{11}{5}$

Exercise:

Problem: $8\left(\frac{15}{16} - \frac{5}{6}\right)$

Exercise:

Problem: $\frac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$

Solution:

1

Exercise:

Problem: $\frac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$

Exercise:

Problem: $\left(\frac{5}{9}+\frac{1}{6}\right)\div\left(\frac{2}{3}-\frac{1}{2}\right)$

Solution:

$\frac{13}{3}$

Exercise:

Problem: $\left(\frac{3}{4}+\frac{1}{6}\right)\div\left(\frac{5}{8}-\frac{1}{3}\right)$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

Exercise:

Problem: $x+\left(-\frac{5}{6}\right)$ when
ⓐ $x=\frac{1}{3}$
ⓑ $x=-\frac{1}{6}$

Solution:

ⓐ $-\frac{1}{2}$ ⓑ -1

Exercise:

Problem: $x+\left(-\frac{11}{12}\right)$ when
ⓐ $x=\frac{11}{12}$
ⓑ $x=\frac{3}{4}$

Exercise:

Problem: $x-\frac{2}{5}$ when
ⓐ $x=\frac{3}{5}$
ⓑ $x=-\frac{3}{5}$

Solution:

ⓐ $\frac{1}{5}$ ⓑ -1

Exercise:

Problem: $x - \frac{1}{3}$ when
ⓐ $x = \frac{2}{3}$
ⓑ $x = -\frac{2}{3}$

Exercise:

Problem: $\frac{7}{10} - w$ when
ⓐ $w = \frac{1}{2}$
ⓑ $w = -\frac{1}{2}$

Solution:

ⓐ $\frac{1}{5}$ ⓑ $\frac{6}{5}$

Exercise:

Problem: $\frac{5}{12} - w$ when
ⓐ $w = \frac{1}{4}$
ⓑ $w = -\frac{1}{4}$

Exercise:

Problem: $2x^2y^3$ when $x = -\frac{2}{3}$ and $y = -\frac{1}{2}$

Solution:

$-\frac{1}{9}$

Exercise:

Problem: $8u^2v^3$ when $u = -\frac{3}{4}$ and $v = -\frac{1}{2}$

Exercise:

Problem: $\frac{a+b}{a-b}$ when $a = -3, b = 8$

Solution:

$-\frac{5}{11}$

Exercise:

Problem: $\frac{r-s}{r+s}$ when $r = 10, s = -5$

Everyday Math

Exercise:

Problem:

Decorating Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{1}{2}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

Solution:

$\frac{7}{8}$ yard

Exercise:

Problem:

Baking Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $\frac{1}{2}$ cup of sugar for the chocolate chip cookies and $\frac{1}{4}$ of sugar for the oatmeal cookies. How much sugar does she need altogether?

Writing Exercises

Exercise:

Problem:

Why do you need a common denominator to add or subtract fractions? Explain.

Solution:

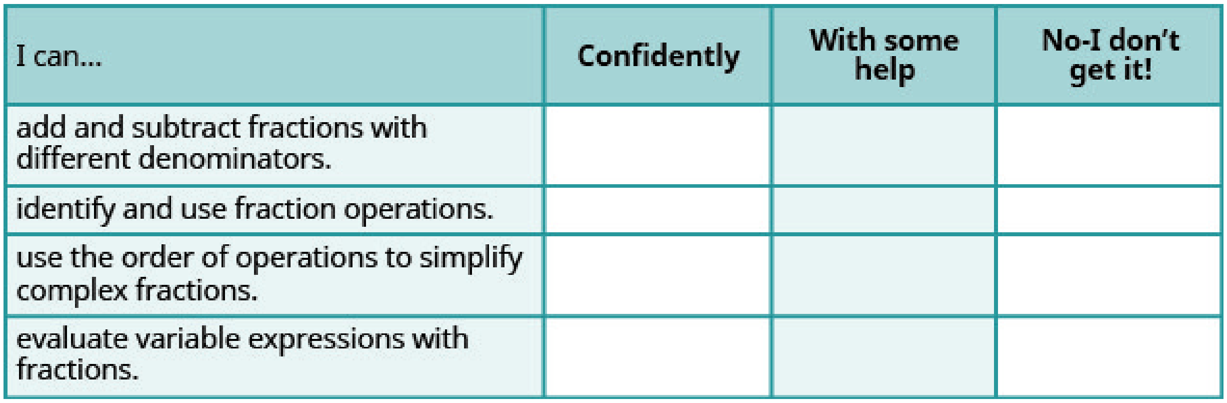
Answers may vary

Exercise:

Problem: How do you find the LCD of 2 fractions?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well-prepared for the next chapter? Why or why not?

Glossary

least common denominator

The least common denominator (LCD) of two fractions is the Least common multiple (LCM) of their denominators.

Decimals
By the end of this section, you will be able to:

- Name and write decimals
- Round decimals
- Add and subtract decimals
- Multiply and divide decimals
- Convert decimals, fractions, and percents

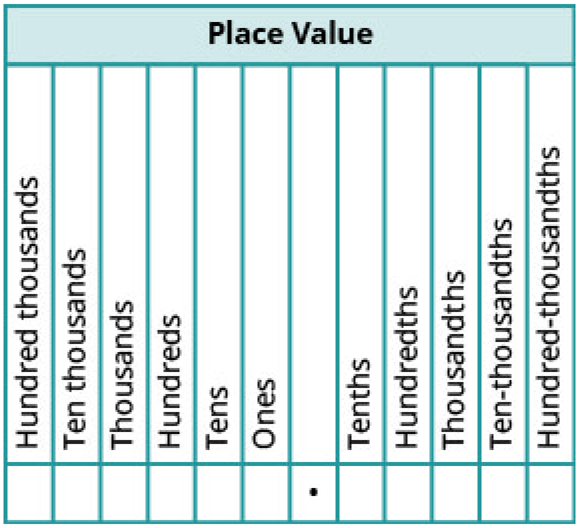
Name and Write Decimals

Decimals are another way of writing fractions whose denominators are powers of 10.
Equation:

$$\begin{array}{rcll} 0.1 & = & \frac{1}{10} & \qquad \text{0.1 is “one tenth”} \\ 0.01 & = & \frac{1}{100} & \qquad \text{0.01 is “one hundredth”} \\ 0.001 & = & \frac{1}{1{,}000} & \qquad \text{0.001 is “one thousandth”} \\ 0.0001 & = & \frac{1}{10{,}000} & \qquad \text{0.0001 is “one ten-thousandth”} \end{array}$$

Notice that “ten thousand” is a number larger than one, but “one ten-thousand**th**” is a number smaller than one. The “th” at the end of the name tells you that the number is smaller than one.

When we name a whole number, the name corresponds to the place value based on the powers of ten. We read 10,000 as “ten thousand” and 10,000,000 as “ten million.” Likewise, the names of the decimal places correspond to their fraction values. [link] shows the names of the place values to the left and right of the decimal point.



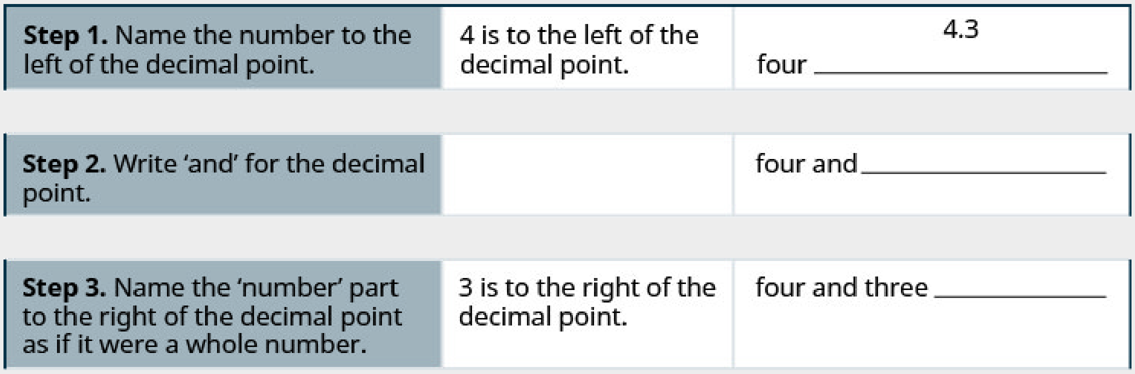
Place Value											
Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
						•					

Place value of decimal numbers are shown to the left and right of the decimal point.

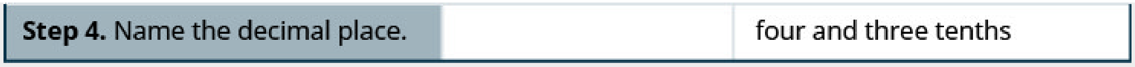
Example:
How to Name Decimals
Exercise:

Problem: **Name the decimal 4.3.

Solution:
Solution



Step 1. Name the number to the left of the decimal point.	4 is to the left of the decimal point.	4.3 four ______________
Step 2. Write 'and' for the decimal point.		four and ______________
Step 3. Name the 'number' part to the right of the decimal point as if it were a whole number.	3 is to the right of the decimal point.	four and three ______________



Step 4. Name the decimal place.		four and three tenths

Note:
Exercise:

TRY IT :: 1.181

Problem: Name the decimal: 6.7.

Solution:

six and seven tenths

Note:
Exercise:

TRY IT :: 1.182

Problem: Name the decimal: 5.8.

Solution:

five and eight tenths

We summarize the steps needed to name a decimal below.

Note:
Name a Decimal.

Name the number to the left of the decimal point.
Write “and” for the decimal point.
Name the “number” part to the right of the decimal point as if it were a whole number.
Name the decimal place of the last digit.

Example:
Exercise:

Problem: **Name the decimal: -15.571.

Solution:
Solution

	-15.571
Name the number to the left of the decimal point.	negative fifteen ________________________________
Write “and” for the decimal point.	negative fifteen and ____________________________
Name the number to the right of the decimal point.	negative fifteen and five hundred seventy-one __________

The 1 is in the thousandths place.	negative fifteen and five hundred seventy-one thousandths

Note:
Exercise:

TRY IT :: 1.183
Problem: Name the decimal: -13.461.

Solution:

negative thirteen and four hundred sixty-one thousandths

Note:
Exercise:

TRY IT :: 1.184
Problem: Name the decimal: -2.053.

Solution:

negative two and fifty-three thousandths

When we write a check we write both the numerals and the name of the number. Let’s see how to write the decimal from the name.

Example:

How to Write Decimals

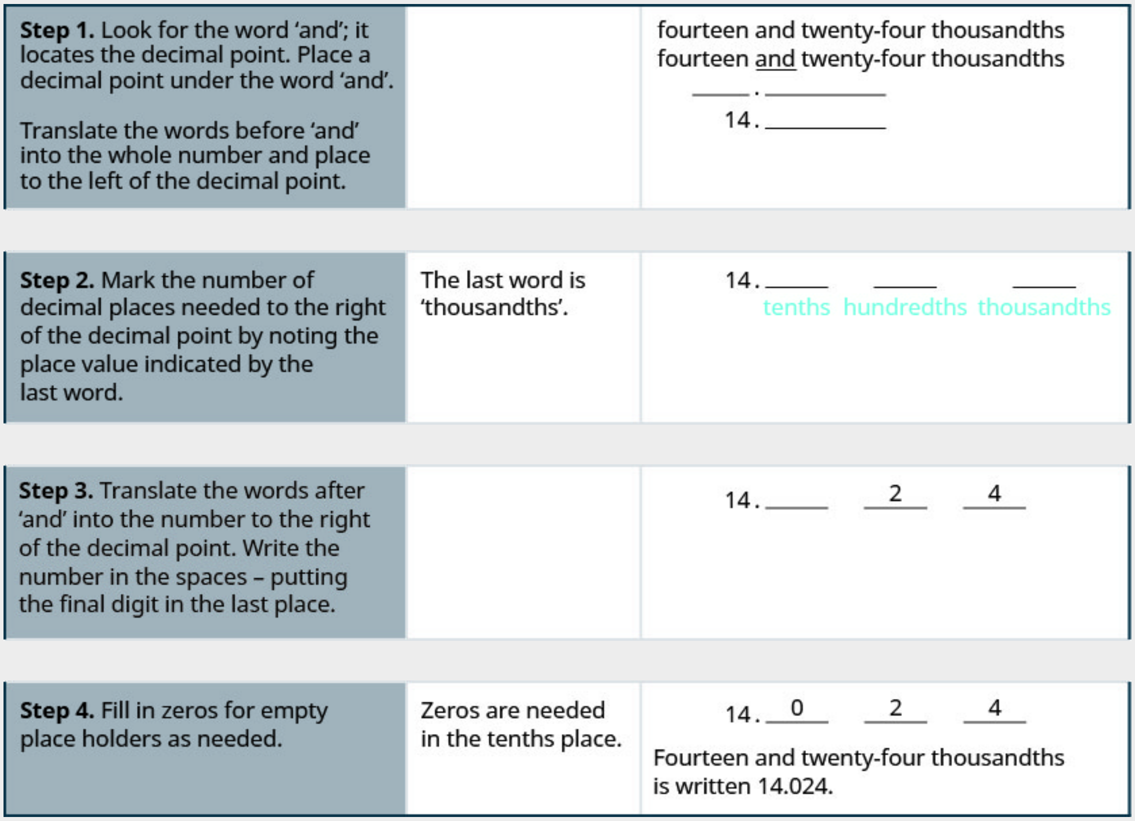
Exercise:

Problem:

**Write “fourteen and twenty-four thousandths” as a decimal.

Solution:

Solution



Step 1. Look for the word ‘and’; it locates the decimal point. Place a decimal point under the word ‘and’. Translate the words before ‘and’ into the whole number and place to the left of the decimal point.		fourteen and twenty-four thousandths fourteen and twenty-four thousandths ____ . ________ 14 . ________
Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.	The last word is ‘thousandths’.	14 . ____ ____ ____ tenths hundredths thousandths
Step 3. Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces – putting the final digit in the last place.		14 . ____ 2 4
Step 4. Fill in zeros for empty place holders as needed.	Zeros are needed in the tenths place.	14 . 0 2 4 Fourteen and twenty-four thousandths is written 14.024.

Note:

Exercise:

TRY IT :: 1.185

Problem: Write as a decimal: thirteen and sixty-eight thousandths.

Solution:

13.068

Note:
Exercise:

TRY IT :: 1.186

Problem: Write as a decimal: five and ninety-four thousandths.

Solution:

5.094

We summarize the steps to writing a decimal.

Note:
Write a decimal.

Look for the word "and"—it locates the decimal point.

- Place a decimal point under the word "and." Translate the words before "and" into the whole number and place it to the left of the decimal point.
- If there is no "and," write a "0" with a decimal point to its right.

Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
Translate the words after "and" into the number to the right of the decimal

point. Write the number in the spaces—putting the final digit in the last place.
Fill in zeros for place holders as needed.

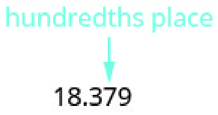
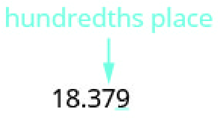
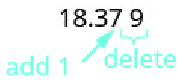
Round Decimals

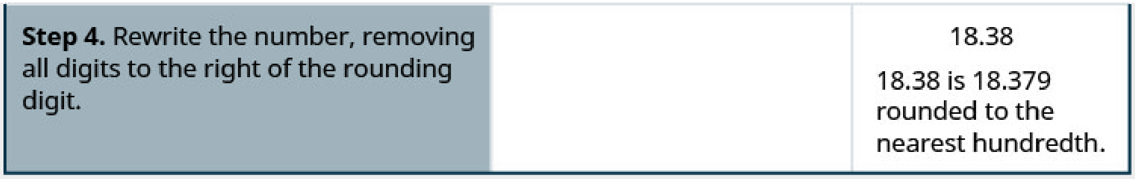
Rounding decimals is very much like rounding whole numbers. We will round decimals with a method based on the one we used to round whole numbers.

Example:
How to Round Decimals
Exercise:

Problem: **Round 18.379 to the nearest hundredth.

Solution:
Solution

Step 1. Locate the given place value and mark it with an arrow.		
Step 2. Underline the digit to the right of the given place value.		
Step 3. Is this digit greater than or equal to 5? **Yes:** Add 1 to the digit in the given place value. **No:** Do not change the digit in the the given place value.	Because 9 is greater than or equal to 5, add 1 to the 7.	



Step 4. Rewrite the number, removing all digits to the right of the rounding digit.		18.38 18.38 is 18.379 rounded to the nearest hundredth.

Note:
Exercise:

TRY IT :: 1.187

Problem: Round to the nearest hundredth: 1.047.

Solution:

1.05

Note:
Exercise:

TRY IT :: 1.188

Problem: Round to the nearest hundredth: 9.173.

Solution:

9.17

We summarize the steps for rounding a decimal here.

Note:
Round Decimals.

Locate the given place value and mark it with an arrow.
Underline the digit to the right of the place value.
Is this digit greater than or equal to 5?

- Yes—add 1 to the digit in the given place value.
- No—do not change the digit in the given place value.

Rewrite the number, deleting all digits to the right of the rounding digit.

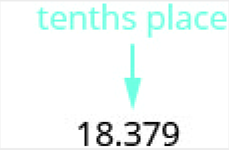
Example:
Exercise:

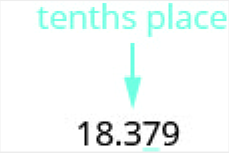
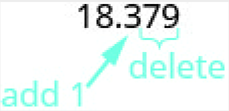
Problem: **Round 18.379 to the nearest ⓐ tenth ⓑ whole number.

Solution:
Solution

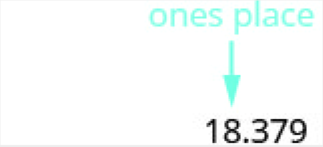
Round 18.379

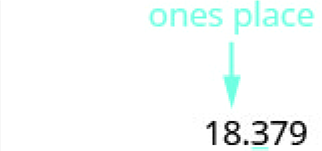
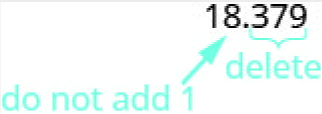
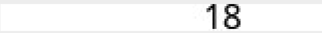
ⓐ to the nearest tenth

Locate the tenths place with an arrow.	

Underline the digit to the right of the given place value.	
Because 7 is greater than or equal to 5, add 1 to the 3.	
Rewrite the number, deleting all digits to the right of the rounding digit.	18.4
Notice that the deleted digits were NOT replaced with zeros.	So, 18.379 rounded to the nearest tenth is 18.4.

ⓑ to the nearest whole number

Locate the ones place with an arrow.	
Underline the digit to the right of the given place value.	

	
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	
Rewrite the number, deleting all digits to the right of the rounding digit.	
	So, 18.379 rounded to the nearest whole number is 18.

Note:
Exercise:

Problem:

TRY IT :: 1.189
Round 6.582 to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Solution:

ⓐ 6.58 ⓑ 6.6 ⓒ 7

Note:
Exercise:

Problem:

TRY IT :: 1.190
Round 15.2175 to the nearest ⓐ thousandth ⓑ hundredth ⓒ tenth.

Solution:

ⓐ 15.218 ⓑ 15.22 ⓒ 15.2

Add and Subtract Decimals

To add or subtract decimals, we line up the decimal points. By lining up the decimal points this way, we can add or subtract the corresponding place values. We then add or subtract the numbers as if they were whole numbers and then place the decimal point in the sum.

Note:
Add or Subtract Decimals.

Write the numbers so the decimal points line up vertically.
Use zeros as place holders, as needed.
Add or subtract the numbers as if they were whole numbers. Then place the decimal point in the answer under the decimal points in the given numbers.

Example:
Exercise:

Problem: **Add: $23.5 + 41.38$.

Solution:
Solution

Write the numbers so the decimal points line up vertically.	$\begin{array}{r} 23.5 \\ \underline{+41.38} \end{array}$
Put 0 as a placeholder after the 5 in 23.5. Remember, $\frac{5}{10} = \frac{50}{100}$ so $0.5 = 0.50$.	$\begin{array}{r} 23.50 \\ \underline{+41.38} \end{array}$
Add the numbers as if they were whole numbers. Then place the decimal point in the sum.	$\begin{array}{r} 23.50 \\ \underline{+41.38} \\ 64.88 \end{array}$

Note:
Exercise:

TRY IT :: 1.191

Problem: Add: $4.8 + 11.69$.

Solution:

16.49

Note:

Exercise:

TRY IT :: 1.192

Problem: Add: $5.123 + 18.47$.

Solution:

23.593

Example:
Exercise:

Problem: **Subtract: $20 - 14.65$.

Solution:
Solution

	$20 - 14.65$
Write the numbers so the decimal points line up vertically. Remember, 20 is a whole number, so place the decimal point after the 0.	$\begin{array}{r} 20. \\ \underline{-14.65} \end{array}$
Put in zeros to the right as placeholders.	$\begin{array}{r} 20.00 \\ \underline{-14.65} \end{array}$
Subtract and place the decimal point in the answer.	

	$\begin{array}{r} \overset{1}{\cancel{2}}\,\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\,.\,\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\,\overset{\cancel{10}}{\cancel{0}} \\ -14.65 \\ \hline 5.35 \end{array}$

Note:
Exercise:

TRY IT :: 1.193

Problem: Subtract: $10 - 9.58$.

Solution:

0.42

Note:
Exercise:

TRY IT :: 1.194

Problem: Subtract: $50 - 37.42$.

Solution:

12.58

Multiply and Divide Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first convert them to fractions and then multiply.

So let’s see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side. Look for a pattern!

Convert to fractions.	$\frac{3}{10} \cdot \frac{7}{10}$ $\qquad$ $\frac{2}{10} \cdot \frac{46}{100}$
Multiply.	$\frac{21}{100}$ $\qquad$ $\frac{92}{1000}$
Convert to decimals.	$0.\underbrace{21}_{\text{2 places}}$ $\qquad$ $0.\underbrace{092}_{\text{3 places}}$

Notice, in the first example, we multiplied two numbers that each had one digit after the decimal point and the product had two decimal places. In the second example, we multiplied a number with one decimal place by a number with two decimal places and the product had three decimal places.

We multiply the numbers just as we do whole numbers, temporarily ignoring the decimal point. We then count the number of decimal points in the factors and that sum tells us the number of decimal places in the product.

The rules for multiplying positive and negative numbers apply to decimals, too, of course!

When *multiplying* two numbers,

- if their signs are the *same* the product is *positive*.
- if their signs are *different* the product is *negative*.

When we multiply signed decimals, first we determine the sign of the product and then multiply as if the numbers were both positive. Finally, we write the product with the appropriate sign.

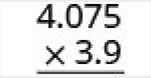
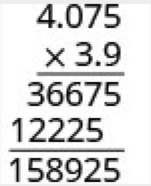
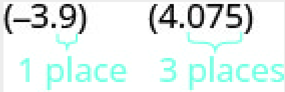
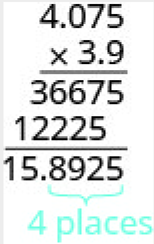
Note:
Multiply Decimals.

Determine the sign of the product.
Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors.
Write the product with the appropriate sign.

Example:
Exercise:

Problem: **Multiply: $(-3.9)\,(4.075)$.

Solution:
Solution

	(−3.9)(4.075)
The signs are different. The product will be negative.	
Write in vertical format, lining up the numbers on the right.	 $\begin{array}{r} 4.075 \\ \times\, 3.9 \\ \hline \end{array}$
Multiply.	 $\begin{array}{r} 4.075 \\ \times\, 3.9 \\ \hline 36675 \\ 12225 \\ \hline 158925 \end{array}$
Add the number of decimal places in the factors (1 + 3).  $(-3.\underbrace{9}_{\text{1 place}})\quad(4.\underbrace{075}_{\text{3 places}})$ Place the decimal point 4 places from the right.	 $\begin{array}{r} 4.075 \\ \times\, 3.9 \\ \hline 36675 \\ 12225 \\ \hline 15.\underbrace{8925}_{\text{4 places}} \end{array}$
The signs are different, so the product is negative.	(−3.9)(4.075) = −15.8925

Note:
Exercise:

TRY IT :: 1.195
Problem: Multiply: $-4.5\,(6.107)$.

Solution:

-27.4815

Note:
Exercise:

TRY IT :: 1.196
Problem: Multiply: $-10.79\,(8.12)$.

Solution:

-87.6148

In many of your other classes, especially in the sciences, you will multiply decimals by powers of 10 (10, 100, 1000, etc.). If you multiply a few products on paper, you may notice a pattern relating the number of zeros in the power of 10 to number of decimal places we move the decimal point to the right to get the product.

Note:
Multiply a Decimal by a Power of Ten.

Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
Add zeros at the end of the number as needed.

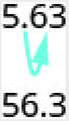
Example:
Exercise:

Problem: **Multiply 5.63 ⓐ by 10 ⓑ by 100 ⓒ by 1,000.

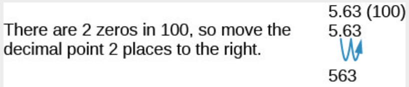
Solution:
Solution

By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

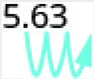
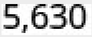
ⓐ

	5.63(10)
There is 1 zero in 10, so move the decimal point 1 place to the right.	

ⓑ

	5.63(100)
There are 2 zeros in 100, so move the decimal point 2 places to the right.	 There are 2 zeros in 100, so move the decimal point 2 places to the right. 5.63 (100) 5.63 563

ⓒ

	5.63(1,000)
There are 3 zeros in 1,000, so move the decimal point 3 places to the right.	 5.63
A zero must be added at the end.	 5,630

Note:

Exercise:

TRY IT :: 1.197

Problem: Multiply 2.58 ⓐ by 10 ⓑ by 100 ⓒ by 1,000.

Solution:

ⓐ 25.8 ⓑ 258 ⓒ 2,580

Note:

Exercise:

TRY IT :: 1.198

Problem: Multiply 14.2 ⓐ by 10 ⓑ by 100 ⓒ by 1,000.

Solution:

ⓐ 142 ⓑ 1,420 ⓒ 14,200

Just as with multiplication, division of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To divide decimals, determine what power of 10 to multiply the denominator by to make it a whole number. Then multiply the numerator by that same power of 10. Because of the equivalent fractions property, we haven’t changed the value of the fraction! The effect is to move the decimal points in the numerator and denominator the same number of places to the right. For example:

Equation:

$$\frac{0.8}{0.4}$$

$$\frac{0.8(10)}{0.4(10)}$$

$$\frac{8}{4}$$

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign.

We review the notation and vocabulary for division:

Equation:

$$\underset{\text{dividend}}{a} \div \underset{\text{divisor}}{b} = \underset{\text{quotient}}{\text{c}} \qquad\qquad \underset{\text{divisor}}{b}\,\overset{\overset{\text{c}}{\text{quotient}}}{\overline{\big)\underset{\text{dividend}}{a}}}$$

We’ll write the steps to take when dividing decimals, for easy reference.

Note:

Divide Decimals.

Determine the sign of the quotient.
Make the divisor a whole number by “moving” the decimal point all the way to the right. “Move” the decimal point in the dividend the same number of places—adding zeros as needed.
Divide. Place the decimal point in the quotient above the decimal point in the dividend.
Write the quotient with the appropriate sign.

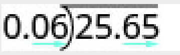
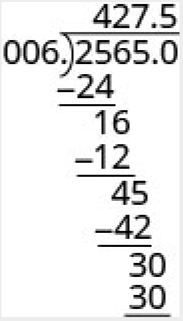
Example:

Exercise:

Problem: **Divide: $-25.56 \div (-0.06)$.

Solution:
Solution

Remember, you can “move” the decimals in the divisor and dividend because of the Equivalent Fractions Property.

	$-25.65 \div (-0.06)$
The signs are the same.	The quotient is positive.
Make the divisor a whole number by “moving” the decimal point all the way to the right.	
“Move” the decimal point in the dividend the same number of places.	
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	

Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

Note:
Exercise:

TRY IT :: 1.199

Problem: Divide: $-23.492 \div (-0.04)$.

Solution:

587.3

Note:
Exercise:

TRY IT :: 1.200

Problem: Divide: $-4.11 \div (-0.12)$.

Solution:

34.25

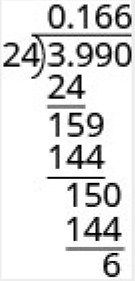
A common application of dividing whole numbers into decimals is when we want to find the price of one item that is sold as part of a multi-pack. For example, suppose a case of 24 water bottles costs $3.99. To find the price of one water bottle, we would divide $3.99 by 24. We show this division in

[link]. In calculations with money, we will round the answer to the nearest cent (hundredth).

Example:
Exercise:

Problem: **Divide (round to the nearest cent): $\$3.99 \div 24$.

Solution:
Solution

	$\$3.99 \div 24$
Place the decimal point in the quotient above the decimal point in the dividend.	
Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth.) To do this, we must carry the division to the thousandths place.	
Round to the nearest cent.	$\$0.166 \approx \$0.17$ $\$3.99 \div 24 \approx \$0.17$

Note:
Exercise:

TRY IT :: 1.201

Problem: Divide (round to the nearest cent): $\$6.99 \div 36$.

Solution:

$0.19

Note:
Exercise:

TRY IT :: 1.202

Problem: Divide (round to the nearest cent): $\$4.99 \div 12$.

Solution:

$0.42

Convert Decimals, Fractions, and Percents

We convert decimals into fractions by identifying the place value of the last (farthest right) digit. In the decimal 0.03 the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.
Equation:

$$0\,0.03 = \frac{3}{100}$$

Notice, when the number to the left of the decimal is zero, we get a fraction whose numerator is less than its denominator. Fractions like this are called proper fractions.

The steps to take to convert a decimal to a fraction are summarized in the procedure box.

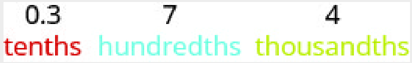
Note:
Convert a Decimal to a Proper Fraction.

Determine the place value of the final digit.
Write the fraction.
- numerator—the “numbers” to the right of the decimal point
- denominator—the place value corresponding to the final digit

Example:
Exercise:

Problem: **Write 0.374 as a fraction.

Solution:
Solution

	0.374
Determine the place value of the final digit.	 0.3 7 4 tenths hundredths thousandths

Write the fraction for 0.374: • The numerator is 374. • The denominator is 1,000.	$\frac{374}{1000}$
Simplify the fraction.	$\frac{2\cdot187}{2\cdot500}$
Divide out the common factors.	$\frac{187}{500}$ so, $0.374 = \frac{187}{500}$

Did you notice that the number of zeros in the denominator of $\frac{374}{1,000}$ is the same as the number of decimal places in 0.374?

Note:
Exercise:

TRY IT :: 1.203

Problem: Write 0.234 as a fraction.

Solution:

$\frac{117}{500}$

Note:
Exercise:

TRY IT :: 1.204

Problem: Write 0.024 as a fraction.

Solution:

$\frac{3}{125}$

We’ve learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar means division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\overline{)4}$. This leads to the following method for converting a fraction to a decimal.

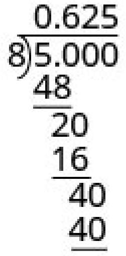
Note:
Convert a Fraction to a Decimal.
To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

Example:
Exercise:

Problem: **Write $-\frac{5}{8}$ as a decimal.

Solution:
Solution

Since a fraction bar means division, we begin by writing $\frac{5}{8}$ as $8\overline{)5}$. Now divide.



so, $-\frac{5}{8} = -0.625$

Note:
Exercise:

TRY IT :: 1.205

Problem: Write $-\frac{7}{8}$ as a decimal.

Solution:

-0.875

Note:
Exercise:

TRY IT :: 1.206

Problem: Write $-\frac{3}{8}$ as a decimal.

Solution:

-0.375

When we divide, we will not always get a zero remainder. Sometimes the quotient ends up with a decimal that repeats. A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly. A bar is placed over the repeating block of digits to indicate it repeats.

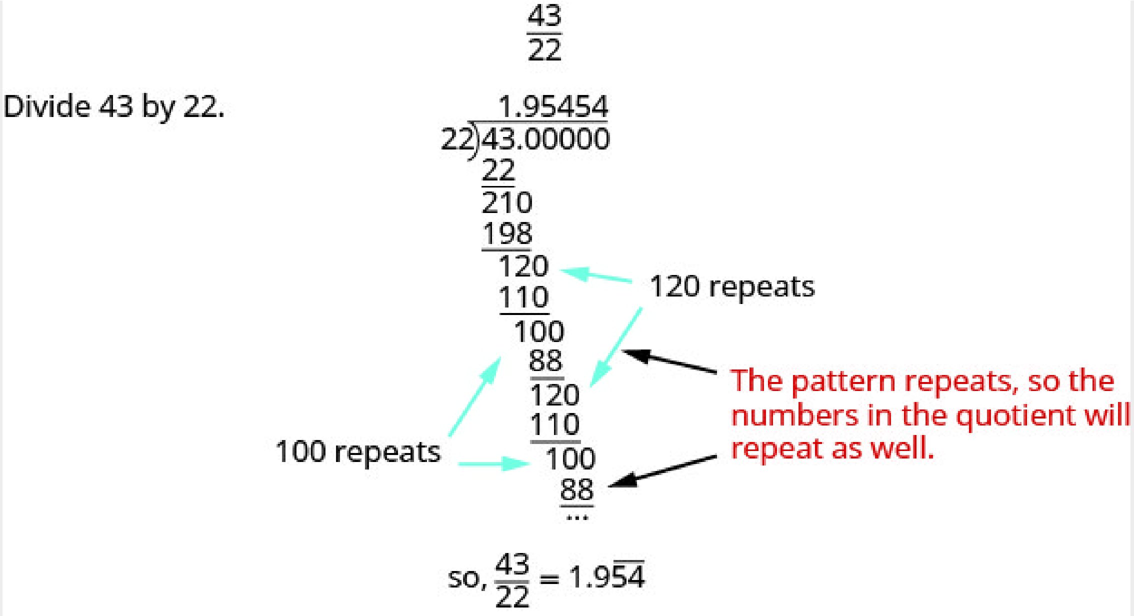
Note:
Repeating Decimal
A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly.

A bar is placed over the repeating block of digits to indicate it repeats.

Example:
Exercise:

Problem: **Write $\frac{43}{22}$ as a decimal.

Solution:
Solution



Note:
Exercise:

TRY IT :: 1.207

Problem: Write $\frac{27}{11}$ as a decimal.

Solution:

$2.\overline{45}$

Note:
Exercise:

TRY IT :: 1.208

Problem: Write $\frac{51}{22}$ as a decimal.

Solution:

$2.3\overline{18}$

Sometimes we may have to simplify expressions with fractions and decimals together.

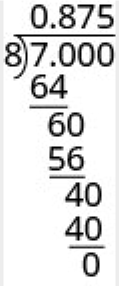
Example:
Exercise:

Problem: **Simplify: $\frac{7}{8} + 6.4$.

Solution:
Solution

First we must change one number so both numbers are in the same form. We can change the fraction to a decimal, or change the decimal to a fraction. Usually it is easier to change the fraction to a decimal.

		$\frac{7}{8} + 6.4$
Change $\frac{7}{8}$ to a decimal.		

		
Add.		$0.875 + 6.4$
		7.275
		So, $\frac{7}{8} + 6.4 = 7.275$

Note:
Exercise:

TRY IT :: 1.209

Problem: Simplify: $\frac{3}{8} + 4.9$.

Solution:

5.275

Note:
Exercise:

TRY IT :: 1.210

Problem: Simplify: $5.7 + \frac{13}{20}$.

Solution:

6.35

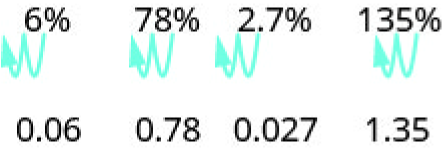
A **percent** is a ratio whose denominator is 100. Percent means per hundred. We use the percent symbol, %, to show percent.

Note:
Percent
A **percent** is a ratio whose denominator is 100.

Since a percent is a ratio, it can easily be expressed as a fraction. Percent means per 100, so the denominator of the fraction is 100. We then change the fraction to a decimal by dividing the numerator by the denominator.

	6%	78%	135%
Write as a ratio with denominator 100.	$\frac{6}{100}$	$\frac{78}{100}$	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06	0.78	1.35

Do you see the pattern? *To convert a percent number to a decimal number, we move the decimal point two places to the left.*



Example:
Exercise:

Problem:

**Convert each percent to a decimal: ⓐ 62% ⓑ 135% ⓒ 5.7%.

Solution:
Solution

ⓐ	
Move the decimal point two places to the left.	0.62

ⓑ	

	135%
Move the decimal point two places to the left.	1.35

ⓒ	
	5.7%
Move the decimal point two places to the left.	0.057

Note:
Exercise:

Problem:

TRY IT :: 1.211
Convert each percent to a decimal: ⓐ 9% ⓑ 87% ⓒ 3.9%.

Solution:

ⓐ 0.09 ⓑ 0.87 ⓒ 0.039

Note:

Exercise:

TRY IT :: 1.212

Problem: Convert each percent to a decimal: ⓐ 3% ⓑ 91% ⓒ 8.3%.

Solution:

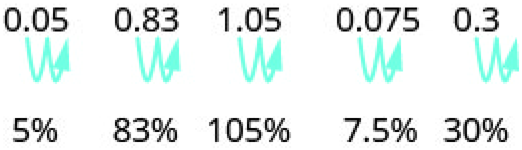
ⓐ 0.03 ⓑ 0.91 ⓒ 0.083

Converting a decimal to a percent makes sense if we remember the definition of percent and keep place value in mind.

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

	0.83	1.05	0.075
Write as a fraction.	$\frac{83}{100}$	$1\frac{5}{100}$	$\frac{75}{1000}$
The denominator is 100.		$\frac{105}{100}$	$\frac{7.5}{100}$
Write the ratio as a percent.	83%	105%	7.5%

Recognize the pattern? *To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.*



Example:
Exercise:

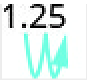
Problem:

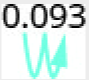
**Convert each decimal to a percent: ⓐ 0.51 ⓑ 1.25 ⓒ 0.093.

Solution:
Solution

ⓐ	
Move the decimal point two places to the right.	51%

ⓑ	

	
Move the decimal point two places to the right.	125%

ⓒ	
	
Move the decimal point two places to the right.	9.3%

Note:
Exercise:

Problem:

TRY IT :: 1.213
Convert each decimal to a percent: ⓐ 0.17 ⓑ 1.75 ⓒ 0.0825.

Solution:

ⓐ 17% ⓑ 175% ⓒ 8.25%

Note:

Exercise:

Problem:

TRY IT :: 1.214
Convert each decimal to a percent: ⓐ 0.41 ⓑ 2.25 ⓒ 0.0925.

Solution:

ⓐ 41% ⓑ 225% ⓒ 9.25%

Key Concepts

- **Name a Decimal**

  Name the number to the left of the decimal point.
  Write ”and” for the decimal point.
  Name the “number” part to the right of the decimal point as if it were a whole number.
  Name the decimal place of the last digit.

- **Write a Decimal**

  Look for the word ‘and’—it locates the decimal point. Place a decimal point under the word ‘and.’ Translate the words before ‘and’ into the whole number and place it to the left of the decimal point. If there is no “and,” write a “0” with a decimal point to its right.
  Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
  Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
  Fill in zeros for place holders as needed.

- **Round a Decimal**

Locate the given place value and mark it with an arrow.
Underline the digit to the right of the place value.
Is this digit greater than or equal to 5? Yes—add 1 to the digit in the given place value. No—do not change the digit in the given place value.
Rewrite the number, deleting all digits to the right of the rounding digit.

- **Add or Subtract Decimals**

Write the numbers so the decimal points line up vertically.
Use zeros as place holders, as needed.
Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

- **Multiply Decimals**

Determine the sign of the product.
Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
Place the decimal point. The number of decimal places in the product is the sum of the decimal places in the factors.
Write the product with the appropriate sign.

- **Multiply a Decimal by a Power of Ten**

Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
Add zeros at the end of the number as needed.

- **Divide Decimals**

Determine the sign of the quotient.
Make the divisor a whole number by “moving” the decimal point all the way to the right. “Move” the decimal point in the dividend the same number of places - adding zeros as needed.

Divide. Place the decimal point in the quotient above the decimal point in the dividend.
Write the quotient with the appropriate sign.

- **Convert a Decimal to a Proper Fraction**

  Determine the place value of the final digit.
  Write the fraction: numerator—the ‘numbers’ to the right of the decimal point; denominator—the place value corresponding to the final digit.

- **Convert a Fraction to a Decimal** Divide the numerator of the fraction by the denominator.

Practice Makes Perfect

Name and Write Decimals

In the following exercises, write as a decimal.
Exercise:

Problem: Twenty-nine and eighty-one hundredths

Solution:

29.81

Exercise:

Problem: Sixty-one and seventy-four hundredths

Exercise:

Problem: Seven tenths

Solution:

0.7

Exercise:

Problem: Six tenths

Exercise:

Problem: Twenty-nine thousandth

Solution:

0.029

Exercise:

Problem: Thirty-five thousandths

Exercise:

Problem: Negative eleven and nine ten-thousandths

Solution:

-11.0009

Exercise:

Problem: Negative fifty-nine and two ten-thousandths

In the following exercises, name each decimal.

Exercise:

Problem: 5.5

Solution:

five and five tenths

Exercise:

Problem: 14.02

Exercise:

Problem: 8.71

Solution:

eight and seventy-one hundredths

Exercise:

Problem: 2.64

Exercise:

Problem: 0.002

Solution:

two thousandths

Exercise:

Problem: 0.479

Exercise:

Problem: -17.9

Solution:

negative seventeen and nine tenths

Exercise:

Problem: -31.4

Round Decimals

In the following exercises, round each number to the nearest tenth.

Exercise:

Problem: 0.67

Solution:

0.7

Exercise:

Problem: 0.49

Exercise:

Problem: 2.84

Solution:

2.8

Exercise:

Problem: 4.63

In the following exercises, round each number to the nearest hundredth.

Exercise:

Problem: 0.845

Solution:

0.85

Exercise:

Problem: 0.761

Exercise:

Problem: 0.299

Solution:

0.30

Exercise:

Problem: 0.697

Exercise:

Problem: 4.098

Solution:

4.10

Exercise:

Problem: 7.096

In the following exercises, round each number to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Exercise:

Problem: 5.781

Solution:

ⓐ 5.78 ⓑ 5.8 ⓒ 6

Exercise:

Problem: 1.6381

Exercise:

Problem: 63.479

Solution:

ⓐ 63.48 ⓑ 63.5 ⓒ 63

Exercise:

Problem: 84.281

Add and Subtract Decimals

In the following exercises, add or subtract.

Exercise:

Problem: $16.92 + 7.56$

Solution:

24.48

Exercise:

Problem: $248.25 - 91.29$

Exercise:

Problem: $21.76 - 30.99$

Solution:

-9.23

Exercise:

Problem: $38.6 + 13.67$

Exercise:

Problem: $-16.53 - 24.38$

Solution:

-40.91

Exercise:

Problem: $-19.47 - 32.58$

Exercise:

Problem: $-38.69 + 31.47$

Solution:

-7.22

Exercise:

Problem: $29.83 + 19.76$

Exercise:

Problem: $72.5 - 100$

Solution:

-27.5

Exercise:

Problem: $86.2 - 100$

Exercise:

Problem: $15 + 0.73$

Solution:

15.73

Exercise:

Problem: $27 + 0.87$

Exercise:

Problem: $91.95 - (-10.462)$

Solution:

102.212

Exercise:

Problem: $94.69 - (-12.678)$

Exercise:

Problem: $55.01 - 3.7$

Solution:

51.31

Exercise:

Problem: $59.08 - 4.6$

Exercise:

Problem: $2.51-7.4$

Solution:

-4.89

Exercise:

Problem: $3.84-6.1$

Multiply and Divide Decimals

In the following exercises, multiply.

Exercise:

Problem: $(0.24)\,(0.6)$

Solution:

0.144

Exercise:

Problem: $(0.81)\,(0.3)$

Exercise:

Problem: $(5.9)\,(7.12)$

Solution:

42.008

Exercise:

Problem: $(2.3)\,(9.41)$

Exercise:

Problem: $(-4.3)\,(2.71)$

Solution:

-11.653

Exercise:

Problem: $(-8.5)\,(1.69)$

Exercise:

Problem: $(-5.18)\,(-65.23)$

Solution:

337.8914

Exercise:

Problem: $(-9.16)\,(-68.34)$

Exercise:

Problem: $(0.06)\,(21.75)$

Solution:

1.305

Exercise:

Problem: $(0.08)\,(52.45)$

Exercise:

Problem: $(9.24)(10)$

Solution:

92.4

Exercise:

Problem: $(6.531)(10)$

Exercise:

Problem: $(55.2)(1000)$

Solution:

55,200

Exercise:

Problem: $(99.4)(1000)$

In the following exercises, divide.

Exercise:

Problem: $4.75 \div 25$

Solution:

0.19

Exercise:

Problem: $12.04 \div 43$

Exercise:

Problem: $\$117.25 \div 48$

Solution:

$\$2.44$

Exercise:

Problem: $\$109.24 \div 36$

Exercise:

Problem: $0.6 \div 0.2$

Solution:

3

Exercise:

Problem: $0.8 \div 0.4$

Exercise:

Problem: $1.44 \div (-0.3)$

Solution:

-4.8

Exercise:

Problem: $1.25 \div (-0.5)$

Exercise:

Problem: $-1.75 \div (-0.05)$

Solution:

35

Exercise:

Problem: $-1.15 \div (-0.05)$

Exercise:

Problem: $5.2 \div 2.5$

Solution:

2.08

Exercise:

Problem: $6.5 \div 3.25$

Exercise:

Problem: $11 \div 0.55$

Solution:

20

Exercise:

Problem: $14 \div 0.35$

Convert Decimals, Fractions and Percents

In the following exercises, write each decimal as a fraction.

Exercise:

Problem: 0.04

Solution:

$\frac{1}{25}$

Exercise:

Problem: 0.19

Exercise:

Problem: 0.52

Solution:

$\frac{13}{25}$

Exercise:

Problem: 0.78

Exercise:

Problem: 1.25

Solution:

$\frac{5}{4}$

Exercise:

Problem: 1.35

Exercise:

Problem: 0.375

Solution:

$\frac{3}{8}$

Exercise:

Problem: 0.464

Exercise:

Problem: 0.095

Solution:

$\frac{19}{200}$

Exercise:

Problem: 0.085

In the following exercises, convert each fraction to a decimal.

Exercise:

Problem: $\frac{17}{20}$

Solution:

0.85

Exercise:

Problem: $\frac{13}{20}$

Exercise:

Problem: $\frac{11}{4}$

Solution:

2.75

Exercise:

Problem: $\frac{17}{4}$

Exercise:

Problem: $-\frac{310}{25}$

Solution:

-12.4

Exercise:

Problem: $-\frac{284}{25}$

Exercise:

Problem: $\frac{15}{11}$

Solution:

$1.\overline{36}$

Exercise:

Problem: $\frac{18}{11}$

Exercise:

Problem: $\frac{15}{111}$

Solution:

$0.1\overline{35}$

Exercise:

Problem: $\frac{25}{111}$

Exercise:

Problem: $2.4+\frac{5}{8}$

Solution:

3.025

Exercise:

Problem: $3.9+\frac{9}{20}$

In the following exercises, convert each percent to a decimal.

Exercise:

Problem: 1%

Solution:

0.011

Exercise:

Problem: 2%

Exercise:

Problem: 63%

Solution:

0.63

Exercise:

Problem: 71%

Exercise:

Problem: 150%

Solution:

1.5

Exercise:

Problem: 250%

Exercise:

Problem: 21.4%

Solution:

0.214

Exercise:

Problem: 39.3%

Exercise:

Problem: 7.8%

Solution:

0.078

Exercise:

Problem: 6.4%

In the following exercises, convert each decimal to a percent.

Exercise:

Problem: 0.01

Solution:

1%

Exercise:

Problem: 0.03

Exercise:

Problem: 1.35

Solution:

135%

Exercise:

Problem: 1.56

Exercise:

Problem: 3

Solution:

300%

Exercise:

Problem: 4

Exercise:

Problem: 0.0875

Solution:

8.75%

Exercise:

Problem: 0.0625

Exercise:

Problem: 2.254

Solution:

225.4%

Exercise:

Problem: 2.317

Everyday Math

Exercise:

Problem:

Salary Increase Danny got a raise and now makes \$58,965.95 a year. Round this number to the nearest
ⓐ dollar
ⓑ thousand dollars
ⓒ ten thousand dollars.

Solution:

ⓐ $58,966 ⓑ $59,000 ⓒ $60,000

Exercise:

Problem:

New Car Purchase Selena’s new car cost $23,795.95. Round this number to the nearest
ⓐ dollar
ⓑ thousand dollars
ⓒ ten thousand dollars.

Exercise:

Problem:

Sales Tax Hyo Jin lives in San Diego. She bought a refrigerator for $1,624.99 and when the clerk calculated the sales tax it came out to exactly $142.186625. Round the sales tax to the nearest
ⓐ penny and
ⓑ dollar.

Solution:

ⓐ $142.19; ⓑ $142

Exercise:

Problem:

Sales Tax Jennifer bought a $1,038.99 dining room set for her home in Cincinnati. She calculated the sales tax to be exactly $67.53435. Round the sales tax to the nearest
ⓐ penny and
ⓑ dollar.

Exercise:

Problem:

Paycheck Annie has two jobs. She gets paid \$14.04 per hour for tutoring at City College and \$8.75 per hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours.
ⓐ How much did she earn?
ⓑ If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?

Solution:

ⓐ \$243.57 ⓑ \$79.35

Exercise:

Problem:

Paycheck Jake has two jobs. He gets paid \$7.95 per hour at the college cafeteria and \$20.25 at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery.
ⓐ How much did he earn?
ⓑ If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Writing Exercises

Exercise:

Problem:

How does knowing about US money help you learn about decimals?

Solution:

Answers may vary

Exercise:

Problem:

Explain how you write “three and nine hundredths” as a decimal.

Exercise:

Problem:

Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

Solution:

Answers may vary

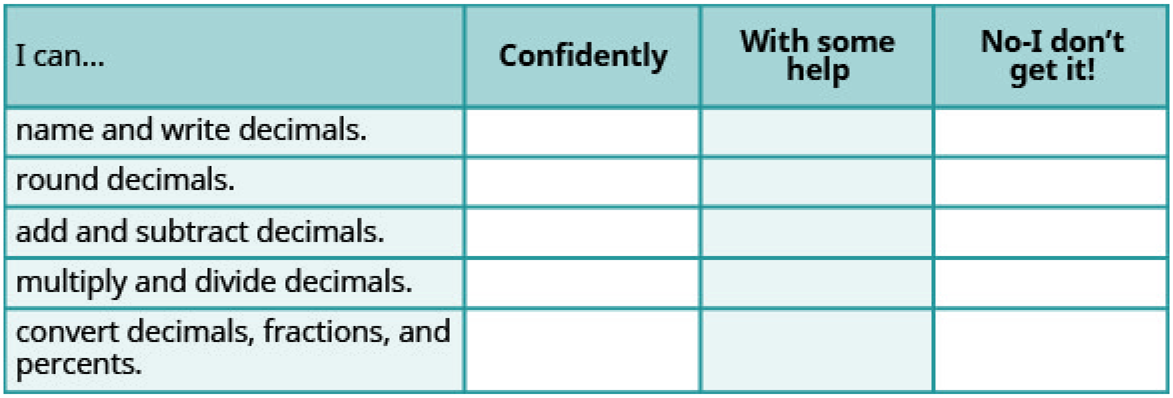
Exercise:

Problem:

When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don’t get it!
name and write decimals.			
round decimals.			
add and subtract decimals.			
multiply and divide decimals.			
convert decimals, fractions, and percents.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

decimal

A decimal is another way of writing a fraction whose denominator is a power of ten.

percent

A percent is a ratio whose denominator is 100.

repeating decimal

A repeating decimal is a decimal in which the last digit or group of digits repeats endlessly.

The Real Numbers
By the end of this section, you will be able to:

- Simplify expressions with square roots
- Identify integers, rational numbers, irrational numbers, and real numbers
- Locate fractions on the number line
- Locate decimals on the number line

Simplify Expressions with Square Roots

Remember that when a number *n* is multiplied by itself, we write n^2 and read it “n squared.” The result is called the **square** of *n*. For example,

Equation:

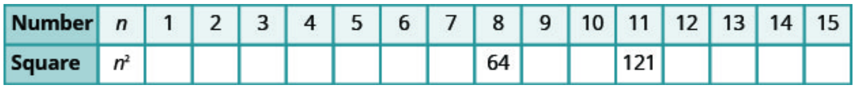
$$\begin{array}{ll} 8^2 & \text{read ‘8 squared’} \\ 64 & 64 \text{ is called the } \textit{square} \text{ of } 8. \end{array}$$

Similarly, 121 is the square of 11, because 11^2 is 121.

Note:
Square of a Number
If $n^2 = m$, then *m* is the **square** of *n*.

Note: Doing the Manipulative Mathematics activity “Square Numbers” will help you develop a better understanding of perfect square numbers.

Complete the following table to show the squares of the counting numbers 1 through 15.



Number	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	n^2								64			121				

The numbers in the second row are called perfect square numbers. It will be helpful to learn to recognize the perfect square numbers.

The squares of the counting numbers are positive numbers. What about the squares of negative numbers? We know that when the signs of two numbers are the same, their product is positive. So the square of any negative number is also positive.

Equation:

$$(-3)^2 = 9 \qquad (-8)^2 = 64 \qquad (-11)^2 = 121 \qquad (-15)^2 = 225$$

Did you notice that these squares are the same as the squares of the positive numbers?

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We also say that 10 is a *square root* of 100. A number whose square is m is called a **square root** of *m*.

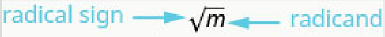
Note:
Square Root of a Number
If $n^2 = m$, then *n* is a **square root** of *m*.

Notice $(-10)^2 = 100$ also, so -10 is also a square root of 100. Therefore, both 10 and -10 are square roots of 100.

So, every positive number has two square roots—one positive and one negative. What if we only wanted the positive square root of a positive number? The radical sign, $\sqrt{m}$, denotes the positive square root. The positive square root is called the principal square root. When we use the radical sign that always means we want the principal square root.

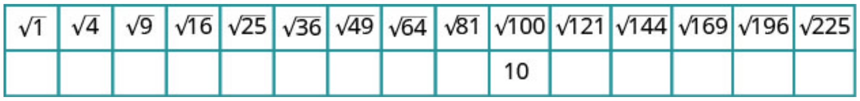
We also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

Note:
Square Root Notation
$\sqrt{m}$ is read “the square root of *m*”



If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.
The square root of *m*, $\sqrt{m}$, is the positive number whose square is *m*.

Since 10 is the principal square root of 100, we write $\sqrt{100} = 10$. You may want to complete the following table to help you recognize square roots.



$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
									10					

Example:
Exercise:

Problem: **Simplify: ⓐ $\sqrt{25}$ ⓑ $\sqrt{121}$.

Solution:
Solution

ⓐ Since $5^2 = 25$	$\sqrt{25}$ 5

ⓑ Since $11^2 = 121$	$\sqrt{121}$ 11

Note:
Exercise:

TRY IT :: 1.215
Problem: Simplify: ⓐ $\sqrt{36}$ ⓑ $\sqrt{169}$.

Solution:

ⓐ 6 ⓑ 13

Note:
Exercise:

TRY IT :: 1.216
Problem: Simplify: ⓐ $\sqrt{16}$ ⓑ $\sqrt{196}$.

Solution:

ⓐ 4 ⓑ 14

We know that every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{100} = 10$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{100} = -10$. We read $-\sqrt{100}$ as “the opposite of the square root of 10.”

Example:
Exercise:

Problem: **Simplify: ⓐ $-\sqrt{9}$ ⓑ $-\sqrt{144}$.

Solution:
Solution

ⓐ The negative is in front of the radical sign.	$-\sqrt{9}$ -3

ⓑ The negative is in front of the radical sign.	$-\sqrt{144}$ -12

Note:
Exercise:

TRY IT :: 1.217

Problem: Simplify: ⓐ $-\sqrt{4}$ ⓑ $-\sqrt{225}$.

Solution:

ⓐ -2 ⓑ -15

Note:
Exercise:

TRY IT :: 1.218

Problem: Simplify: ⓐ $-\sqrt{81}$ ⓑ $-\sqrt{100}$.

Solution:

ⓐ -9 ⓑ -10

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

We have already described numbers as *counting numbers*, *whole numbers*, and *integers*. What is the difference between these types of numbers?
Equation:

$$\begin{array}{ll} \text{Counting numbers} & 1, 2, 3, 4, \ldots \\ \text{Whole numbers} & 0, 1, 2, 3, 4, \ldots \\ \text{Integers} & \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \end{array}$$

What type of numbers would we get if we started with all the integers and then included all the fractions? The numbers we would have form the set of rational numbers. A **rational number** is a number that can be written as a ratio of two integers.

Note:
Rational Number
A **rational number** is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
A rational number can be written as the ratio of two integers.

All signed fractions, such as $\frac{4}{5}, -\frac{7}{8}, \frac{13}{4}, -\frac{20}{3}$ are rational numbers. Each numerator and each denominator is an integer.

Are integers rational numbers? To decide if an integer is a rational number, we try to write it as a ratio of two integers. Each integer can be written as a ratio of integers in many ways. For example, 3 is equivalent to $\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5} \ldots$

An easy way to write an integer as a ratio of integers is to write it as a fraction with denominator one.
Equation:

$$3 = \frac{3}{1} \qquad -8 = -\frac{8}{1} \qquad 0 = \frac{0}{1}$$

Since any integer can be written as the ratio of two integers, *all integers are rational numbers*! Remember that the counting numbers and the whole numbers are also integers, and so they, too, are rational.

What about decimals? Are they rational? Let’s look at a few to see if we can write each of them as the ratio of two integers.

We’ve already seen that integers are rational numbers. The integer -8 could be written as the decimal -8.0. So, clearly, some decimals are rational.

Think about the decimal 7.3. Can we write it as a ratio of two integers? Because 7.3 means $7\frac{3}{10}$, we can write it as an improper fraction, $\frac{73}{10}$. So 7.3 is the ratio of the integers 73 and 10. It is a rational number.

In general, any decimal that ends after a number of digits (such as 7.3 or -1.2684) is a rational number. We can use the place value of the last digit as the denominator when writing the decimal as a fraction.

Example:
Exercise:

Problem: **Write as the ratio of two integers: ⓐ -27 ⓑ 7.31.

Solution:
Solution

ⓐ Write it as a fraction with denominator 1.	-27 $\frac{-27}{1}$
ⓑ Write it as a mixed number. Remember, 7 is the whole number and the decimal part, 0.31, indicates hundredths. Convert to an improper fraction.	7.31 $7\frac{31}{100}$ $\frac{731}{100}$

So we see that -27 and 7.31 are both rational numbers, since they can be written as the ratio of two integers.

Note:
Exercise:

TRY IT :: 1.219
Problem: Write as the ratio of two integers: ⓐ -24 ⓑ 3.57.

Solution:

ⓐ $\frac{-24}{1}$ ⓑ $\frac{357}{100}$

Note:
Exercise:

TRY IT :: 1.220
Problem: Write as the ratio of two integers: ⓐ -19 ⓑ 8.41.

Solution:

ⓐ $\frac{-19}{1}$ ⓑ $\frac{841}{100}$

Let's look at the decimal form of the numbers we know are rational.

We have seen that *every integer is a rational number*, since $a = \frac{a}{1}$ for any integer, a. We can also change any integer to a decimal by adding a decimal point and a zero.

Equation:

Integer	-2	-1	0	1	2	3
Decimal form	-2.0	-1.0	0.0	1.0	2.0	3.0

Equation:

$$\text{These decimal numbers stop.}$$

We have also seen that *every fraction is a rational number*. Look at the decimal form of the fractions we considered above.

Equation:

Ratio of integers	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$
The decimal form	0.8	-0.875	3.25	$-6.666\ldots$ $-6.\overline{6}$

Equation:

These decimals either stop or repeat.

What do these examples tell us?

Every rational number can be written both as a ratio of integers, ($\frac{p}{q}$, *where p and q are integers and* $q \neq 0$), *and as a decimal that either stops or repeats.*

Here are the numbers we looked at above expressed as a ratio of integers and as a decimal:

Fractions					**Integers**					
Number	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$	-2	-1	0	1	2	3
Ratio of Integers	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$	$-\frac{2}{1}$	$-\frac{1}{1}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$
Decimal Form	0.8	-0.875	3.25	$-6.\overline{6}$	-2.0	-1.0	0.0	1.0	2.0	3.0

Note:
Rational Number
A **rational number** is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Its decimal form stops or repeats.

Are there any decimals that do not stop or repeat? Yes!

The number π (the Greek letter *pi*, pronounced “pie”), which is very important in describing circles, has a decimal form that does not stop or repeat.
Equation:

$$\pi = 3.141592654...$$

We can even create a decimal pattern that does not stop or repeat, such as
Equation:

$$2.01001000100001...$$

Numbers whose decimal form does not stop or repeat cannot be written as a fraction of integers. We call these numbers irrational.

Note:
Irrational Number
An **irrational number** is a number that cannot be written as the ratio of two integers.

Its decimal form does not stop and does not repeat.

Let’s summarize a method we can use to determine whether a number is rational or irrational.

Note:
Rational or Irrational?
If the decimal form of a number

- *repeats or stops*, the number is **rational**.
- *does not repeat and does not stop*, the number is **irrational**.

Example:
Exercise:

Problem:

**Given the numbers $0.58\overline{3}, 0.47, 3.605551275\ldots$ list the ⓐ rational numbers ⓑ irrational numbers.

Solution:
Solution

ⓐ Look for decimals that repeat or stop.	The 3 repeats in $0.58\overline{3}$. The decimal 0.47 stops after the 7. So $0.58\overline{3}$ and 0.47 are rational.
ⓑ Look for decimals that neither stop nor repeat.	$3.605551275\ldots$ has no repeating block of digits and it does not stop. So $3.605551275\ldots$ is irrational.

Note:
Exercise:

Problem:

TRY IT :: 1.221
For the given numbers list the ⓐ rational numbers ⓑ irrational numbers: $0.29, 0.81\overline{6}, 2.515115111\ldots$.

Solution:

ⓐ $0.29, 0.81\overline{6}$ ⓑ $2.515115111\ldots$

Note:
Exercise:

Problem:

TRY IT :: 1.222
For the given numbers list the ⓐ rational numbers ⓑ irrational numbers: $2.6\bar{3}, 0.125, 0.418302...$

Solution:

ⓐ $2.6\bar{3}, 0.125$ ⓑ $0.418302...$

Example:
Exercise:

Problem: **For each number given, identify whether it is rational or irrational: ⓐ $\sqrt{36}$ ⓑ $\sqrt{44}$.

Solution:
Solution

ⓐ Recognize that 36 is a perfect square, since $6^2 = 36$. So $\sqrt{36} = 6$, therefore $\sqrt{36}$ is rational.
ⓑ Remember that $6^2 = 36$ and $7^2 = 49$, so 44 is not a perfect square. Therefore, the decimal form of $\sqrt{44}$ will never repeat and never stop, so $\sqrt{44}$ is irrational.

Note:
Exercise:

TRY IT :: 1.223
Problem: For each number given, identify whether it is rational or irrational: ⓐ $\sqrt{81}$ ⓑ $\sqrt{17}$.

Solution:

ⓐ rational ⓑ irrational

Note:
Exercise:

TRY IT :: 1.224
Problem: For each number given, identify whether it is rational or irrational: ⓐ $\sqrt{116}$ ⓑ $\sqrt{121}$.

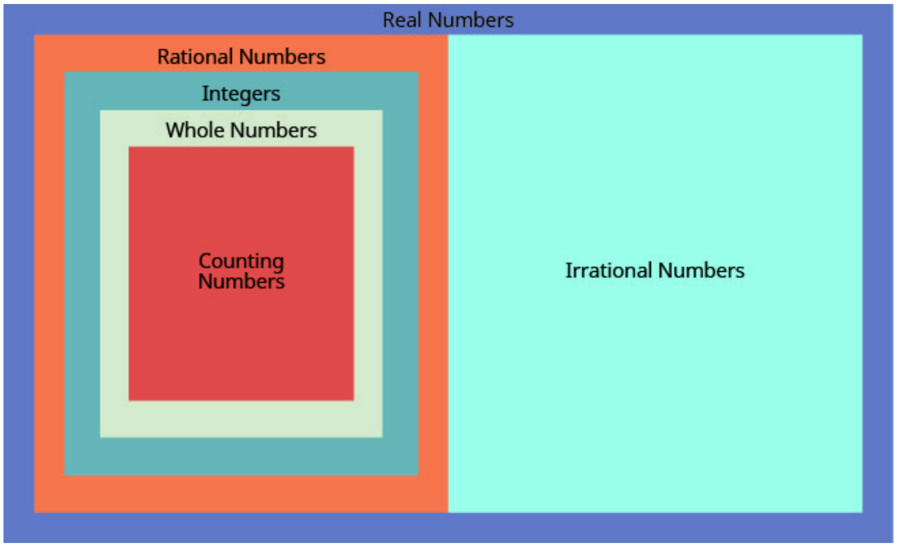
Solution:

ⓐ irrational ⓑ rational

We have seen that all counting numbers are whole numbers, all whole numbers are integers, and all integers are rational numbers. The irrational numbers are numbers whose decimal form does not stop and does not repeat. When we put together the rational numbers and the irrational numbers, we get the set of **real numbers**.

Note:
Real Number
A **real number** is a number that is either rational or irrational.

All the numbers we use in elementary algebra are real numbers. [link] illustrates how the number sets we've discussed in this section fit together.



This chart shows the number sets that make up the set of real numbers. Does the term "real numbers" seem strange to you? Are there any numbers that are not "real," and, if so, what could they be?

Can we simplify $\sqrt{-25}$? Is there a number whose square is -25?
Equation:

$$(\quad)^2 = -25?$$

None of the numbers that we have dealt with so far has a square that is -25. Why? Any positive number squared is positive. Any negative number squared is positive. So we say there is no real number equal to $\sqrt{-25}$.

The square root of a negative number is not a real number.

Example:
Exercise:

Problem:

**For each number given, identify whether it is a real number or not a real number: ⓐ $\sqrt{-169}$ ⓑ $-\sqrt{64}$.

Solution:
Solution

ⓐ There is no real number whose square is -169. Therefore, $\sqrt{-169}$ is not a real number.
ⓑ Since the negative is in front of the radical, $-\sqrt{64}$ is -8, Since -8 is a real number, $-\sqrt{64}$ is a real number.

Note:
Exercise:

Problem:

TRY IT :: 1.225
For each number given, identify whether it is a real number or not a real number: ⓐ $\sqrt{-196}$ ⓑ $-\sqrt{81}$.

Solution:

ⓐ not a real number ⓑ real number

Note:
Exercise:

Problem:

TRY IT :: 1.226
For each number given, identify whether it is a real number or not a real number: ⓐ $-\sqrt{49}$ ⓑ $\sqrt{-121}$.

Solution:

ⓐ real number ⓑ not a real number

Example:
Exercise:

Problem:

**Given the numbers $-7, \frac{14}{5}, 8, \sqrt{5}, 5.9, -\sqrt{64}$, list the ⓐ whole numbers ⓑ integers ⓒ rational numbers ⓓ irrational numbers ⓔ real numbers.

Solution:
Solution

ⓐRemember, the whole numbers are $0, 1, 2, 3, \ldots$ and 8 is the only whole number given.

ⓑ The integers are the whole numbers, their opposites, and 0. So the whole number 8 is an integer, and -7 is the opposite of a whole number so it is an integer, too. Also, notice that 64 is the square of 8 so $-\sqrt{64} = -8$. So the integers are $-7, 8, -\sqrt{64}$.
ⓒ Since all integers are rational, then $-7, 8, -\sqrt{64}$ are rational. Rational numbers also include fractions and decimals that repeat or stop, so $\frac{14}{5}$ and 5.9 are rational. So the list of rational numbers is $-7, \frac{14}{5}, 8, 5.9, -\sqrt{64}$.
ⓓ Remember that 5 is not a perfect square, so $\sqrt{5}$ is irrational.
ⓔ All the numbers listed are real numbers.

Note:
Exercise:

Problem:

TRY IT :: 1.227
For the given numbers, list the ⓐ whole numbers ⓑ integers ⓒ rational numbers ⓓ irrational numbers ⓔ real numbers: $-3, -\sqrt{2}, 0.\bar{3}, \frac{9}{5}, 4, \sqrt{49}$.

Solution:

ⓐ $4, \sqrt{49}$ ⓑ $-3, 4, \sqrt{49}$ ⓒ $-3, 0.\bar{3}, \frac{9}{5}, 4, \sqrt{49}$ ⓓ $-\sqrt{2}$ ⓔ $-3, -\sqrt{2}, 0.\bar{3}, \frac{9}{5}, 4, \sqrt{49}$

Note:
Exercise:

Problem:

TRY IT :: 1.228
For the given numbers, list the ⓐ whole numbers ⓑ integers ⓒ rational numbers ⓓ irrational numbers ⓔ real numbers: $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}, 2.041975...$

Solution:

ⓐ $6, \sqrt{121}$ ⓑ $-\sqrt{25}, -1, 6, \sqrt{121}$ ⓒ $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}$ ⓓ $2.041975...$ ⓔ $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}, 2.041975...$

Locate Fractions on the Number Line

The last time we looked at the number line, it only had positive and negative integers on it. We now want to include fractions and decimals on it.

Note: Doing the Manipulative Mathematics activity “Number Line Part 3” will help you develop a better understanding of the location of fractions on the number line.

Let's start with fractions and locate $\frac{1}{5}, -\frac{4}{5}, 3, \frac{7}{4}, -\frac{9}{2}, -5,$ and $\frac{8}{3}$ on the number line.

We'll start with the whole numbers 3 and $-5.$ because they are the easiest to plot. See [link].

The proper fractions listed are $\frac{1}{5}$ and $-\frac{4}{5}$. We know the proper fraction $\frac{1}{5}$ has value less than one and so would be located between 0 and 1. The denominator is 5, so we divide the unit from 0 to 1 into 5 equal parts $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$. We plot $\frac{1}{5}$. See [link].

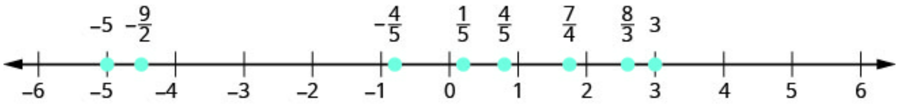
Similarly, $-\frac{4}{5}$ is between 0 and -1. After dividing the unit into 5 equal parts we plot $-\frac{4}{5}$. See [link].

Finally, look at the improper fractions $\frac{7}{4}, -\frac{9}{2}, \frac{8}{3}$. These are fractions in which the numerator is greater than the denominator. Locating these points may be easier if you change each of them to a mixed number. See [link].

Equation:

$$\frac{7}{4} = 1\frac{3}{4} \qquad -\frac{9}{2} = -4\frac{1}{2} \qquad \frac{8}{3} = 2\frac{2}{3}$$

[link] shows the number line with all the points plotted.



Example:
Exercise:

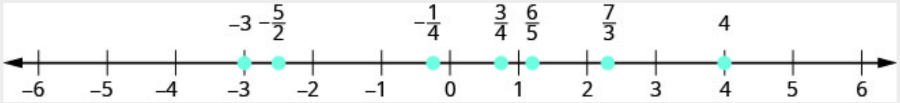
Problem: **Locate and label the following on a number line: $4, \frac{3}{4}, -\frac{1}{4}, -3, \frac{6}{5}, -\frac{5}{2},$ and $\frac{7}{3}$.

Solution:
Solution

Locate and plot the integers, $4, -3$.

Locate the proper fraction $\frac{3}{4}$ first. The fraction $\frac{3}{4}$ is between 0 and 1. Divide the distance between 0 and 1 into four equal parts then, we plot $\frac{3}{4}$. Similarly plot $-\frac{1}{4}$.

Now locate the improper fractions $\frac{6}{5}, -\frac{5}{2}, \frac{7}{3}$. It is easier to plot them if we convert them to mixed numbers and then plot them as described above: $\frac{6}{5} = 1\frac{1}{5}, -\frac{5}{2} = -2\frac{1}{2}, \frac{7}{3} = 2\frac{1}{3}$.

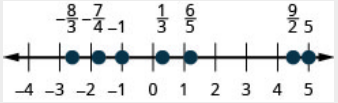


Note:
Exercise:

TRY IT :: 1.229

Problem: Locate and label the following on a number line: $-1, \frac{1}{3}, \frac{6}{5}, -\frac{7}{4}, \frac{9}{2}, 5, -\frac{8}{3}$.

Solution:

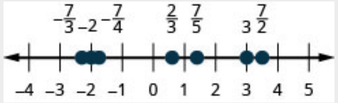


Note:
Exercise:

TRY IT :: 1.230

Problem: Locate and label the following on a number line: $-2, \frac{2}{3}, \frac{7}{5}, -\frac{7}{4}, \frac{7}{2}, 3, -\frac{7}{3}$.

Solution:



In [link], we'll use the inequality symbols to order fractions. In previous chapters we used the number line to order numbers.

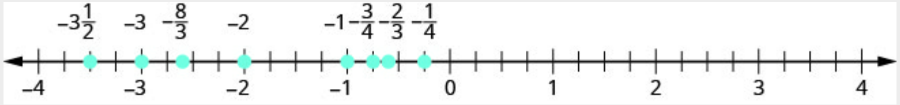
- $a < b$ "*a* is less than *b*" when *a* is to the left of *b* on the number line
- $a > b$ "*a* is greater than *b*" when *a* is to the right of *b* on the number line

As we move from left to right on a number line, the values increase.

Example:
Exercise:

Problem: **Order each of the following pairs of numbers, using < or >. It may be helpful to refer [link].

ⓐ $-\frac{2}{3}$ ___ -1 ⓑ $-3\frac{1}{2}$ ___ -3 ⓒ $-\frac{3}{4}$ ___ $-\frac{1}{4}$ ⓓ -2 ___ $-\frac{8}{3}$



Solution:
Solution

ⓐ $-\frac{2}{3}$ is to the right of -1 on the number line.	$-\frac{2}{3}$ ___ -1 $-\frac{2}{3} > -1$
ⓑ $-3\frac{1}{2}$ is to the right of -3 on the number line.	$-3\frac{1}{2}$ ___ -3 $-3\frac{1}{2} < -3$
ⓒ $-\frac{3}{4}$ is to the right of $-\frac{1}{4}$ on the number line.	$-\frac{3}{4}$ ___ $-\frac{1}{4}$ $-\frac{3}{4} < -\frac{1}{4}$
ⓓ -2 is to the right of $-\frac{8}{3}$ on the number line.	-2 ___ $-\frac{8}{3}$ $-2 > -\frac{8}{3}$

Note:
Exercise:

TRY IT :: 1.231

Problem: Order each of the following pairs of numbers, using < or >:

ⓐ $-\frac{1}{3}$ ___ -1 ⓑ $-1\frac{1}{2}$ ___ -2 ⓒ $-\frac{2}{3}$ ___ $-\frac{1}{3}$ ⓓ -3 ___ $-\frac{7}{3}$.

Solution:

ⓐ > ⓑ > ⓒ < ⓓ <

Note:
Exercise:

TRY IT :: 1.232

Problem: Order each of the following pairs of numbers, using < or >:

ⓐ -1 ___ $-\frac{2}{3}$ ⓑ $-2\frac{1}{4}$ ___ -2 ⓒ $-\frac{3}{5}$ ___ $-\frac{4}{5}$ ⓓ -4 ___ $-\frac{10}{3}$.

Solution:

ⓐ < ⓑ < ⓒ > ⓓ <

Locate Decimals on the Number Line

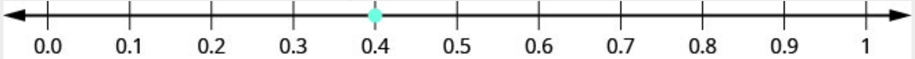
Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

Example:
Exercise:

Problem: **Locate 0.4 on the number line.

Solution:
Solution

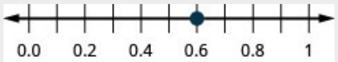
A proper fraction has value less than one. The decimal number 0.4 is equivalent to $\frac{4}{10}$, a proper fraction, so 0.4 is located between 0 and 1. On a number line, divide the interval between 0 and 1 into 10 equal parts. Now label the parts 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. We write 0 as 0.0 and 1 and 1.0, so that the numbers are consistently in tenths. Finally, mark 0.4 on the number line. See [link].



Note:
Exercise:

TRY IT :: 1.233
Problem: Locate on the number line: 0.6.

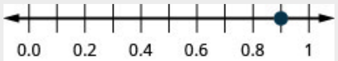
Solution:



Note:
Exercise:

TRY IT :: 1.234
Problem: Locate on the number line: 0.9.

Solution:

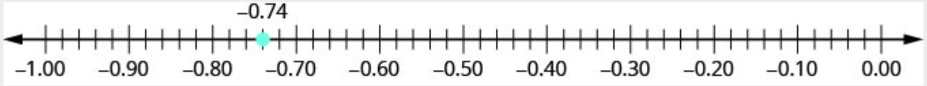


Example:
Exercise:

Problem: **Locate -0.74 on the number line.

Solution:
Solution

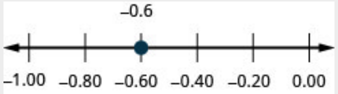
The decimal -0.74 is equivalent to $-\frac{74}{100}$, so it is located between 0 and -1. On a number line, mark off and label the hundredths in the interval between 0 and -1. See [link].



Note:
Exercise:

TRY IT :: 1.235
Problem: Locate on the number line: -0.6.

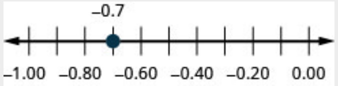
Solution:



Note:
Exercise:

TRY IT :: 1.236
Problem: Locate on the number line: -0.7.

Solution:



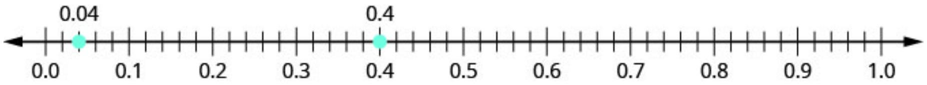
Which is larger, 0.04 or 0.40? If you think of this as money, you know that $0.40 (forty cents) is greater than $0.04 (four cents). So,

$$0.40 > 0.04$$

Again, we can use the number line to order numbers.

- $a < b$ “*a* is less than *b*” when *a* is to the left of *b* on the number line
- $a > b$ “*a* is greater than *b*” when *a* is to the right of *b* on the number line

Where are 0.04 and 0.40 located on the number line? See [link].



We see that 0.40 is to the right of 0.04 on the number line. This is another way to demonstrate that 0.40 > 0.04.

How does 0.31 compare to 0.308? This doesn’t translate into money to make it easy to compare. But if we convert 0.31 and 0.308 into fractions, we can tell which is larger.

	0.31	0.308
Convert to fractions.	$\frac{31}{100}$	$\frac{308}{1000}$
We need a common denominator to compare them.	$\frac{31 \cdot 10}{100 \cdot 10}$	$\frac{308}{1000}$
	$\frac{310}{1000}$	$\frac{308}{1000}$

Because 310 > 308, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, 0.31 > 0.308.

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives 0.310. So 0.31 is equivalent to 0.310. Writing zeros at the end of a decimal does not change its value!
Equation:

$$\frac{31}{100} = \frac{310}{1000} \quad \text{and} \quad 0.31 = 0.310$$

We say 0.31 and 0.310 are **equivalent decimals**.

Note:
Equivalent Decimals
Two decimals are equivalent if they convert to equivalent fractions.

We use equivalent decimals when we order decimals.

The steps we take to order decimals are summarized here.

Note:
Order Decimals.

Write the numbers one under the other, lining up the decimal points.
Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
Compare the numbers as if they were whole numbers.
Order the numbers using the appropriate inequality sign.

Example:
Exercise:

Problem: **Order 0.64 ___ 0.6 using $<$ or $>$.

Solution:
Solution

Write the numbers one under the other, lining up the decimal points.	0.64 $\ 0.6$
Add a zero to 0.6 to make it a decimal with 2 decimal places. Now they are both hundredths.	0.64 0.60
64 is greater than 60.	$64 > 60$
64 hundredths is greater than 60 hundredths.	$0.64 > 0.60$
	$0.64 > 0.6$

Note:
Exercise:

TRY IT :: 1.237
Problem: Order each of the following pairs of numbers, using $<$ or $>$: 0.42 ___ 0.4.

Solution:

$>$

Note:
Exercise:

TRY IT :: 1.238
Problem: Order each of the following pairs of numbers, using $<$ or $>$: 0.18 ___ 0.1.

Solution:

$>$

Example:
Exercise:

Problem: **Order 0.83 ___ 0.803 using $<$ or $>$.

Solution:
Solution

	0.83 ___ 0.803
Write the numbers one under the other, lining up the decimals.	0.83 0.803
They do not have the same number of digits. Write one zero at the end of 0.83.	0.830 0.803
Since $830 > 803$, 830 thousandths is greater than 803 thousandths.	$0.830 > 0.803$
	$0.83 > 0.803$

Note:
Exercise:

TRY IT :: 1.239
Problem: Order the following pair of numbers, using $<$ or $>$: 0.76 ___ 0.706.

Solution:

$>$

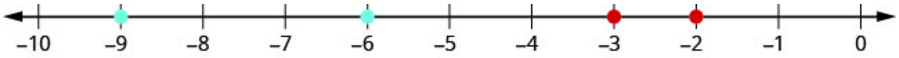
Note:
Exercise:

TRY IT :: 1.240
Problem: Order the following pair of numbers, using $<$ or $>$: 0.305 ___ 0.35.

Solution:

$<$

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because -2 lies to the right of -3 on the number line, we know that $-2 > -3$. Similarly, smaller numbers lie to the left on the number line. For example, because -9 lies to the left of -6 on the number line, we know that $-9 < -6$. See [link].



If we zoomed in on the interval between 0 and -1, as shown in [link], we would see in the same way that $-0.2 > -0.3$ and $-0.9 < -0.6$.

Example:
Exercise:

Problem: **Use $<$ or $>$ to order -0.1 ___ -0.8.

Solution:
Solution

	-0.1 ___ -0.8
Write the numbers one under the other, lining up the decimal points. They have the same number of digits.	-0.1 -0.8
Since $-1 > -8$, −1 tenth is greater than −8 tenths.	$-0.1 > -0.8$

Note:
Exercise:

TRY IT :: 1.241
Problem: Order the following pair of numbers, using < or >: -0.3 ___ -0.5.

Solution:

>

Note:
Exercise:

TRY IT :: 1.242
Problem: Order the following pair of numbers, using < or >: -0.6 ___ -0.7.

Solution:

>

Key Concepts

- **Square Root Notation**
  $\sqrt{m}$ is read 'the square root of m.' If $m=n^2$, then $\sqrt{m}=n$, for $n \geq 0$.
- **Order Decimals**

  Write the numbers one under the other, lining up the decimal points.
  Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
  Compare the numbers as if they were whole numbers.
  Order the numbers using the appropriate inequality sign.

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.
Exercise:

Problem: $\sqrt{36}$

Solution:

6

Exercise:

Problem: $\sqrt{4}$

Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{169}$

Exercise:

Problem: $\sqrt{9}$

Solution:

3

Exercise:

Problem: $\sqrt{16}$

Exercise:

Problem: $\sqrt{100}$

Solution:

10

Exercise:

Problem: $\sqrt{144}$

Exercise:

Problem: $-\sqrt{4}$

Solution:

-2

Exercise:

Problem: $-\sqrt{100}$

Exercise:

Problem: $-\sqrt{1}$

Solution:

-1

Exercise:

Problem: $-\sqrt{121}$

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

In the following exercises, write as the ratio of two integers.

Exercise:

Problem: ⓐ 5 ⓑ 3.19

Solution:

ⓐ $\frac{5}{1}$ ⓑ $\frac{319}{100}$

Exercise:

Problem: ⓐ 8 ⓑ 1.61

Exercise:

Problem: ⓐ -12 ⓑ 9.279

Solution:

ⓐ $\frac{-12}{1}$ ⓑ $\frac{9297}{1000}$

Exercise:

Problem: ⓐ -16 ⓑ 4.399

In the following exercises, list the ⓐ rational numbers, ⓑ irrational numbers

Exercise:

Problem: $0.75, 0.22\overline{3}, 1.39174$

Solution:

ⓐ $0.75, 0.22\overline{3}$ ⓑ $1.39174\ldots$

Exercise:

Problem: $0.36, 0.94729\ldots, 2.52\overline{8}$

Exercise:

Problem: $0.4\overline{5}, 1.919293\ldots, 3.59$

Solution:

ⓐ $0.4\overline{5}, 3.59$ ⓑ $1.919293\ldots$

Exercise:

Problem: $0.1\overline{3}, 0.42982\ldots, 1.875$

In the following exercises, identify whether each number is rational or irrational.

Exercise:

Problem: ⓐ $\sqrt{25}$ ⓑ $\sqrt{30}$

Solution:

ⓐ rational ⓑ irrational

Exercise:

Problem: ⓐ $\sqrt{44}$ ⓑ $\sqrt{49}$

Exercise:

Problem: ⓐ $\sqrt{164}$ ⓑ $\sqrt{169}$

Solution:

ⓐ irrational ⓑ rational

Exercise:

Problem: ⓐ $\sqrt{225}$ ⓑ $\sqrt{216}$

In the following exercises, identify whether each number is a real number or not a real number.

Exercise:

Problem: ⓐ $-\sqrt{81}$ ⓑ $\sqrt{-121}$

Solution:

ⓐ real number ⓑ not a real number

Exercise:

Problem: ⓐ $-\sqrt{64}$ ⓑ $\sqrt{-9}$

Exercise:

Problem: ⓐ $\sqrt{-36}$ ⓑ $-\sqrt{144}$

Solution:

ⓐ not a real number ⓑ real number

Exercise:

Problem: ⓐ $\sqrt{-49}$ ⓑ $-\sqrt{144}$

In the following exercises, list the ⓐ whole numbers, ⓑ integers, ⓒ rational numbers, ⓓ irrational numbers, ⓔ real numbers for each set of numbers.

Exercise:

Problem: $-8, 0, 1.95286..., \frac{12}{5}, \sqrt{36}, 9$

Solution:

ⓐ $0, \sqrt{36}, 9$ ⓑ $-8, \sqrt{36}, 9$ ⓒ $-8, 0, \frac{12}{5}, \sqrt{36}, 9$ ⓓ $1.95286...$ ⓔ $-8, 0, 1.95286..., \frac{12}{5}, \sqrt{36}, 9$

Exercise:

Problem: $-9, -3\frac{4}{9}, -\sqrt{9}, 0.40\bar{9}, \frac{11}{6}, 7$

Exercise:

Problem: $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$

Solution:

ⓐ none ⓑ $-\sqrt{100}, -7, -1$ ⓒ $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$ ⓓ none ⓔ $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$

Exercise:

Problem: $-6, -\frac{5}{2}, 0, 0.7\overline{14285}, 2\frac{1}{5}, \sqrt{14}$

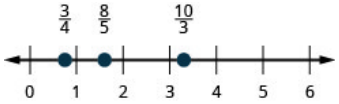
Locate Fractions on the Number Line

In the following exercises, locate the numbers on a number line.

Exercise:

Problem: $\frac{3}{4}, \frac{8}{5}, \frac{10}{3}$

Solution:



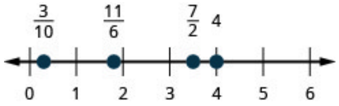
Exercise:

Problem: $\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$

Exercise:

Problem: $\frac{3}{10}, \frac{7}{2}, \frac{11}{6}, 4$

Solution:



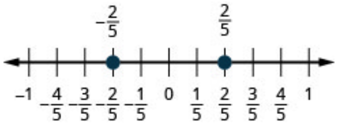
Exercise:

Problem: $\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$

Exercise:

Problem: $\frac{2}{5}, -\frac{2}{5}$

Solution:



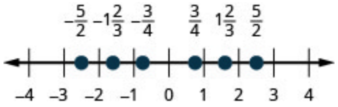
Exercise:

Problem: $\frac{3}{4}, -\frac{3}{4}$

Exercise:

Problem: $\frac{3}{4}, -\frac{3}{4}, 1\frac{2}{3}, -1\frac{2}{3}, \frac{5}{2}, -\frac{5}{2}$

Solution:



Exercise:

Problem: $\frac{1}{5}, -\frac{2}{5}, 1\frac{3}{4}, -1\frac{3}{4}, \frac{8}{3}, -\frac{8}{3}$

In the following exercises, order each of the pairs of numbers, using < or >.

Exercise:

Problem: -1 ___ $-\frac{1}{4}$

Solution:

<

Exercise:

Problem: -1 ___ $-\frac{1}{3}$

Exercise:

Problem: $-2\frac{1}{2}$ ___ -3

Solution:

>

Exercise:

Problem: $-1\frac{3}{4}$ ___ -2

Exercise:

Problem: $-\frac{5}{12}$ ___ $-\frac{7}{12}$

Solution:

$>$

Exercise:

Problem: $-\frac{9}{10}$ ___ $-\frac{3}{10}$

Exercise:

Problem: -3 ___ $-\frac{13}{5}$

Solution:

$<$

Exercise:

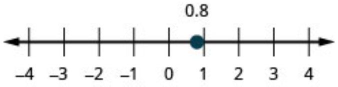
Problem: -4 ___ $-\frac{23}{6}$

Locate Decimals on the Number Line In the following exercises, locate the number on the number line.

Exercise:

Problem: 0.8

Solution:



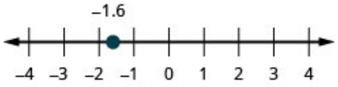
Exercise:

Problem: -0.9

Exercise:

Problem: -1.6

Solution:



Exercise:

Problem: 3.1

In the following exercises, order each pair of numbers, using < or >.

Exercise:

Problem: 0.37 ___ 0.63

Solution:

$<$

Exercise:

Problem: 0.86 ___ 0.69

Exercise:

Problem: 0.91 ___ 0.901

Solution:

$>$

Exercise:

Problem: 0.415 ___ 0.41

Exercise:

Problem: -0.5 ___ -0.3

Solution:

$<$

Exercise:

Problem: -0.1 ___ -0.4

Exercise:

Problem: -0.62 ___ -0.619

Solution:

$<$

Exercise:

Problem: -7.31 ___ -7.3

Everyday Math

Exercise:

Problem:

Field trip All the 5th graders at Lincoln Elementary School will go on a field trip to the science museum. Counting all the children, teachers, and chaperones, there will be 147 people. Each bus holds 44 people.

ⓐ How many busses will be needed?
ⓑ Why must the answer be a whole number?
ⓒ Why shouldn’t you round the answer the usual way, by choosing the whole number closest to the exact answer?

Solution:

ⓐ 4 busses ⓑ answers may vary ⓒ answers may vary

Exercise:

Problem:

Child care Serena wants to open a licensed child care center. Her state requires there be no more than 12 children for each teacher. She would like her child care center to serve 40 children.

ⓐ How many teachers will be needed?
ⓑ Why must the answer be a whole number?
ⓒ Why shouldn’t you round the answer the usual way, by choosing the whole number closest to the exact answer?

Writing Exercises

Exercise:

Problem: In your own words, explain the difference between a rational number and an irrational number.

Solution:

Answers may vary

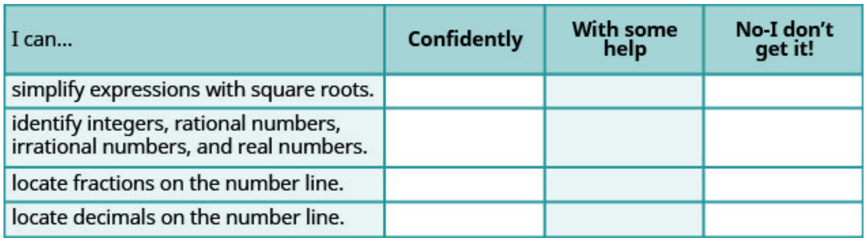
Exercise:

Problem:

Explain how the sets of numbers (counting, whole, integer, rational, irrationals, reals) are related to each other.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.



I can...	Confidently	With some help	No-I don’t get it!
simplify expressions with square roots.			
identify integers, rational numbers, irrational numbers, and real numbers.			
locate fractions on the number line.			
locate decimals on the number line.			

ⓑ On a scale of $1-10$, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

equivalent decimals
Two decimals are equivalent if they convert to equivalent fractions.

irrational number

An irrational number is a number that cannot be written as the ratio of two integers. Its decimal form does not stop and does not repeat.

rational number

A rational number is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. A rational number can be written as the ratio of two integers. Its decimal form stops or repeats.

radical sign

A radical sign is the symbol $\sqrt{m}$ that denotes the positive square root.

real number

A real number is a number that is either rational or irrational.

square and square root

If $n^2 = m$, then m is the square of n and n is a square root of m.

Properties of Real Numbers

By the end of this section, you will be able to:

- Use the commutative and associative properties
- Use the identity and inverse properties of addition and multiplication
- Use the properties of zero
- Simplify expressions using the distributive property

Use the Commutative and Associative Properties

Think about adding two numbers, say 5 and 3. The order we add them doesn't affect the result, does it?

Equation:

$$\begin{array}{cc} 5+3 & 3+5 \\ 8 & 8 \end{array}$$

Equation:

$$5+3=3+5$$

The results are the same.

As we can see, the order in which we add does not matter!

What about multiplying $5\text{ and }3$?

Equation:

$$\begin{array}{cc} 5\cdot 3 & 3\cdot 5 \\ 15 & 15 \end{array}$$

Equation:

$$5\cdot 3=3\cdot 5$$

Again, the results are the same!

The order in which we multiply does not matter!

These examples illustrate the **commutative property**. When adding or multiplying, changing the *order* gives the same result.

Note:
Commutative Property
Equation:

$$\begin{array}{lllll} \textbf{of Addition} & \text{If } a, b \text{ are real numbers, then} & a+b & = & b+a \\ \textbf{of Multiplication} & \text{If } a, b \text{ are real numbers, then} & a \cdot b & = & b \cdot a \end{array}$$

When adding or multiplying, changing the *order* gives the same result.

The commutative property has to do with order. If you change the order of the numbers when adding or multiplying, the result is the same.

What about subtraction? Does order matter when we subtract numbers? Does $7-3$ give the same result as $3-7$?
Equation:

$$\begin{array}{cc} 7-3 & 3-7 \\ 4 & -4 \end{array}$$

$$4 \neq -4$$
$$7-3 \neq 3-7$$

The results are not the same.

Since changing the order of the subtraction did not give the same result, we know that *subtraction is not commutative*.

Let’s see what happens when we divide two numbers. Is division commutative?
Equation:

$$\begin{array}{cc} 12 \div 4 & 4 \div 12 \\ \frac{12}{4} & \frac{4}{12} \\ 3 & \frac{1}{3} \end{array}$$

$$3 \neq \frac{1}{3}$$
$$12 \div 4 \neq 4 \div 12$$

The results are not the same.

Since changing the order of the division did not give the same result, *division is not commutative*. The commutative properties only apply to addition and multiplication!

- Addition and multiplication *are* commutative.
- Subtraction and Division *are not* commutative.

If you were asked to simplify this expression, how would you do it and what would your answer be?
Equation:

$$7+8+2$$

Some people would think $7+8$ is 15 and then $15+2$ is 17. Others might start with $8+2$ makes 10 and then $7+10$ makes 17.

Either way gives the same result. Remember, we use parentheses as grouping symbols to indicate which operation should be done first.

Add $7+8$. Add.	$(7+8)+2$ $15+2$ 17
Add $8+2$. Add.	$7+(8+2)$ $7+10$ 17
	$(7+8)+2=7+(8+2)$

When adding three numbers, changing the grouping of the numbers gives the same result.

This is true for multiplication, too.

Multiply. $5\cdot\frac{1}{3}$ Multiply.	$\left(5\cdot\frac{1}{3}\right)\cdot 3$ $\frac{5}{3}\cdot 3$ 5
Multiply. $\frac{1}{3}\cdot 3$. Multiply.	$5\cdot\left(\frac{1}{3}\cdot 3\right)$ $5\cdot 1$ 5
	$\left(5\cdot\frac{1}{3}\right)\cdot 3=5\cdot\left(\frac{1}{3}\cdot 3\right)$

When multiplying three numbers, changing the grouping of the numbers gives the same result.

You probably know this, but the terminology may be new to you. These examples illustrate the **associative property**.

Note:
Associative Property
Equation:

of Addition	If a, b, c are real numbers, then $(a+b)+c=a+(b+c)$
of Multiplication	If a, b, c are real numbers, then $(a\cdot b)\cdot c=a\cdot(b\cdot c)$

When adding or multiplying, changing the *grouping* gives the same result.

Let’s think again about multiplying $5\cdot\frac{1}{3}\cdot 3$. We got the same result both ways, but which way was easier? Multiplying $\frac{1}{3}$ and 3 first, as shown above on the right side, eliminates the fraction in the first step. Using the associative property can make the math easier!

The associative property has to do with grouping. If we change how the numbers are grouped, the result will be the same. Notice it is the same three numbers in the same order—the only difference is the grouping.

We saw that subtraction and division were not commutative. They are not associative either.

When simplifying an expression, it is always a good idea to plan what the steps will be. In order to combine like terms in the next example, we will use the commutative property of addition to write the like terms together.

Example:
Exercise:

Problem: **Simplify: $18p + 6q + 15p + 5q$.

Solution:
Solution

	$18p + 6q + 15p + 5q$
Use the commutative property of addition to re-order so that like terms are together.	$18p + 15p + 6q + 5q$
Add like terms.	$33p + 11q$

Note:
Exercise:

TRY IT :: 1.243
Problem: Simplify: $23r + 14s + 9r + 15s$.

Solution:

$32r + 29s$

Note:

Exercise:

TRY IT :: 1.244

Problem: Simplify: $37m + 21n + 4m - 15n$.

Solution:

$41m + 6n$

When we have to simplify algebraic expressions, we can often make the work easier by applying the commutative or associative property first, instead of automatically following the order of operations. When adding or subtracting fractions, combine those with a common denominator first.

Example:
Exercise:

Problem: **Simplify: $\left(\frac{5}{13} + \frac{3}{4}\right) + \frac{1}{4}$.

Solution:
Solution

	$\left(\frac{5}{13} + \frac{3}{4}\right) + \frac{1}{4}$
Notice that the last 2 terms have a common denominator, so change the grouping.	$\frac{5}{13} + \left(\frac{3}{4} + \frac{1}{4}\right)$
Add in parentheses first.	$\frac{5}{13} + \left(\frac{4}{4}\right)$
Simplify the fraction.	$\frac{5}{13} + 1$
Add.	$1\frac{5}{13}$

Convert to an improper fraction.	$\frac{18}{13}$

Note:
Exercise:

TRY IT :: 1.245

Problem: Simplify: $\left(\frac{7}{15}+\frac{5}{8}\right)+\frac{3}{8}$.

Solution:

$1\frac{7}{15}$ or $\frac{22}{15}$

Note:
Exercise:

TRY IT :: 1.246

Problem: Simplify: $\left(\frac{2}{9}+\frac{7}{12}\right)+\frac{5}{12}$.

Solution:

$1\frac{2}{9}$ or $\frac{11}{9}$

Example:
Exercise:

Problem: **Use the associative property to simplify $6\left(3x\right)$.

Solution:
Solution

	$6\left(3x\right)$
Change the grouping.	$(6\cdot 3)x$
Multiply in the parentheses.	$18x$

Notice that we can multiply $6\cdot 3$ but we could not multiply $3x$ without having a value for x.

Note:
Exercise:

TRY IT :: 1.247
Problem: Use the associative property to simplify $8(4x)$.

Solution:

$32x$

Note:
Exercise:

TRY IT :: 1.248
Problem: Use the associative property to simplify $-9\left(7y\right)$.

Solution:

$-63y$

Use the Identity and Inverse Properties of Addition and Multiplication

What happens when we add 0 to any number? Adding 0 doesn’t change the value. For this reason, we call 0 the **additive identity**.

For example,

Equation:

$$\begin{array}{ccc} 13+0 & -14+0 & 0+(-8) \\ 13 & -14 & -8 \end{array}$$

These examples illustrate the **Identity Property of Addition** that states that for any real number a, $a+0=a$ and $0+a=a$.

What happens when we multiply any number by one? Multiplying by 1 doesn’t change the value. So we call 1 the **multiplicative identity**.

For example,
Equation:

$$\begin{array}{ccc} 43\cdot 1 & -27\cdot 1 & 1\cdot \frac{3}{5} \\ 43 & -27 & \frac{3}{5} \end{array}$$

These examples illustrate the **Identity Property of Multiplication** that states that for any real number a, $a\cdot 1=a$ and $1\cdot a=a$.

We summarize the Identity Properties below.

Note:
Identity Property
Equation:

$$\begin{array}{lll} \textbf{of addition}\text{ For any real number } a\text{:} & a+0=a & 0+a=a \\ \quad \mathbf{0}\text{ is the }\textbf{additive identity} & & \\ \textbf{of multiplication}\text{ For any real number } a\text{:} & a\cdot 1=a & 1\cdot a=a \\ \quad \mathbf{1}\text{ is the }\textbf{multiplicative identity} & & \end{array}$$

What number added to 5 gives the additive identity, 0?

$5 + \_\_\_\_\_ = 0$ We know $5 + (-5) = 0$

What number added to −6 gives the additive identity, 0?

$-6 + \_\_\_\_\_ = 0$ We know $-6 + 6 = 0$

Notice that in each case, the missing number was the opposite of the number!

We call $-a$. the **additive inverse** of a. *The opposite of a number is its additive inverse.* A number and its opposite add to zero, which is the additive identity. This leads to the **Inverse Property of Addition** that states for any real number $a, a + (-a) = 0$. Remember, a number and its opposite add to zero.

What number multiplied by $\frac{2}{3}$ gives the multiplicative identity, 1? In other words, $\frac{2}{3}$ times what results in 1?

$\frac{2}{3} \cdot \_\_\_\_ = 1$ We know $\frac{2}{3} \cdot \frac{3}{2} = 1$

What number multiplied by 2 gives the multiplicative identity, 1? In other words 2 times what results in 1?

$2 \cdot \_\_\_\_ = 1$ We know $2 \cdot \frac{1}{2} = 1$

Notice that in each case, the missing number was the reciprocal of the number!

We call $\frac{1}{a}$ the **multiplicative inverse** of a. *The reciprocal of a number is its multiplicative inverse.* A number and its reciprocal multiply to one, which is the multiplicative identity. This leads to the **Inverse Property of Multiplication** that states that for any real number $a, a \neq 0, a \cdot \frac{1}{a} = 1$.

We'll formally state the inverse properties here:

Note:
Inverse Property

of addition	For any real number a, $-a$ is the **additive inverse** of a.	$a + (-a) = 0$

	A number and its opposite add to zero.	
of multiplication	For any real number a, $\frac{1}{a}$ is the **multiplicative inverse** of a. A number and its reciprocal multiply to one.	$a \cdot \frac{1}{a} = 1$

Example:
Exercise:

Problem: **Find the additive inverse of ⓐ $\frac{5}{8}$ ⓑ 0.6 ⓒ -8 ⓓ $-\frac{4}{3}$.

Solution:
Solution

To find the additive inverse, we find the opposite.

ⓐ The additive inverse of $\frac{5}{8}$ is the opposite of $\frac{5}{8}$. The additive inverse of $\frac{5}{8}$ is $-\frac{5}{8}$.

ⓑ The additive inverse of 0.6 is the opposite of 0.6. The additive inverse of 0.6 is -0.6.

ⓒ The additive inverse of -8 is the opposite of -8. We write the opposite of -8 as $-(-8)$, and then simplify it to 8. Therefore, the additive inverse of -8 is 8.

ⓓ The additive inverse of $-\frac{4}{3}$ is the opposite of $-\frac{4}{3}$. We write this as $-\left(-\frac{4}{3}\right)$, and then simplify to $\frac{4}{3}$. Thus, the additive inverse of $-\frac{4}{3}$ is $\frac{4}{3}$.

Note:
Exercise:

TRY IT :: 1.249
Problem: Find the additive inverse of: ⓐ $\frac{7}{9}$ ⓑ 1.2 ⓒ -14 ⓓ $-\frac{9}{4}$.

Solution:

ⓐ $-\frac{7}{9}$ ⓑ -1.2 ⓒ 14 ⓓ $\frac{9}{4}$

Note:
Exercise:

TRY IT :: 1.250

Problem: Find the additive inverse of: ⓐ $\frac{7}{13}$ ⓑ 8.4 ⓒ -46 ⓓ $-\frac{5}{2}$.

Solution:

ⓐ $-\frac{7}{13}$ ⓑ -8.4 ⓒ 46 ⓓ $\frac{5}{2}$

Example:
Exercise:

Problem: **Find the multiplicative inverse of ⓐ 9 ⓑ $-\frac{1}{9}$ ⓒ 0.9.

Solution:
Solution

To find the multiplicative inverse, we find the reciprocal.

ⓐ The multiplicative inverse of 9 is the reciprocal of 9, which is $\frac{1}{9}$. Therefore, the multiplicative inverse of 9 is $\frac{1}{9}$.
ⓑ The multiplicative inverse of $-\frac{1}{9}$ is the reciprocal of $-\frac{1}{9}$, which is -9. Thus, the multiplicative inverse of $-\frac{1}{9}$ is -9.
ⓒ To find the multiplicative inverse of 0.9, we first convert 0.9 to a fraction, $\frac{9}{10}$. Then we find the reciprocal of the fraction. The reciprocal of $\frac{9}{10}$ is $\frac{10}{9}$. So the multiplicative inverse of 0.9 is $\frac{10}{9}$.

Note:

Exercise:

TRY IT :: 1.251

Problem: Find the multiplicative inverse of ⓐ 4 ⓑ $-\frac{1}{7}$ ⓒ 0.3

Solution:

ⓐ $\frac{1}{4}$ ⓑ -7 ⓒ $\frac{10}{3}$

Note:
Exercise:

TRY IT :: 1.252

Problem: Find the multiplicative inverse of ⓐ 18 ⓑ $-\frac{4}{5}$ ⓒ 0.6.

Solution:

ⓐ $\frac{1}{18}$ ⓑ $-\frac{5}{4}$ ⓒ $\frac{5}{3}$

Use the Properties of Zero

The identity property of addition says that when we add 0 to any number, the result is that same number. What happens when we multiply a number by 0? Multiplying by 0 makes the product equal zero.

Note:
Multiplication by Zero
For any real number *a*.
Equation:

$$a \cdot 0 = 0 \qquad\qquad 0 \cdot a = 0$$

The product of any real number and 0 is 0.

What about division involving zero? What is $0 \div 3$? Think about a real example: If there are no cookies in the cookie jar and 3 people are to share them, how many cookies does each person get? There are no cookies to share, so each person gets 0 cookies. So,

Equation:

$$0 \div 3 = 0$$

We can check division with the related multiplication fact.

Equation:

$$12 \div 6 = 2 \text{ because } 2 \cdot 6 = 12.$$

So we know $0 \div 3 = 0$ because $0 \cdot 3 = 0$.

Note:

Division of Zero

For any real number a, except 0, $\frac{0}{a} = 0$ and $0 \div a = 0$.

Zero divided by any real number except zero is zero.

Now think about dividing *by* zero. What is the result of dividing 4 by 0? Think about the related multiplication fact: $4 \div 0 = ?$ means $? \cdot 0 = 4$. Is there a number that multiplied by 0 gives 4? Since any real number multiplied by 0 gives 0, there is no real number that can be multiplied by 0 to obtain 4.

We conclude that there is no answer to $4 \div 0$ and so we say that division by 0 is undefined.

Note:

Division by Zero

For any real number a, except 0, $\frac{a}{0}$ and $a \div 0$ are undefined.

Division by zero is undefined.

We summarize the properties of zero below.

Note:
Properties of Zero
Multiplication by Zero: For any real number a,

$a \cdot 0 = 0 \quad 0 \cdot a = 0$	The product of any number and 0 is 0.

Division of Zero, Division by Zero: For any real number $a, a \neq 0$

$\frac{0}{a} = 0$	Zero divided by any real number except itself is zero.
$\frac{a}{0}$ is undefined	Division by zero is undefined.

Example:
Exercise:

Problem: **Simplify: ⓐ $-8 \cdot 0$ ⓑ $\frac{0}{-2}$ ⓒ $\frac{-32}{0}$.

Solution:
Solution

ⓐ The product of any real number and 0 is 0.	$-8 \cdot 0$ 0
ⓑ	

The product of any real number and 0 is 0.	$\frac{0}{-2}$ 0
ⓒ Division by 0 is undefined.	$\frac{-32}{0}$ Undefined

Note:
Exercise:

TRY IT :: 1.253

Problem: Simplify: ⓐ $-14 \cdot 0$ ⓑ $\frac{0}{-6}$ ⓒ $\frac{-2}{0}$.

Solution:

ⓐ 0 ⓑ 0 ⓒ undefined

Note:
Exercise:

TRY IT :: 1.254

Problem: Simplify: ⓐ $0\left(-17\right)$ ⓑ $\frac{0}{-10}$ ⓒ $\frac{-5}{0}$.

Solution:

ⓐ 0 ⓑ 0 ⓒ undefined

We will now practice using the properties of identities, inverses, and zero to simplify expressions.

Example:
Exercise:

Problem: **Simplify: $-84n + (-73n) + 84n$.

Solution:
Solution

	$-84n + (-73n) + 84n$
Notice that the first and third terms are opposites; use the commutative property of addition to re-order the terms.	$-84n + 84n + (-73n)$
Add left to right.	$0 + (-73)$
Add.	$-73n$

Note:
Exercise:

TRY IT :: 1.255
Problem: Simplify: $-27a + (-48a) + 27a$.

Solution:

$-48a$

Note:
Exercise:

TRY IT :: 1.256
Problem: Simplify: $39x + (-92x) + (-39x)$.

Solution:

$-92x$

Now we will see how recognizing reciprocals is helpful. Before multiplying left to right, look for reciprocals—their product is 1.

Example:
Exercise:

Problem: **Simplify: $\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$.

Solution:
Solution

	$\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$
Notice that the first and third terms are reciprocals, so use the commutative property of multiplication to re-order the factors.	$\frac{7}{15} \cdot \frac{15}{7} \cdot \frac{8}{23}$
Multiply left to right.	$1 \cdot \frac{8}{23}$
Multiply.	$\frac{8}{23}$

Note:
Exercise:

TRY IT :: 1.257

Problem: Simplify: $\frac{9}{16} \cdot \frac{5}{49} \cdot \frac{16}{9}$.

Solution:

$\frac{5}{49}$

Note:

Exercise:

TRY IT :: 1.258

Problem: Simplify: $\frac{6}{17} \cdot \frac{11}{25} \cdot \frac{17}{6}$.

Solution:

$\frac{11}{25}$

Example:

Exercise:

Problem: **Simplify: ⓐ $\frac{0}{n+5}$, where $n \neq -5$ ⓑ $\frac{10-3p}{0}$, where $10 - 3p \neq 0$.

Solution:

Solution

ⓐ Zero divided by any real number except itself is 0.	$\frac{0}{n+5}$ 0
ⓑ Division by 0 is undefined.	$\frac{10-3p}{0}$ Undefined

Note:
Exercise:

TRY IT :: 1.259

Problem: Simplify: ⓐ $\frac{0}{m+7}$, where $m \neq -7$ ⓑ $\frac{18-6c}{0}$, where $18-6c \neq 0$.

Solution:

ⓐ 0 ⓑ undefined

Note:
Exercise:

TRY IT :: 1.260

Problem: Simplify: ⓐ $\frac{0}{d-4}$, where $d \neq 4$ ⓑ $\frac{15-4q}{0}$, where $15-4q \neq 0$.

Solution:

ⓐ 0 ⓑ undefined

Example:
Exercise:

Problem: **Simplify: $\frac{3}{4} \cdot \frac{4}{3}(6x+12)$.

Solution:
Solution

	$\frac{3}{4} \cdot \frac{4}{3}(6x + 12)$
There is nothing to do in the parentheses, so multiply the two fractions first—notice, they are reciprocals.	$1\,(6x + 12)$
Simplify by recognizing the multiplicative identity.	$6x + 12$

Note:
Exercise:

TRY IT :: 1.261

Problem: Simplify: $\frac{2}{5} \cdot \frac{5}{2}(20y + 50)$.

Solution:

$20y + 50$

Note:
Exercise:

TRY IT :: 1.262

Problem: Simplify: $\frac{3}{8} \cdot \frac{8}{3}(12z + 16)$.

Solution:

$12z + 16$

Simplify Expressions Using the Distributive Property

Suppose that three friends are going to the movies. They each need $9.25—that's 9 dollars and 1 quarter—to pay for their tickets. How much money do they need all together?

You can think about the dollars separately from the quarters. They need 3 times $9 so $27, and 3 times 1 quarter, so 75 cents. In total, they need $27.75. If you think about doing the math in this way, you are using the **distributive property**.

Note:
Distributive Property
Equation:

$$\begin{array}{rl} \text{If } a, b, c \text{ are real numbers, then} & a(b+c)=ab+ac \\ \text{Also,} & (b+c)a=ba+ca \\ & a(b-c)=ab-ac \\ & (b-c)a=ba-ca \end{array}$$

Back to our friends at the movies, we could find the total amount of money they need like this:
Equation:

$$\begin{array}{c} 3(9.25) \\ 3(9 \quad + \quad 0.25) \\ 3(9) \quad + \quad 3(0.25) \\ 27 \quad + \quad 0.75 \\ 27.75 \end{array}$$

In algebra, we use the **distributive property** to remove parentheses as we simplify expressions.

For example, if we are asked to simplify the expression $3\left(x+4\right)$, the order of operations says to work in the parentheses first. But we cannot add *x* and 4, since they are not like terms. So we use the distributive property, as shown in [link].

Example:
Exercise:

Problem: **Simplify: $3\left(x+4\right)$.

Solution:
Solution

	$3\left(x+4\right)$
Distribute.	$3\cdot x+3\cdot 4$
Multiply.	$3x+12$

Note:
Exercise:

TRY IT :: 1.263
Problem: Simplify: $4\left(x+2\right)$.

Solution:

$4x+8$

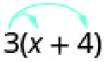
Note:
Exercise:

TRY IT :: 1.264
Problem: Simplify: $6\left(x+7\right)$.

Solution:

$6x+42$

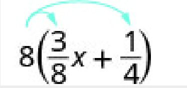
Some students find it helpful to draw in arrows to remind them how to use the distributive property. Then the first step in [link] would look like this:


$3(x+4)$

Example:
Exercise:

Problem: **Simplify: $8\left(\frac{3}{8}x+\frac{1}{4}\right)$.

Solution:
Solution

	 $8\left(\frac{3}{8}x+\frac{1}{4}\right)$
Distribute.	$8\cdot\frac{3}{8}x+8\cdot\frac{1}{4}$
Multiply.	$3x+2$

Note:
Exercise:

TRY IT :: 1.265
Problem: Simplify: $6\left(\frac{5}{6}y+\frac{1}{2}\right)$.

Solution:

$5y+3$

Note:
Exercise:

TRY IT :: 1.266

Problem: Simplify: $12\left(\frac{1}{3}n+\frac{3}{4}\right)$.

Solution:

$4n+9$

Using the distributive property as shown in [link] will be very useful when we solve money applications in later chapters.

Example:
Exercise:

Problem: **Simplify: $100\left(0.3+0.25q\right)$.

Solution:
Solution

	$100(0.3+0.25q)$
Distribute.	$100(0.3)+100(0.25q)$

Multiply.	$30 + 25q$

Note:
Exercise:

TRY IT :: 1.267
Problem: Simplify: $100\left(0.7 + 0.15p\right)$.

Solution:

$70 + 15p$

Note:
Exercise:

TRY IT :: 1.268
Problem: Simplify: $100\left(0.04 + 0.35d\right)$.

Solution:

$4 + 35d$

When we distribute a negative number, we need to be extra careful to get the signs correct!

Example:
Exercise:

Problem: **Simplify: $-2\left(4y + 1\right)$.

Solution:
Solution

	$-2(4y+1)$
Distribute.	$-2\cdot 4y+(-2)\cdot 1$
Multiply.	$-8y-2$

Note:
Exercise:

TRY IT :: 1.269
Problem: Simplify: $-3\left(6m+5\right)$.

Solution:

$-18m-15$

Note:
Exercise:

TRY IT :: 1.270
Problem: Simplify: $-6\left(8n+11\right)$.

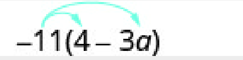
Solution:

$-48n - 66$

Example:
Exercise:

Problem: **Simplify: $-11\,(4 - 3a)$.

Solution:
Solution

Distribute.	 $-11(4 - 3a)$
Multiply.	$-11 \cdot 4 - (-11) \cdot 3a$ $-44 - (-33a)$
Simplify.	$-44 + 33a$

Notice that you could also write the result as $33a - 44$. Do you know why?

Note:
Exercise:

TRY IT :: 1.271
Problem: Simplify: $-5\,(2 - 3a)$.

Solution:

$-10 + 15a$

Note:
Exercise:

TRY IT :: 1.272
Problem: Simplify: $-7\left(8-15y\right)$.

Solution:

$-56+105y$

[link] will show how to use the distributive property to find the opposite of an expression.

Example:
Exercise:

Problem: **Simplify: $-\left(y+5\right)$.

Solution:
Solution

	$\left(y+5\right)$
Multiplying by −1 results in the opposite.	$-1\left(y+5\right)$
Distribute.	$-1\cdot y+\left(-1\right)\cdot 5$
Simplify.	$-y+\left(-5\right)$
	$-y-5$

Note:
Exercise:

TRY IT :: 1.273

Problem: Simplify: $-\left(z-11\right)$.

Solution:

$-z+11$

Note:
Exercise:

TRY IT :: 1.274

Problem: Simplify: $-\left(x-4\right)$.

Solution:

$-x+4$

There will be times when we'll need to use the distributive property as part of the order of operations. Start by looking at the parentheses. If the expression inside the parentheses cannot be simplified, the next step would be multiply using the distributive property, which removes the parentheses. The next two examples will illustrate this.

Example:
Exercise:

Problem: **Simplify: $8-2\left(x+3\right)$.

Be sure to follow the order of operations. Multiplication comes before subtraction, so we will distribute the 2 first and then subtract.

Solution:
Solution

	$8-2(x+3)$
Distribute.	$8-2\cdot x-2\cdot 3$
Multiply.	$8-2x-6$
Combine like terms.	$-2x+2$

Note:
Exercise:

TRY IT :: 1.275
Problem: Simplify: $9-3(x+2)$.

Solution:

$3-3x$

Note:
Exercise:

TRY IT :: 1.276
Problem: Simplify: $7x-5(x+4)$.

Solution:

$2x-20$

Example:
Exercise:

Problem: **Simplify: $4(x-8)-(x+3)$.

Solution:

Solution

	$4\left(x-8\right)-\left(x+3\right)$
Distribute.	$4x-32-x-3$
Combine like terms.	$3x-35$

Note:
Exercise:

TRY IT :: 1.277

Problem: Simplify: $6\left(x-9\right)-\left(x+12\right)$.

Solution:

$5x-66$

Note:
Exercise:

TRY IT :: 1.278

Problem: Simplify: $8\left(x-1\right)-\left(x+5\right)$.

Solution:

$7x-13$

All the properties of real numbers we have used in this chapter are summarized in [link].

Commutative Property	
of addition If a, b are real numbers, then **of multiplication** If a, b are real numbers, then	$a+b=b+a$ $a \cdot b=b \cdot a$
Associative Property	
of addition If a, b, c are real numbers, then **of multiplication** If a, b, c are real numbers, then	$(a+b)+c=a+(b+c)$ $(a \cdot b) \cdot c=a \cdot (b \cdot c)$
Distributive Property	
If a, b, c are real numbers, then	$a\,(b+c)=ab+ac$
Identity Property	
of addition For any real number a: 0 is the **additive identity** **of multiplication** For any real number a: **1** is the **multiplicative identity**	$a+0=a$ $0+a=a$ $a \cdot 1=a$ $1 \cdot a=a$
Inverse Property	
of addition For any real number a, $-a$ is the **additive inverse** of a **of multiplication** For any real number $a, a \neq 0$ $\frac{1}{a}$ is the **multiplicative inverse** of a.	$a+(-a)=0$ $a \cdot \frac{1}{a}=1$
Properties of Zero	
For any real number a, For any real number $a, a \neq 0$	$a \cdot 0=0$ $0 \cdot a=0$ $\frac{0}{a}=0$

For any real number $a, a \neq 0$	$\frac{a}{0}$ is undefined

Key Concepts

- **Commutative Property of**
  - **Addition:** If a, b are real numbers, then $a + b = b + a$.
  - **Multiplication:** If a, b are real numbers, then $a \cdot b = b \cdot a$. When adding or multiplying, changing the *order* gives the same result.
- **Associative Property of**
  - **Addition:** If a, b, c are real numbers, then $(a + b) + c = a + (b + c)$.
  - **Multiplication:** If a, b, c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. When adding or multiplying, changing the *grouping* gives the same result.
- **Distributive Property:** If a, b, c are real numbers, then
  - $a\,(b + c) = ab + ac$
  - $(b + c)a = ba + ca$
  - $a\,(b - c) = ab - ac$
  - $(b - c)a = ba - ca$
- **Identity Property**
  - **of Addition:** For any real number a: $a + 0 = a \;\; 0 + a = a$
    0 is the **additive identity**
  - **of Multiplication:** For any real number a: $a \cdot 1 = a \;\; 1 \cdot a = a$
    1 is the **multiplicative identity**
- **Inverse Property**
  - **of Addition:** For any real number $a, a + (-a) = 0$. A number and its *opposite* add to zero. $-a$ is the **additive inverse** of a.
  - **of Multiplication:** For any real number $a, (a \neq 0)\; a \cdot \frac{1}{a} = 1$. A number and its *reciprocal* multiply to one. $\frac{1}{a}$ is the **multiplicative inverse** of a.
- **Properties of Zero**
  - For any real number a,
    $a \cdot 0 = 0 \;\; 0 \cdot a = 0$ – The product of any real number and 0 is 0.
  - $\frac{0}{a} = 0$ for $a \neq 0$ – Zero divided by any real number except zero is zero.

- $\frac{a}{0}$ is undefined – Division by zero is undefined.

Practice Makes Perfect

Use the Commutative and Associative Properties

In the following exercises, use the associative property to simplify.

Exercise:

Problem: $3(4x)$

Solution:

$12x$

Exercise:

Problem: $4(7m)$

Exercise:

Problem: $(y+12)+28$

Solution:

$y+40$

Exercise:

Problem: $(n+17)+33$

In the following exercises, simplify.

Exercise:

Problem: $\frac{1}{2}+\frac{7}{8}+\left(-\frac{1}{2}\right)$

Solution:

$\frac{7}{8}$

Exercise:

Problem: $\frac{2}{5} + \frac{5}{12} + \left(-\frac{2}{5}\right)$

Exercise:

Problem: $\frac{3}{20} \cdot \frac{49}{11} \cdot \frac{20}{3}$

Solution:

$\frac{49}{11}$

Exercise:

Problem: $\frac{13}{18} \cdot \frac{25}{7} \cdot \frac{18}{13}$

Exercise:

Problem: $-24.7 \cdot \frac{3}{8}$

Solution:

-63

Exercise:

Problem: $-36 \cdot 11 \cdot \frac{4}{9}$

Exercise:

Problem: $\left(\frac{5}{6} + \frac{8}{15}\right) + \frac{7}{15}$

Solution:

$1\,\frac{5}{6}$

Exercise:

Problem: $\left(\frac{11}{12} + \frac{4}{9}\right) + \frac{5}{9}$

Exercise:

Problem: $17(0.25)(4)$

Solution:

17

Exercise:

Problem: 36(0.2)(5)

Exercise:

Problem: [2.48(12)](0.5)

Solution:

14.88

Exercise:

Problem: [9.731(4)](0.75)

Exercise:

Problem: $7(4a)$

Solution:

$28a$

Exercise:

Problem: $9(8w)$

Exercise:

Problem: $-15\left(5m\right)$

Solution:

$-75m$

Exercise:

Problem: $-23\left(2n\right)$

Exercise:

Problem: $12\left(\frac{5}{6}p\right)$

Solution:

$10p$

Exercise:

Problem: $20\left(\frac{3}{5}q\right)$

Exercise:

Problem: $43m + (-12n) + (-16m) + (-9n)$

Solution:

$27m + (-21n)$

Exercise:

Problem: $-22p + 17q + (-35p) + (-27q)$

Exercise:

Problem: $\frac{3}{8}g + \frac{1}{12}h + \frac{7}{8}g + \frac{5}{12}h$

Solution:

$\frac{5}{4}g + \frac{1}{2}h$

Exercise:

Problem: $\frac{5}{6}a + \frac{3}{10}b + \frac{1}{6}a + \frac{9}{10}b$

Exercise:

Problem: $6.8p + 9.14q + (-4.37p) + (-0.88q)$

Solution:

$2.43p + 8.26q$

Exercise:

Problem: $9.6m + 7.22n + (-2.19m) + (-0.65n)$

Use the Identity and Inverse Properties of Addition and Multiplication

In the following exercises, find the additive inverse of each number.

Exercise:

ⓐ $\frac{2}{5}$
ⓑ 4.3
ⓒ -8
Problem: ⓓ $-\frac{10}{3}$

Solution:

ⓐ $-\frac{2}{5}$ ⓑ -4.3 ⓒ 8 ⓓ $\frac{10}{3}$

Exercise:

ⓐ $\frac{5}{9}$
ⓑ 2.1
ⓒ -3
Problem: ⓓ $-\frac{9}{5}$

Exercise:

ⓐ $-\frac{7}{6}$
ⓑ -0.075
ⓒ 23
Problem: ⓓ $\frac{1}{4}$

Solution:

ⓐ $\frac{7}{6}$ ⓑ 0.075 ⓒ -23 ⓓ $-\frac{1}{4}$

Exercise:

ⓐ $-\frac{8}{3}$
ⓑ -0.019
ⓒ 52
Problem: ⓓ $\frac{5}{6}$

In the following exercises, find the multiplicative inverse of each number.

Exercise:

Problem: ⓐ 6 ⓑ $-\frac{3}{4}$ ⓒ 0.7

Solution:

ⓐ $\frac{1}{6}$ ⓑ $-\frac{4}{3}$ ⓒ $\frac{10}{7}$

Exercise:

Problem: ⓐ 12 ⓑ $-\frac{9}{2}$ ⓒ 0.13

Exercise:

Problem: ⓐ $\frac{11}{12}$ ⓑ -1.1 ⓒ -4

Solution:

ⓐ $\frac{12}{11}$ ⓑ $-\frac{10}{11}$ ⓒ $-\frac{1}{4}$

Exercise:

Problem: ⓐ $\frac{17}{20}$ ⓑ -1.5 ⓒ -3

Use the Properties of Zero

In the following exercises, simplify.

Exercise:

Problem: $\frac{0}{6}$

Solution:

0

Exercise:

Problem: $\frac{3}{0}$

Exercise:

Problem: $0 \div \frac{11}{12}$

Solution:

0

Exercise:

Problem: $\frac{6}{0}$

Exercise:

Problem: $\frac{0}{3}$

Solution:

0

Exercise:

Problem: $0 \cdot \frac{8}{15}$

Exercise:

Problem: $(-3.14)\,(0)$

Solution:

0

Exercise:

Problem: $\frac{\frac{1}{10}}{0}$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $19a + 44 - 19a$

Solution:

44

Exercise:

Problem: $27c+16-27c$

Exercise:

Problem: $10(0.1d)$

Solution:

id

Exercise:

Problem: $100\left(0.01p\right)$

Exercise:

Problem: $\frac{0}{u-4.99}$, where $u\ne 4.99$

Solution:

0

Exercise:

Problem: $\frac{0}{v-65.1}$, where $v\ne 65.1$

Exercise:

Problem: $0\div\left(x-\ \frac{1}{2}\right)$, where $x\ne\ \frac{1}{2}$

Solution:

0

Exercise:

Problem: $0\div\left(y-\ \frac{1}{6}\right)$, where $x\ne\ \frac{1}{6}$

Exercise:

Problem: $\frac{32-5a}{0}$, where $32-5a\ne 0$

Solution:

undefined

Exercise:

Problem: $\frac{28-9b}{0}$, where $28-9b\neq 0$

Exercise:

Problem: $\left(\frac{3}{4}+\frac{9}{10}m\right)\div 0$ where $\frac{3}{4}+\frac{9}{10}m\neq 0$

Solution:

undefined

Exercise:

Problem: $\left(\frac{5}{16}n-\ \frac{3}{7}\right)\div 0$ where $\frac{5}{16}n-\ \frac{3}{7}\neq 0$

Exercise:

Problem: $15\cdot\frac{3}{5}(4d+10)$

Solution:

$36d+90$

Exercise:

Problem: $18\cdot\frac{5}{6}(15h+24)$

Simplify Expressions Using the Distributive Property

In the following exercises, simplify using the distributive property.

Exercise:

Problem: $8\left(4y+9\right)$

Solution:

$32y+72$

Exercise:

Problem: $9\left(3w+7\right)$

Exercise:

Problem: $6\left(c-13\right)$

Solution:

$6c-78$

Exercise:

Problem: $7\left(y-13\right)$

Exercise:

Problem: $\frac{1}{4}(3q+12)$

Solution:

$\frac{3}{4}q+3$

Exercise:

Problem: $\frac{1}{5}(4m+20)$

Exercise:

Problem: $9\left(\frac{5}{9}y-\ \frac{1}{3}\right)$

Solution:

$5y-3$

Exercise:

Problem: $10\left(\frac{3}{10}x-\ \frac{2}{5}\right)$

Exercise:

Problem: $12\left(\frac{1}{4}+\frac{2}{3}r\right)$

Solution:

$3+8r$

Exercise:

Problem: $12\left(\frac{1}{6}+\frac{3}{4}s\right)$

Exercise:

Problem: $r\left(s-18\right)$

Solution:

$rs-18r$

Exercise:

Problem: $u\left(v-10\right)$

Exercise:

Problem: $(y+4)p$

Solution:

$yp+4p$

Exercise:

Problem: $(a+7)x$

Exercise:

Problem: $-7\left(4p+1\right)$

Solution:

$-28p-7$

Exercise:

Problem: $-9\left(9a+4\right)$

Exercise:

Problem: $-3\left(x-6\right)$

Solution:

$-3x+18$

Exercise:

Problem: $-4\left(q-7\right)$

Exercise:

Problem: $-\left(3x-7\right)$

Solution:

$-3x+7$

Exercise:

Problem: $-\left(5p-4\right)$

Exercise:

Problem: $16-3\left(y+8\right)$

Solution:

$-3y-8$

Exercise:

Problem: $18-4\left(x+2\right)$

Exercise:

Problem: $4-11\left(3c-2\right)$

Solution:

$-33c+26$

Exercise:

Problem: $9-6\left(7n-5\right)$

Exercise:

Problem: $22-\left(a+3\right)$

Solution:

$-a+19$

Exercise:

Problem: $8-\left(r-7\right)$

Exercise:

Problem: $\left(5m-3\right)-\left(m+7\right)$

Solution:

$4m-10$

Exercise:

Problem: $\left(4y-1\right)-\left(y-2\right)$

Exercise:

Problem: $5\left(2n+9\right)+12\left(n-3\right)$

Solution:

$22n+9$

Exercise:

Problem: $9\left(5u+8\right)+2\left(u-6\right)$

Exercise:

Problem: $9\left(8x-3\right)-\left(-2\right)$

Solution:

$72x-25$

Exercise:

Problem: $4\left(6x-1\right)-\left(-8\right)$

Exercise:

Problem: $14\left(c-1\right)-8\left(c-6\right)$

Solution:

$6c+34$

Exercise:

Problem: $11\left(n-7\right)-5\left(n-1\right)$

Exercise:

Problem: $6\left(7y+8\right)-\left(30y-15\right)$

Solution:

$12y+63$

Exercise:

Problem: $7\left(3n+9\right)-\left(4n-13\right)$

Everyday Math

Exercise:

Problem:

Insurance copayment Carrie had to have 5 fillings done. Each filling cost $80. Her dental insurance required her to pay 20% of the cost as a copay. Calculate Carrie’s copay:

ⓐ First, by multiplying 0.20 by 80 to find her copay for each filling and then multiplying your answer by 5 to find her total copay for 5 fillings.
ⓑ Next, by multiplying [5(0.20)](80)

ⓒ Which of the properties of real numbers says that your answers to parts (a), where you multiplied 5[(0.20)(80)] and (b), where you multiplied [5(0.20)](80), should be equal?

Solution:

ⓐ $80 ⓑ $80 ⓒ answers will vary

Exercise:

Problem:

Cooking time Helen bought a 24-pound turkey for her family’s Thanksgiving dinner and wants to know what time to put the turkey in to the oven. She wants to allow 20 minutes per pound cooking time. Calculate the length of time needed to roast the turkey:

ⓐ First, by multiplying $24 \cdot 20$ to find the total number of minutes and then multiplying the answer by $\frac{1}{60}$ to convert minutes into hours.
ⓑ Next, by multiplying $24\left(20 \cdot \frac{1}{60}\right)$.
ⓒ Which of the properties of real numbers says that your answers to parts (a), where you multiplied $(24 \cdot 20)\frac{1}{60}$, and (b), where you multiplied $24\left(20 \cdot \frac{1}{60}\right)$, should be equal?

Exercise:

Problem:

Buying by the case Trader Joe’s grocery stores sold a bottle of wine they called “Two Buck Chuck” for $1.99. They sold a case of 12 bottles for $23.88. To find the cost of 12 bottles at $1.99, notice that 1.99 is $2 - 0.01$.

ⓐ Multiply 12(1.99) by using the distributive property to multiply $12\left(2 - 0.01\right)$.
ⓑ Was it a bargain to buy “Two Buck Chuck” by the case?

Solution:

ⓐ $23.88 ⓑ no, the price is the same

Exercise:

Problem:

Multi-pack purchase Adele’s shampoo sells for \$3.99 per bottle at the grocery store. At the warehouse store, the same shampoo is sold as a 3 pack for \$10.49. To find the cost of 3 bottles at \$3.99, notice that 3.99 is $4 - 0.01$.

ⓐ Multiply 3(3.99) by using the distributive property to multiply $3\left(4 - 0.01\right)$.
ⓑ How much would Adele save by buying 3 bottles at the warehouse store instead of at the grocery store?

Writing Exercises

Exercise:

Problem: In your own words, state the commutative property of addition.

Solution:

Answers may vary

Exercise:

Problem:

What is the difference between the additive inverse and the multiplicative inverse of a number?

Exercise:

Problem:

Simplify $8\left(x - \frac{1}{4}\right)$ using the distributive property and explain each step.

Solution:

Answers may vary

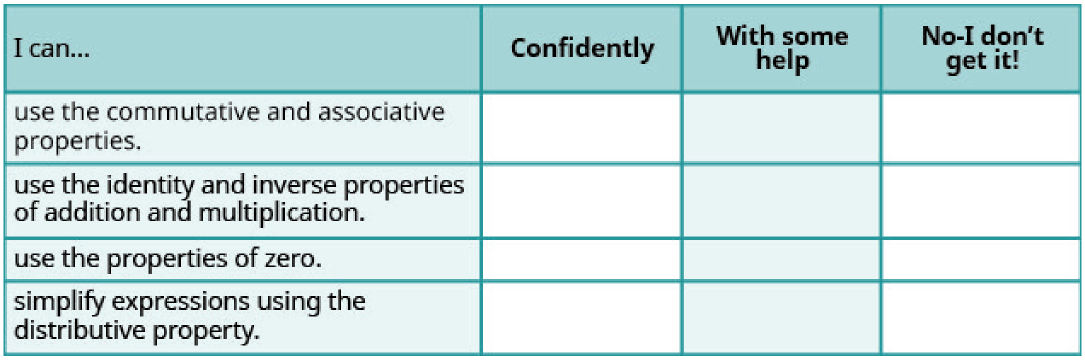
Exercise:

Problem:

Explain how you can multiply 4(\$5.97) without paper or calculator by thinking of \$5.97 as $6 - 0.03$ and then using the distributive property.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
use the commutative and associative properties.			
use the identity and inverse properties of addition and multiplication.			
use the properties of zero.			
simplify expressions using the distributive property.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

additive identity

The additive identity is the number 0; adding 0 to any number does not change its value.

additive inverse

The opposite of a number is its additive inverse. A number and it additive inverse add to 0.

multiplicative identity

The multiplicative identity is the number 1; multiplying 1 by any number does not change the value of the number.

multiplicative inverse

The reciprocal of a number is its multiplicative inverse. A number and its multiplicative inverse multiply to one.

Systems of Measurement
By the end of this section, you will be able to:

- Make unit conversions in the US system
- Use mixed units of measurement in the US system
- Make unit conversions in the metric system
- Use mixed units of measurement in the metric system
- Convert between the US and the metric systems of measurement
- Convert between Fahrenheit and Celsius temperatures

Make Unit Conversions in the U.S. System

There are two systems of measurement commonly used around the world. Most countries use the metric system. The U.S. uses a different system of measurement, usually called the **U.S. system**. We will look at the U.S. system first.

The U.S. system of measurement uses units of inch, foot, yard, and mile to measure length and pound and ton to measure weight. For capacity, the units used are cup, pint, quart, and gallons. Both the U.S. system and the metric system measure time in seconds, minutes, and hours.

The equivalencies of measurements are shown in [link]. The table also shows, in parentheses, the common abbreviations for each measurement.

U.S. System of Measurement							
Length	1 foot (ft.)	=	12 inches (in.)	**Volume**	3 teaspoons (t)	=	1 tablespoo
	1 yard (yd.)	=	3 feet (ft.)		16 tablespoons (T)	=	1 cup (C)
	1 mile (mi.)	=	5,280 feet (ft.)		1 cup (C)	=	8 fluid oun
					1 pint (pt.)	=	2 cups (C)
					1 quart (qt.)	=	2 pints (pt.
					1 gallon (gal)	=	4 quarts (q
Weight	1 pound (lb.)	=	16 ounces (oz.)	**Time**	1 minute (min)	=	60 seconds (sec
	1 ton	=	2000 pounds (lb.)		1 hour (hr)	=	60 minutes (mi
					1 day	=	24 hours (hr)
					1 week (wk)	=	7 days
					1 year (yr)	=	365 days

In many real-life applications, we need to convert between units of measurement, such as feet and yards, minutes and seconds, quarts and gallons, etc. We will use the identity property of multiplication to do these conversions. We'll restate the identity property of multiplication here for easy reference.

Note:
Identity Property of Multiplication
For any real number a : $a \cdot 1 = a \qquad 1 \cdot a = a$
$\mathbf{1}$ is the **multiplicative identity**

To use the identity property of multiplication, we write 1 in a form that will help us convert the units. For example, suppose we want to change inches to feet. We know that 1 foot is equal to 12 inches, so we will write 1 as the fraction $\frac{1\text{ foot}}{12\text{ inches}}$. When we multiply by this fraction we do not change the value, but just change the units.

But $\frac{12\text{ inches}}{1\text{ foot}}$ also equals 1. How do we decide whether to multiply by $\frac{1\text{ foot}}{12\text{ inches}}$ or $\frac{12\text{ inches}}{1\text{ foot}}$? We choose the fraction that will make the units we want to convert *from* divide out. Treat the unit words like factors and “divide out” common units like we do common factors. If we want to convert 66 inches to feet, which multiplication will eliminate the inches?

$$66\text{ inches}\cdot\frac{1\text{ foot}}{12\text{ inches}} \quad \text{or} \quad \cancel{66\text{ inches}\cdot\frac{12\text{ inches}}{1\text{ foot}}}$$

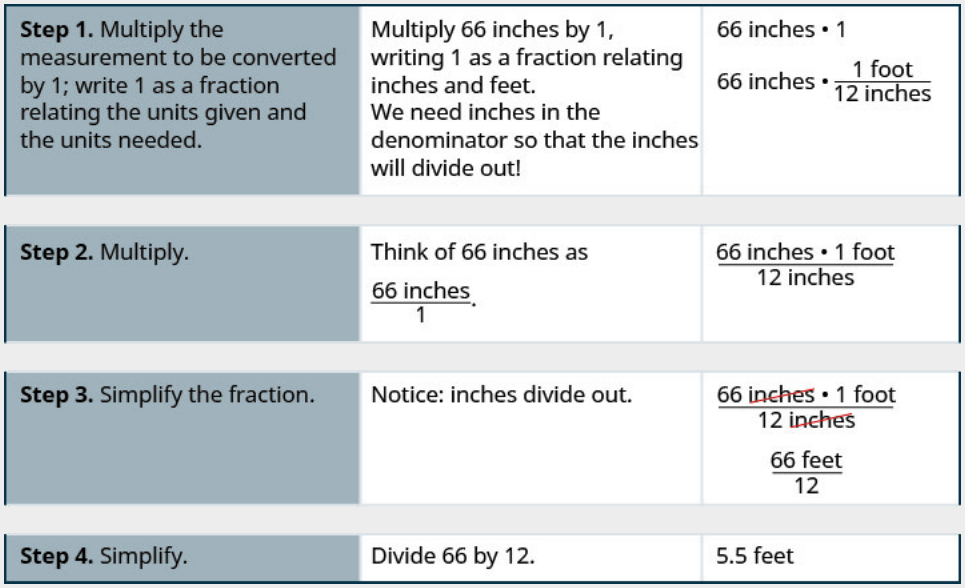
The first form works since $66\ \cancel{\text{inches}}\cdot\frac{1\text{ foot}}{12\ \cancel{\text{inches}}}$.

The inches divide out and leave only feet. The second form does not have any units that will divide out and so will not help us.

Example:
How to Make Unit Conversions
Exercise:

Problem: **MaryAnne is 66 inches tall. Convert her height into feet.

Solution:
Solution



Step 1. Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.	Multiply 66 inches by 1, writing 1 as a fraction relating inches and feet. We need inches in the denominator so that the inches will divide out!	$66\text{ inches}\cdot 1$ $66\text{ inches}\cdot\frac{1\text{ foot}}{12\text{ inches}}$
Step 2. Multiply.	Think of 66 inches as $\frac{66\text{ inches}}{1}$.	$\frac{66\text{ inches}\cdot 1\text{ foot}}{12\text{ inches}}$
Step 3. Simplify the fraction.	Notice: inches divide out.	$\frac{66\ \cancel{\text{inches}}\cdot 1\text{ foot}}{12\ \cancel{\text{inches}}}$ $\frac{66\text{ feet}}{12}$
Step 4. Simplify.	Divide 66 by 12.	5.5 feet

Note:
Exercise:

TRY IT :: 1.279
Problem: Lexie is 30 inches tall. Convert her height to feet.

Solution:

2.5 feet

Note:
Exercise:

TRY IT :: 1.280
Problem: Rene bought a hose that is 18 yards long. Convert the length to feet.

Solution:

54 feet

Note:
Make Unit Conversions.

Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.
Multiply.
Simplify the fraction.
Simplify.

When we use the identity property of multiplication to convert units, we need to make sure the units we want to change from will divide out. Usually this means we want the conversion fraction to have those units in the denominator.

Example:
Exercise:

Problem:

**Ndula, an elephant at the San Diego Safari Park, weighs almost 3.2 tons. Convert her weight to pounds.

Solution:
Solution

We will convert 3.2 tons into pounds. We will use the identity property of multiplication, writing 1 as the fraction $\frac{2000\,\text{pounds}}{1\,\text{ton}}$.

	3.2 tons
Multiply the measurement to be converted, by 1.	$3.2 \text{ tons} \cdot 1$
Write 1 as a fraction relating tons and pounds.	$3.2 \text{ tons} \cdot \frac{2{,}000 \text{ pounds}}{1 \text{ ton}}$
Simplify.	$\frac{3.2 \cancel{\text{tons}} \cdot 2{,}000 \text{ pounds}}{1 \cancel{\text{ton}}}$
Multiply.	6,400 pounds
	Ndula weighs almost 6,400 pounds.

Note:
Exercise:

TRY IT :: 1.281
Problem: Arnold’s SUV weighs about 4.3 tons. Convert the weight to pounds.

Solution:

8,600 pounds

Note:
Exercise:

TRY IT :: 1.282
Problem: The Carnival *Destiny* cruise ship weighs 51,000 tons. Convert the weight to pounds.

Solution:

102,000,000 pounds

Sometimes, to convert from one unit to another, we may need to use several other units in between, so we will need to multiply several fractions.

Example:
Exercise:

Problem:

**Juliet is going with her family to their summer home. She will be away from her boyfriend for 9 weeks. Convert the time to minutes.

Solution:
Solution

To convert weeks into minutes we will convert weeks into days, days into hours, and then hours into minutes. To do this we will multiply by conversion factors of 1.

	9 weeks
Write 1 as $\frac{7\text{ days}}{1\text{ week}}$, $\frac{24\text{ hours}}{1\text{ day}}$, and $\frac{60\text{ minutes}}{1\text{ hour}}$.	$\frac{9\text{ wk}}{1}\cdot\frac{7\text{ days}}{1\text{ wk}}\cdot\frac{24\text{ hr}}{1\text{ day}}\cdot\frac{60\text{ min}}{1\text{ hr}}$
Divide out the common units.	$\frac{9\,\cancel{\text{wk}}}{1}\cdot\frac{7\,\cancel{\text{days}}}{1\,\cancel{\text{wk}}}\cdot\frac{24\,\cancel{\text{hr}}}{1\,\cancel{\text{day}}}\cdot\frac{60\text{ min}}{1\,\cancel{\text{hr}}}$
Multiply.	$\frac{9\cdot7\cdot24\cdot60\text{ min}}{1\cdot1\cdot1\cdot1}$
Multiply.	90,720 min

Juliet and her boyfriend will be apart for 90,720 minutes (although it may seem like an eternity!).

Note:
Exercise:

TRY IT :: 1.283

Problem: The distance between the earth and the moon is about 250,000 miles. Convert this length to yards.

Solution:

440,000,000 yards

Note:
Exercise:

Problem:

TRY IT :: 1.284

The astronauts of Expedition 28 on the International Space Station spend 15 weeks in space. Convert the time to minutes.

Solution:

151,200 minutes

Example:
Exercise:

Problem: **How many ounces are in 1 gallon?

Solution:
Solution

We will convert gallons to ounces by multiplying by several conversion factors. Refer to [link].

	1 gallon
Multiply the measurement to be converted by 1.	$\frac{1\text{ gallon}}{1}\cdot\frac{4\text{ quarts}}{1\text{ gallon}}\cdot\frac{2\text{ pints}}{1\text{ quart}}\cdot\frac{2\text{ cups}}{1\text{ pint}}\cdot\frac{8\text{ ounces}}{1\text{ cup}}$
Use conversion factors to get to the right unit. Simplify.	$\frac{1\cancel{\text{ gallon}}}{1}\cdot\frac{4\cancel{\text{ quarts}}}{1\cancel{\text{ gallon}}}\cdot\frac{2\cancel{\text{ pints}}}{1\cancel{\text{ quart}}}\cdot\frac{2\cancel{\text{ cups}}}{1\cancel{\text{ pint}}}\cdot\frac{8\text{ ounces}}{1\cancel{\text{ cup}}}$
Multiply.	$\frac{1\cdot4\cdot2\cdot2\cdot8\text{ ounces}}{1\cdot1\cdot1\cdot1\cdot1}$
Simplify.	128 ounces There are 128 ounces in a gallon.

Note:
Exercise:

TRY IT :: 1.285
Problem: How many cups are in 1 gallon?

Solution:

16 cups

Note:
Exercise:

TRY IT :: 1.286
Problem: How many teaspoons are in 1 cup?

Solution:

48 teaspoons

Use Mixed Units of Measurement in the U.S. System

We often use mixed units of measurement in everyday situations. Suppose Joe is 5 feet 10 inches tall, stays at work for 7 hours and 45 minutes, and then eats a 1 pound 2 ounce steak for dinner—all these measurements have mixed units.

Performing arithmetic operations on measurements with mixed units of measures requires care. Be sure to add or subtract like units!

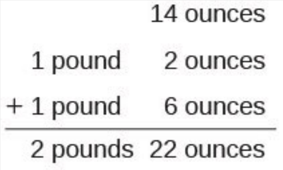
Example:
Exercise:

Problem:

**Seymour bought three steaks for a barbecue. Their weights were 14 ounces, 1 pound 2 ounces and 1 pound 6 ounces. How many total pounds of steak did he buy?

Solution:
Solution

We will add the weights of the steaks to find the total weight of the steaks.

Add the ounces. Then add the pounds.	 14 ounces 1 pound 2 ounces + 1 pound 6 ounces 2 pounds 22 ounces
Convert 22 ounces to pounds and ounces.	2 pounds + 1 pound, 6 ounces
Add the pounds.	3 pounds, 6 ounces
	Seymour bought 3 pounds 6 ounces of steak.

Note:
Exercise:

Problem:

TRY IT :: 1.287
Laura gave birth to triplets weighing 3 pounds 12 ounces, 3 pounds 3 ounces, and 2 pounds 9 ounces. What was the total birth weight of the three babies?

Solution:

9 lbs. 8 oz

Note:
Exercise:

Problem:

TRY IT :: 1.288
Stan cut two pieces of crown molding for his family room that were 8 feet 7 inches and 12 feet 11 inches. What was the total length of the molding?

Solution:

21 ft. 6 in.

Example:
Exercise:

Problem:

**Anthony bought four planks of wood that were each 6 feet 4 inches long. What is the total length of the wood he purchased?

Solution:
Solution

We will multiply the length of one plank to find the total length.

Multiply the inches and then the feet.	$\begin{array}{ll} \text{6 feet} & \text{4 inches} \\ \times & 4 \\ \hline \text{24 feet} & \text{16 inches} \end{array}$
Convert the 16 inches to feet. Add the feet.	$\begin{array}{l} \text{24 feet + 1 foot 4 inches} \\ \hline \text{25 feet 4 inches} \end{array}$
	Anthony bought 25 feet and 4 inches of wood.

Note:
Exercise:

Problem:

TRY IT :: 1.289
Henri wants to triple his spaghetti sauce recipe that uses 1 pound 8 ounces of ground turkey. How many pounds of ground turkey will he need?

Solution:

4 lbs. 8 oz.

Note:
Exercise:

Problem:

TRY IT :: 1.290
Joellen wants to double a solution of 5 gallons 3 quarts. How many gallons of solution will she have in all?

Solution:

11 gallons 2 qt.

Make Unit Conversions in the Metric System

In the **metric system**, units are related by powers of 10. The roots words of their names reflect this relation. For example, the basic unit for measuring length is a meter. One kilometer is 1,000 meters; the prefix *kilo* means *thousand*. One centimeter is $\frac{1}{100}$ of a meter, just like one cent is $\frac{1}{100}$ of one dollar.

The equivalencies of measurements in the metric system are shown in [link]. The common abbreviations for each measurement are given in parentheses.

Metric System of Measurement		
Length	**Mass**	**Capacity**

Metric System of Measurement		
Length	**Mass**	**Capacity**
1 kilometer (km) = 1,000 m	1 kilogram (kg) = 1,000 g	1 kiloliter (kL) = 1,000 L
1 hectometer (hm) = 100 m	1 hectogram (hg) = 100 g	1 hectoliter (hL) = 100 L
1 dekameter (dam) = 10 m	1 dekagram (dag) = 10 g	1 dekaliter (daL) = 10 L
1 meter (m) = 1 m	1 gram (g) = 1 g	1 liter (L) = 1 L
1 decimeter (dm) = 0.1 m	1 decigram (dg) = 0.1 g	1 deciliter (dL) = 0.1 L
1 centimeter (cm) = 0.01 m	1 centigram (cg) = 0.01 g	1 centiliter (cL) = 0.01 L
1 millimeter (mm) = 0.001 m	1 milligram (mg) = 0.001 g	1 milliliter (mL) = 0.001 L
1 meter = 100 centimeters	1 gram = 100 centigrams	1 liter = 100 centiliters
1 meter = 1,000 millimeters	1 gram = 1,000 milligrams	1 liter = 1,000 milliliters

To make conversions in the metric system, we will use the same technique we did in the US system. Using the identity property of multiplication, we will multiply by a conversion factor of one to get to the correct units.

Have you ever run a 5K or 10K race? The length of those races are measured in kilometers. The metric system is commonly used in the United States when talking about the length of a race.

Example:
Exercise:

Problem: **Nick ran a 10K race. How many meters did he run?

Solution:
Solution

We will convert kilometers to meters using the identity property of multiplication.

	10 kilometers
Multiply the measurement to be converted by 1.	$10 \text{ kilometers} \cdot 1$
Write 1 as a fraction relating kilometers and meters.	$10 \text{ kilometers} \cdot \frac{1{,}000 \text{ meters}}{1 \text{ kilometers}}$
Simplify.	

	$\frac{10\ \cancel{\text{kilometers}} \cdot 1{,}000\ \text{m}}{1\ \cancel{\text{kilometers}}}$
Multiply.	10,000 meters
	Nick ran 10,000 meters.

Note:
Exercise:

TRY IT :: 1.291

Problem: Sandy completed her first 5K race! How many meters did she run?

Solution:

5,000 meters

Note:
Exercise:

TRY IT :: 1.292

Problem: Herman bought a rug 2.5 meters in length. How many centimeters is the length?

Solution:

250 centimeters

Example:
Exercise:

Problem: **Eleanor’s newborn baby weighed 3,200 grams. How many kilograms did the baby weigh?

Solution:
Solution

We will convert grams into kilograms.

	$3{,}200\ \text{grams}$
Multiply the measurement to be converted by 1.	$3{,}200\ \text{grams} \cdot 1$

Write 1 as a function relating kilograms and grams.	$3{,}200\text{ grams}\cdot\frac{1\text{ kg}}{1{,}000\text{ grams}}$
Simplify.	$3{,}200\ \cancel{\text{grams}}\cdot\frac{1\text{ kg}}{1{,}000\ \cancel{\text{grams}}}$
Multiply.	$\frac{3{,}200\text{ kilograms}}{1{,}000}$
Divide.	3.2 kilograms The baby weighed 3.2 kilograms.

Note:
Exercise:

TRY IT :: 1.293
Problem: Kari’s newborn baby weighed 2,800 grams. How many kilograms did the baby weigh?

Solution:

2.8 kilograms

Note:
Exercise:

Problem:

TRY IT :: 1.294
Anderson received a package that was marked 4,500 grams. How many kilograms did this package weigh?

Solution:

4.5 kilograms

As you become familiar with the metric system you may see a pattern. Since the system is based on multiples of ten, the calculations involve multiplying by multiples of ten. We have learned how to simplify these calculations by just moving the decimal.

To multiply by 10, 100, or 1,000, we move the decimal to the right one, two, or three places, respectively. To multiply by 0.1, 0.01, or 0.001, we move the decimal to the left one, two, or three places, respectively.

We can apply this pattern when we make measurement conversions in the metric system. In [link], we changed 3,200 grams to kilograms by multiplying by $\frac{1}{1000}$ (or 0.001). This is the same as moving the decimal three places to the left.

$3{,}200 \cdot \frac{1}{1{,}000}$	$3{,}200.$
3.2	3.2

Example:
Exercise:

Problem: **Convert ⓐ 350 L to kiloliters ⓑ 4.1 L to milliliters.

Solution:
Solution

ⓐ We will convert liters to kiloliters. In [link], we see that 1 kiloliter $=$ 1,000 liters.

	350 L
Multiply by 1, writing 1 as a fraction relating liters to kiloliters.	$350\text{ L} \cdot \frac{1\text{ kL}}{1{,}000\text{ L}}$
Simplify.	$350\,\cancel{\text{L}} \cdot \frac{1\text{ kL}}{1{,}000\,\cancel{\text{L}}}$
Move the decimal 3 units to the left. ($350.$)	0.35 kL

ⓑ We will convert liters to milliliters. From [link] we see that 1 liter $=$ 1,000 milliliters.

	4.1 L
Multiply by 1, writing 1 as a fraction relating liters to milliliters.	$4.1\text{ L} \cdot \frac{1{,}000\text{ mL}}{1\text{ L}}$
Simplify.	$4.1\,\cancel{\text{L}} \cdot \frac{1{,}000\text{ mL}}{1\,\cancel{\text{L}}}$
Move the decimal 3 units to the right.	4.100 mL

	4,100 mL

Note:
Exercise:

TRY IT :: 1.295
Problem: Convert: ⓐ 725 L to kiloliters ⓑ 6.3 L to milliliters

Solution:

ⓐ 0.725 kiloliters ⓑ 6,300 milliliters

Note:
Exercise:

TRY IT :: 1.296
Problem: Convert: ⓐ 350 hL to liters ⓑ 4.1 L to centiliters

Solution:

ⓐ 35,000 liters ⓑ 410 centiliters

Use Mixed Units of Measurement in the Metric System

Performing arithmetic operations on measurements with mixed units of measures in the metric system requires the same care we used in the US system. But it may be easier because of the relation of the units to the powers of 10. Make sure to add or subtract like units.

Example:
Exercise:

Problem:

**Ryland is 1.6 meters tall. His younger brother is 85 centimeters tall. How much taller is Ryland than his younger brother? Write the answer is meters.

Solution:
Solution

We can convert both measurements to either centimeters or meters. Since meters is the larger unit, we will subtract the lengths in meters. We convert 85 centimeters to meters by moving the decimal 2 places to the left.

Write the 85 centimeters as meters.	$$\begin{array}{r} 1.60\text{ m} \\ \underline{-0.85\text{ m}} \\ 0.75\text{ m} \end{array}$$

Ryland is 0.75 m taller than his brother.

Note:
Exercise:

Problem:

TRY IT :: 1.297
Mariella is 1.58 meters tall. Her daughter is 75 centimeters tall. How much taller is Mariella than her daughter? Write the answer in centimeters.

Solution:

83 centimeters

Note:
Exercise:

Problem:

TRY IT :: 1.298
The fence around Hank’s yard is 2 meters high. Hank is 96 centimeters tall. How much shorter than the fence is Hank? Write the answer in meters.

Solution:

1.04 meters

Example:
Exercise:

Problem:

**Dena’s recipe for lentil soup calls for 150 milliliters of olive oil. Dena wants to triple the recipe. How many liters of olive oil will she need?

Solution:
Solution

We will find the amount of olive oil in millileters then convert to liters.

	Triple 150 mL
Translate to algebra.	$3 \cdot 150\ \text{mL}$
Multiply.	$450\ \text{mL}$
Convert to liters.	$450 \cdot \frac{0.001\ \text{L}}{1\ \text{mL}}$
Simplify.	$0.45\ \text{L}$
	Dena needs 0.45 liters of olive oil.

Note:
Exercise:

Problem:

TRY IT :: 1.299
A recipe for Alfredo sauce calls for 250 milliliters of milk. Renata is making pasta with Alfredo sauce for a big party and needs to multiply the recipe amounts by 8. How many liters of milk will she need?

Solution:

2 liters

Note:
Exercise:

Problem:

TRY IT :: 1.300
To make one pan of baklava, Dorothea needs 400 grams of filo pastry. If Dorothea plans to make 6 pans of baklava, how many kilograms of filo pastry will she need?

Solution:

2.4 kilograms

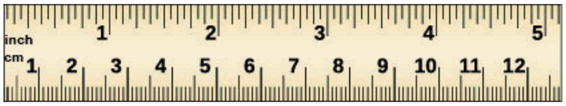
Convert Between the U.S. and the Metric Systems of Measurement

Many measurements in the United States are made in metric units. Our soda may come in 2-liter bottles, our calcium may come in 500-mg capsules, and we may run a 5K race. To work easily in both systems, we need to be able to convert between the two systems.

[link] shows some of the most common conversions.

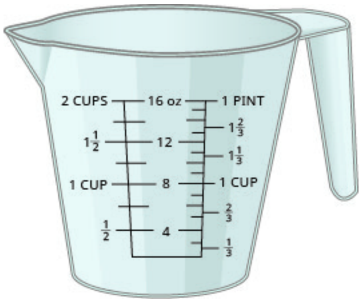
Conversion Factors Between U.S. and Metric Systems		
Length	**Mass**	**Capacity**
1 in. = 2.54 cm 1 ft. = 0.305 m 1 yd. = 0.914 m 1 mi. = 1.61 km 1 m = 3.28 ft.	1 lb. = 0.45 kg 1 oz. = 28 g 1 kg = 2.2 lb.	1 qt. = 0.95 L 1 fl. oz. = 30 mL 1 L = 1.06 qt.

[link] shows how inches and centimeters are related on a ruler.



This ruler shows inches and centimeters.

[link] shows the ounce and milliliter markings on a measuring cup.



This measuring cup shows ounces and milliliters.

[link] shows how pounds and kilograms marked on a bathroom scale.

This scale shows pounds and kilograms.

We make conversions between the systems just as we do within the systems—by multiplying by unit conversion factors.

Example:
Exercise:

Problem:

**Lee's water bottle holds 500 mL of water. How many ounces are in the bottle? Round to the nearest tenth of an ounce.

Solution:
Solution

	500 mL
Multiply by a unit conversion factor relating mL and ounces.	$500\text{ milliliters}\cdot\frac{1\text{ ounce}}{30\text{ milliliters}}$
Simplify.	$\frac{50\text{ ounce}}{30}$
Divide.	16.7 ounces.
	The water bottle has 16.7 ounces.

Note:
Exercise:

TRY IT :: 1.301
Problem: How many quarts of soda are in a 2-L bottle?

Solution:

2.12 quarts

Note:
Exercise:

TRY IT :: 1.302
Problem: How many liters are in 4 quarts of milk?

Solution:

3.8 liters

Example:
Exercise:

Problem:

**Soleil was on a road trip and saw a sign that said the next rest stop was in 100 kilometers. How many miles until the next rest stop (rounded to the nearest mile)?

Solution:
Solution

	100 kilometers
Multiply by a unit conversion factor relating km and mi.	$100 \text{ kilometers} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometer}}$
Simplify.	$\frac{100 \text{ miles}}{1.61}$
Divide.	$62 \text{ miles}.$
	Soleil will travel 62 miles.

Note:
Exercise:

TRY IT :: 1.303
Problem: The height of Mount Kilimanjaro is 5,895 meters. Convert the height to feet.

Solution:

19,335.6 feet

Note:
Exercise:

Problem:

TRY IT :: 1.304
The flight distance from New York City to London is 5,586 kilometers. Convert the distance to miles (rounded to the nearest mile).

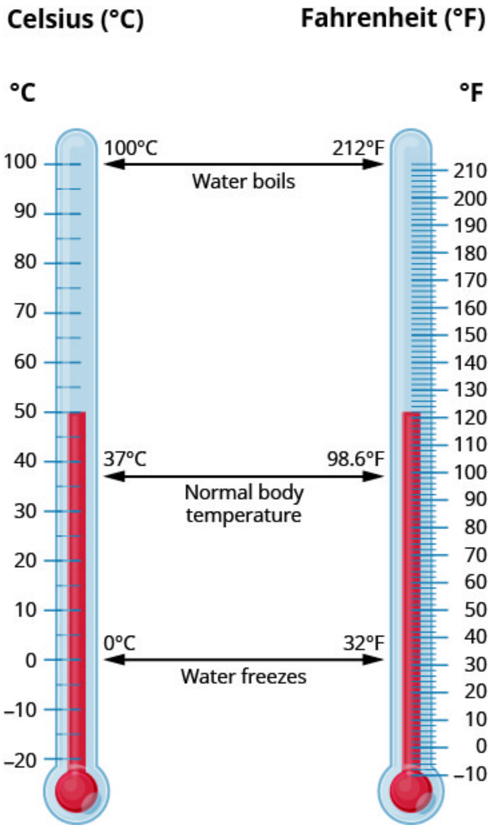
Solution:

8,993.46 km

Convert between Fahrenheit and Celsius Temperatures

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for 22°C, what does that mean?

The U.S. and metric systems use different scales to measure temperature. The U.S. system uses degrees Fahrenheit, written $^\circ\text{F}$. The metric system uses degrees Celsius, written $^\circ\text{C}$. [link] shows the relationship between the two systems.



The diagram shows normal body temperature, along with the freezing and boiling temperatures of water in degrees Fahrenheit and degrees Celsius.

Note:
Temperature Conversion
To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula
Equation:

$$C = \frac{5}{9}(F - 32).$$

To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula
Equation:

$$F = \frac{9}{5}C + 32.$$

Example:
Exercise:

Problem: **Convert 50˚ Fahrenheit into degrees Celsius.

Solution:
Solution

We will substitute 50˚F into the formula to find C.

	$C = \frac{5}{9}(F - 32)$
Substitute 50 for F.	$C = \frac{5}{9}(50 - 32)$
Simplify in parentheses.	$C = \frac{5}{9}(18)$
Multiply.	$C = 10$

	So we found that 50°F is equivalent to 10°C.

Note:
Exercise:

TRY IT :: 1.305
Problem: Convert the Fahrenheit temperature to degrees Celsius: 59° Fahrenheit.

Solution:

$15^{\circ}\mathrm{C}$

Note:
Exercise:

TRY IT :: 1.306
Problem: Convert the Fahrenheit temperature to degrees Celsius: 41° Fahrenheit.

Solution:

$5^{\circ}\mathrm{C}$

Example:
Exercise:

Problem:

**While visiting Paris, Woody saw the temperature was 20° Celsius. Convert the temperature into degrees Fahrenheit.

Solution:
Solution

We will substitute $20^{\circ}\mathrm{C}$ into the formula to find F.

	$F=\frac{9}{5}C+32$
Substitute 20 for C.	$F=\frac{9}{5}(20)+32$

Multiply.	$F = 36 + 32$
Add.	$F = 68$
	So we found that 20°C is equivalent to 68°F.

Note:
Exercise:

Problem:

TRY IT :: 1.307
Convert the Celsius temperature to degrees Fahrenheit: the temperature in Helsinki, Finland, was 15° Celsius.

Solution:

59°F

Note:
Exercise:

Problem:

TRY IT :: 1.308
Convert the Celsius temperature to degrees Fahrenheit: the temperature in Sydney, Australia, was 10° Celsius.

Solution:

50°F

Key Concepts

- **Metric System of Measurement**
  - **Length**
    1 kilometer (km) = 1,000 m
    1 hectometer (hm) = 100 m
    1 dekameter (dam) = 10 m
    1 meter (m) = 1 m
    1 decimeter (dm) = 0.1 m
    1 centimeter (cm) = 0.01 m
    1 millimeter (mm) = 0.001 m
    1 meter = 100 centimeters
    1 meter = 1,000 millimeters

  - **Mass**
    1 kilogram (kg) = 1,000 g
    1 hectogram (hg) = 100 g
    1 dekagram (dag) = 10 g
    1 gram (g) = 1 g
    1 decigram (dg) = 0.1 g
    1 centigram (cg) = 0.01 g
    1 milligram (mg) = 0.001 g
    1 gram = 100 centigrams
    1 gram = 1,000 milligrams
  - **Capacity**
    1 kiloliter (kL) = 1,000 L
    1 hectoliter (hL) = 100 L
    1 dekaliter (daL) = 10 L
    1 liter (L) = 1 L
    1 deciliter (dL) = 0.1 L
    1 centiliter (cL) = 0.01 L
    1 milliliter (mL) = 0.001 L
    1 liter = 100 centiliters
    1 liter = 1,000 milliliters

- **Temperature Conversion**
  - To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula $\mathrm{C} = \frac{5}{9}(\mathrm{F} - 32)$
  - To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula $\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$

Section Exercises

Practice Makes Perfect

Make Unit Conversions in the U.S. System

In the following exercises, convert the units.

Exercise:

Problem: A park bench is 6 feet long. Convert the length to inches.

Solution:

72 inches

Exercise:

Problem: A floor tile is 2 feet wide. Convert the width to inches.

Exercise:

Problem: A ribbon is 18 inches long. Convert the length to feet.

Solution:

1.5 feet

Exercise:

Problem: Carson is 45 inches tall. Convert his height to feet.

Exercise:

Problem: A football field is 160 feet wide. Convert the width to yards.

Solution:

$53\frac{1}{3}$ yards

Exercise:

Problem:

On a baseball diamond, the distance from home plate to first base is 30 yards. Convert the distance to feet.

Exercise:

Problem: Ulises lives 1.5 miles from school. Convert the distance to feet.

Solution:

7,920 feet

Exercise:

Problem: Denver, Colorado, is 5,183 feet above sea level. Convert the height to miles.

Exercise:

Problem: A killer whale weighs 4.6 tons. Convert the weight to pounds.

Solution:

9,200 pounds

Exercise:

Problem: Blue whales can weigh as much as 150 tons. Convert the weight to pounds.

Exercise:

Problem: An empty bus weighs 35,000 pounds. Convert the weight to tons.

Solution:

$17\frac{1}{2}$ tons

Exercise:

Problem: At take-off, an airplane weighs 220,000 pounds. Convert the weight to tons.

Exercise:

Problem: Rocco waited $1\frac{1}{2}$ hours for his appointment. Convert the time to seconds.

Solution:

5,400 s

Exercise:

Problem: Misty’s surgery lasted $2\frac{1}{4}$ hours. Convert the time to seconds.

Exercise:

Problem: How many teaspoons are in a pint?

Solution:

96 teaspoons

Exercise:

Problem: How many tablespoons are in a gallon?

Exercise:

Problem: JJ’s cat, Posy, weighs 14 pounds. Convert her weight to ounces.

Solution:

224 ounces

Exercise:

Problem: April’s dog, Beans, weighs 8 pounds. Convert his weight to ounces.

Exercise:

Problem: Crista will serve 20 cups of juice at her son’s party. Convert the volume to gallons.

Solution:

$1\frac{1}{4}$ gallons

Exercise:

Problem: Lance needs 50 cups of water for the runners in a race. Convert the volume to gallons.

Exercise:

Problem: Jon is 6 feet 4 inches tall. Convert his height to inches.

Solution:

26 in.

Exercise:

Problem: Faye is 4 feet 10 inches tall. Convert her height to inches.

Exercise:

Problem: The voyage of the *Mayflower* took 2 months and 5 days. Convert the time to days.

Solution:

65 days

Exercise:

Problem: Lynn’s cruise lasted 6 days and 18 hours. Convert the time to hours.

Exercise:

Problem: Baby Preston weighed 7 pounds 3 ounces at birth. Convert his weight to ounces.

Solution:

115 ounces

Exercise:

Problem: Baby Audrey weighted 6 pounds 15 ounces at birth. Convert her weight to ounces.

Use Mixed Units of Measurement in the U.S. System

In the following exercises, solve.

Exercise:

Problem:

Eli caught three fish. The weights of the fish were 2 pounds 4 ounces, 1 pound 11 ounces, and 4 pounds 14 ounces. What was the total weight of the three fish?

Solution:

8 lbs. 13 oz.

Exercise:

Problem:

Judy bought 1 pound 6 ounces of almonds, 2 pounds 3 ounces of walnuts, and 8 ounces of cashews. How many pounds of nuts did Judy buy?

Exercise:

Problem:

One day Anya kept track of the number of minutes she spent driving. She recorded 45, 10, 8, 65, 20, and 35. How many hours did Anya spend driving?

Solution:

3.05 hours

Exercise:

Problem:

Last year Eric went on 6 business trips. The number of days of each was 5, 2, 8, 12, 6, and 3. How many weeks did Eric spend on business trips last year?

Exercise:

Problem:

Renee attached a 6 feet 6 inch extension cord to her computer’s 3 feet 8 inch power cord. What was the total length of the cords?

Solution:

10 ft. 2 in.

Exercise:

Problem:

Fawzi’s SUV is 6 feet 4 inches tall. If he puts a 2 feet 10 inch box on top of his SUV, what is the total height of the SUV and the box?

Exercise:

Problem:

Leilani wants to make 8 placemats. For each placemat she needs 18 inches of fabric. How many yards of fabric will she need for the 8 placemats?

Solution:

4 yards

Exercise:

Problem:

Mireille needs to cut 24 inches of ribbon for each of the 12 girls in her dance class. How many yards of ribbon will she need altogether?

Make Unit Conversions in the Metric System

In the following exercises, convert the units.

Exercise:

Problem: Ghalib ran 5 kilometers. Convert the length to meters.

Solution:

5,000 meters

Exercise:

Problem: Kitaka hiked 8 kilometers. Convert the length to meters.

Exercise:

Problem: Estrella is 1.55 meters tall. Convert her height to centimeters.

Solution:

155 centimeters

Exercise:

Problem: The width of the wading pool is 2.45 meters. Convert the width to centimeters.

Exercise:

Problem: Mount Whitney is 3,072 meters tall. Convert the height to kilometers.

Solution:

3.072 kilometers

Exercise:

Problem: The depth of the Mariana Trench is 10,911 meters. Convert the depth to kilometers.

Exercise:

Problem: June’s multivitamin contains 1,500 milligrams of calcium. Convert this to grams.

Solution:

1.5 grams

Exercise:

Problem: A typical ruby-throated hummingbird weights 3 grams. Convert this to milligrams.

Exercise:

Problem: One stick of butter contains 91.6 grams of fat. Convert this to milligrams.

Solution:

91,600 milligrams

Exercise:

Problem: One serving of gourmet ice cream has 25 grams of fat. Convert this to milligrams.

Exercise:

Problem: The maximum mass of an airmail letter is 2 kilograms. Convert this to grams.

Solution:

2,000 grams

Exercise:

Problem: Dimitri’s daughter weighed 3.8 kilograms at birth. Convert this to grams.

Exercise:

Problem: A bottle of wine contained 750 milliliters. Convert this to liters.

Solution:

0.75 liters

Exercise:

Problem: A bottle of medicine contained 300 milliliters. Convert this to liters.

Use Mixed Units of Measurement in the Metric System

In the following exercises, solve.

Exercise:

Problem: Matthias is 1.8 meters tall. His son is 89 centimeters tall. How much taller is Matthias than his son?

Solution:

91 centimeters

Exercise:

Problem:

Stavros is 1.6 meters tall. His sister is 95 centimeters tall. How much taller is Stavros than his sister?

Exercise:

Problem:

A typical dove weighs 345 grams. A typical duck weighs 1.2 kilograms. What is the difference, in grams, of the weights of a duck and a dove?

Solution:

855 grams

Exercise:

Problem:

Concetta had a 2-kilogram bag of flour. She used 180 grams of flour to make biscotti. How many kilograms of flour are left in the bag?

Exercise:

Problem:

Harry mailed 5 packages that weighed 420 grams each. What was the total weight of the packages in kilograms?

Solution:

2.1 kilograms

Exercise:

Problem:

One glass of orange juice provides 560 milligrams of potassium. Linda drinks one glass of orange juice every morning. How many grams of potassium does Linda get from her orange juice in 30 days?

Exercise:

Problem:

Jonas drinks 200 milliliters of water 8 times a day. How many liters of water does Jonas drink in a day?

Solution:

1.6 liters

Exercise:

Problem:

One serving of whole grain sandwich bread provides 6 grams of protein. How many milligrams of protein are provided by 7 servings of whole grain sandwich bread?

Convert Between the U.S. and the Metric Systems of Measurement

In the following exercises, make the unit conversions. Round to the nearest tenth.

Exercise:

Problem: Bill is 75 inches tall. Convert his height to centimeters.

Solution:

190.5 centimeters

Exercise:

Problem: Frankie is 42 inches tall. Convert his height to centimeters.

Exercise:

Problem: Marcus passed a football 24 yards. Convert the pass length to meters

---

Solution:

21.9 meters

Exercise:

Problem: Connie bought 9 yards of fabric to make drapes. Convert the fabric length to meters.

Exercise:

Problem:

Each American throws out an average of 1,650 pounds of garbage per year. Convert this weight to kilograms.

Solution:

742.5 kilograms

Exercise:

Problem:

An average American will throw away 90,000 pounds of trash over his or her lifetime. Convert this weight to kilograms.

Exercise:

Problem: A 5K run is 5 kilometers long. Convert this length to miles.

Solution:

3.1 miles

Exercise:

Problem: Kathryn is 1.6 meters tall. Convert her height to feet.

Exercise:

Problem: Dawn's suitcase weighed 20 kilograms. Convert the weight to pounds.

Solution:

44 pounds

Exercise:

Problem: Jackson's backpack weighed 15 kilograms. Convert the weight to pounds.

Exercise:

Problem: Ozzie put 14 gallons of gas in his truck. Convert the volume to liters.

Solution:

53.2 liters

Exercise:

Problem: Bernard bought 8 gallons of paint. Convert the volume to liters.

Convert between Fahrenheit and Celsius Temperatures

In the following exercises, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.

Exercise:

Problem: 86° Fahrenheit

Solution:

$30^\circ C$

Exercise:

Problem: 77° Fahrenheit

Exercise:

Problem: 104° Fahrenheit

Solution:

$40^\circ C$

Exercise:

Problem: 14° Fahrenheit

Exercise:

Problem: 72° Fahrenheit

Solution:

22.2°C

Exercise:

Problem: 4° Fahrenheit

Exercise:

Problem: 0° Fahrenheit

Solution:

-17.8°C

Exercise:

Problem: 120° Fahrenheit

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

Exercise:

Problem: 5° Celsius

Solution:

41°F

Exercise:

Problem: 25° Celsius

Exercise:

Problem: -10° Celsius

Solution:

14°F

Exercise:

Problem: -15° Celsius

Exercise:

Problem: 22° Celsius

Solution:

71.6°F

Exercise:

Problem: 8° Celsius

Exercise:

Problem: 43° Celsius

Solution:

$109.4^\circ F$

Exercise:

Problem: 16° Celsius

Everyday Math

Exercise:

Problem:

Nutrition Julian drinks one can of soda every day. Each can of soda contains 40 grams of sugar. How many kilograms of sugar does Julian get from soda in 1 year?

Solution:

14.6 kilograms

Exercise:

Problem:

Reflectors The reflectors in each lane-marking stripe on a highway are spaced 16 yards apart. How many reflectors are needed for a one mile long lane-marking stripe?

Writing Exercises

Exercise:

Problem: Some people think that 65° to 75° Fahrenheit is the ideal temperature range.

ⓐ What is your ideal temperature range? Why do you think so?
ⓑ Convert your ideal temperatures from Fahrenheit to Celsius.

Solution:

Answers may vary.

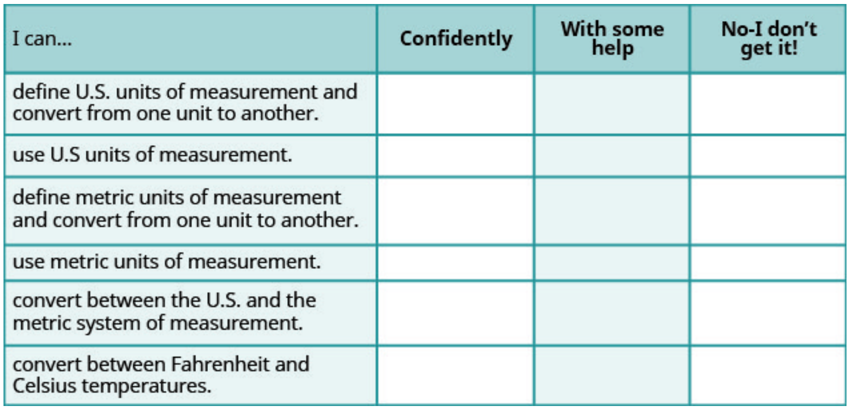
Exercise:

Problem:

ⓐ Did you grow up using the U.S. or the metric system of measurement?
ⓑ Describe two examples in your life when you had to convert between the two systems of measurement.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
define U.S. units of measurement and convert from one unit to another.			
use U.S units of measurement.			
define metric units of measurement and convert from one unit to another.			
use metric units of measurement.			
convert between the U.S. and the metric system of measurement.			
convert between Fahrenheit and Celsius temperatures.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Review & Practice Test
By the end of this section, you will be prepared to take your Chapter 1 Assessment.

Chapter Review Exercises

Introduction to Whole Numbers

Use Place Value with Whole Number

In the following exercises find the place value of each digit.

Exercise:

Problem: 26,915

ⓐ 1
ⓑ 2
ⓒ 9
ⓓ 5
ⓔ 6

Solution:

ⓐ tens ⓑ ten thousands ⓒ hundreds ⓓ ones ⓔ thousands

Exercise:

Problem: 359,417

ⓐ 9
ⓑ 3
ⓒ 4
ⓓ 7
ⓔ 1

Exercise:

Problem: 58,129,304

ⓐ 5
ⓑ 0
ⓒ 1
ⓓ 8
ⓔ 2

Solution:

ⓐ ten millions ⓑ tens ⓒ hundred thousands ⓓ millions ⓔ ten thousands

Exercise:

Problem: 9,430,286,157

ⓐ 6
ⓑ 4
ⓒ 9
ⓓ 0
ⓔ 5

In the following exercises, name each number.

Exercise:

Problem: 6,104

Solution:

six thousand, one hundred four

Exercise:

Problem: 493,068

Exercise:

Problem: 3,975,284

Solution:

three million, nine hundred seventy-five thousand, two hundred eighty-four

Exercise:

Problem: 85,620,435

In the following exercises, write each number as a whole number using digits.

Exercise:

Problem: three hundred fifteen

Solution:

315

Exercise:

Problem: sixty-five thousand, nine hundred twelve

Exercise:

Problem: ninety million, four hundred twenty-five thousand, sixteen

---

Solution:

90,425,016

Exercise:

Problem:

one billion, forty-three million, nine hundred twenty-two thousand, three hundred eleven

In the following exercises, round to the indicated place value.

Exercise:

Problem: Round to the nearest ten.

ⓐ 407 ⓑ 8,564

Solution:

ⓐ 410 ⓑ 8,560

Exercise:

Problem: Round to the nearest hundred.

ⓐ 25,846 ⓑ 25,864

In the following exercises, round each number to the nearest ⓐ hundred ⓑ thousand ⓒ ten thousand.

Exercise:

Problem: 864,951

Solution:

ⓐ 865,000 ⓑ 865,000 ⓒ 860,000

Exercise:

Problem: 3,972,849

Identify Multiples and Factors

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, by 3, by 5, by 6, and by 10.

Exercise:

Problem: 168

Solution:

by $2, 3, 6$

Exercise:

Problem: 264

Exercise:

Problem: 375

Solution:

by $3, 5$

Exercise:

Problem: 750

Exercise:

Problem: 1430

Solution:

by $2, 5, 10$

Exercise:

Problem: 1080

Find Prime Factorizations and Least Common Multiples

In the following exercises, find the prime factorization.

Exercise:

Problem: 420

Solution:

$2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$

Exercise:

Problem: 115

Exercise:

Problem: 225

Solution:

$3 \cdot 3 \cdot 5 \cdot 5$

Exercise:

Problem: 2475

Exercise:

Problem: 1560

Solution:

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 13$

Exercise:

Problem: 56

Exercise:

Problem: 72

Solution:

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

Exercise:

Problem: 168

Exercise:

Problem: 252

Solution:

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$

Exercise:

Problem: 391

In the following exercises, find the least common multiple of the following numbers using the multiples method.

Exercise:

Problem: 6,15

Solution:

30

Exercise:

Problem: 60, 75

In the following exercises, find the least common multiple of the following numbers using the prime factors method.

Exercise:

Problem: 24, 30

Solution:

120

Exercise:

Problem: 70, 84

Use the Language of Algebra

Use Variables and Algebraic Symbols

In the following exercises, translate the following from algebra to English.

Exercise:

Problem: $25-7$

Solution:

25 minus 7, the difference of twenty-five and seven

Exercise:

Problem: $5\cdot 6$

Exercise:

Problem: $45 \div 5$

Solution:

45 divided by 5, the quotient of forty-five and five

Exercise:

Problem: $x + 8$

Exercise:

Problem: $42 \geq 27$

Solution:

forty-two is greater than or equal to twenty-seven

Exercise:

Problem: $3n = 24$

Exercise:

Problem: $3 \leq 20 \div 4$

Solution:

3 is less than or equal to 20 divided by 4, three is less than or equal to the quotient of twenty and four

Exercise:

Problem: $a \neq 7 \cdot 4$

In the following exercises, determine if each is an expression or an equation.

Exercise:

Problem: $6 \cdot 3 + 5$

Solution:

expression

Exercise:

Problem: $y - 8 = 32$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify each expression.

Exercise:

Problem: 3^5

Solution:

243

Exercise:

Problem: 10^8

In the following exercises, simplify

Exercise:

Problem: $6 + 10/2 + 2$

Solution:

13

Exercise:

Problem: $9 + 12/3 + 4$

Exercise:

Problem: $20 \div (4 + 6) \cdot 5$

Solution:

10

Exercise:

Problem: $33 \div (3 + 8) \cdot 2$

Exercise:

Problem: $4^2 + 5^2$

Solution:

41

Exercise:

Problem: $(4 + 5)^2$

Evaluate an Expression

In the following exercises, evaluate the following expressions.

Exercise:

Problem: $9x + 7$ when $x = 3$

Solution:

34

Exercise:

Problem: $5x - 4$ when $x = 6$

Exercise:

Problem: x^4 when $x = 3$

Solution:

81

Exercise:

Problem: 3^x when $x = 3$

Exercise:

Problem: $x^2 + 5x - 8$ when $x = 6$

Solution:

58

Exercise:

Problem: $2x + 4y - 5$ when $x = 7, y = 8$

Simplify Expressions by Combining Like Terms

In the following exercises, identify the coefficient of each term.

Exercise:

Problem: $12n$

Solution:

12

Exercise:

Problem: $9x^2$

In the following exercises, identify the like terms.

Exercise:

Problem: $3n, n^2, 12, 12p^2, 3, 3n^2$

Solution:

12 and 3, n^2 and $3n^2$

Exercise:

Problem: $5, 18r^2, 9s, 9r, 5r^2, 5s$

In the following exercises, identify the terms in each expression.

Exercise:

Problem: $11x^2 + 3x + 6$

Solution:

$11x^2, 3x, 6$

Exercise:

Problem: $22y^3 + y + 15$

In the following exercises, simplify the following expressions by combining like terms.

Exercise:

Problem: $17a+9a$

Solution:

$26a$

Exercise:

Problem: $18z+9z$

Exercise:

Problem: $9x+3x+8$

Solution:

$12x+8$

Exercise:

Problem: $8a+5a+9$

Exercise:

Problem: $7p+6+5p-4$

Solution:

$12p+2$

Exercise:

Problem: $8x+7+4x-5$

Translate an English Phrase to an Algebraic Expression

In the following exercises, translate the following phrases into algebraic expressions.

Exercise:

Problem: the sum of 8 and 12

Solution:

$8+12$

Exercise:

Problem: the sum of 9 and 1

Exercise:

Problem: the difference of x and 4

Solution:

$x-4$

Exercise:

Problem: the difference of x and 3

Exercise:

Problem: the product of 6 and y

Solution:

$6y$

Exercise:

Problem: the product of 9 and y

Exercise:

Problem:

Adele bought a skirt and a blouse. The skirt cost \$15 more than the blouse. Let b represent the cost of the blouse. Write an expression for the cost of the skirt.

Solution:

$b+15$

Exercise:

Problem:

Marcella has 6 fewer boy cousins than girl cousins. Let g represent the number of girl cousins. Write an expression for the number of boy cousins.

Add and Subtract Integers

Use Negatives and Opposites of Integers

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise:

ⓐ 6 ____ 2
ⓑ -7 ____ 4
ⓒ -9 ____ -1
Problem: ⓓ 9 ____ -3

Solution:

ⓐ > ⓑ < ⓒ < ⓓ >

Exercise:

ⓐ -5 ___ 1
ⓑ -4 ___ -9
ⓒ 6 ___ 10
Problem: ⓓ 3 ___ -8

In the following exercises,, find the opposite of each number.
Exercise:

Problem: ⓐ -8 ⓑ 1

Solution:

ⓐ 8 ⓑ -1

Exercise:

Problem: ⓐ -2 ⓑ 6

In the following exercises, simplify.
Exercise:

Problem: $-(-19)$

Solution:

19

Exercise:

Problem: $-(-53)$

In the following exercises, simplify.
Exercise:

Problem: $-m$ when
ⓐ $m = 3$
ⓑ $m = -3$

Solution:

ⓐ -3 ⓑ 3

Exercise:

Problem: $-p$ when
ⓐ $p = 6$
ⓑ $p = -6$

Simplify Expressions with Absolute Value

In the following exercises,, simplify.

Exercise:

Problem: ⓐ $|7|$ ⓑ $|-25|$ ⓒ $|0|$

Solution:

ⓐ 7 ⓑ 25 ⓒ 0

Exercise:

Problem: ⓐ $|5|$ ⓑ $|0|$ ⓒ $|-19|$

In the following exercises, fill in <, >, or = for each of the following pairs of numbers.

Exercise:

Problem: ⓐ $-8 \; \underline{\hspace{2em}} \; |-8|$
ⓑ $-|-2| \; \underline{\hspace{2em}} \; -2$

Solution:

ⓐ < ⓑ =

Exercise:

Problem: ⓐ $|-3| \; \underline{\hspace{2em}} \; -|-3|$
ⓑ $4 \; \underline{\hspace{2em}} \; -|-4|$

In the following exercises, simplify.

Exercise:

Problem: $|8-4|$

Solution:

4

Exercise:

Problem: $|9-6|$

Exercise:

Problem: $8\left(14-2\left|-2\right|\right)$

Solution:

80

Exercise:

Problem: $6\left(13-4\left|-2\right|\right)$

In the following exercises, evaluate.

Exercise:

Problem: ⓐ $\left|x\right|$ when $x=-28$ ⓑ

Solution:

ⓐ 28 ⓑ 15

Exercise:

ⓐ $\left|y\right|$ when $y=-37$
Problem: ⓑ $\left|-z\right|$ when $z=-24$

Add Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-200+65$

Solution:

-135

Exercise:

Problem: $-150+45$

Exercise:

Problem: $2+\left(-8\right)+6$

Solution:

0

Exercise:

Problem: $4+(-9)+7$

Exercise:

Problem: $140+(-75)+67$

Solution:

132

Exercise:

Problem: $-32+24+(-6)+10$

Subtract Integers

In the following exercises, simplify.

Exercise:

Problem: $9-3$

Solution:

6

Exercise:

Problem: $-5-(-1)$

Exercise:

Problem: ⓐ $15-6$ ⓑ $15+(-6)$

Solution:

ⓐ 9 ⓑ 9

Exercise:

Problem: ⓐ $12 - 9$ ⓑ $12 + (-9)$

Exercise:

Problem: ⓐ $8 - (-9)$ ⓑ $8 + 9$

Solution:

ⓐ 17 ⓑ 17

Exercise:

Problem: ⓐ $4 - (-4)$ ⓑ $4 + 4$

In the following exercises, simplify each expression.

Exercise:

Problem: $10 - (-19)$

Solution:

29

Exercise:

Problem: $11 - (-18)$

Exercise:

Problem: $31 - 79$

Solution:

-48

Exercise:

Problem: $39 - 81$

Exercise:

Problem: $-31 - 11$

Solution:

-42

Exercise:

Problem: $-32 - 18$

Exercise:

Problem: $-15 - (-28) + 5$

Solution:

18

Exercise:

Problem: $71 + (-10) - 8$

Exercise:

Problem: $-16 - (-4 + 1) - 7$

Solution:

-20

Exercise:

Problem: $-15-(-6+4)-3$

Multiply and Divide Integers

Multiply Integers

In the following exercises, multiply.

Exercise:

Problem: $-5\left(7\right)$

Solution:

-35

Exercise:

Problem: $-8\left(6\right)$

Exercise:

Problem: $-18\left(-2\right)$

Solution:

36

Exercise:

Problem: $-10\left(-6\right)$

Divide Integers

In the following exercises, divide.

Exercise:

Problem: $-28 \div 7$

Solution:

-4

Exercise:

Problem: $56 \div (-7)$

Exercise:

Problem: $-120 \div (-20)$

Solution:

6

Exercise:

Problem: $-200 \div 25$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-8\,(-2) - 3\,(-9)$

Solution:

43

Exercise:

Problem: $-7\left(-4\right)-5\left(-3\right)$

Exercise:

Problem: $\left(-5\right)^{3}$

Solution:

-125

Exercise:

Problem: $\left(-4\right)^{3}$

Exercise:

Problem: $-4\cdot 2\cdot 11$

Solution:

-88

Exercise:

Problem: $-5\cdot 3\cdot 10$

Exercise:

Problem: $-10\left(-4\right)\div\left(-8\right)$

Solution:

-5

Exercise:

Problem: $-8\left(-6\right)\div\left(-4\right)$

Exercise:

Problem: $31-4\left(3-9\right)$

Solution:

55

Exercise:

Problem: $24-3\left(2-10\right)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem:
$x+8$ when
ⓐ $x=-26$
ⓑ $x=-95$

Solution:

ⓐ -18 ⓑ -87

Exercise:

Problem:
$y+9$ when
ⓐ $y=-29$
ⓑ $y=-84$

Exercise:

Problem:
When $b=-11$, evaluate:
ⓐ $b+6$
ⓑ $-b+6$

Solution:

ⓐ -5 ⓑ 17

Exercise:

Problem: When $c=-9$, evaluate:
ⓐ $c+(-4)$
ⓑ $-c+(-4)$

Exercise:

Problem: p^2-5p+2 when $p=-1$

Solution:

8

Exercise:

Problem: q^2-2q+9 when $q=-2$

Exercise:

Problem: $6x-5y+15$ when $x=3$ and $y=-1$

Solution:

38

Exercise:

Problem: $3p-2q+9$ when $p=8$ and $q=-2$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise:

Problem: the sum of -4 and -17, increased by 32

Solution:

$(-4+(-17))+32; 11$

Exercise:

Problem: ⓐ the difference of 15 and -7 ⓑ subtract 15 from -7

Exercise:

Problem: the quotient of -45 and -9

Solution:

$\frac{-45}{-9}; 5$

Exercise:

Problem: the product of -12 and the difference of c and d

Use Integers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature The high temperature one day in Miami Beach, Florida, was 76°. That same day, the high temperature in Buffalo, New York was -8°. What was the difference between the temperature in Miami Beach and the temperature in Buffalo?

Solution:

84 degrees

Exercise:

Problem:

Checking Account Adrianne has a balance of $-\$22$ in her checking account. She deposits $301 to the account. What is the new balance?

Visualize Fractions

Find Equivalent Fractions

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise:

Problem: $\frac{1}{4}$

Solution:

$\frac{2}{8}, \frac{3}{12}, \frac{4}{16}$ answers may vary

Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{5}{6}$

Solution:

$\frac{10}{12}, \frac{15}{18}, \frac{20}{24}$ answers may vary

Exercise:

Problem: $\frac{2}{7}$

Simplify Fractions

In the following exercises, simplify.

Exercise:

Problem: $\frac{7}{21}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{8}{24}$

Exercise:

Problem: $\frac{15}{20}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{12}{18}$

Exercise:

Problem: $-\frac{168}{192}$

Solution:

$-\frac{7}{8}$

Exercise:

Problem: $-\frac{140}{224}$

Exercise:

Problem: $\frac{11x}{11y}$

Solution:

$\frac{x}{y}$

Exercise:

Problem: $\frac{15a}{15b}$

Multiply Fractions

In the following exercises, multiply.

Exercise:

Problem: $\frac{2}{5} \cdot \frac{1}{3}$

Solution:

$\frac{2}{15}$

Exercise:

Problem: $\frac{1}{2} \cdot \frac{3}{8}$

Exercise:

Problem: $\frac{7}{12}\left(-\frac{8}{21}\right)$

Solution:

$-\frac{2}{9}$

Exercise:

Problem: $\frac{5}{12}\left(-\frac{8}{15}\right)$

Exercise:

Problem: $-28p\left(-\frac{1}{4}\right)$

Solution:

$7p$

Exercise:

Problem: $-51q\left(-\frac{1}{3}\right)$

Exercise:

Problem: $\frac{14}{5}(-15)$

Solution:

-42

Exercise:

Problem: $-1\left(-\frac{3}{8}\right)$

Divide Fractions

In the following exercises, divide.

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{4}$

Solution:

2

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{8}$

Exercise:

Problem: $-\frac{4}{5} \div \frac{4}{7}$

Solution:

$-\frac{7}{5}$

Exercise:

Problem: $-\frac{3}{4} \div \frac{3}{5}$

Exercise:

Problem: $\frac{5}{8} \div \frac{a}{10}$

Solution:

$\frac{25}{4a}$

Exercise:

Problem: $\frac{5}{6} \div \frac{c}{15}$

Exercise:

Problem: $\frac{7p}{12} \div \frac{21p}{8}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{5q}{12} \div \frac{15q}{8}$

Exercise:

Problem: $\frac{2}{5} \div (-10)$

Solution:

$-\frac{1}{25}$

Exercise:

Problem: $-18 \div -\left(\frac{9}{2}\right)$

In the following exercises, simplify.

Exercise:

Problem: $\frac{\frac{2}{3}}{\frac{8}{9}}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{\frac{4}{5}}{\frac{8}{15}}$

Exercise:

Problem: $\frac{-\frac{9}{10}}{3}$

Solution:

$-\frac{3}{10}$

Exercise:

Problem: $\frac{2}{\frac{5}{8}}$

Exercise:

Problem: $\frac{\frac{r}{5}}{\frac{s}{3}}$

Solution:

$\frac{3r}{5s}$

Exercise:

Problem: $\frac{-\frac{x}{6}}{-\frac{8}{9}}$

Simplify Expressions Written with a Fraction Bar

In the following exercises, simplify.

Exercise:

Problem: $\frac{4+11}{8}$

Solution:

$\frac{15}{8}$

Exercise:

Problem: $\frac{9+3}{7}$

Exercise:

Problem: $\frac{30}{7-12}$

Solution:

-6

Exercise:

Problem: $\frac{15}{4-9}$

Exercise:

Problem: $\frac{22-14}{19-13}$

Solution:

$\frac{4}{3}$

Exercise:

Problem: $\frac{15+9}{18+12}$

Exercise:

Problem: $\frac{5\cdot 8}{-10}$

Solution:

-4

Exercise:

Problem: $\frac{3\cdot 4}{-24}$

Exercise:

Problem: $\frac{15\cdot 5-5^2}{2\cdot 10}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{12\cdot 9-3^2}{3\cdot 18}$

Exercise:

Problem: $\frac{2+4(3)}{-3-2^2}$

Solution:

-2

Exercise:

Problem: $\frac{7+3(5)}{-2-3^2}$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

Exercise:

Problem: the quotient of c and the sum of d and 9.

Solution:

$\frac{c}{d+9}$

Exercise:

Problem: the quotient of the difference of h and k, and -5.

Add and Subtract Fractions

Add and Subtract Fractions with a Common Denominator

In the following exercises, add.

Exercise:

Problem: $\frac{4}{9}+\frac{1}{9}$

Solution:

$\frac{5}{9}$

Exercise:

Problem: $\frac{2}{9}+\frac{5}{9}$

Exercise:

Problem: $\frac{y}{3}+\frac{2}{3}$

Solution:

$\frac{y+2}{3}$

Exercise:

Problem: $\frac{7}{p}+\frac{9}{p}$

Exercise:

Problem: $-\frac{1}{8}+\left(-\frac{3}{8}\right)$

Solution:

$-\frac{1}{2}$

Exercise:

Problem: $-\frac{1}{8}+\left(-\frac{5}{8}\right)$

In the following exercises, subtract.

Exercise:

Problem: $\frac{4}{5}-\frac{1}{5}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{4}{5}-\frac{3}{5}$

Exercise:

Problem: $\frac{y}{17}-\frac{9}{17}$

Solution:

$\frac{y-9}{17}$

Exercise:

Problem: $\frac{x}{19} - \frac{8}{19}$

Exercise:

Problem: $-\frac{8}{d} - \frac{3}{d}$

Solution:

$-\frac{11}{d}$

Exercise:

Problem: $-\frac{7}{c} - \frac{7}{c}$

Add or Subtract Fractions with Different Denominators

In the following exercises, add or subtract.

Exercise:

Problem: $\frac{1}{3} + \frac{1}{5}$

Solution:

$\frac{8}{15}$

Exercise:

Problem: $\frac{1}{4} + \frac{1}{5}$

Exercise:

Problem: $\frac{1}{5} - \left(-\frac{1}{10}\right)$

Solution:

$\frac{3}{10}$

Exercise:

Problem: $\frac{1}{2} - \left(- \frac{1}{6}\right)$

Exercise:

Problem: $\frac{2}{3} + \frac{3}{4}$

Solution:

$\frac{17}{12}$

Exercise:

Problem: $\frac{3}{4} + \frac{2}{5}$

Exercise:

Problem: $\frac{11}{12} - \frac{3}{8}$

Solution:

$\frac{13}{24}$

Exercise:

Problem: $\frac{5}{8} - \frac{7}{12}$

Exercise:

Problem: $- \frac{9}{16} - \left(- \frac{4}{5}\right)$

Solution:

$\frac{19}{80}$

Exercise:

Problem: $-\frac{7}{20}-\left(-\frac{5}{8}\right)$

Exercise:

Problem: $1+\frac{5}{6}$

Solution:

$\frac{11}{6}$

Exercise:

Problem: $1-\frac{5}{9}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

Exercise:

Problem: $\frac{\left(\frac{1}{5}\right)^2}{2+3^2}$

Solution:

$\frac{1}{275}$

Exercise:

Problem: $\frac{\left(\frac{1}{3}\right)^2}{5+2^2}$

Exercise:

Problem: $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{2}{3}}$

Solution:

14

Exercise:

Problem: $\frac{\frac{3}{4}+\frac{1}{2}}{\frac{5}{6}-\frac{2}{3}}$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

Exercise:

Problem: $x+\frac{1}{2}$ when
ⓐ $x=-\frac{1}{8}$
ⓑ $x=-\frac{1}{2}$

Solution:

ⓐ $\frac{3}{8}$ ⓑ 0

Exercise:

Problem: $x+\frac{2}{3}$ when
ⓐ $x=-\frac{1}{6}$
ⓑ $x=-\frac{5}{3}$

Exercise:

Problem: $4p^2q$ when $p=-\frac{1}{2}$ and $q=\frac{5}{9}$

Solution:

$\frac{5}{9}$

Exercise:

Problem: $5m^2n$ when $m = -\frac{2}{5}$ and $n = \frac{1}{3}$

Exercise:

Problem: $\frac{u+v}{w}$ when $u = -4, v = -8, w = 2$

Solution:

-6

Exercise:

Problem: $\frac{m+n}{p}$ when $m = -6, n = -2, p = 4$

Decimals

Name and Write Decimals

In the following exercises, write as a decimal.

Exercise:

Problem: Eight and three hundredths

Solution:

8.03

Exercise:

Problem: Nine and seven hundredths

Exercise:

Problem: One thousandth

Solution:

0.001

Exercise:

Problem: Nine thousandths

In the following exercises, name each decimal.

Exercise:

Problem: 7.8

Solution:

seven and eight tenths

Exercise:

Problem: 5.01

Exercise:

Problem: 0.005

Solution:

five thousandths

Exercise:

Problem: 0.381

Round Decimals

In the following exercises, round each number to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Exercise:

Problem: 5.7932

Solution:

ⓐ 5.79 ⓑ 5.8 ⓒ 6

Exercise:

Problem: 3.6284

Exercise:

Problem: 12.4768

Solution:

ⓐ 12.48 ⓑ 12.5 ⓒ 12

Exercise:

Problem: 25.8449

Add and Subtract Decimals

In the following exercises, add or subtract.

Exercise:

Problem: $18.37 + 9.36$

Solution:

27.73

Exercise:

Problem: $256.37 - 85.49$

Exercise:

Problem: $15.35 - 20.88$

Solution:

-5.53

Exercise:

Problem: $37.5 + 12.23$

Exercise:

Problem: $-4.2 + (-9.3)$

Solution:

-13.5

Exercise:

Problem: $-8.6 + (-8.6)$

Exercise:

Problem: $100 - 64.2$

Solution:

35.8

Exercise:

Problem: $100 - 65.83$

Exercise:

Problem: $2.51 + 40$

Solution:

42.51

Exercise:

Problem: $9.38 + 60$

Multiply and Divide Decimals

In the following exercises, multiply.

Exercise:

Problem: $(0.3)\,(0.4)$

Solution:

0.12

Exercise:

Problem: $(0.6)\,(0.7)$

Exercise:

Problem: $(8.52)\,(3.14)$

Solution:

26.7528

Exercise:

Problem: $(5.32)\,(4.86)$

Exercise:

Problem: $(0.09)\,(24.78)$

Solution:

2.2302

Exercise:

Problem: $(0.04)\,(36.89)$

In the following exercises, divide.

Exercise:

Problem: $0.15 \div 5$

Solution:

0.03

Exercise:

Problem: $0.27 \div 3$

Exercise:

Problem: $\$8.49 \div 12$

Solution:

$0.71

Exercise:

Problem: $\$16.99 \div 9$

Exercise:

Problem: $12 \div 0.08$

Solution:

150

Exercise:

Problem: $5 \div 0.04$

Convert Decimals, Fractions, and Percents

In the following exercises, write each decimal as a fraction.

Exercise:

Problem: 0.08

Solution:

$\frac{2}{25}$

Exercise:

Problem: 0.17

Exercise:

Problem: 0.425

Solution:

$\frac{17}{40}$

Exercise:

Problem: 0.184

Exercise:

Problem: 1.75

Solution:

$\frac{7}{4}$

Exercise:

Problem: 0.035

In the following exercises, convert each fraction to a decimal.

Exercise:

Problem: $\frac{2}{5}$

Solution:

0.4

Exercise:

Problem: $\frac{4}{5}$

Exercise:

Problem: $-\frac{3}{8}$

Solution:

-0.375

Exercise:

Problem: $-\frac{5}{8}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

$0.\overline{5}$

Exercise:

Problem: $\frac{2}{9}$

Exercise:

Problem: $\frac{1}{2}+6.5$

Solution:

7

Exercise:

Problem: $\frac{1}{4}+10.75$

In the following exercises, convert each percent to a decimal.

Exercise:

Problem: 5%

Solution:

0.05

Exercise:

Problem: 9%

Exercise:

Problem: 40%

Solution:

0.4

Exercise:

Problem: 50%

Exercise:

Problem: 115%

Solution:

1.15

Exercise:

Problem: 125%

In the following exercises, convert each decimal to a percent.

Exercise:

Problem: 0.18

Solution:

18%

Exercise:

Problem: 0.15

Exercise:

Problem: 0.009

Solution:

0.9%

Exercise:

Problem: 0.008

Exercise:

Problem: 1.5

Solution:

150%

Exercise:

Problem: 2.2

The Real Numbers

Simplify Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{144}$

Exercise:

Problem: $-\sqrt{25}$

Solution:

-5

Exercise:

Problem: $-\sqrt{81}$

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

In the following exercises, write as the ratio of two integers.

Exercise:

Problem: ⓐ 9 ⓑ 8.47

Solution:

ⓐ $\frac{9}{1}$ ⓑ $\frac{847}{100}$

Exercise:

Problem: ⓐ -15 ⓑ 3.591

In the following exercises, list the ⓐ rational numbers, ⓑ irrational numbers.

Exercise:

Problem: $0.84, 0.79132\ldots, 1.\overline{3}$

Solution:

ⓐ $0.84, 1.\overline{3}$ ⓑ $0.79132\ldots,$

Exercise:

Problem: $2.3\overline{8}, 0.572, 4.93814\ldots$

In the following exercises, identify whether each number is rational or irrational.

Exercise:

Problem: ⓐ $\sqrt{121}$ ⓑ $\sqrt{48}$

Solution:

ⓐ rational ⓑ irrational

Exercise:

Problem: ⓐ $\sqrt{56}$ ⓑ $\sqrt{16}$

In the following exercises, identify whether each number is a real number or not a real number.

Exercise:

Problem: ⓐ $\sqrt{-9}$ ⓑ $-\sqrt{169}$

Solution:

ⓐ not a real number ⓑ real number

Exercise:

Problem: ⓐ $\sqrt{-64}$ ⓑ $-\sqrt{81}$

In the following exercises, list the ⓐ whole numbers, ⓑ integers, ⓒ rational numbers, ⓓ irrational numbers, ⓔ real numbers for each set of numbers.

Exercise:

Problem: $-4, 0, \frac{5}{6}, \sqrt{16}, \sqrt{18}, 5.2537...$

Solution:

ⓐ $0, \sqrt{16}$ ⓑ $-4, 0, \sqrt{16}$ ⓒ $-4, 0, \frac{5}{6}, \sqrt{16}$ ⓓ $\sqrt{18}, 5.2537...$ ⓔ $-4, 0, \frac{5}{6}, \sqrt{16}, \sqrt{18}, 5.2537...$

Exercise:

Problem: $-\sqrt{4}, 0.\overline{36}, \frac{13}{3}, 6.9152..., \sqrt{48}, 10\frac{1}{2}$

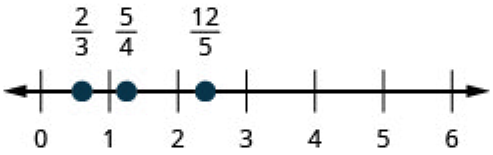
Locate Fractions on the Number Line

In the following exercises, locate the numbers on a number line.

Exercise:

Problem: $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

Solution:



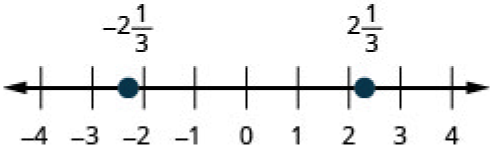
Exercise:

Problem: $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$

Exercise:

Problem: $2\frac{1}{3}, -2\frac{1}{3}$

Solution:



Exercise:

Problem: $1\frac{3}{5}, -1\frac{3}{5}$

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise:

Problem: -1 ___ $-\frac{1}{8}$

Solution:

<

Exercise:

Problem: $-3\frac{1}{4}$ ___ -4

Exercise:

Problem: $-\frac{7}{9}$ ___ $-\frac{4}{9}$

Solution:

$>$

Exercise:

Problem: -2 ___ $-\frac{19}{8}$

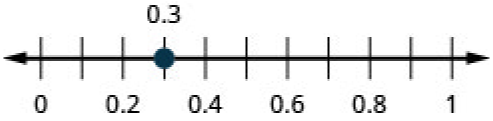
Locate Decimals on the Number Line

In the following exercises, locate on the number line.

Exercise:

Problem: 0.3

Solution:



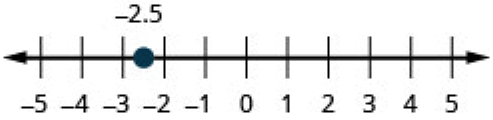
Exercise:

Problem: -0.2

Exercise:

Problem: -2.5

Solution:



Exercise:

Problem: 2.7

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise:

Problem: 0.9 ___ 0.6

Solution:

>

Exercise:

Problem: 0.7 ___ 0.8

Exercise:

Problem: -0.6 ___ -0.59

Solution:

>

Exercise:

Problem: -0.27 ___ -0.3

Properties of Real Numbers

Use the Commutative and Associative Properties

In the following exercises, use the Associative Property to simplify.

Exercise:

Problem: $-12\left(4m\right)$

Solution:

$-48m$

Exercise:

Problem: $30\left(\frac{5}{6}q\right)$

Exercise:

Problem: $\left(a+16\right)+31$

Solution:

$a+47$

Exercise:

Problem: $\left(c+0.2\right)+0.7$

In the following exercises, simplify.

Exercise:

Problem: $6y+37+\left(-6y\right)$

Solution:

37

Exercise:

Problem: $\frac{1}{4}+\frac{11}{15}+\left(-\frac{1}{4}\right)$

Exercise:

Problem: $\frac{14}{11} \cdot \frac{35}{9} \cdot \frac{14}{11}$

Solution:

$\frac{35}{9}$

Exercise:

Problem: $-18 \cdot 15 \cdot \frac{2}{9}$

Exercise:

Problem: $\left(\frac{7}{12} + \frac{4}{5}\right) + \frac{1}{5}$

Solution:

$1\,\frac{7}{12}$

Exercise:

Problem: $(3.98d + 0.75d) + 1.25d$

Exercise:

Problem: $11x + 8y + 16x + 15y$

Solution:

$27x + 23y$

Exercise:

Problem: $52m + (-20n) + (-18m) + (-5n)$

Use the Identity and Inverse Properties of Addition and Multiplication

In the following exercises, find the additive inverse of each number.

Exercise:

Problem: ⓐ $\frac{1}{3}$
ⓑ 5.1
ⓒ -14
ⓓ $-\frac{8}{5}$

Solution:

ⓐ $-\frac{1}{3}$ ⓑ -5.1 ⓒ 14 ⓓ $\frac{8}{5}$

Exercise:

Problem: ⓐ $-\frac{7}{8}$
ⓑ -0.03
ⓒ 17
ⓓ $\frac{12}{5}$

In the following exercises, find the multiplicative inverse of each number.

Exercise:

Problem: ⓐ 10 ⓑ $-\frac{4}{9}$ ⓒ 0.6

Solution:

ⓐ $\frac{1}{10}$ ⓑ $-\frac{9}{4}$ ⓒ $\frac{5}{3}$

Exercise:

Problem: ⓐ $-\frac{9}{2}$ ⓑ -7 ⓒ 2.1

Use the Properties of Zero

In the following exercises, simplify.

Exercise:

Problem: $83 \cdot 0$

Solution:

0

Exercise:

Problem: $\frac{0}{9}$

Exercise:

Problem: $\frac{5}{0}$

Solution:

undefined

Exercise:

Problem: $0 \div \frac{2}{3}$

In the following exercises, simplify.

Exercise:

Problem: $43 + 39 + (-43)$

Solution:

39

Exercise:

Problem: $(n + 6.75) + 0.25$

Exercise:

Problem: $\frac{5}{13} \cdot 57 \cdot \frac{13}{5}$

Solution:

57

Exercise:

Problem: $\frac{1}{6} \cdot 17 \cdot 12$

Exercise:

Problem: $\frac{2}{3} \cdot 28 \cdot \frac{3}{7}$

Solution:

8

Exercise:

Problem: $9\left(6x - 11\right) + 15$

Simplify Expressions Using the Distributive Property

In the following exercises, simplify using the Distributive Property.

Exercise:

Problem: $7\left(x + 9\right)$

Solution:

$7x + 63$

Exercise:

Problem: $9\left(u-4\right)$

Exercise:

Problem: $-3\left(6m-1\right)$

Solution:

$-18m+3$

Exercise:

Problem: $-8\left(-7a-12\right)$

Exercise:

Problem: $\frac{1}{3}\left(15n-6\right)$

Solution:

$5n-2$

Exercise:

Problem: $\left(y+10\right)\cdot p$

Exercise:

Problem: $\left(a-4\right)-\left(6a+9\right)$

Solution:

$-5a-13$

Exercise:

Problem: $4\left(x+3\right)-8\left(x-7\right)$

Systems of Measurement

1.1 Define U.S. Units of Measurement and Convert from One Unit to Another

In the following exercises, convert the units. Round to the nearest tenth.

Exercise:

Problem: A floral arbor is 7 feet tall. Convert the height to inches.

Solution:

84 inches

Exercise:

Problem: A picture frame is 42 inches wide. Convert the width to feet.

Exercise:

Problem: Kelly is 5 feet 4 inches tall. Convert her height to inches.

Solution:

64 inches

Exercise:

Problem: A playground is 45 feet wide. Convert the width to yards.

Exercise:

Problem:

The height of Mount Shasta is 14,179 feet. Convert the height to miles.

Solution:

2.7 miles

Exercise:

Problem: Shamu weights 4.5 tons. Convert the weight to pounds.

Exercise:

The play lasted

$$1\frac{3}{4}$$

Problem: hours. Convert the time to minutes.

Solution:

105 minutes

Exercise:

Problem: How many tablespoons are in a quart?

Exercise:

Problem:

Naomi’s baby weighed 5 pounds 14 ounces at birth. Convert the weight to ounces.

Solution:

94 ounces

Exercise:

Problem:

Trinh needs 30 cups of paint for her class art project. Convert the volume to gallons.

Use Mixed Units of Measurement in the U.S. System.

In the following exercises, solve.

Exercise:

Problem:

John caught 4 lobsters. The weights of the lobsters were 1 pound 9 ounces, 1 pound 12 ounces, 4 pounds 2 ounces, and 2 pounds 15 ounces. What was the total weight of the lobsters?

Solution:

10 lbs. 6 oz.

Exercise:

Problem:

Every day last week Pedro recorded the number of minutes he spent reading. The number of minutes were 50, 25, 83, 45, 32, 60, 135. How many hours did Pedro spend reading?

Exercise:

Problem:

Fouad is 6 feet 2 inches tall. If he stands on a rung of a ladder 8 feet 10 inches high, how high off the ground is the top of Fouad’s head?

Solution:

15 feet

Exercise:

Problem:

Dalila wants to make throw pillow covers. Each cover takes 30 inches of fabric. How many yards of fabric does she need for 4 covers?

Make Unit Conversions in the Metric System

In the following exercises, convert the units.

Exercise:

Problem: Donna is 1.7 meters tall. Convert her height to centimeters.

Solution:

170 centimeters

Exercise:

Problem:

Mount Everest is 8,850 meters tall. Convert the height to kilometers.

Exercise:

Problem:

One cup of yogurt contains 488 milligrams of calcium. Convert this to grams.

Solution:

0.488 grams

Exercise:

Problem:

One cup of yogurt contains 13 grams of protein. Convert this to milligrams.

Exercise:

Problem:

Sergio weighed 2.9 kilograms at birth. Convert this to grams.

Solution:

2,900 grams

Exercise:

Problem:

A bottle of water contained 650 milliliters. Convert this to liters.

Use Mixed Units of Measurement in the Metric System

In the following exerices, solve.

Exercise:

Problem:

Minh is 2 meters tall. His daughter is 88 centimeters tall. How much taller is Minh than his daughter?

Solution:

1.12 meter

Exercise:

Problem:

Selma had a 1 liter bottle of water. If she drank 145 milliliters, how much water was left in the bottle?

Exercise:

Problem:

One serving of cranberry juice contains 30 grams of sugar. How many kilograms of sugar are in 30 servings of cranberry juice?

Solution:

0.9 kilograms

Exercise:

Problem:

One ounce of tofu provided 2 grams of protein. How many milligrams of protein are provided by 5 ounces of tofu?

Convert between the U.S. and the Metric Systems of Measurement

In the following exercises, make the unit conversions. Round to the nearest tenth.

Exercise:

Problem: Majid is 69 inches tall. Convert his height to centimeters.

Solution:

175.3 centimeters

Exercise:

Problem:

A college basketball court is 84 feet long. Convert this length to meters.

Exercise:

Problem:

Caroline walked 2.5 kilometers. Convert this length to miles.

Solution:

1.6 miles

Exercise:

Problem: Lucas weighs 78 kilograms. Convert his weight to pounds.

Exercise:

Problem: Steve’s car holds 55 liters of gas. Convert this to gallons.

Solution:

14.6 gallons

Exercise:

Problem:

A box of books weighs 25 pounds. Convert the weight to kilograms.

Convert between Fahrenheit and Celsius Temperatures

In the following exercises, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.

Exercise:

Problem: 95° Fahrenheit

Solution:

35° C

Exercise:

Problem: 23° Fahrenheit

Exercise:

Problem: 20° Fahrenheit

Solution:

–6.7° C

Exercise:

Problem: 64° Fahrenheit

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

Exercise:

Problem: 30° Celsius

Solution:

86° F

Exercise:

Problem: –5° Celsius

Exercise:

Problem: –12° Celsius

Solution:

10.4° F

Exercise:

Problem: 24° Celsius

Chapter Practice Test

Exercise:

Problem:

Write as a whole number using digits: two hundred five thousand, six hundred seventeen.

Solution:

205,617

Exercise:

Problem: Find the prime factorization of 504.

Exercise:

Problem: Find the Least Common Multiple of 18 and 24.

Solution:

72

Exercise:

Problem: Combine like terms: $5n + 8 + 2n - 1$.

In the following exercises, evaluate.

Exercise:

Problem: $-\left|x\right|$ when $x = -2$

Solution:

-2

Exercise:

Problem: $11 - a$ when $a = -3$

Exercise:

Problem:

Translate to an algebraic expression and simplify: twenty less than negative 7.

Solution:

$-7-20; -27$

Exercise:

Problem:

Monique has a balance of $-\$18$ in her checking account. She deposits $\$152$ to the account. What is the new balance?

Exercise:

Problem: Round 677.1348 to the nearest hundredth.

Solution:

677.13

Exercise:

Problem: Convert $\frac{4}{5}$ to a decimal.

Exercise:

Problem: Convert 1.85 to a percent.

Solution:

185%

Exercise:

Problem: Locate $\frac{2}{3}, -1.5, \text{and } \frac{9}{4}$ on a number line.

In the following exercises, simplify each expression.

Exercise:

Problem: $4+10\left(3+9\right)-5^2$

Solution:

99

Exercise:

Problem: $-85+42$

Exercise:

Problem: $-19-25$

Solution:

-44

Exercise:

Problem: $\left(-2\right)^4$

Exercise:

Problem: $-5\left(-9\right)\div 15$

Solution:

3

Exercise:

Problem: $\frac{3}{8}\cdot\frac{11}{12}$

Exercise:

Problem: $\frac{4}{5}\div\frac{9}{20}$

Solution:

$\frac{16}{9}$

Exercise:

Problem: $\frac{12+3\cdot5}{15-6}$

Exercise:

Problem: $\frac{m}{7}+\frac{10}{7}$

Solution:

$\frac{m+10}{7}$

Exercise:

Problem: $\frac{7}{12}-\frac{3}{8}$

Exercise:

Problem: $-5.8+(-4.7)$

Solution:

-10.5

Exercise:

Problem: $100-64.25$

Exercise:

Problem: $(0.07)\,(31.95)$

Solution:

2.2365

Exercise:

Problem: $9 \div 0.05$

Exercise:

Problem: $-14\left(\frac{5}{7}p\right)$

Solution:

$-10p$

Exercise:

Problem: $(u+8)-9$

Exercise:

Problem: $6x+(-4y)+9x+8y$

Solution:

$15x+4y$

Exercise:

Problem: $\frac{0}{23}$

Exercise:

Problem: $\frac{75}{0}$

Solution:

undefined

Exercise:

Problem: $-2\left(13q-5\right)$

Exercise:

Problem:

A movie lasted $1\frac{2}{3}$ hours. How many minutes did it last? *(1 hour = 60 minutes)*

Solution:

100 minutes

Exercise:

Problem:

Mike’s SUV is 5 feet 11 inches tall. He wants to put a rooftop cargo bag on the the SUV. The cargo bag is 1 foot 6 inches tall. What will the total height be of the SUV with the cargo bag on the roof? *(1 foot = 12 inches)*

Exercise:

Problem:

Jennifer ran 2.8 miles. Convert this length to kilometers. *(1 mile = 1.61 kilometers)*

Solution:

4.508 km

Introduction class="introduction"

The rocks in this formation must remain perfectly balanced around the center for the formation to hold its shape.

If we carefully placed more rocks of equal weight on both sides of this formation, it would still balance. Similarly, the expressions in an equation remain balanced when we add the same quantity to both sides of the equation. In this chapter, we will solve equations, remembering that what we do to one side of the equation, we must also do to the other side.

Solve Equations Using the Subtraction and Addition Properties of Equality
By the end of this section, you will be able to:

- Verify a solution of an equation
- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Note:
Before you get started, take this readiness quiz.

1. Evaluate $x+4$ when $x=-3$.
   If you missed this problem, review [link].
2. Evaluate $15-y$ when $y=-5$.
   If you missed this problem, review [link].
3. Simplify $4(4n+1)-15n$.
   If you missed this problem, review [link].
4. Translate into algebra “5 less than x.”
   If you missed this problem, review [link].

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same – so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Note:

Solution of an equation
A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

Note:
To determine whether a number is a solution to an equation.

Substitute the number in for the variable in the equation.
Simplify the expressions on both sides of the equation.
Determine whether the resulting equation is true (the left side is equal to the right side)

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

Example:
Exercise:

Problem:

**Determine whether $x = \frac{3}{2}$ is a solution of $4x - 2 = 2x + 1$.

Solution:
Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

	$4x-2=2x+1$
Substitute $\frac{3}{2}$ for x.	$4\left(\frac{3}{2}\right)-2\overset{?}{=}2\left(\frac{3}{2}\right)+1$
Multiply.	$6-2\overset{?}{=}3+1$
Subtract.	$4=4\checkmark$

Since $x=\frac{3}{2}$ results in a true equation (4 is in fact equal to 4), $\frac{3}{2}$ is a solution to the equation $4x-2=2x+1$.

Note:
Exercise:

TRY IT :: 2.1

Problem: Is $y=\frac{4}{3}$ a solution of $9y+2=6y+3$?

Solution:

no

Note:
Exercise:

TRY IT :: 2.2

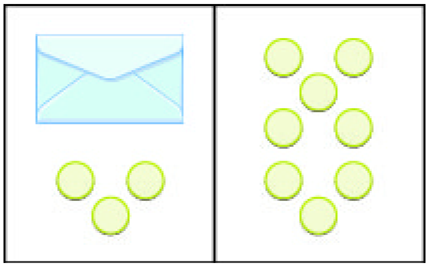
Problem: Is $y=\frac{7}{5}$ a solution of $5y+3=10y-4$?

Solution:

yes

Solve Equations Using the Subtraction and Addition Properties of Equality

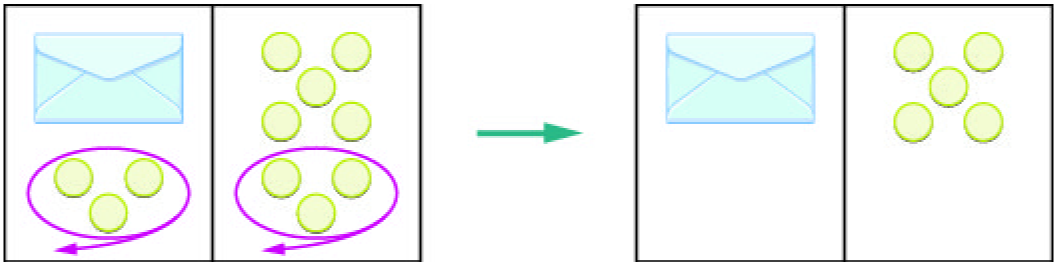
We are going to use a model to clarify the process of solving an equation. An envelope represents the variable – since its contents are unknown – and each counter represents one. We will set out one envelope and some counters on our workspace, as shown in [link]. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?



The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

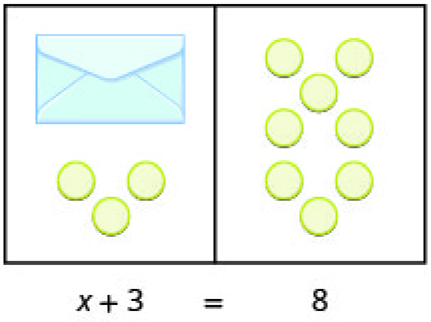
What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: “I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right and so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope.” See [link] for an illustration of this process.



The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? In [link] each side of the workspace represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x.



The illustration shows a model for the equation $x + 3 = 8$.

Let’s write algebraically the steps we took to discover how many counters were in the envelope:

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Check:

Five in the envelope plus three more does equal eight!

Equation:

$$5+3=8$$

Our model has given us an idea of what we need to do to solve one kind of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the **Subtraction Property of Equality**.

Note:
Subtraction Property of Equality
For any numbers *a*, *b*, and *c*,
Equation:

$$\begin{array}{lrcl} \text{If} & a & = & b, \\ \text{then} & a-c & = & b-c \end{array}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

Note: Doing the Manipulative Mathematics activity “Subtraction Property of Equality” will help you develop a better understanding of how to solve equations by using the Subtraction Property of Equality.

Let’s see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

Example:
Exercise:

Problem: **Solve: $y+37=-13$.

Solution:
Solution

To get *y* by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

		$y+37=-13$
Subtract 37 from each side to ‘undo’ the addition.		$y+37-37=-13-37$
Simplify.		$y=-50$
Check:	$y+37=-13$	
Substitute $y=-50$	$-50+37=-13$	
	$-13\overset{?}{=}-13\checkmark$	

Since $y=-50$ makes $y+37=-13$ a true statement, we have the solution to this equation.

Note:
Exercise:

TRY IT :: 2.3

Problem: Solve: $x + 19 = -27$.

Solution:

$x = -46$

Note:
Exercise:

TRY IT :: 2.4

Problem: Solve: $x + 16 = -34$.

Solution:

$x = -50$

What happens when an equation has a number subtracted from the variable, as in the equation $x - 5 = 8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to ‘undo’ the subtraction we will add the number to both sides. We use the **Addition Property of Equality**.

Note:
Addition Property of Equality
For any numbers *a*, *b*, and *c*,
Equation:

$$\begin{aligned} \text{If} \qquad\qquad a &= b, \\ \text{then} \qquad a + c &= b + c \end{aligned}$$

When you add the same quantity to both sides of an equation, you still have equality.

In [link], 37 was added to the *y* and so we subtracted 37 to ‘undo’ the addition. In [link], we will need to ‘undo’ subtraction by using the Addition Property of Equality.

Example:
Exercise:

Problem: **Solve: $a - 28 = -37$.

Solution:
Solution

	$a - 28 = -37$
Add 28 to each side to ‘undo’ the subtraction.	$a - 28 + 28 = -37 + 28$
Simplify.	$a = -9$

Check:	$a - 28 = -37$	
Substitute $a = -9$	$-9 - 28 = -37$	
	$-37 \overset{?}{=} -37$ ✓	
		The solution to $a - 28 = -37$ is $a = -9$.

Note:
Exercise:

TRY IT :: 2.5

Problem: Solve: $n - 61 = -75$.

Solution:

$n = -14$

Note:
Exercise:

TRY IT :: 2.6

Problem: Solve: $p - 41 = -73$.

Solution:

$p = -32$

Example:
Exercise:

Problem: **Solve: $x - \frac{5}{8} = \frac{3}{4}$.

Solution:
Solution

		$x - \frac{5}{8} = \frac{3}{4}$
Use the Addition Property of Equality.		$x - \frac{5}{8} + \frac{5}{8} = \frac{3}{4} + \frac{5}{8}$
Find the LCD to add the fractions on the right.		$x - \frac{5}{8} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8}$
Simplify.		$x = \frac{11}{8}$
Check:	$x - \frac{5}{8} = \frac{3}{4}$	
Substitute		

$x = \frac{11}{8}$.	$\frac{11}{8} - \frac{5}{8} \overset{?}{=} \frac{3}{4}$	
Subtract.	$\frac{6}{8} \overset{?}{=} \frac{3}{4}$	
Simplify.	$\frac{3}{4} = \frac{3}{4}$ ✓	
		The solution to $x - \frac{5}{8} = \frac{3}{4}$ is $x = \frac{11}{8}$.

Note:
Exercise:

TRY IT :: 2.7

Problem: Solve: $p - \frac{2}{3} = \frac{5}{6}$.

Solution:

$p = \frac{9}{6}$ $p = \frac{3}{2}$

Note:
Exercise:

TRY IT :: 2.8

Problem: Solve: $q - \frac{1}{2} = \frac{5}{6}$.

Solution:

$q=\frac{8}{6}$ $q=\frac{4}{3}$

The next example will be an equation with decimals.

Example:
Exercise:

Problem: **Solve: $n-0.63=-4.2.$

Solution:
Solution

		$n-0.63=-4.2$
Use the Addition Property of Equality.		$n-0.63+0.63=-4.2+0.63$
Add.		$n=-3.57$
Check:	$n=-3.57$	

Let $n = -3.57$.	$-3.57 - 0.63 \overset{?}{=} -4.2$	
	$-4.2 = -4.2 \checkmark$	

Note:
Exercise:

TRY IT :: 2.9
Problem: Solve: $b - 0.47 = -2.1$.

Solution:

$b = -1.63$

Note:
Exercise:

TRY IT :: 2.10
Problem: Solve: $c - 0.93 = -4.6$.

Solution:

$c = -3.67$

Solve Equations That Require Simplification

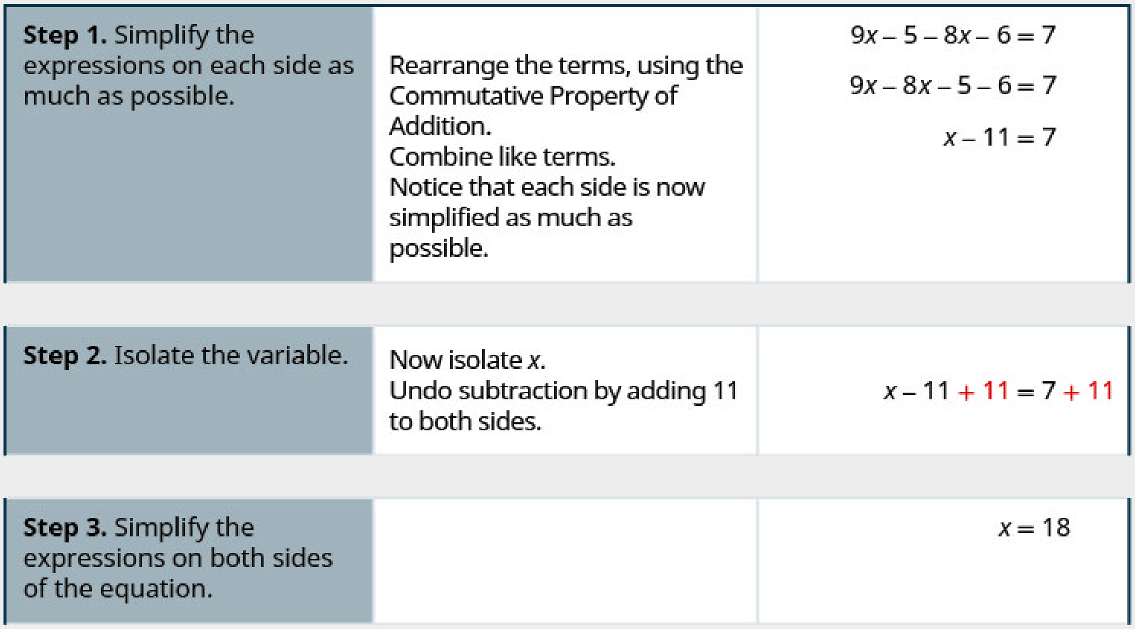
In the previous examples, we were able to isolate the variable with just one operation. Most of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality.

You should always simplify as much as possible before you try to isolate the variable. Remember that to simplify an expression means to do all the operations in the expression. Simplify one side of the equation at a time. Note that simplification is different from the process used to solve an equation in which we apply an operation to both sides.

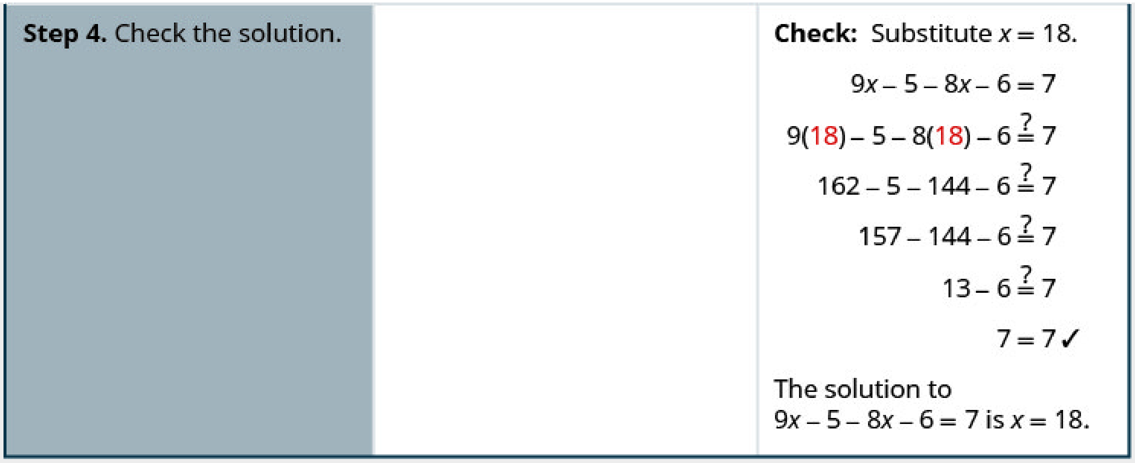
Example:
How to Solve Equations That Require Simplification
Exercise:

Problem: **Solve: $9x - 5 - 8x - 6 = 7.$

Solution:
Solution



Step 1. Simplify the expressions on each side as much as possible.	Rearrange the terms, using the Commutative Property of Addition. Combine like terms. Notice that each side is now simplified as much as possible.	$9x - 5 - 8x - 6 = 7$ $9x - 8x - 5 - 6 = 7$ $x - 11 = 7$
Step 2. Isolate the variable.	Now isolate *x*. Undo subtraction by adding 11 to both sides.	$x - 11 + 11 = 7 + 11$
Step 3. Simplify the expressions on both sides of the equation.		$x = 18$



Step 4. Check the solution.		**Check:** Substitute $x = 18$. $9x - 5 - 8x - 6 = 7$ $9(18) - 5 - 8(18) - 6 \overset{?}{=} 7$ $162 - 5 - 144 - 6 \overset{?}{=} 7$ $157 - 144 - 6 \overset{?}{=} 7$ $13 - 6 \overset{?}{=} 7$ $7 = 7 ✓$ The solution to $9x - 5 - 8x - 6 = 7$ is $x = 18$.

Note:
Exercise:

TRY IT :: 2.11

Problem: Solve: $8y - 4 - 7y - 7 = 4$.

Solution:

$y = 15$

Note:
Exercise:

TRY IT :: 2.12

Problem: Solve: $6z + 5 - 5z - 4 = 3$.

Solution:

$z = 2$

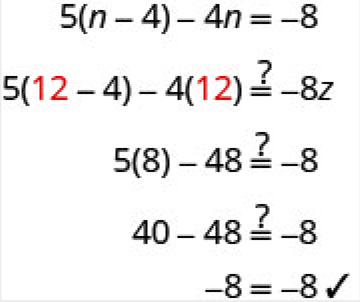
Example:
Exercise:

Problem: **Solve: $5(n-4)-4n=-8$.

Solution:
Solution

We simplify both sides of the equation as much as possible before we try to isolate the variable.

	$5(n-4)-4n=-8$
Distribute on the left.	$5n-20-4n=-8$
Use the Commutative Property to rearrange terms.	$5n-4n-20=-8$
Combine like terms.	$n-20=-8$
Each side is as simplified as possible. Next, isolate n.	
Undo subtraction by using the Addition Property of Equality.	$n-20+20=-8+20$
Add.	$n=12$

Check. Substitute $n = 12$.  $5(n-4)-4n=-8$ $5(12-4)-4(12)\overset{?}{=}-8z$ $5(8)-48\overset{?}{=}-8$ $40-48\overset{?}{=}-8$ $-8=-8$ ✓	
	The solution to $5(n-4)-4n=-8$ is $n=12$.

Note:
Exercise:

TRY IT :: 2.13

Problem: Solve: $5(p-3)-4p=-10$.

Solution:

$p=5$

Note:
Exercise:

TRY IT :: 2.14

Problem: Solve: $4(q+2)-3q=-8$.

Solution:

$q = -16$

Example:
Exercise:

Problem: **Solve: $3(2y - 1) - 5y = 2\left(y + 1\right) - 2\left(y + 3\right)$.

Solution:
Solution

We simplify both sides of the equation before we isolate the variable.

	$3(2y - 1) - 5y = 2(y + 1) - 2(y + 3)$
Distribute on both sides.	$6y - 3 - 5y = 2y + 2 - 2y - 6$
Use the Commutative Property of Addition.	$6y - 5y - 3 = 2y - 2y + 2 - 6$
Combine like terms.	$y - 3 = -4$

Each side is as simplified as possible. Next, isolate y.	
Undo subtraction by using the Addition Property of Equality.	$y - 3 + 3 = -4 + 3$
Add.	$y = -1$
Check. Let $y = -1$. $3(2y - 1) - 5y = 2(y + 1) - 2(y + 3)$ $3(2(-1) - 1) - 5(-1) \overset{?}{=} 2(-1 + 1) - 2(-1 + 3)$ $3(-2 - 1) + 5 \overset{?}{=} 2(0) - 2(2)$ $3(-3) + 5 \overset{?}{=} -4$ $-9 + 5 \overset{?}{=} -4$ $-4 = -4$ ✓	
	The solution to $3(2y - 1) - 5y = 2\left(y + 1\right) - 2\left(y + 3\right)$ is $y = -1$.

Note:
Exercise:

TRY IT :: 2.15

Problem: Solve: $4(2h - 3) - 7h = 6\left(h - 2\right) - 6\left(h - 1\right)$.

Solution:

$h = 6$

Note:
Exercise:

TRY IT :: 2.16

Problem: Solve: $2(5x + 2) - 9x = 3\,(x - 2) - 3\,(x - 4)$.

Solution:

$x = 2$

Translate to an Equation and Solve

To solve applications algebraically, we will begin by translating from English sentences into equations. Our first step is to look for the word (or words) that would translate to the equals sign. [link] shows us some of the words that are commonly used.

Equals =

Equals =
is is equal to is the same as the result is gives was will be

The steps we use to translate a sentence into an equation are listed below.

Note:
Translate an English sentence to an algebraic equation.

Locate the “equals” word(s). Translate to an equals sign (=).
Translate the words to the left of the “equals” word(s) into an algebraic expression.
Translate the words to the right of the “equals” word(s) into an algebraic expression.

Example:
Exercise:

Problem: **Translate and solve: Eleven more than *x* is equal to 54.

Solution:
Solution

Translate.	$\underbrace{\text{Eleven more than } x}_{x+11}\quad \underbrace{\text{is equal to}}_{=}\quad \underbrace{54}_{54}$
Subtract 11 from both sides.	$x+11-11 \quad = \quad 54-11$
Simplify.	$x \quad = \quad 43$
Check: Is 54 eleven more than 43? $43+11 \overset{?}{=} 54$ $54 = 54 \checkmark$	

Note:
Exercise:

TRY IT :: 2.17

Problem: Translate and solve: Ten more than *x* is equal to 41.

Solution:

$x+10=41; x=31$

Note:
Exercise:

TRY IT :: 2.18

Problem: Translate and solve: Twelve less than *x* is equal to 51.

Solution:

$x - 12 = 51; x = 63$

Example:
Exercise:

Problem:

**Translate and solve: The difference of $12t$ and $11t$ is -14.

Solution:
Solution

Translate.	$\underbrace{\text{The difference of } 12t \text{ and } 11t}_{12t-11t}\ \underbrace{\text{is}}_{=}\ \underbrace{-14}_{-14}$
Simplify.	$t = -14$
Check:	

$$\begin{aligned} 12(-14)-11(-14) &\overset{?}{=} -14 \\ -168+154 &\overset{?}{=} -14 \\ -14 &= -14\checkmark \end{aligned}$$

Note:
Exercise:

TRY IT :: 2.19

Problem: Translate and solve: The difference of $4x$ and $3x$ is 14.

Solution:

$4x-3x=14; x=14$

Note:
Exercise:

TRY IT :: 2.20

Problem: Translate and solve: The difference of $7a$ and $6a$ is -8.

Solution:

$7a-6a=-8; a=-8$

Translate and Solve Applications

Most of the time a question that requires an algebraic solution comes out of a real life question. To begin with that question is asked in English (or the language of the person asking) and not in math symbols. Because of this, it is an important skill to be able to translate an everyday situation into algebraic language.

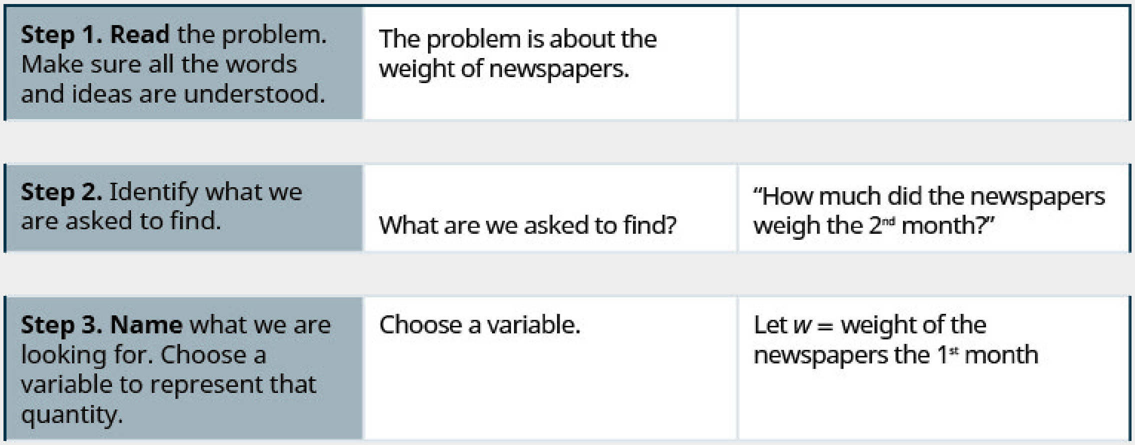
We will start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for. For example, you might use *q* for the number of quarters if you were solving a problem about coins.

Example:
How to Solve Translate and Solve Applications
Exercise:

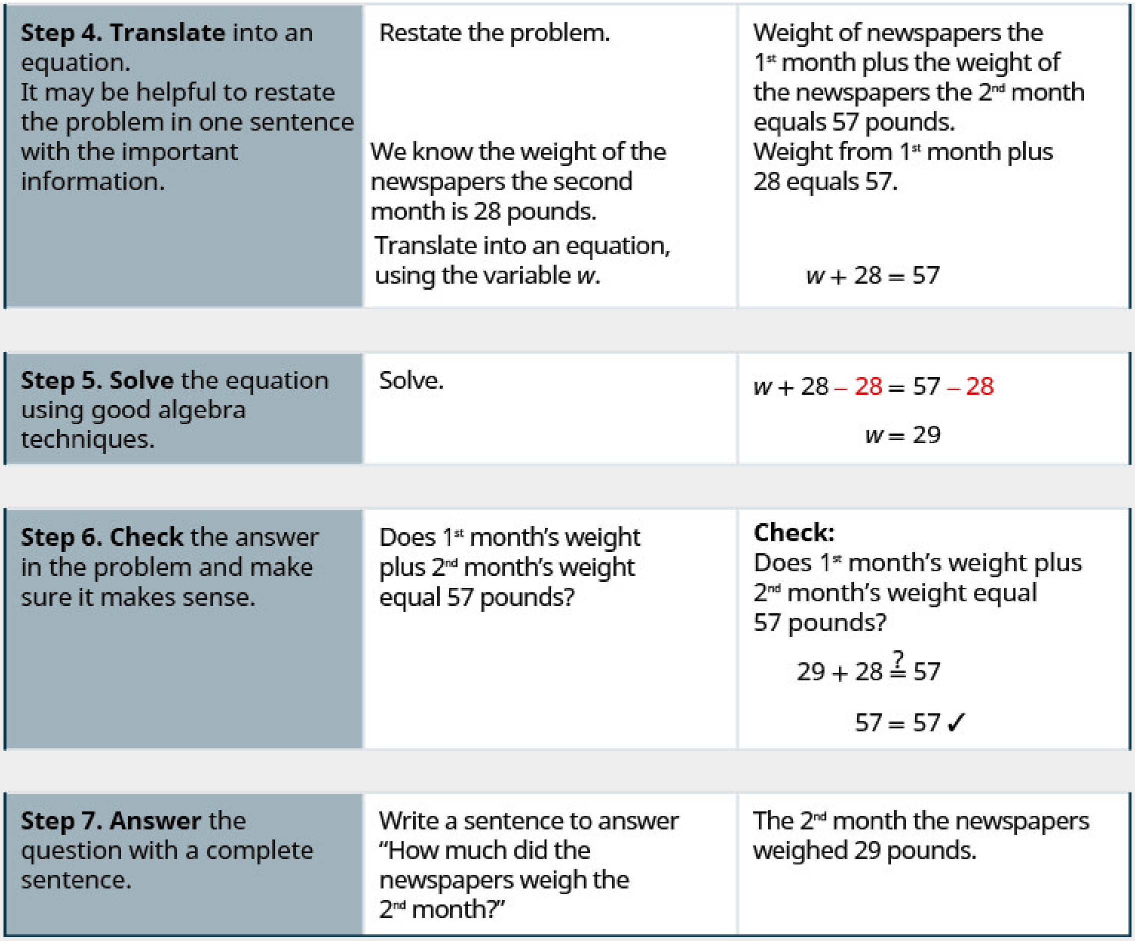
Problem:

**The MacIntyre family recycled newspapers for two months. The two months of newspapers weighed a total of 57 pounds. The second month, the newspapers weighed 28 pounds. How much did the newspapers weigh the first month?

Solution:
Solution



Step 1. Read the problem. Make sure all the words and ideas are understood.	The problem is about the weight of newspapers.	
Step 2. Identify what we are asked to find.	What are we asked to find?	"How much did the newspapers weigh the 2nd month?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Choose a variable.	Let *w* = weight of the newspapers the 1st month



Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.	Restate the problem. We know the weight of the newspapers the second month is 28 pounds. Translate into an equation, using the variable *w*.	Weight of newspapers the 1st month plus the weight of the newspapers the 2nd month equals 57 pounds. Weight from 1st month plus 28 equals 57. $w + 28 = 57$
Step 5. Solve the equation using good algebra techniques.	Solve.	$w + 28 - 28 = 57 - 28$ $w = 29$
Step 6. Check the answer in the problem and make sure it makes sense.	Does 1st month's weight plus 2nd month's weight equal 57 pounds?	**Check:** Does 1st month's weight plus 2nd month's weight equal 57 pounds? $29 + 28 \overset{?}{=} 57$ $57 = 57 \checkmark$
Step 7. Answer the question with a complete sentence.	Write a sentence to answer "How much did the newspapers weigh the 2nd month?"	The 2nd month the newspapers weighed 29 pounds.

Note:
Exercise:

TRY IT :: 2.21

Problem: Translate into an algebraic equation and solve:

The Pappas family has two cats, Zeus and Athena. Together, they weigh 23 pounds. Zeus weighs 16 pounds. How much does Athena weigh?

Solution:

7 pounds

Note:
Exercise:

TRY IT :: 2.22

Problem: Translate into an algebraic equation and solve:

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Solution:

42 books

Note:
Solve an application.

Read the problem. Make sure all the words and ideas are understood.
Identify what we are looking for.
Name what we are looking for. Choose a variable to represent that quantity.
Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
Solve the equation using good algebra techniques.
Check the answer in the problem and make sure it makes sense.
Answer the question with a complete sentence.

Example:
Exercise:

Problem:

**Randell paid $28,675 for his new car. This was $875 less than the sticker price. What was the sticker price of the car?

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	"What was the sticker price of the car?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let $s =$ the sticker price of the car.
Step 4. Translate into an equation. Restate the problem in one sentence.	$28,675 is $875 less than the sticker price
Step 5. Solve the equation.	$28,675 is $875 less than s mtd mtd mtd mtd mtd
Step 6. Check the answer. Is $875 less than $29,550	

equal to $28,675? $$\begin{aligned} 29{,}550 - 875 &\overset{?}{=} 28{,}675 \\ 28{,}675 &= 28{,}675\checkmark \end{aligned}$$	
Step 7. Answer the question with a complete sentence.	The sticker price of the car was $29,550.

Note:
Exercise:

TRY IT :: 2.23
Problem: Translate into an algebraic equation and solve:

Eddie paid $19,875 for his new car. This was $1,025 less than the sticker price. What was the sticker price of the car?

Solution:

$20,900

Note:
Exercise:

TRY IT :: 2.24
Problem: Translate into an algebraic equation and solve:

The admission price for the movies during the day is $7.75. This is $3.25 less the price at night. How much does the movie cost at night?

Solution:

$11.00

Key Concepts

- **To Determine Whether a Number is a Solution to an Equation**

  Substitute the number in for the variable in the equation.
  Simplify the expressions on both sides of the equation.
  Determine whether the resulting statement is true.
    - If it is true, the number is a solution.
    - If it is not true, the number is not a solution.

- **Addition Property of Equality**
    - For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.

- **Subtraction Property of Equality**
    - For any numbers a, b, and c, if $a = b$, then $a - c = b - c$.

- **To Translate a Sentence to an Equation**

  Locate the “equals” word(s). Translate to an equal sign (=).
  Translate the words to the left of the “equals” word(s) into an algebraic expression.
  Translate the words to the right of the “equals” word(s) into an algebraic expression.

- **To Solve an Application**

  Read the problem. Make sure all the words and ideas are understood.
  Identify what we are looking for.
  Name what we are looking for. Choose a variable to represent that

quantity.
Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
Solve the equation using good algebra techniques.
Check the answer in the problem and make sure it makes sense.
Answer the question with a complete sentence.

Practice Makes Perfect

Verify a Solution of an Equation

In the following exercises, determine whether the given value is a solution to the equation.

Exercise:

Problem: Is $y=\frac{5}{3}$ a solution of $6y+10=12y$?

Solution:

yes

Exercise:

Problem: Is $x=\frac{9}{4}$ a solution of $4x+9=8x$?

Exercise:

Problem: Is $u=-\frac{1}{2}$ a solution of $8u-1=6u$?

Solution:

no

Exercise:

Problem: Is $v = -\frac{1}{3}$ a solution of $9v - 2 = 3v$?

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction and Addition Properties of Equality.

Exercise:

Problem: $x + 24 = 35$

Solution:

$x = 11$

Exercise:

Problem: $x + 17 = 22$

Exercise:

Problem: $y + 45 = -66$

Solution:

$y = -111$

Exercise:

Problem: $y + 39 = -83$

Exercise:

Problem: $b + \frac{1}{4} = \frac{3}{4}$

Solution:

$b = \frac{1}{2}$

Exercise:

Problem: $a + \frac{2}{5} = \frac{4}{5}$

Exercise:

Problem: $p + 2.4 = -9.3$

Solution:

$p = -11.7$

Exercise:

Problem: $m + 7.9 = 11.6$

Exercise:

Problem: $a - 45 = 76$

Solution:

$a = 121$

Exercise:

Problem: $a - 30 = 57$

Exercise:

Problem: $m - 18 = -200$

Solution:

$m = -182$

Exercise:

Problem: $m - 12 = -12$

Exercise:

Problem: $x - \frac{1}{3} = 2$

Solution:

$x = \frac{7}{3}$

Exercise:

Problem: $x - \frac{1}{5} = 4$

Exercise:

Problem: $y - 3.8 = 10$

Solution:

$y = 13.8$

Exercise:

Problem: $y - 7.2 = 5$

Exercise:

Problem: $x - 165 = -420$

Solution:

$x = -255$

Exercise:

Problem: $z - 101 = -314$

Exercise:

Problem: $z + 0.52 = -8.5$

Solution:

$z = -9.02$

Exercise:

Problem: $x + 0.93 = -4.1$

Exercise:

Problem: $q + \frac{3}{4} = \frac{1}{2}$

Solution:

$q = -\frac{1}{4}$

Exercise:

Problem: $p + \frac{1}{3} = \frac{5}{6}$

Exercise:

Problem: $p - \frac{2}{5} = \frac{2}{3}$

Solution:

$p = \frac{16}{15}$

Exercise:

Problem: $y - \frac{3}{4} = \frac{3}{5}$

Solve Equations that Require Simplification

In the following exercises, solve each equation.

Exercise:

Problem: $c + 31 - 10 = 46$

Solution:

$c = 25$

Exercise:

Problem: $m + 16 - 28 = 5$

Exercise:

Problem: $9x + 5 - 8x + 14 = 20$

Solution:

$x = 1$

Exercise:

Problem: $6x + 8 - 5x + 16 = 32$

Exercise:

Problem: $-6x - 11 + 7x - 5 = -16$

Solution:

$x = 0$

Exercise:

Problem: $-8n-17+9n-4=-41$

Exercise:

Problem: $5\left(y-6\right)-4y=-6$

Solution:

$y=8y=24$

Exercise:

Problem: $9\left(y-2\right)-8y=-16$

Exercise:

Problem: $8\left(u+1.5\right)-7u=4.9$

Solution:

$u=-7.1$

Exercise:

Problem: $5\left(w+2.2\right)-4w=9.3$

Exercise:

Problem: $6a-5\left(a-2\right)+9=-11$

Solution:

$a=-30$

Exercise:

Problem: $8c-7\left(c-3\right)+4=-16$

Exercise:

Problem: $6\left(y-2\right)-5y=4\left(y+3\right)-4\left(y-1\right)$

Solution:

$y=28$

Exercise:

Problem: $9\left(x-1\right)-8x=-3\left(x+5\right)+3\left(x-5\right)$

Exercise:

Problem: $3(5n-1)-14n+9=10\left(n-4\right)-6n-4\left(n+1\right)$

Solution:

$n=-50$

Exercise:

Problem: $2(8m+3)-15m-4=9\left(m+6\right)-2\left(m-1\right)-7m$

Exercise:

Problem: $-\left(j+2\right)+2j-1=5$

Solution:

$j=8$

Exercise:

Problem: $-\left(k+7\right)+2k+8=7$

Exercise:

Problem: $-\left(\frac{1}{4}a-\frac{3}{4}\right)+\frac{5}{4}a=-2$

Solution:

$a=-\frac{11}{4}$

Exercise:

Problem: $-\left(\frac{2}{3}d-\frac{1}{3}\right)+\frac{5}{3}d=-4$

Exercise:

Problem: $$\begin{array}{l}8(4x+5)-5(6x)-x\\ =53-6(x+1)+3(2x+2)\end{array}$$

Solution:

$x=13$

Exercise:

Problem: $$\begin{array}{l}6(9y-1)-10(5y)-3y\\ =22-4\left(2y-12\right)+8\left(y-6\right)\end{array}$$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve it.

Exercise:

Problem: Nine more than x is equal to 52.

Solution:

$x + 9 = 52; x = 43$

Exercise:

Problem: The sum of *x* and -15 is 23.

Exercise:

Problem: Ten less than *m* is -14.

Solution:

$m - 10 = -14; m = -4$

Exercise:

Problem: Three less than *y* is -19.

Exercise:

Problem: The sum of *y* and -30 is 40.

Solution:

$y + (-30) = 40; y = 70$

Exercise:

Problem: Twelve more than *p* is equal to 67.

Exercise:

Problem: The difference of $9x$ and $8x$ is 107.

Solution:

$9x - 8x = 107; 107$

Exercise:

Problem: The difference of $5c$ and $4c$ is 602.

Exercise:

Problem: The difference of n and $\frac{1}{6}$ is $\frac{1}{2}$.

Solution:

$n - \frac{1}{6} = \frac{1}{2}; \frac{2}{3}$

Exercise:

Problem: The difference of f and $\frac{1}{3}$ is $\frac{1}{12}$.

Exercise:

Problem: The sum of $-4n$ and $5n$ is -82.

Solution:

$-4n + 5n = -82; -82$

Exercise:

Problem: The sum of $-9m$ and $10m$ is -95.

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise:

Problem:

Distance Avril rode her bike a total of 18 miles, from home to the library and then to the beach. The distance from Avril's house to the library is 7 miles. What is the distance from the library to the beach?

Solution:

11 miles

Exercise:

Problem:

Reading Jeff read a total of 54 pages in his History and Sociology textbooks. He read 41 pages in his History textbook. How many pages did he read in his Sociology textbook?

Exercise:

Problem:

Age Eva’s daughter is 15 years younger than her son. Eva’s son is 22 years old. How old is her daughter?

Solution:

7 years old

Exercise:

Problem:

Age Pablo’s father is 3 years older than his mother. Pablo’s mother is 42 years old. How old is his father?

Exercise:

Problem:

Groceries For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday turkey weighed 16 pounds. How much did the Thanksgiving turkey weigh?

Solution:

21 pounds

Exercise:

Problem:

Weight Allie weighs 8 pounds less than her twin sister Lorrie. Allie weighs 124 pounds. How much does Lorrie weigh?

Exercise:

Problem:

Health Connor’s temperature was 0.7 degrees higher this morning than it had been last night. His temperature this morning was 101.2 degrees. What was his temperature last night?

Solution:

100.5 degrees

Exercise:

Problem:

Health The nurse reported that Tricia’s daughter had gained 4.2 pounds since her last checkup and now weighs 31.6 pounds. How much did Tricia’s daughter weigh at her last checkup?

Exercise:

Problem:

Salary Ron’s paycheck this week was $17.43 less than his paycheck last week. His paycheck this week was $103.76. How much was Ron’s paycheck last week?

Solution:

$121.19

Exercise:

Problem:

Textbooks Melissa’s math book cost $22.85 less than her art book cost. Her math book cost $93.75. How much did her art book cost?

Everyday Math

Exercise:

Problem:

Construction Miguel wants to drill a hole for a $\frac{5}{8}$ inch screw. The hole should be $\frac{1}{12}$ inch smaller than the screw. Let d equal the size of the hole he should drill. Solve the equation $d-\frac{1}{12}=\frac{5}{8}$ to see what size the hole should be.

Solution:

$d=\frac{17}{24}$ inch

Exercise:

Problem:

Baking Kelsey needs $\frac{2}{3}$ cup of sugar for the cookie recipe she wants to make. She only has $\frac{3}{8}$ cup of sugar and will borrow the rest from her neighbor. Let s equal the amount of sugar she will borrow. Solve the equation $\frac{3}{8}+s=\frac{2}{3}$ to find the amount of sugar she should ask to borrow.

Writing Exercises

Exercise:

Problem:

Is -8 a solution to the equation $3x=16-5x$? How do you know?

Solution:

No. Justifications will vary.

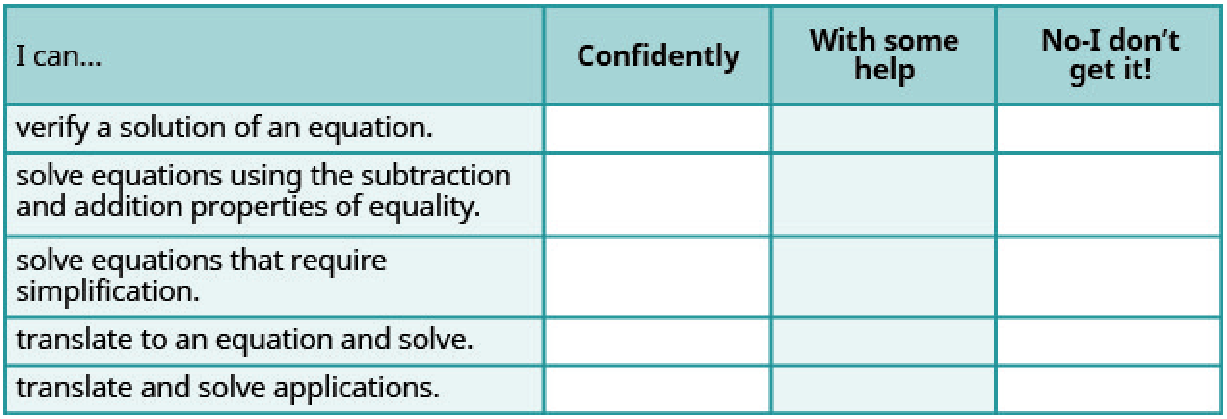
Exercise:

Problem:

What is the first step in your solution to the equation $10x+2=4x+26$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
verify a solution of an equation.			
solve equations using the subtraction and addition properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved your goals in this section! Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific!

…with some help. This must be addressed quickly as topics you do not master become potholes in your road to success. Math is sequential - every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is critical and you must not ignore it. You need to get help immediately or you will quickly be overwhelmed. See your instructor as soon as possible to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

Solve Equations using the Division and Multiplication Properties of Equality
By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

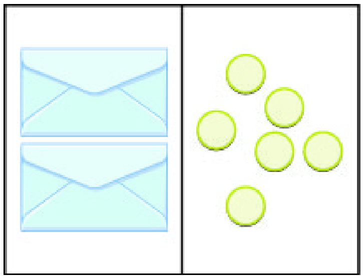
Note:
Before you get started, take this readiness quiz.

1. Simplify: $-7\left(\frac{1}{-7}\right)$.
   If you missed this problem, review [link].
2. Evaluate $9x+2$ when $x=-3$.
   If you missed this problem, review [link].

Solve Equations Using the Division and Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x+a=b$ or $x-a=b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant and so will require division to isolate the variable.

Let's look at our puzzle again with the envelopes and counters in [link].

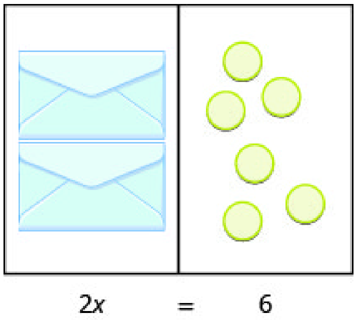


The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

In the illustration there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are “hidden” in the envelopes. So how many counters are in each envelope?

How do we determine the number? We have to separate the counters on the right side into two groups of the same size to correspond with the two envelopes on the left side. The 6 counters divided into 2 equal groups gives 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in [link]? There are two envelopes, and each contains x counters. Together, the two envelopes must contain a total of 6 counters.



The illustration shows a model of the equation $2x = 6$.

	$2x = 6$

If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2}=\frac{6}{2}$
we get:	$x=3$

We found that each envelope contains 3 counters. Does this check? We know $2\cdot 3=6$, so it works! Three counters in each of two envelopes does equal six!

This example leads to the **Division Property of Equality**.

Note:
The Division Property of Equality
For any numbers a, b, and c, and $c\neq 0$,
Equation:

$$\begin{array}{lccc} \text{If} & a & = & b, \\ \text{then} & \frac{a}{c} & = & \frac{b}{c} \end{array}$$

When you divide both sides of an equation by any non-zero number, you still have equality.

Note: Doing the Manipulative Mathematics activity “Division Property of Equality” will help you develop a better understanding of how to solve equations by using the Division Property of Equality.

The goal in solving an equation is to ‘undo’ the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to ‘undo’ the multiplication.

Example:
Exercise:

Problem: **Solve: $5x=-27$.

Solution:
Solution

To isolate x, “undo” the multiplication by 5.		$5x = -27$
Divide to ‘undo’ the multiplication.		$\frac{5x}{5} = -\frac{27}{5}$
Simplify.		$x = -\frac{27}{5}$
Check:	$5x = -27$	
Substitute $-\frac{27}{5}$ for x.	$5\left(-\frac{27}{5}\right) \overset{?}{=} -27$	
	$-27 = -27$ ✓	
		Since this is a true statement, $x = -\frac{27}{5}$ is the solution to $5x = -27$.

Note:
Exercise:

TRY IT :: 2.25

Problem: Solve: $3y = -41$.

Solution:

$y = \frac{-41}{3}$

Note:
Exercise:

TRY IT :: 2.26

Problem: Solve: $4z = -55$.

Solution:

$z = \frac{-55}{4}$

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to "undo" the division, we will need to multiply by 4. The **Multiplication Property of Equality** will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

Note:
The Multiplication Property of Equality
For any numbers *a*, *b*, and *c*,
Equation:

$$\begin{array}{lrcl} \text{If} & a & = & b, \\ \text{then} & ac & = & bc \end{array}$$

If you multiply both sides of an equation by the same number, you still have equality.

Example:
Exercise:

Problem: **Solve: $\frac{y}{-7} = -14$.

Solution:

Solution

Here y is divided by -7. We must multiply by -7 to isolate y.

		$\frac{y}{-7}=-14$
Multiply both sides by -7.		$-7\left(\frac{y}{-7}\right)=-7(-14)$
Multiply.		$\frac{-7y}{7}=98$
Simplify.		$y=98$
Check: $\frac{y}{-7}=-14$		
Substitute $y=98$.	$\frac{98}{-7}\stackrel{?}{=}-14$	
Divide.	$-14=-14$ ✓	

Note:
Exercise:

TRY IT :: 2.27

Problem: Solve: $\frac{a}{-7}=-42$.

Solution:

$a = 294$

Note:
Exercise:

TRY IT :: 2.28

Problem: Solve: $\frac{b}{-6} = -24$.

Solution:

$b = 144$

Example:
Exercise:

Problem: **Solve: $-n = 9$.

Solution:
Solution

	$-n = 9$
Remember $-n$ is equivalent to $-1n$.	$-1n = 9$
Divide both sides by -1.	$\frac{-1n}{-1} = \frac{9}{-1}$
Divide.	$n = -9$

<table>
<tr><td colspan="3">Notice that there are two other ways to solve $-n = 9$. We can also solve this equation by multiplying both sides by -1 and also by taking the opposite of both sides.</td></tr>
<tr><td>Check:</td><td>$-n = 9$</td><td></td></tr>
<tr><td>Substitute $n = -9$.</td><td>$-(-9) \overset{?}{=} 9$</td><td></td></tr>
<tr><td>Simplify.</td><td>$9 = 9 \checkmark$</td><td></td></tr>
</table>

Note:
Exercise:

TRY IT :: 2.29
Problem: Solve: $-k = 8$.

Solution:

$k = -8$

Note:
Exercise:

TRY IT :: 2.30
Problem: Solve: $-g = 3$.

Solution:

$g = -3$

Example:

Exercise:

Problem: **Solve: $\frac{3}{4}x = 12$.

Solution:
Solution

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

<table>
<tr><td></td><td colspan="2">$\frac{3}{4}x = 12$</td></tr>
<tr><td>Multiply by the reciprocal of $\frac{3}{4}$.</td><td colspan="2">$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 12$</td></tr>
<tr><td>Reciprocals multiply to 1.</td><td colspan="2">$1x = \frac{4}{3} \cdot \frac{12}{1}$</td></tr>
<tr><td>Multiply.</td><td colspan="2">$x = 16$</td></tr>
<tr><td colspan="3">Notice that we could have divided both sides of the equation $\frac{3}{4}x = 12$ by $\frac{3}{4}$ to isolate x. While this would work, most people would find multiplying by the reciprocal easier.</td></tr>
<tr><td>Check:</td><td>$\frac{3}{4}x = 12$</td><td></td></tr>
<tr><td>Substitute $x = 16$.</td><td>$\frac{3}{4} \cdot 16 \stackrel{?}{=} 12$</td><td></td></tr>
<tr><td></td><td>$12 = 12 ✓$</td><td></td></tr>
</table>

Note:
Exercise:

TRY IT :: 2.31
Problem: Solve: $\frac{2}{5}n = 14$.

Solution:

$n = 35$

Note:
Exercise:

TRY IT :: 2.32
Problem: Solve: $\frac{5}{6}y = 15$.

Solution:

$y = 18$

In the next example, all the variable terms are on the right side of the equation. As always, our goal in solving the equation is to isolate the variable.

Example:
Exercise:

Problem: **Solve: $\frac{8}{15} = -\frac{4}{5}x$.

Solution:
Solution

		$\frac{8}{15}=-\frac{4}{5}x$
Multiply by the reciprocal of $-\frac{4}{5}$.		$\left(-\frac{5}{4}\right)\left(\frac{8}{15}\right)=\left(-\frac{5}{4}\right)\left(-\frac{4}{5}x\right)$
Reciprocals multiply to 1.		$-\frac{\cancel{5}\cdot\cancel{4}\cdot 2}{\cancel{4}\cdot 3\cdot\cancel{5}}=1x$
Multiply.		$-\frac{2}{3}=x$
Check:	$\frac{8}{15}=-\frac{4}{5}x$	
Let $x=-\frac{2}{3}$.	$\frac{8}{15}=-\frac{4}{5}\left(-\frac{2}{3}\right)$	
	$\frac{8}{15}=\frac{8}{15}$ ✓	

Note:
Exercise:

TRY IT :: 2.33

Problem: Solve: $\frac{9}{25}=-\frac{4}{5}z$.

Solution:

$z=-\frac{9}{20}$

Note:
Exercise:

TRY IT :: 2.34

Problem: Solve: $\frac{5}{6} = -\frac{8}{3}\, r$.

Solution:

$r = -\frac{5}{16}$

Solve Equations That Require Simplification

Many equations start out more complicated than the ones we have been working with.

With these more complicated equations the first step is to simplify both sides of the equation as much as possible. This usually involves combining like terms or using the distributive property.

Example:
Exercise:

Problem: **Solve: $14 - 23 = 12y - 4y - 5y$.

Solution:
Solution

Begin by simplifying each side of the equation.

	$14 - 23 = 12y - 4y - 5y$
Simplify each side.	$-9 = 3y$

Divide both sides by 3 to isolate y.		$\frac{-9}{3}=\frac{3y}{3}$
Divide.		$-3=y$
Check:	$14-23=12y-4y-5y$	
Substitute $y=-3$.	$14-23\overset{?}{=}12(-3)-4(-3)-5(-3)$	
	$14-23\overset{?}{=}-36+12+15$	
	$-9=-9\checkmark$	

Note:
Exercise:

TRY IT :: 2.35

Problem: Solve: $18-27=15c-9c-3c$.

Solution:

$c=-3$

Note:
Exercise:

TRY IT :: 2.36

Problem: Solve:$18-22=12x-x-4x$.

Solution:

$x = -\frac{4}{7}$

Example:
Exercise:

Problem: **Solve: $-4(a-3)-7=25$.

Solution:
Solution

Here we will simplify each side of the equation by using the distributive property first.

		$-4(a-3)-7=25$
Distribute.		$-4a+12-7=25$
Simplify.		$-4a+5=25$
Simplify.		$-4a=20$
Divide both sides by -4 to isolate a.		$\frac{-4a}{-4}=\frac{20}{-4}$
Divide.		$a=-5$
Check:	$-4(a-3)-7=25$	
Substitute $a=-5$.		

	$-4(-5-3)-7 \overset{?}{=} 25$	
	$-4(-8)-7 \overset{?}{=} 25$	
	$32-7 \overset{?}{=} 25$	
	$25=25$ ✓	

Note:
Exercise:

TRY IT :: 2.37
Problem: Solve: $-4(q-2)-8=24.$

Solution:

$q=-6$

Note:
Exercise:

TRY IT :: 2.38
Problem: Solve: $-6(r-2)-12=30.$

Solution:

$r=-5$

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We’ll list them all together here for easy reference.

Note:
Properties of Equality
Equation:

Subtraction Property of Equality

For any real numbers $a, b,$ and $c,$

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a - c & = & b - c. \end{array}$$

Addition Property of Equality

For any real numbers $a, b,$ and $c,$

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a + c & = & b + c. \end{array}$$

Division Property of Equality

For any numbers $a, b,$ and $c,$ and $c \neq 0,$

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & \frac{a}{c} & = & \frac{b}{c}. \end{array}$$

Multiplication Property of Equality

For any numbers $a, b,$ and $c,$

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & ac & = & bc. \end{array}$$

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Translate to an Equation and Solve

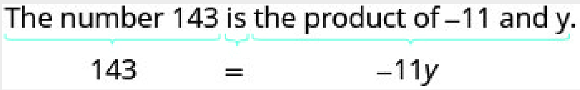
In the next few examples, we will translate sentences into equations and then solve the equations. You might want to review the translation table in the previous chapter.

Example:
Exercise:

Problem: **Translate and solve: The number 143 is the product of -11 and y.

Solution:
Solution

Begin by translating the sentence into an equation.

Translate.	 $\underbrace{\text{The number 143}}_{143}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the product of }-11\text{ and }y.}_{-11y}$

Divide by -11.	$\frac{143}{-11} = \frac{-11y}{-11}$
Simplify.	$-13 = y$
Check: $143 = -11y$ $143 \overset{?}{=} -11(-13)$ $143 = 143\checkmark$	

Note:
Exercise:

TRY IT :: 2.39

Problem: Translate and solve: The number 132 is the product of -12 and y.

Solution:

$132 = -12y; y = -11$

Note:
Exercise:

TRY IT :: 2.40

Problem: Translate and solve: The number 117 is the product of -13 and z.

Solution:

$117 = -13z; z = -9$

Example:

Exercise:

Problem: **Translate and solve: n divided by 8 is -32.

Solution:
Solution

Begin by translating the sentence into an equation. Translate.		$\underbrace{n \text{ divided by } 8}_{\frac{n}{8}}\ \underbrace{\text{is}}_{=}\ \underbrace{-32.}_{-32}$
Multiple both sides by 8.		$8 \cdot \frac{n}{8} = 8(-32)$
Simplify.		$n = -256$
Check:	Is n divided by 8 equal to −32?	
Let $n = -256$.	Is -256 divided by 8 equal to -32?	
Translate.	$\frac{-256}{8} \overset{?}{=} -32$	
Simplify.	$-32 = -32✓$	

Note:
Exercise:

TRY IT :: 2.41

Problem: Translate and solve: n divided by 7 is equal to -21.

Solution:

$\frac{n}{7} = -21; n = -147$

Note:
Exercise:

TRY IT :: 2.42
Problem: Translate and solve: n divided by 8 is equal to -56.

Solution:

$\frac{n}{8} = -56; n = -448$

Example:
Exercise:

Problem: **Translate and solve: The quotient of y and -4 is 68.

Solution:
Solution

Begin by translating the sentence into an equation.

Translate.	$\underbrace{\text{The quotient of } y \text{ and } -4}_{\frac{y}{-4}}\ \underbrace{\text{is}}_{=}\ \underbrace{68.}_{68}$
Multiply both sides by -4.	$-4\left(\frac{y}{-4}\right) = -4(68)$
Simplify.	$y = -272$

Check:	Is the quotient of y and -4 equal to 68?	
Let $y = -272$.	Is the quotient of -272 and -4 equal to 68?	
Translate.	$\frac{-272}{-4} \overset{?}{=} 68$	
Simplify.	$68 = 68\checkmark$	

Note:
Exercise:

TRY IT :: 2.43
Problem: Translate and solve: The quotient of q and -8 is 72.

Solution:

$\frac{q}{-8} = 72; q = -576$

Note:
Exercise:

TRY IT :: 2.44
Problem: Translate and solve: The quotient of p and -9 is 81.

Solution:

$\frac{p}{-9} = 81; p = -729$

Example:
Exercise:

Problem: **Translate and solve: Three-fourths of p is 18.

Solution:
Solution

Begin by translating the sentence into an equation. Remember, “of” translates into multiplication.

Translate.		$\underbrace{\text{Three-fourths of } p}_{\frac{3}{4}p}\ \underbrace{\text{is}}_{=}\ \underbrace{18.}_{18}$
Multiply both sides by $\frac{4}{3}$.		$\frac{4}{3}\cdot\frac{3}{4}p=\frac{4}{3}\cdot 18$
Simplify.		$p=24$
Check:	Is three-fourths of p equal to 18?	
Let $p=24$.	Is three-fourths of 24 equal to 18?	
Translate.	$\frac{3}{4}\cdot 24 \overset{?}{=} 18$	
Simplify.	$18=18\checkmark$	

Note:
Exercise:

TRY IT :: 2.45

Problem: Translate and solve: Two-fifths of f is 16.

Solution:

$\frac{2}{5}f = 16; f = 40$

Note:
Exercise:

TRY IT :: 2.46
Problem: Translate and solve: Three-fourths of f is 21.

Solution:

$\frac{3}{4}f = 21; f = 28$

Example:
Exercise:

Problem: **Translate and solve: The sum of three-eighths and x is one-half.

Solution:
Solution

Begin by translating the sentence into an equation.

Translate.	$\underbrace{\text{The sum of three – eighths and } x}_{\frac{3}{8} + x} \underbrace{\text{is}}_{=} \underbrace{\frac{1}{2}}_{\frac{1}{2}}$
Subtract $\frac{3}{8}$ from each side.	$\frac{3}{8} - \frac{3}{8} + x = \frac{1}{2} - \frac{3}{8}$
Simplify and rewrite fractions with common denominators.	$x = \frac{4}{8} - \frac{3}{8}$

Simplify.			$x = \frac{1}{8}$
Check:		Is the sum of three-eighths and x equal to one-half?	
Let $x = \frac{1}{8}$.		Is the sum of three-eighths and one-eighth equal to one-half?	
Translate.		$\frac{3}{8} + \frac{1}{8} \overset{?}{=} \frac{1}{2}$	
Simplify.		$\frac{4}{8} \overset{?}{=} \frac{1}{2}$	
Simplify.		$\frac{1}{2} = \frac{1}{2} \checkmark$	

Note:
Exercise:

TRY IT :: 2.47
Problem: Translate and solve: The sum of five-eighths and *x* is one-fourth.

Solution:

$\frac{5}{8} + x = \frac{1}{4}; x = -\frac{3}{8}$

Note:
Exercise:

TRY IT :: 2.48
Problem: Translate and solve: The sum of three-fourths and *x* is five-sixths.

Solution:

$\frac{3}{4} + x = \frac{5}{6}; x = \frac{1}{12}$

Translate and Solve Applications

To solve applications using the Division and Multiplication Properties of Equality, we will follow the same steps we used in the last section. We will restate the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve.

Example:
Exercise:

Problem:

**Denae bought 6 pounds of grapes for $10.74. What was the cost of one pound of grapes?

Solution:
Solution

What are you asked to find?	The cost of 1 pound of grapes
Assign a variable.	Let c = the cost of one pound.
Write a sentence that gives the information to find it.	The cost of 6 pounds is $10.74.
Translate into an equation.	$6c = 10.74$
Solve.	$\frac{6c}{6} = \frac{10.74}{6}$ $c = 1.79$
	The grapes cost $1.79 per pound.
Check: If one pound costs $1.79, do 6 pounds cost #10.74? $6(1.79) \overset{?}{=} 10.74$ $10.74 = 10.74\checkmark$	

Note:
Exercise:

TRY IT :: 2.49

Problem: Translate and solve:

Arianna bought a 24-pack of water bottles for \$9.36. What was the cost of one water bottle?

Solution:

\$0.39

Note:
Exercise:

TRY IT :: 2.50

Problem: Translate and solve:

At JB's Bowling Alley, 6 people can play on one lane for \$34.98. What is the cost for each person?

Solution:

\$5.83

Example:
Exercise:

Problem:

**Andreas bought a used car for \$12,000. Because the car was 4-years old, its price was $\frac{3}{4}$ of the original price, when the car was new. What was the original price of the car?

Solution:

Solution

What are you asked to find?	The original price of the car
Assign a variable.	Let p = the original price.
Write a sentence that gives the information to find it.	\$12,000 is $\frac{3}{4}$ of the original price.
Translate into an equation.	$12{,}000 = \frac{3}{4}p$
Solve.	$\frac{4}{3}(12{,}000) = \frac{4}{3} \cdot \frac{3}{4}p$ $16{,}000 = p$
	The original cost of the car was \$16,000.
Check: Is $\frac{3}{4}$ of \$16,000 equal to \$12,000? $\frac{3}{4} \cdot 16{,}000 \overset{?}{=} 12{,}000$ $12{,}000 = 12{,}000✓$	

Note:
Exercise:

TRY IT :: 2.51

Problem: Translate and solve:

The annual property tax on the Mehta’s house is \$1,800, calculated as $\frac{15}{1{,}000}$ of the assessed value of the house. What is the assessed value of the Mehta’s house?

Solution:

\$120,000

Note:
Exercise:

TRY IT :: 2.52

Problem: Translate and solve:

Stella planted 14 flats of flowers in $\frac{2}{3}$ of her garden. How many flats of flowers would she need to fill the whole garden?

Solution:

21 flats

Key Concepts

- **The Division Property of Equality—**For any numbers a, b, and c, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
  When you divide both sides of an equation by any non-zero number, you still have equality.
- **The Multiplication Property of Equality—**For any numbers a, b, and c, if $a = b$, then $ac = bc$.
  If you multiply both sides of an equation by the same number, you still have equality.

Practice Makes Perfect

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the Division and Multiplication Properties of Equality and check the solution.

Exercise:

Problem: $8x = 56$

Solution:

$x = 7$

Exercise:

Problem: $7p = 63$

Exercise:

Problem: $-5c = 55$

Solution:

$c = -11$

Exercise:

Problem: $-9x = -27$

Exercise:

Problem: $-809 = 15y$

Solution:

$y = -\frac{809}{15}$

Exercise:

Problem: $-731 = 19y$

Exercise:

Problem: $-37p = -541$

Solution:

$p = -\frac{541}{37}$

Exercise:

Problem: $-19m = -586$

Exercise:

Problem: $0.25z = 3.25$

Solution:

$z = 13$

Exercise:

Problem: $0.75a = 11.25$

Exercise:

Problem: $-13x = 0$

Solution:

$x = 0$

Exercise:

Problem: $24x = 0$

Exercise:

Problem: $\frac{x}{4} = 35$

Solution:

$x = 140$

Exercise:

Problem: $\frac{z}{2} = 54$

Exercise:

Problem: $-20 = \frac{q}{-5}$

Solution:

$q = 100$

Exercise:

Problem: $\frac{c}{-3} = -12$

Exercise:

Problem: $\frac{y}{9} = -16$

Solution:

$y = -144$

Exercise:

Problem: $\frac{q}{6} = -38$

Exercise:

Problem: $\frac{m}{-12} = 45$

Solution:

$m = -540$

Exercise:

Problem: $-24 = \frac{p}{-20}$

Exercise:

Problem: $-y = 6$

Solution:

$y = -6$

Exercise:

Problem: $-u = 15$

Exercise:

Problem: $-v = -72$

Solution:

$v = 72$

Exercise:

Problem: $-x = -39$

Exercise:

Problem: $\frac{2}{3}y = 48$

Solution:

$y = 72$

Exercise:

Problem: $\frac{3}{5}r = 75$

Exercise:

Problem: $-\frac{5}{8}w = 40$

Solution:

$w = -64$

Exercise:

Problem: $24 = -\frac{3}{4}x$

Exercise:

Problem: $-\frac{2}{5} = \frac{1}{10}a$

Solution:

$a = -4$

Exercise:

Problem: $-\frac{1}{3}q = -\frac{5}{6}$

Exercise:

Problem: $-\frac{7}{10}x = -\frac{14}{3}$

Solution:

$x = \frac{20}{3}$

Exercise:

Problem: $\frac{3}{8}y = -\frac{1}{4}$

Exercise:

Problem: $\frac{7}{12} = -\frac{3}{4}p$

Solution:

$p = -\frac{7}{9}$

Exercise:

Problem: $\frac{11}{18} = -\frac{5}{6}q$

Exercise:

Problem: $-\frac{5}{18} = -\frac{10}{9}u$

Solution:

$u = \frac{1}{4}$

Exercise:

Problem: $-\frac{7}{20} = -\frac{7}{4}v$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

Exercise:

Problem: $100 - 16 = 4p - 10p - p$

Solution:

$p = -12$

Exercise:

Problem: $-18 - 7 = 5t - 9t - 6t$

Exercise:

Problem: $\frac{7}{8}n - \frac{3}{4}n = 9 + 2$

Solution:

$n = 88$

Exercise:

Problem: $\frac{5}{12}q + \frac{1}{2}q = 25 - 3$

Exercise:

Problem: $0.25d + 0.10d = 6 - 0.75$

Solution:

$d = 15$

Exercise:

Problem: $0.05p - 0.01p = 2 + 0.24$

Exercise:

Problem: $-10(q - 4) - 57 = 93$

Solution:

$q = -11$

Exercise:

Problem: $-12(d - 5) - 29 = 43$

Exercise:

Problem: $-10(x + 4) - 19 = 85$

Solution:

$x = -\frac{72}{5}$

Exercise:

Problem: $-15(z + 9) - 11 = 75$

Mixed Practice

In the following exercises, solve each equation.

Exercise:

Problem: $\frac{9}{10}x = 90$

Solution:

$x = 100$

Exercise:

Problem: $\frac{5}{12}y = 60$

Exercise:

Problem: $y + 46 = 55$

Solution:

$y = 9$

Exercise:

Problem: $x + 33 = 41$

Exercise:

Problem: $\frac{w}{-2} = 99$

Solution:

$w = -198$

Exercise:

Problem: $\frac{s}{-3} = -60$

Exercise:

Problem: $27 = 6a$

Solution:

$a = \frac{9}{2}$

Exercise:

Problem: $-a = 7$

Exercise:

Problem: $-x = 2$

Solution:

$x = -2$

Exercise:

Problem: $z - 16 = -59$

Exercise:

Problem: $m - 41 = -14$

Solution:

$m = 27$

Exercise:

Problem: $0.04r = 52.60$

Exercise:

Problem: $63.90 = 0.03p$

Solution:

$p = 2130$

Exercise:

Problem: $-15x = -120$

Exercise:

Problem: $84 = -12z$

Solution:

$y = -7$

Exercise:

Problem: $19.36 = x - 0.2x$

Exercise:

Problem: $c - 0.3c = 35.70$

Solution:

$c = 51$

Exercise:

Problem: $-y = -9$

Exercise:

Problem: $-x = -8$

Solution:

$x = 8$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

Exercise:

Problem: 187 is the product of -17 and *m*.

Exercise:

Problem: 133 is the product of -19 and *n*.

Solution:

$133 = -19n; n = -7$

Exercise:

Problem: -184 is the product of 23 and *p*.

Exercise:

Problem: -152 is the product of 8 and *q*.

Solution:

$-152 = 8q; q = -19$

Exercise:

Problem: u divided by 7 is equal to -49.

Exercise:

Problem: r divided by 12 is equal to -48.

Solution:

$\frac{r}{12} = -48; r = -576$

Exercise:

Problem: h divided by -13 is equal to -65.

Exercise:

Problem: j divided by -20 is equal to -80.

Solution:

$\frac{j}{-20} = -80; j = 1{,}600$

Exercise:

Problem: The quotient c and -19 is 38.

Exercise:

Problem: The quotient of b and -6 is 18.

Solution:

$\frac{b}{-6} = 18; b = -108$

Exercise:

Problem: The quotient of h and 26 is -52.

Exercise:

Problem: The quotient k and 22 is -66.

Solution:

$\frac{k}{22}=-66; k=-1{,}452$

Exercise:

Problem: Five-sixths of *y* is 15.

Exercise:

Problem: Three-tenths of *x* is 15.

Solution:

$\frac{3}{10}x=15; x=50$

Exercise:

Problem: Four-thirds of *w* is 36.

Exercise:

Problem: Five-halves of *v* is 50.

Solution:

$\frac{5}{2}v=50; v=20$

Exercise:

Problem: The sum of nine-tenths and *g* is two-thirds.

Exercise:

Problem: The sum of two-fifths and *f* is one-half.

Solution:

$\frac{2}{5}+f=\frac{1}{2}; f=\frac{1}{10}$

Exercise:

Problem: The difference of *p* and one-sixth is two-thirds.

Exercise:

Problem: The difference of *q* and one-eighth is three-fourths.

Solution:

$q-\frac{1}{8}=\frac{3}{4};q=\frac{7}{8}$

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise:

Problem:

Kindergarten Connie’s kindergarten class has 24 children. She wants them to get into 4 equal groups. How many children will she put in each group?

Exercise:

Problem:

Balloons Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. How many balloons did she use in each bunch?

Solution:

6 balloons

Exercise:

Problem:

Tickets Mollie paid $36.25 for 5 movie tickets. What was the price of each ticket?

Exercise:

Problem:

Shopping Serena paid $12.96 for a pack of 12 pairs of sport socks. What was the price of pair of sport socks?

Solution:

$1.08

Exercise:

Problem:

Sewing Nancy used 14 yards of fabric to make flags for one-third of the drill team. How much fabric, would Nancy need to make flags for the whole team?

Exercise:

Problem:

MPG John’s SUV gets 18 miles per gallon (mpg). This is half as many mpg as his wife’s hybrid car. How many miles per gallon does the hybrid car get?

Solution:

36 mpg

Exercise:

Problem:

Height Aiden is 27 inches tall. He is $\frac{3}{8}$ as tall as his father. How tall is his father?

Exercise:

Problem:

Real estate Bea earned $11,700 commission for selling a house, calculated as $\frac{6}{100}$ of the selling price. What was the selling price of the house?

Solution:

$195,000

Everyday Math

Exercise:

Problem:

Commission Every week Perry gets paid $150 plus 12% of his total sales amount. Solve the equation $840 = 150 + 0.12(a - 1250)$ for a, to find the total amount Perry must sell in order to be paid $840 one week.

Exercise:

Problem:

Stamps Travis bought $9.45 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 5 less than the number of 49-cent stamps. Solve the equation $0.49s + 0.21(s - 5) = 9.45$ for s, to find the number of 49-cent stamps Travis bought.

Solution:

15 49-cent stamps

Writing Exercises

Exercise:

Problem:

Frida started to solve the equation $-3x=36$ by adding 3 to both sides. Explain why Frida’s method will not solve the equation.

Exercise:

Problem:

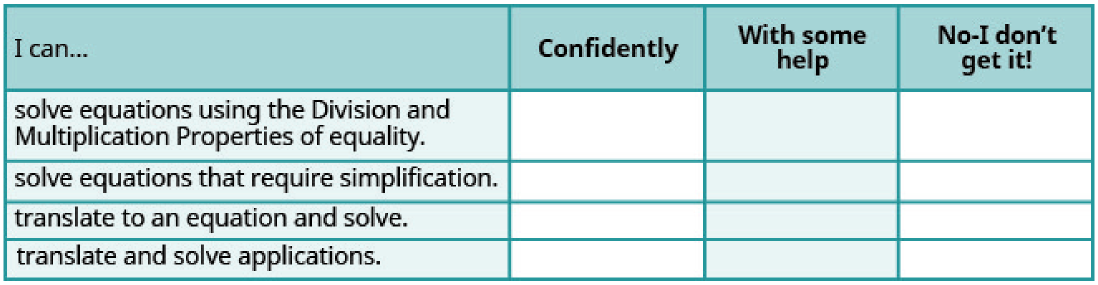
Emiliano thinks $x=40$ is the solution to the equation $\frac{1}{2}x=80$. Explain why he is wrong.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
solve equations using the Division and Multiplication Properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Solve Equations with Variables and Constants on Both Sides
By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides

Note:
Before you get started, take this readiness quiz.

1. Simplify: $4y-9+9$.
   If you missed this problem, review [link].

Solve Equations with Constants on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we will learn to solve equations in which the variable terms, or constant terms, or both are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the “variable side”, and the other side of the equation to be the “constant side.” Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax=b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Example:
Exercise:

Problem: **Solve: $7x + 8 = -13$.

Solution:
Solution

In this equation, the variable is found only on the left side. It makes sense to call the left side the “variable” side. Therefore, the right side will be the “constant” side. We will write the labels above the equation to help us remember what goes where.

$$\overset{\text{variable}}{7x + 8} = \overset{\text{constant}}{-13}$$

Since the left side is the “x”, or variable side, the 8 is out of place. We must “undo” adding 8 by subtracting 8, and to keep the equality we must subtract 8 from both sides.

	$\overset{\text{variable}}{7x + 8} = \overset{\text{constant}}{-14\ 3}$
Use the Subtraction Property of Equality.	$7x + 8 - 8 = -13 - 8$
Simplify.	$7x = -21$
Now all the variables are on the left and the constant on the right.	

The equation looks like those you learned to solve earlier.		
Use the Division Property of Equality.		$\frac{7x}{7}=\frac{-21}{7}$
Simplify.		$x=-3$
Check:	$7x+8=-13$	
Let $x=-3$.	$7(-3)+8\overset{?}{=}-13$	
	$-21+8\overset{?}{=}-13$ $-13=-13$ ✓	

Note:
Exercise:

TRY IT :: 2.53
Problem: Solve: $3x+4=-8$.

Solution:

$x=-4$

Note:
Exercise:

TRY IT :: 2.54

Problem: Solve: $5a + 3 = -37$.

Solution:

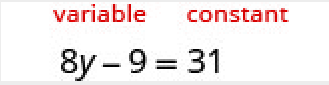
$a = -8$

Example:
Exercise:

Problem: **Solve: $8y - 9 = 31$.

Solution:
Solution

Notice, the variable is only on the left side of the equation, so we will call this side the “variable” side, and the right side will be the “constant” side. Since the left side is the “variable” side, the 9 is out of place. It is subtracted from the $8y$, so to “undo” subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

Add 9 to both sides.		$8y-9+9=31+9$
Simplify.		$8y=40$
		The variables are now on one side and the constants on the other. We continue from here as we did earlier.
Divide both sides by 8.		$\frac{8y}{8}=\frac{40}{8}$
Simplify.		$y=5$
Check:	$8y-9=31$	
Let $y=5$.	$8\cdot5-9\overset{?}{=}31$	
	$40-9\overset{?}{=}31$	
	$31=31$ ✓	

Note:
Exercise:

TRY IT :: 2.55

Problem: Solve: $5y - 9 = 16$.

Solution:

$y = 5$

Note:
Exercise:

TRY IT :: 2.56

Problem: Solve: $3m - 8 = 19$.

Solution:

$m = 9$

Solve Equations with Variables on Both Sides

What if there are variables on both sides of the equation? For equations like this, begin as we did above—choose a “variable” side and a “constant” side, and then use the subtraction and addition properties of equality to collect all variables on one side and all constants on the other side.

Example:
Exercise:

Problem: **Solve: $9x = 8x - 6$.

Solution:
Solution

Here the variable is on both sides, but the constants only appear on the right side, so let’s make the right side the “constant” side. Then the left side will be the “variable” side.

	$\overset{\text{variable}}{9x} = \overset{\text{constant}}{8x - 6}$
We don’t want any x’s on the right, so subtract the $8x$ from both sides.	$9x - 8x = 8x - 8x - 6$
Simplify.	$x = -6$
We succeeded in getting the variables on one side and the constants on the other, and have obtained the solution.	

Check:	$9x = 8x - 6$	
Let $x = -6$.	$9(-6) \overset{?}{=} 8(-6) - 6$	
	$-54 \overset{?}{=} -48 - 6$	
	$-54 = -54 \checkmark$	

Note:
Exercise:

TRY IT :: 2.57

Problem: Solve: $6n = 5n - 10$.

Solution:

$n = -10$

Note:
Exercise:

TRY IT :: 2.58

Problem: Solve: $-6c = -7c - 1$.

Solution:

$c = -1$

Example:
Exercise:

Problem: **Solve: $5y - 9 = 8y$.

Solution:
Solution

The only constant is on the left and the y’s are on both sides. Let’s leave the constant on the left and get the variables to the right.

		$5y \overset{\text{constant}}{-9} = \overset{\text{variable}}{8y}$
Subtract $5y$ from both sides.		$5y - 5y - 9 = 8y - 5y$
Simplify.		$-9 = 3y$
We have the y’s on the right and the constants on the left. Divide both sides by 3.		$\frac{-9}{3} = \frac{3y}{3}$
Simplify.		$-3 = y$
Check:	$5y - 9 = 8y$	
Let $y = -3$.	$5(-3) - 9 \overset{?}{=} 8(-3)$	
	$-15 - 9 \overset{?}{=} -24$	

	$-24 = -24$ ✓	

Note:
Exercise:

TRY IT :: 2.59

Problem: Solve: $3p - 14 = 5p$.

Solution:

$p = -7$

Note:
Exercise:

TRY IT :: 2.60

Problem: Solve: $8m + 9 = 5m$.

Solution:

$m = -3$

Example:
Exercise:

Problem: **Solve: $12x = -x + 26$.

Solution:
Solution

The only constant is on the right, so let the left side be the “variable” side.

	$\overset{\text{variable}}{12x} = \overset{\text{constant}}{-x + 26}$
Remove the $-x$ from the right side by adding x to both sides.	$12x + x = -x + x + 26$
Simplify.	$13x = 26$
All the x’s are on the left and the constants are on the right. Divide both sides by 13.	$\frac{13x}{13} = \frac{26}{13}$
Simplify.	$x = 2$

Note:
Exercise:

TRY IT :: 2.61

Problem: Solve: $12j = -4j + 32$.

Solution:

$j = 2$

Note:
Exercise:

TRY IT :: 2.62

Problem: Solve: $8h = -4h + 12.$

Solution:

$h = 1$

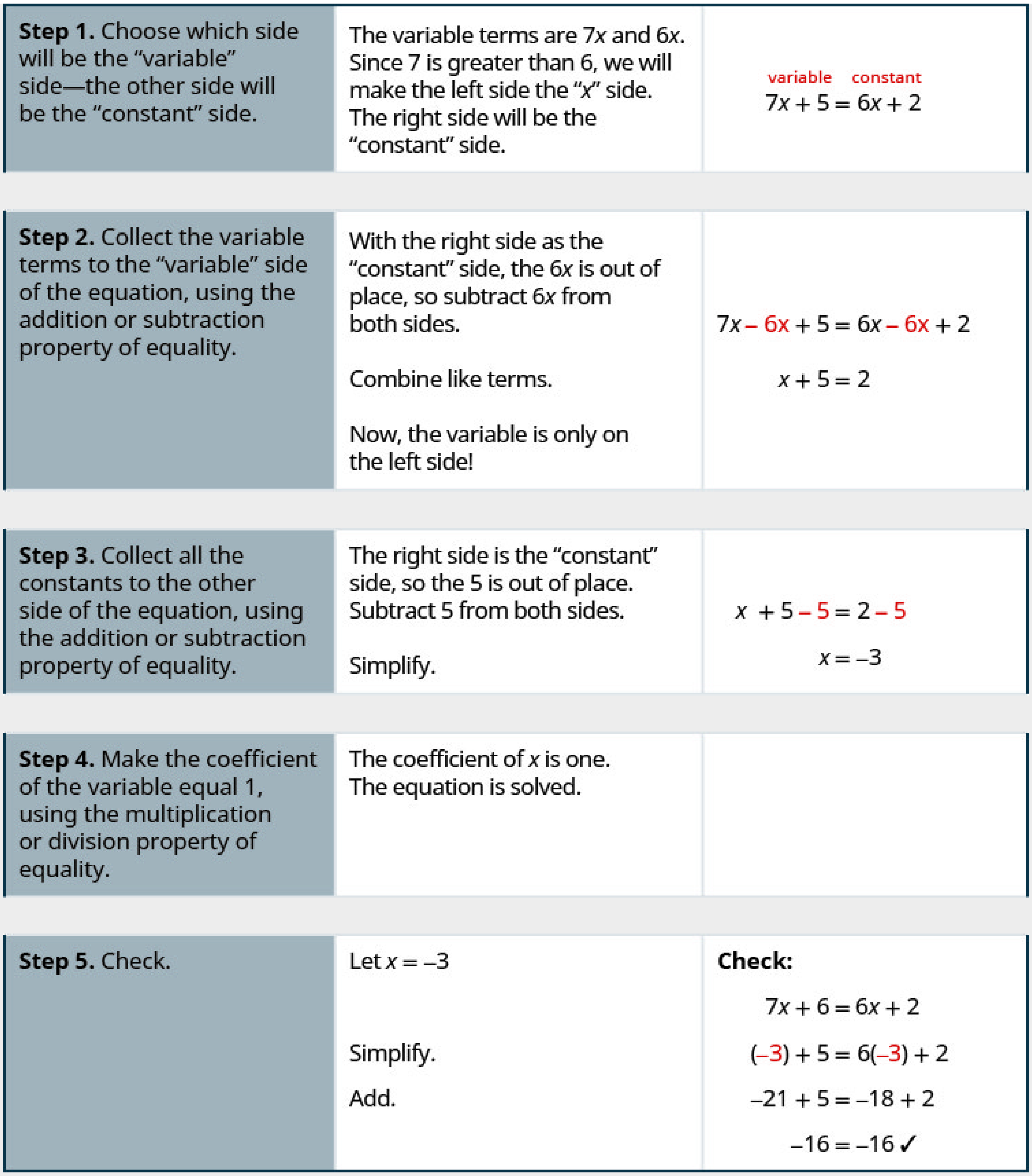
Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Example:
How to Solve Equations with Variables and Constants on Both Sides
Exercise:

Problem: **Solve: $7x + 5 = 6x + 2.$

Solution:
Solution



Step 1. Choose which side will be the “variable” side—the other side will be the “constant” side.	The variable terms are 7*x* and 6*x*. Since 7 is greater than 6, we will make the left side the “*x*” side. The right side will be the “constant” side.	$\overset{\text{variable}}{7x+5} = \overset{\text{constant}}{6x+2}$
Step 2. Collect the variable terms to the “variable” side of the equation, using the addition or subtraction property of equality.	With the right side as the “constant” side, the 6*x* is out of place, so subtract 6*x* from both sides. Combine like terms. Now, the variable is only on the left side!	$7x - 6x + 5 = 6x - 6x + 2$ $x + 5 = 2$
Step 3. Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.	The right side is the “constant” side, so the 5 is out of place. Subtract 5 from both sides. Simplify.	$x + 5 - 5 = 2 - 5$ $x = -3$
Step 4. Make the coefficient of the variable equal 1, using the multiplication or division property of equality.	The coefficient of *x* is one. The equation is solved.	
Step 5. Check.	Let $x = -3$ Simplify. Add.	**Check:** $7x + 6 = 6x + 2$ $(-3) + 5 = 6(-3) + 2$ $-21 + 5 = -18 + 2$ $-16 = -16$ ✓

Note:

Exercise:

TRY IT :: 2.63

Problem: Solve: $12x + 8 = 6x + 2$.

Solution:

$x = -1$

Note:
Exercise:

TRY IT :: 2.64

Problem: Solve: $9y + 4 = 7y + 12$.

Solution:

$y = 4$

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

Note:
Beginning Strategy for Solving Equations with Variables and Constants on Both Sides of the Equation.

Choose which side will be the "variable" side—the other side will be the "constant" side.
Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.

Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the “variable” side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

Example:
Exercise:

Problem: **Solve: $8n-4=-2n+6$.

Solution:
Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $8>-2$, make the left side the “variable” side.	$\overset{\text{variable}}{8n-4} = \overset{\text{constant}}{-2n+6}$
We don’t want variable terms on the right side—add $2n$ to both sides to leave only constants on the right.	$8n+2n-4=-2n+2n+6$
Combine like terms.	$10n-4=6$
We don’t want any constants on the left side, so add	$10n-4+4=6+4$

4 to both sides.		
Simplify.		$10n = 10$
The variable term is on the left and the constant term is on the right. To get the coefficient of n to be one, divide both sides by 10.		$\frac{10n}{10} = \frac{10}{10}$
Simplify.		$n = 1$
Check:	$8n - 4 = -2n + 6$	
Let $n = 1$.	$8 \cdot 1 - 4 \overset{?}{=} -2 \cdot 1 + 6$	
	$8 - 4 \overset{?}{=} -2 + 6$	
	$4 = 4 \checkmark$	

Note:
Exercise:

TRY IT :: 2.65

Problem: Solve: $8q - 5 = -4q + 7$.

Solution:

$q = 1$

Note:
Exercise:

TRY IT :: 2.66

Problem: Solve: $7n - 3 = n + 3$.

Solution:

$n = 1$

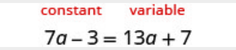
Example:
Exercise:

Problem: **Solve: $7a - 3 = 13a + 7$.

Solution:
Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $13 > 7$, make the right side the “variable” side and the left side the “constant” side.

Subtract $7a$ from both sides to remove the variable term from the left.		$7a-7a-3=13a-7a+7$
Combine like terms.		$-3=6a+7$
Subtract 7 from both sides to remove the constant from the right.		$-3-7=6a+7-7$
Simplify.		$-10=6a$
Divide both sides by 6 to make 1 the coefficient of a.		$\frac{-10}{6}=\frac{6a}{6}$
Simplify.		$-\frac{5}{3}=a$
Check:	$7a-3=13a+7$	
Let $a=-\frac{5}{3}$.	$7\left(-\frac{5}{3}\right)-3\overset{?}{=}13\left(-\frac{5}{3}\right)+7$	
	$-\frac{35}{3}-\frac{9}{3}\overset{?}{=}-\frac{65}{3}+\frac{21}{3}$	
	$-\frac{54}{3}=-\frac{54}{3}$ ✓	

Note:
Exercise:

TRY IT :: 2.67

Problem: Solve: $2a - 2 = 6a + 18$.

Solution:

$a = -5$

Note:
Exercise:

TRY IT :: 2.68

Problem: Solve: $4k - 1 = 7k + 17$.

Solution:

$k = -6$

In the last example, we could have made the left side the “variable” side, but it would have led to a negative coefficient on the variable term. (Try it!) While we could work with the negative, there is less chance of errors when working with positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

Example:
Exercise:

Problem: **Solve: $\frac{5}{4}x+6=\frac{1}{4}x-2$.

Solution:
Solution

Since $\frac{5}{4}>\frac{1}{4}$, make the left side the “variable” side and the right side the “constant” side.

	$\overset{\text{variable}}{\frac{5}{4}x+6}=\overset{\text{constant}}{\frac{1}{4}x-2}$
Subtract $\frac{1}{4}x$ from both sides.	$\frac{5}{4}x-\frac{1}{4}x+6=\frac{1}{4}x-\frac{1}{4}x-2$
Combine like terms.	$x+6=-2$
Subtract 6 from both sides.	$x+6-6=-2-6$
Simplify.	$x=-8$

Check:

Let $x=-8$.

$$\begin{aligned}\frac{5}{4}x+6&=\frac{1}{4}x-2\\\frac{5}{4}(-8)+6&\overset{?}{=}\frac{1}{4}(-8)-2\\-10+6&\overset{?}{=}-2-2\\-4&=-4\checkmark\end{aligned}$$

Note:
Exercise:

TRY IT :: 2.69

Problem: Solve: $\frac{7}{8}x - 12 = -\frac{1}{8}x - 2$.

Solution:

$x = 10$

Note:
Exercise:

TRY IT :: 2.70

Problem: Solve: $\frac{7}{6}y + 11 = \frac{1}{6}y + 8$.

Solution:

$y = -3$

We will use the same strategy to find the solution for an equation with decimals.

Example:
Exercise:

Problem: **Solve: $7.8x + 4 = 5.4x - 8$.

Solution:
Solution

Since $7.8 > 5.4$, make the left side the “variable” side and the right side the “constant” side.

	$\overset{\text{variable side}}{7.8x + 4} = \overset{\text{constant side}}{5.4x - 8}$
Subtract $5.4x$ from both sides.	$7.8x - 5.4x + 4 = 5.4x - 5.4x - 8$
Combine like terms.	$2.4x + 4 = -8$
Subtract 4 from both sides.	$2.4x + 4 - 4 = -8 - 4$
Simplify.	$2.4x = -12$
Use the Division Propery of Equality.	$\frac{2.4x}{2.4} = \frac{-12}{2.4}$
Simplify.	$x = -5$

Check:	$7.8x + 4 = 5.4x - 5$	
Let $x = -5.$	$7.8(-5) + 4 = 5.4(-5) - 8$	
	$-39 + 4 \overset{?}{=} -27 - 8$	
	$-35 = -35$ ✓	

Note:
Exercise:

TRY IT :: 2.71

Problem: Solve: $2.8x + 12 = -1.4x - 9.$

Solution:

$x = -5$

Note:
Exercise:

TRY IT :: 2.72

Problem: Solve: $3.6y + 8 = 1.2y - 4.$

Solution:

$y = -5$

Key Concepts

- **Beginning Strategy for Solving an Equation with Variables and Constants on Both Sides of the Equation**

  Choose which side will be the “variable” side—the other side will be the “constant” side.
  Collect the variable terms to the “variable” side of the equation, using the Addition or Subtraction Property of Equality.
  Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
  Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
  Check the solution by substituting it into the original equation.

Practice Makes Perfect

Solve Equations with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

Exercise:

Problem: $9x - 3 = 60$

Exercise:

Problem: $12x - 8 = 64$

Solution:

$x = 6$

Exercise:

Problem: $14w + 5 = 117$

Exercise:

Problem: $15y + 7 = 97$

Solution:

$y = 6$

Exercise:

Problem: $2a + 8 = -28$

Exercise:

Problem: $3m + 9 = -15$

Solution:

$m = -8$

Exercise:

Problem: $-62 = 8n - 6$

Exercise:

Problem: $-77 = 9b - 5$

Solution:

$b = -8$

Exercise:

Problem: $35=-13y+9$

Exercise:

Problem: $60=-21x-24$

Solution:

$x=-4$

Exercise:

Problem: $-12p-9=9$

Exercise:

Problem: $-14q-2=16$

Solution:

$q=-\frac{9}{7}$

Solve Equations with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

Exercise:

Problem: $19z=18z-7$

Exercise:

Problem: $21k=20k-11$

Solution:

$k = -11$

Exercise:

Problem: $9x + 36 = 15x$

Exercise:

Problem: $8x + 27 = 11x$

Solution:

$x = 9$

Exercise:

Problem: $c = -3c - 20$

Exercise:

Problem: $b = -4b - 15$

Solution:

$b = -3$

Exercise:

Problem: $9q = 44 - 2q$

Exercise:

Problem: $5z = 39 - 8z$

Solution:

$z = 3$

Exercise:

Problem: $6y + \frac{1}{2} = 5y$

Exercise:

Problem: $4x + \frac{3}{4} = 3x$

Solution:

$x = -\frac{3}{4}$

Exercise:

Problem: $-18a - 8 = -22a$

Exercise:

Problem: $-11r - 8 = -7r$

Solution:

$r = -2$

Solve Equations with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

Exercise:

Problem: $8x - 15 = 7x + 3$

Exercise:

Problem: $6x - 17 = 5x + 2$

Solution:

$x = 19$

Exercise:

Problem: $26 + 13d = 14d + 11$

Exercise:

Problem: $21 + 18f = 19f + 14$

Solution:

$f = 7$

Exercise:

Problem: $2p - 1 = 4p - 33$

Exercise:

Problem: $12q - 5 = 9q - 20$

Solution:

$q = -5$

Exercise:

Problem: $4a + 5 = -a - 40$

Exercise:

Problem: $8c + 7 = -3c - 37$

Solution:

$c = -4$

Exercise:

Problem: $5y-30=-5y+30$

Exercise:

Problem: $7x-17=-8x+13$

Solution:

$x=2$

Exercise:

Problem: $7s+12=5+4s$

Exercise:

Problem: $9p+14=6+4p$

Solution:

$p=-\frac{8}{5}$

Exercise:

Problem: $2z-6=23-z$

Exercise:

Problem: $3y-4=12-y$

Solution:

$y=4$

Exercise:

Problem: $\frac{5}{3}c-3=\frac{2}{3}c-16$

Exercise:

Problem: $\frac{7}{4}m-7=\frac{3}{4}m-13$

Solution:

$m=-6$

Exercise:

Problem: $8-\frac{2}{5}q=\frac{3}{5}q+6$

Exercise:

Problem: $11-\frac{1}{5}a=\frac{4}{5}a+4$

Solution:

$a=7$

Exercise:

Problem: $\frac{4}{3}n+9=\frac{1}{3}n-9$

Exercise:

Problem: $\frac{5}{4}a+15=\frac{3}{4}a-5$

Solution:

$a=-40$

Exercise:

Problem: $\frac{1}{4}y+7=\frac{3}{4}y-3$

Exercise:

Problem: $\frac{3}{5}p + 2 = \frac{4}{5}p - 1$

Solution:

$p = 15$

Exercise:

Problem: $14n + 8.25 = 9n + 19.60$

Exercise:

Problem: $13z + 6.45 = 8z + 23.75$

Solution:

$z = 3.46$

Exercise:

Problem: $2.4w - 100 = 0.8w + 28$

Exercise:

Problem: $2.7w - 80 = 1.2w + 10$

Solution:

$w = 60$

Exercise:

Problem: $5.6r + 13.1 = 3.5r + 57.2$

Exercise:

Problem: $6.6x - 18.9 = 3.4x + 54.7$

Solution:

$x = 23$

Everyday Math

Exercise:

Problem:

Concert tickets At a school concert the total value of tickets sold was \$1506. Student tickets sold for \$6 and adult tickets sold for \$9. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, *s*, by solving the equation $6s + 27s - 45 = 1506$.

Exercise:

Problem:

Making a fence Jovani has 150 feet of fencing to make a rectangular garden in his backyard. He wants the length to be 15 feet more than the width. Find the width, *w*, by solving the equation $150 = 2w + 30 + 2w$.

Solution:

30 feet

Writing Exercises

Exercise:

Problem:

Solve the equation $\frac{6}{5}y - 8 = \frac{1}{5}y + 7$ explaining all the steps of your solution as in the examples in this section.

Exercise:

Problem:

Solve the equation $10x + 14 = -2x + 38$ explaining all the steps of your solution as in the examples in this section.

Solution:

$x = 2$ Justifications will vary.

Exercise:

Problem:

When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient of x to be the “variable” side?

Exercise:

Problem:

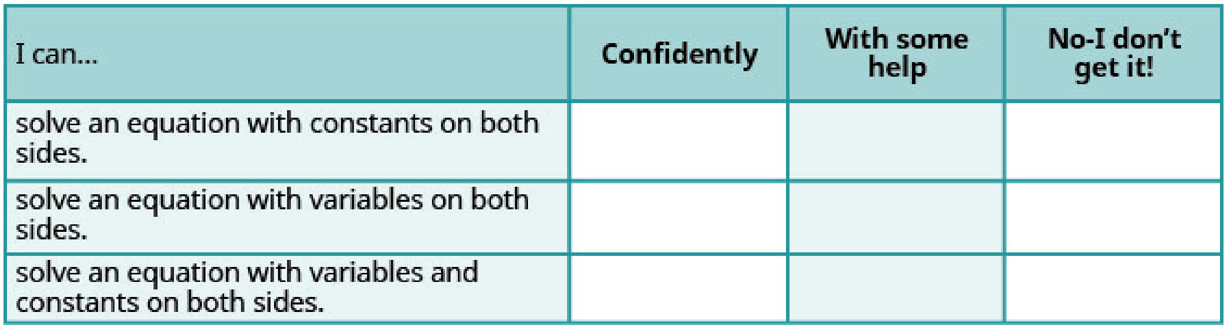
Is $x = -2$ a solution to the equation $5 - 2x = -4x + 1$? How do you know?

Solution:

Yes. Justifications will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Use a General Strategy to Solve Linear Equations
By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

Note:
Before you get started, take this readiness quiz.

1. Simplify: $-(a-4)$.
   If you missed this problem, review [link].
2. Multiply: $\frac{3}{2}(12x+20)$.
   If you missed this problem, review [link].
3. Simplify: $5-2(n+1)$.
   If you missed this problem, review [link].
4. Multiply: $3(7y+9)$.
   If you missed this problem, review [link].
5. Multiply: $(2.5)\,(6.4)$.
   If you missed this problem, review [link].

Solve Equations Using the General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

Example:

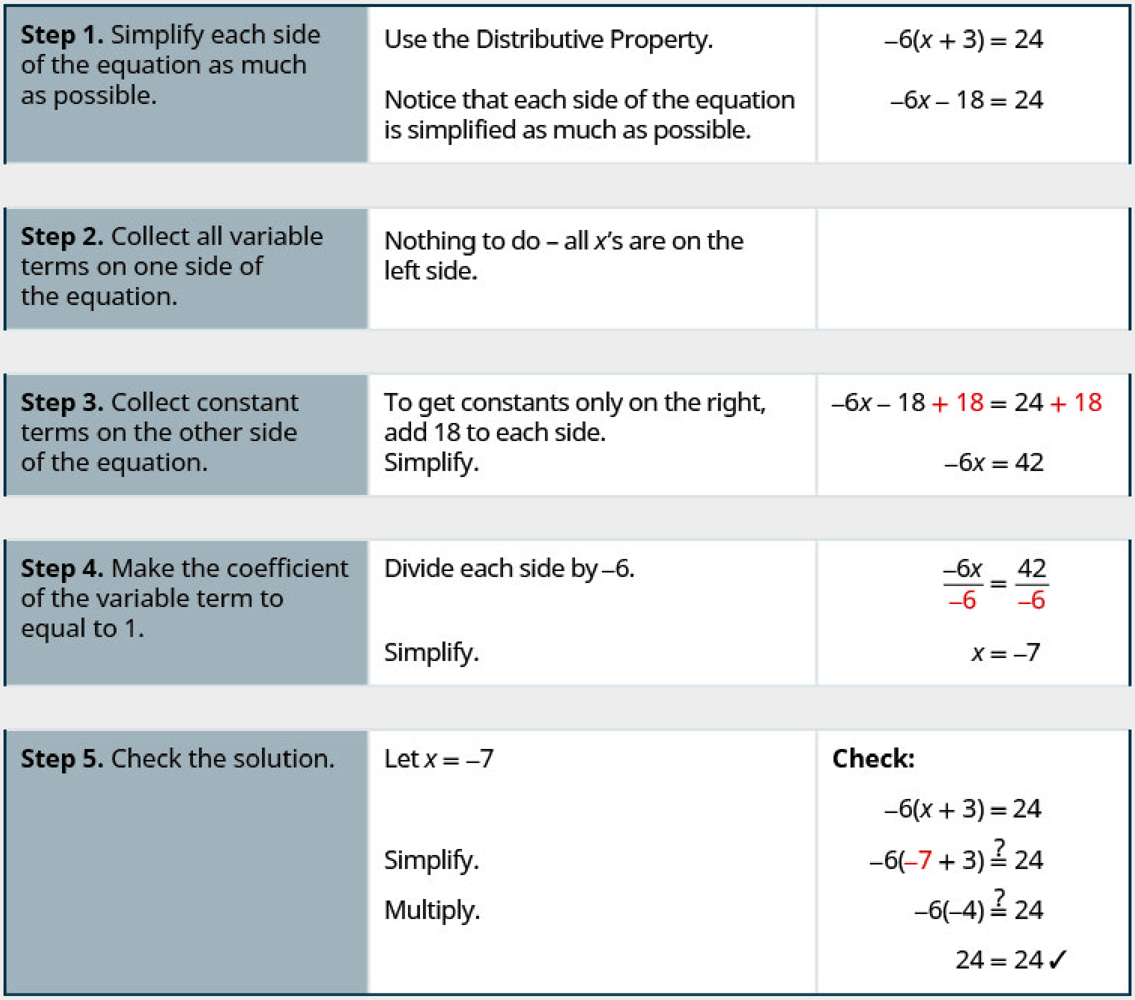
How to Solve Linear Equations Using the General Strategy

Exercise:

Problem: **Solve: $-6\left(x+3\right)=24.$

Solution:

Solution



Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x+3)=24$ $-6x-18=24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all *x*'s are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side. Simplify.	$-6x-18+18=24+18$ $-6x=42$
Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by –6. Simplify.	$\frac{-6x}{-6}=\frac{42}{-6}$ $x=-7$
Step 5. Check the solution.	Let $x=-7$ Simplify. Multiply.	**Check:** $-6(x+3)=24$ $-6(-7+3)\stackrel{?}{=}24$ $-6(-4)\stackrel{?}{=}24$ $24=24$ ✓

Note:

Exercise:

TRY IT :: 2.73

Problem: Solve: $5\left(x+3\right)=35$.

Solution:

$x=4$

Note:
Exercise:

TRY IT :: 2.74

Problem: Solve: $6\left(y-4\right)=-18$.

Solution:

$y=1$

Note:
General strategy for solving linear equations.

Simplify each side of the equation as much as possible.	Use the Distributive Property to remove any parentheses.	Combine like terms.
Collect all the variable terms on one side of the equation.	Use the Addition or Subtraction Property of Equality.	
Collect all the constant terms on the other side of the equation.	Use the Addition or Subtraction Property of Equality.	
Make the coefficient of the variable term to equal to 1.	Use the Multiplication or Division Property of Equality.	State the solution to the equation.
Check the solution.	Substitute the solution into the original equation to make sure the result is a true statement.	

Example:
Exercise:

Problem: **Solve: $-(y+9)=8$.

Solution:
Solution

	$-(y+9)=8$
Simplify each side of the equation as much as possible by distributing.	$-y-9=8$
The only y term is on the left side, so all variable terms are on the left side of the equation.	
Add 9 to both sides to get all constant terms on the right side of the equation.	$-y-9+9=8+9$
Simplify.	$-y=17$
Rewrite $-y$ as $-1y$.	$-1y=17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by -1.	$\frac{-1y}{-1}=\frac{17}{-1}$

Simplify.		$y=-17$
Check:	$-(y+9)=8$	
Let $y=-17$.	$-(-17+9)\overset{?}{=}8$	
	$-(-8)\overset{?}{=}8$	
	$8=8$ ✓	

Note:
Exercise:

TRY IT :: 2.75

Problem: Solve: $-(y+8)=-2$.

Solution:

$y=-6$

Note:
Exercise:

TRY IT :: 2.76

Problem: Solve: $-(z+4)=-12$.

Solution:

$z=8$

Example:
Exercise:

Problem: **Solve: $5(a-3)+5=-10$.

Solution:
Solution

	$5(a-3)+5=-10$
Simplify each side of the equation as much as possible.	
Distribute.	$5a-15+5=-10$
Combine like terms.	$5a-10=-10$
The only a term is on the left side, so all	

variable terms are on one side of the equation.		
Add 10 to both sides to get all constant terms on the other side of the equation.		$5a-10+10=-10+10$
Simplify.		$5a=0$
Make the coefficient of the variable term to equal to 1 by dividing both sides by 5.		$\frac{5a}{5}=\frac{0}{5}$
Simplify.		$a=0$
Check:	$5(a-3)+5=-10$	
Let $a=0$.	$5(0-3)+5\overset{?}{=}-10$	
	$5(-3)+5\overset{?}{=}-10$	
	$-15+5\overset{?}{=}-10$	
	$-10=-10$ ✓	

Note:

Exercise:

TRY IT :: 2.77

Problem: Solve: $2\left(m-4\right)+3=-1.$

Solution:

$m=2$

Note:

Exercise:

TRY IT :: 2.78

Problem: Solve: $7\left(n-3\right)-8=-15.$

Solution:

$n=2$

Example:

Exercise:

Problem: **Solve: $\frac{2}{3}\left(6m-3\right)=8-m.$

Solution:

Solution

		$\frac{2}{3}(6m-3)=8-m$
Distribute.		$4m-2=8-m$
Add m to get the variables only to the left.		$4m+m-2=8-m+m$
Simplify.		$5m-2=8$
Add 2 to get constants only on the right.		$5m-2+2=8+2$
Simplify.		$5m=10$
Divide by 5.		$\frac{5m}{5}=\frac{10}{5}$
Simplify.		$m=2$
Check:	$\frac{2}{3}(6m-3)=8-m$	
Let $m=2$.	$\frac{2}{3}(6\cdot 2-3)\overset{?}{=}8-2$	

	$\frac{2}{3}(12-3)\overset{?}{=}6$	
	$\frac{2}{3}(9)\overset{?}{=}6$	
	$6=6\checkmark$	

Note:
Exercise:

TRY IT :: 2.79

Problem: Solve: $\frac{1}{3}(6u+3)=7-u$.

Solution:

$u=2$

Note:
Exercise:

TRY IT :: 2.80

Problem: Solve: $\frac{2}{3}(9x-12)=8+2x$.

Solution:

$x=4$

Example:
Exercise:

Problem: **Solve: $8-2\left(3y+5\right)=0$.

Solution:
Solution

	$8-2(3y+5)=0$
Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by -6.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$

Check: Let $y = -\frac{1}{3}$.

$$8 - 2(3y + 5) = 0$$

$$8 - 2\left[3\left(-\frac{1}{3}\right) + 5\right] = 0$$

$$8 - 2(-1 + 5) \overset{?}{=} 0$$

$$8 - 2(4) \overset{?}{=} 0$$

$$8 - 8 \overset{?}{=} 0$$

$$0 = 0 \checkmark$$

Note:
Exercise:

TRY IT :: 2.81

Problem: Solve: $12 - 3\left(4j + 3\right) = -17$.

Solution:

$j = \frac{5}{3}$

Note:
Exercise:

TRY IT :: 2.82

Problem: Solve: $-6-8(k-2)=-10$.

Solution:

$k=\frac{5}{2}$

Example:
Exercise:

Problem: **Solve: $4(x-1)-2=5(2x+3)+6$.

Solution:
Solution

	$4(x-1)-2=5(2x+3)+6$
Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract $4x$ to get the variables only on the right side since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$

Subtract 21 to get the constants on left.		$-6-21=6x+21-21$
Simplify.		$-27=6x$
Divide by 6.		$\frac{-27}{6}=\frac{6x}{6}$
Simplify.		$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$	
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\overset{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$	
	$4\left(-\frac{11}{2}\right)-2\overset{?}{=}5(-9+3)+6$	
	$-22-2\overset{?}{=}5(-6)+6$	
	$-24\overset{?}{=}-30+6$	
	$-24=-24$ ✓	

Note:
Exercise:

TRY IT :: 2.83

Problem: Solve: $6(p-3)-7=5(4p+3)-12$.

Solution:

$p=-2$

Note:
Exercise:

TRY IT :: 2.84

Problem: Solve: $8(q+1)-5=3(2q-4)-1$.

Solution:

$q=-8$

Example:
Exercise:

Problem: **Solve: $10[3-8(2s-5)]=15(40-5s)$.

Solution:
Solution

		$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add 160*s* to get the s’s to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Substitute	$10[3-8(2(-2)-5)]\stackrel{?}{=}15(40-5(-2))$	

$s=-2.$		
	$10[3-8(-4-5)] \stackrel{?}{=} 15(40+10)$	
	$10[3-8(-9)] \stackrel{?}{=} 15(50)$	
	$10[3+72] \stackrel{?}{=} 750$	
	$10[75] \stackrel{?}{=} 750$	
	$750 = 750$ ✓	

Note:
Exercise:

TRY IT :: 2.85

Problem: Solve: $6\left[4-2\left(7y-1\right)\right]=8\left(13-8y\right)$.

Solution:

$y=-\frac{17}{5}$

Note:
Exercise:

TRY IT :: 2.86

Problem: Solve: $12\left[1-5\left(4z-1\right)\right]=3\left(24+11z\right)$.

Solution:

$z=0$

Example:

Exercise:

Problem: **Solve: $0.36(100n+5)=0.6(30n+15)$.

Solution:

Solution

	$0.36(100n+5)=0.6(30n+15)$
Distribute.	$36n+1.8=18n+9$
Subtract $18n$ to get the variables to the left.	$36n-18n+1.8=18n-18n+9$
Simplify.	$18n+1.8=9$
Subtract 1.8 to get the constants to	$18n+1.8-1.8=9-1.8$

the right.		
Simplify.		$18n = 7.2$
Divide.		$\frac{18n}{18} = \frac{7.2}{18}$
Simplify.		$n = 0.4$
Check:	$0.36(100n + 5) = 0.6(30n + 15)$	
Let $n = 0.4$.	$0.36(100(0.4) + 5) \overset{?}{=} 0.6(30(0.4) + 15)$	
	$0.36(40 + 5) \overset{?}{=} 0.6(12 + 15)$	
	$0.36(45) \overset{?}{=} 0.6(27)$	
	$16.2 = 16.2$ ✓	

Note:
Exercise:

TRY IT :: 2.87

Problem: Solve: $0.55(100n+8)=0.6(85n+14)$.

Solution:

$n=1$

Note:
Exercise:

TRY IT :: 2.88

Problem: Solve: $0.15(40m-120)=0.5(60m+12)$.

Solution:

$m=-1$

Classify Equations

Consider the equation we solved at the start of the last section, $7x+8=-13$. The solution we found was $x=-3$. This means the equation $7x+8=-13$ is true when we replace the variable, *x*, with the value -3. We showed this when we checked the solution $x=-3$ and evaluated $7x+8=-13$ for $x=-3$.

$$7(-3)+8\overset{?}{=}-13$$

$$-21+8\overset{?}{=}-13$$

$$-13=-13\ \checkmark$$

If we evaluate $7x+8$ for a different value of x, the left side will not be -13.

The equation $7x+8=-13$ is true when we replace the variable, x, with the value -3, but not true when we replace x with any other value. Whether or not the equation $7x+8=-13$ is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

Note:
Conditional equation
An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let’s consider the equation $2y+6=2\left(y+3\right)$. Do you recognize that the left side and the right side are equivalent? Let’s see what happens when we solve for y.

	$2y+6=2(y+3)$
Distribute.	$2y+6=2y+6$
Subtract $2y$ to get the y’s to one side.	$2y-2y+6=2y-2y+6$

Simplify—the y’s are gone!	$6=6$

But $6=6$ is true.

This means that the equation $2y+6=2\left(y+3\right)$ is true for any value of y. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable like this is called an identity.

Note:
Identity
An equation that is true for any value of the variable is called an **identity**. The solution of an identity is all real numbers.

What happens when we solve the equation $5z=5z-1$?

	$5z=5z-1$
Subtract $5z$ to get the constant alone on the right.	$5z-5z=5z-5z-1$
Simplify—the z’s are gone!	$0\ne-1$

But $0\ne-1$.

Solving the equation $5z = 5z - 1$ led to the false statement $0 = -1$. The equation $5z = 5z - 1$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a contradiction.

Note:
Contradiction
An equation that is false for all values of the variable is called a **contradiction**.
A contradiction has no solution.

Example:
Exercise:

Problem:

**Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$$

Solution:
Solution

	$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$
Distribute.	$12n - 6 + 3 = 2n - 8 + 10n + 5$

Combine like terms.	$12n - 3 = 12n - 3$
Subtract $12n$ to get the n’s to one side.	$12n - 12n - 3 = 12n - 12n - 3$
Simplify.	$-3 = -3$
This is a true statement.	The equation is an identity. The solution is all real numbers.

Note:
Exercise:

Problem:

TRY IT :: 2.89
Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$$4 + 9(3x - 7) = -42x - 13 + 23(3x - 2)$$

Solution:

identity; all real numbers

Note:
Exercise:

Problem:

TRY IT :: 2.90
Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$$8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$$

Solution:

identity; all real numbers

Example:
Exercise:

Problem:

**Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$$10+4(p-5)=0$$

Solution:
Solution

	$10+4(p-5)=0$
Distribute.	$10+4p-20=0$

Combine like terms.	$4p - 10 = 0$
Add 10 to both sides.	$4p - 10 + 10 = 0 + 10$
Simplify.	$4p = 10$
Divide.	$\frac{4p}{4} = \frac{10}{4}$
Simplify.	$p = \frac{5}{2}$
The equation is true when $p = \frac{5}{2}$.	This is a conditional equation. The solution is $p = \frac{5}{2}$.

Note:

Exercise:

Problem:

TRY IT :: 2.91

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3) - 5 = 19$

Solution:

conditional equation; $q = -\frac{9}{11}$

Note:
Exercise:

Problem:

TRY IT :: 2.92
Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6 + 14(k - 8) = 95$

Solution:

conditional equation; $k = \frac{201}{14}$

Example:
Exercise:

Problem:

**Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m + 3\left(9 + 3m\right) = 2\left(7m - 11\right)$

Solution:
Solution

	$5m+3(9+3m)=2(7m-11)$
Distribute.	$5m+27+9m=14m-22$
Combine like terms.	$14m+27=14m-22$
Subtract $14m$ from both sides.	$14m+27-14m=14m-22-14m$
Simplify.	$27\neq-22$
But $27\neq-22$.	The equation is a contradiction. It has no solution.

Note:
Exercise:

Problem:

TRY IT :: 2.93
Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$$12c+5(5+3c)=3(9c-4)$$

Solution:

contradiction; no solution

Note:
Exercise:

Problem:

TRY IT :: 2.94
Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$$4(7d+18)=13(3d-2)-11d$$

Solution:

contradiction; no solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Key Concepts

- **General Strategy for Solving Linear Equations**

Simplify each side of the equation as much as possible.	Use the Distributive Property to remove any parentheses.	Combine like terms.
Collect all the variable terms on one side of the equation.	Use the Addition or Subtraction Property of Equality.	
Collect all the constant terms on the other side of the equation.	Use the Addition or Subtraction Property of Equality.	
Make the coefficient of the variable term to equal to 1.	Use the Multiplication or Division Property of Equality.	State the solution to the equation.
Check the solution.	Substitute the solution into the original equation.	

Practice Makes Perfect

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

Exercise:

Problem: $15\left(y-9\right)=-60$

Exercise:

Problem: $21\left(y-5\right)=-42$

Solution:

$y=3$

Exercise:

Problem: $-9\left(2n+1\right)=36$

Exercise:

Problem: $-16\left(3n+4\right)=32$

Solution:

$n = -2$

Exercise:

Problem: $8\left(22 + 11r\right) = 0$

Exercise:

Problem: $5\left(8 + 6p\right) = 0$

Solution:

$p = -\frac{4}{3}$

Exercise:

Problem: $-\left(w - 12\right) = 30$

Exercise:

Problem: $-\left(t - 19\right) = 28$

Solution:

$t = -9$

Exercise:

Problem: $9\left(6a + 8\right) + 9 = 81$

Exercise:

Problem: $8\left(9b - 4\right) - 12 = 100$

Solution:

$b = 2$

Exercise:

Problem: $32 + 3(z + 4) = 41$

Exercise:

Problem: $21 + 2(m - 4) = 25$

Solution:

$m = 6$

Exercise:

Problem: $51 + 5(4 - q) = 56$

Exercise:

Problem: $-6 + 6(5 - k) = 15$

Solution:

$k = \frac{3}{2}$

Exercise:

Problem: $2(9s - 6) - 62 = 16$

Exercise:

Problem: $8(6t - 5) - 35 = -27$

Solution:

$t = 1$

Exercise:

Problem: $3(10-2x)+54=0$

Exercise:

Problem: $-2(11-7x)+54=4$

Solution:

$x=-2$

Exercise:

Problem: $\frac{2}{3}(9c-3)=22$

Exercise:

Problem: $\frac{3}{5}(10x-5)=27$

Solution:

$x=5$

Exercise:

Problem: $\frac{1}{5}(15c+10)=c+7$

Exercise:

Problem: $\frac{1}{4}(20d+12)=d+7$

Solution:

$d=1$

Exercise:

Problem: $18-(9r+7)=-16$

Exercise:

Problem: $15 - (3r + 8) = 28$

Solution:

$r = -7$

Exercise:

Problem: $5 - (n - 1) = 19$

Exercise:

Problem: $-3 - (m - 1) = 13$

Solution:

$m = -15$

Exercise:

Problem: $11 - 4(y - 8) = 43$

Exercise:

Problem: $18 - 2(y - 3) = 32$

Solution:

$y = -4$

Exercise:

Problem: $24 - 8(3v + 6) = 0$

Exercise:

Problem: $35-5(2w+8)=-10$

Solution:

$w=\frac{1}{2}$

Exercise:

Problem: $4(a-12)=3\left(a+5\right)$

Exercise:

Problem: $-2(a-6)=4\left(a-3\right)$

Solution:

$a=4$

Exercise:

Problem: $2(5-u)=-3\left(2u+6\right)$

Exercise:

Problem: $5(8-r)=-2\left(2r-16\right)$

Solution:

$r=8$

Exercise:

Problem: $3(4n-1)-2=8n+3$

Exercise:

Problem: $9(2m-3)-8=4m+7$

Solution:

$m = 3$

Exercise:

Problem: $12 + 2(5 - 3y) = -9\left(y - 1\right) - 2$

Exercise:

Problem: $-15 + 4(2 - 5y) = -7\left(y - 4\right) + 4$

Solution:

$y = -3$

Exercise:

Problem: $8\left(x - 4\right) - 7x = 14$

Exercise:

Problem: $5\left(x - 4\right) - 4x = 14$

Solution:

$x = 34$

Exercise:

Problem: $5 + 6(3s - 5) = -3 + 2\left(8s - 1\right)$

Exercise:

Problem: $-12 + 8(x - 5) = -4 + 3\left(5x - 2\right)$

Solution:

$x = -6$

Exercise:

Problem: $4\left(u-1\right)-8=6\left(3u-2\right)-7$

Exercise:

Problem: $7\left(2n-5\right)=8\left(4n-1\right)-9$

Solution:

$n=-1$

Exercise:

Problem: $4\left(p-4\right)-\left(p+7\right)=5\left(p-3\right)$

Exercise:

Problem: $3\left(a-2\right)-\left(a+6\right)=4\left(a-1\right)$

Solution:

$a=-4$

Exercise:

Problem: $$\begin{array}{l}-(9y+5)-(3y-7)\\ =16-(4y-2)\end{array}$$

Exercise:

Problem: $$\begin{array}{l}-\left(7m+4\right)-\left(2m-5\right)\\ =14-\left(5m-3\right)\end{array}$$

Solution:

$m=-4$

Exercise:

Problem: $4\left[5-8\left(4c-3\right)\right]=12\left(1-13c\right)-8$

Exercise:

Problem: $5\left[9-2\left(6d-1\right)\right]=11\left(4-10d\right)-139$

Solution:

$d=-3$

Exercise:

Problem: $3\left[-9+8\left(4h-3\right)\right]=2\left(5-12h\right)-19$

Exercise:

Problem: $3\left[-14+2\left(15k-6\right)\right]=8\left(3-5k\right)-24$

Solution:

$k=\frac{3}{5}$

Exercise:

Problem: $5\left[2\left(m+4\right)+8\left(m-7\right)\right]=2\left[3\left(5+m\right)-\left(21-3m\right)\right]$

Exercise:

Problem: $10\left[5\left(n+1\right)+4\left(n-1\right)\right]=11[7\left(5+n\right)-\left(25-3n\right)]$

Solution:

$n=-5$

Exercise:

Problem: $5\left(1.2u-4.8\right)=-12$

Exercise:

Problem: $4\left(2.5v-0.6\right)=7.6$

Solution:

$v=1$

Exercise:

Problem: $0.25(q-6)=0.1\left(q+18\right)$

Exercise:

Problem: $0.2(p-6)=0.4\left(p+14\right)$

Solution:

$p=-34$

Exercise:

Problem: $0.2\left(30n+50\right)=28$

Exercise:

Problem: $0.5\left(16m+34\right)=-15$

Solution:

$m=-4$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

Exercise:

Problem: $23z+19=3\left(5z-9\right)+8z+46$

Exercise:

Problem: $15y+32=2\left(10y-7\right)-5y+46$

Solution:

identity; all real numbers

Exercise:

Problem: $5\left(b-9\right)+4\left(3b+9\right)=6\left(4b-5\right)-7b+21$

Exercise:

Problem: $9\left(a-4\right)+3\left(2a+5\right)=7\left(3a-4\right)-6a+7$

Solution:

identity; all real numbers

Exercise:

Problem: $18(5j-1)+29=47$

Exercise:

Problem: $24(3d-4)+100=52$

Solution:

conditional equation; $d = \frac{2}{3}$

Exercise:

Problem: $22(3m - 4) = 8\left(2m + 9\right)$

Exercise:

Problem: $30(2n - 1) = 5\left(10n + 8\right)$

Solution:

conditional equation; $n = 7$

Exercise:

Problem: $7v + 42 = 11\left(3v + 8\right) - 2\left(13v - 1\right)$

Exercise:

Problem: $18u - 51 = 9\left(4u + 5\right) - 6\left(3u - 10\right)$

Solution:

contradiction; no solution

Exercise:

Problem: $3\left(6q - 9\right) + 7\left(q + 4\right) = 5\left(6q + 8\right) - 5\left(q + 1\right)$

Exercise:

Problem: $5\left(p + 4\right) + 8\left(2p - 1\right) = 9\left(3p - 5\right) - 6\left(p - 2\right)$

Solution:

contradiction; no solution

Exercise:

Problem: $12(6h-1)=8\left(8h+5\right)-4$

Exercise:

Problem: $9(4k-7)=11\left(3k+1\right)+4$

Solution:

conditional equation; $k=26$

Exercise:

Problem: $45(3y-2)=9\left(15y-6\right)$

Exercise:

Problem: $60(2x-1)=15\left(8x+5\right)$

Solution:

contradiction; no solution

Exercise:

Problem: $16(6n+15)=48\left(2n+5\right)$

Exercise:

Problem: $36(4m+5)=12\left(12m+15\right)$

Solution:

identity; all real numbers

Exercise:

Problem: $9(14d+9)+4d=13\left(10d+6\right)+3$

Exercise:

Problem: $11(8c+5)-8c=2\,(40c+25)+5$

Solution:

identity; all real numbers

Everyday Math

Exercise:

Problem:

Fencing Micah has 44 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, L, by solving the equation $2L+2(L-2.5)=44$.

Exercise:

Problem:

Coins Rhonda has $1.90 in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n+0.10(2n-1)=1.90$.

Solution:

8 nickels

Writing Exercises

Exercise:

Problem:

Using your own words, list the steps in the general strategy for solving linear equations.

Exercise:

Problem:

Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Solution:

Answers will vary.

Exercise:

Problem:

What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

Exercise:

Problem:

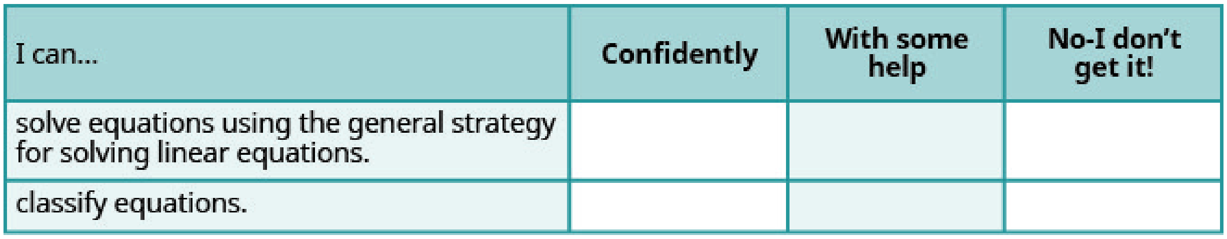
Solve the equation $\frac{1}{4}(8x+20)=3x-4$ explaining all the steps of your solution as in the examples in this section.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.



I can…	Confidently	With some help	No-I don’t get it!
solve equations using the general strategy for solving linear equations.			
classify equations.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction

An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity

An equation that is true for any value of the variable is called an identity. The solution of an identity is all real numbers.

Solve Equations with Fractions or Decimals

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Note:
Before you get started, take this readiness quiz.

1. Multiply: $8 \cdot \frac{3}{8}$.
   If you missed this problem, review [link].
2. Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$.
   If you missed this problem, review [link].
3. Multiply 4.78 by 100.
   If you missed this problem, review [link].

Solve Equations with Fraction Coefficients

Let's use the general strategy for solving linear equations introduced earlier to solve the equation, $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

	$\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x + \frac{1}{2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2}$

Simplify the left side.	$\frac{1}{8}x=\frac{1}{4}-\frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x=\frac{1}{4}-\frac{2}{4}$
Subtract.	$\frac{1}{8}x=-\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1}\cdot\frac{1}{8}x=\frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x=-2$

This method worked fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called “clearing” the equation of fractions.

Let’s solve a similar equation, but this time use the method that eliminates the fractions.

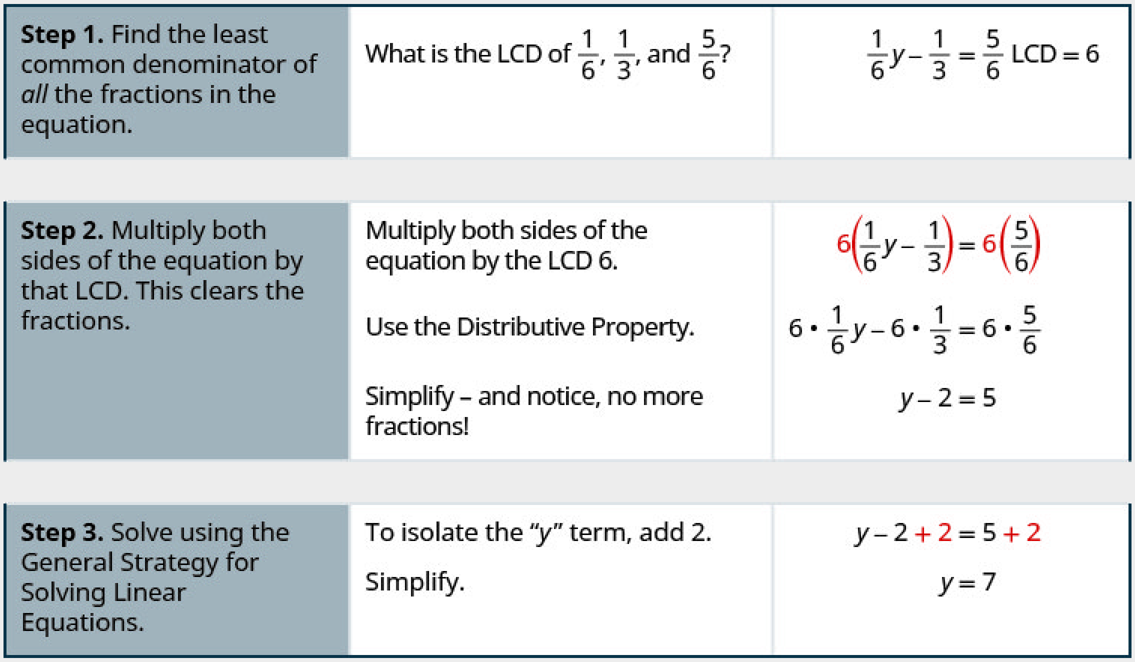
Example:
How to Solve Equations with Fraction Coefficients

Exercise:

Problem: **Solve: $\frac{1}{6}y - \frac{1}{3} = \frac{5}{6}$.

Solution:

Solution



Step 1. Find the least common denominator of *all* the fractions in the equation.	What is the LCD of $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$?	$\frac{1}{6}y - \frac{1}{3} = \frac{5}{6}$ LCD = 6
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.	Multiply both sides of the equation by the LCD 6. Use the Distributive Property. Simplify – and notice, no more fractions!	$6\left(\frac{1}{6}y - \frac{1}{3}\right) = 6\left(\frac{5}{6}\right)$ $6 \cdot \frac{1}{6}y - 6 \cdot \frac{1}{3} = 6 \cdot \frac{5}{6}$ $y - 2 = 5$
Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the "*y*" term, add 2. Simplify.	$y - 2 + 2 = 5 + 2$ $y = 7$

Note:

Exercise:

TRY IT :: 2.95

Problem: Solve: $\frac{1}{4}x + \frac{1}{2} = \frac{5}{8}$.

Solution:

$x = \frac{1}{2}$

Note:
Exercise:

TRY IT :: 2.96

Problem: Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Solution:

$x = -2$

Notice in [link], once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

Note:
Strategy to solve equations with fraction coefficients.

Find the least common denominator of*all*the fractions in the equation.
Multiply both sides of the equation by that LCD. This clears the fractions.
Solve using the General Strategy for Solving Linear Equations.

Example:
Exercise:

Problem: **Solve: $6 = \frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v$.

Solution:
Solution

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

<table>
<tr><td colspan="2">Find the LCD of all fractions in the equation.</td><td>$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$</td></tr>
<tr><td colspan="2">The LCD is 20.</td><td></td></tr>
<tr><td colspan="2">Multiply both sides of the equation by 20.</td><td>$20(6)=20\cdot\left(\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v\right)$</td></tr>
<tr><td colspan="2">Distribute.</td><td>$20(6)=20\cdot\frac{1}{2}v+20\cdot\frac{2}{5}v-20\cdot\frac{3}{4}v$</td></tr>
<tr><td colspan="2">Simplify—notice, no more fractions!</td><td>$120=10v+8v-15v$</td></tr>
<tr><td colspan="2">Combine like terms.</td><td>$120=3v$</td></tr>
<tr><td colspan="2">Divide by 3.</td><td>$\frac{120}{3}=\frac{3v}{3}$</td></tr>
<tr><td colspan="2">Simplify.</td><td>$40=v$</td></tr>
<tr><td>Check:</td><td>$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$</td><td></td></tr>
</table>

Let $v = 40$.	$6 \overset{?}{=} \frac{1}{2}(40) + \frac{2}{5}(40) - \frac{3}{4}(40)$	
	$6 \overset{?}{=} 20 + 16 - 30$	
	$6 = 6 \checkmark$	

Note:
Exercise:

TRY IT :: 2.97

Problem: Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$.

Solution:

$x = 12$

Note:
Exercise:

TRY IT :: 2.98

Problem: Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$.

Solution:

$u = -12$

In the next example, we again have variables on both sides of the equation.

Example:
Exercise:

Problem: **Solve: $a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$.

Solution:
Solution

	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$
Find the LCD of all fractions in the equation. The LCD is 8.	
Multiply both sides by the LCD.	$8\left(a+\frac{3}{4}\right)=8\left(\frac{3}{8}a-\frac{1}{2}\right)$
Distribute.	$8\cdot a+8\cdot\frac{3}{4}=8\cdot\frac{3}{8}a-8\cdot\frac{1}{2}$
Simplify—no more fractions.	$8a+6=3a-4$
Subtract $3a$ from both sides.	$8a-3a+6=3a-3a-4$

Simplify.		$5a + 6 = -4$
Subtract 6 from both sides.		$5a + 6 - 6 = -4 - 6$
Simplify.		$5a = -10$
Divide by 5.		$\frac{5a}{5} = \frac{-10}{5}$
Simplify.		$a = -2$
Check:	$a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$	
Let $a = -2$.	$-2 + \frac{3}{4} \overset{?}{=} \frac{3}{8}(-2) - \frac{1}{2}$	
	$-\frac{8}{4} + \frac{3}{4} \overset{?}{=} -\frac{16}{8} - \frac{4}{8}$	
	$-\frac{5}{4} = -\frac{10}{8}$	
	$-\frac{5}{4} = -\frac{5}{4} \checkmark$	

Note:
Exercise:

TRY IT :: 2.99

Problem: Solve: $x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}$.

Solution:

$x=-1$

Note:
Exercise:

TRY IT :: 2.100

Problem: Solve: $c+\frac{3}{4}=\frac{1}{2}c-\frac{1}{4}$.

Solution:

$c=-2$

In the next example, we start by using the Distributive Property. This step clears the fractions right away.

Example:
Exercise:

Problem: **Solve: $-5 = \frac{1}{4}(8x + 4)$.

Solution:
Solution

	$-5 = \frac{1}{4}(8x + 4)$
Distribute.	$-5 = \frac{1}{4} \cdot 8x + \frac{1}{4} \cdot 4$
Simplify. Now there are no fractions.	$-5 = 2x + 1$
Subtract 1 from both sides.	$-5 - 1 = 2x + 1 - 1$
Simplify.	$-6 = 2x$
Divide by 2.	$\frac{-6}{2} = \frac{2x}{2}$
Simplify.	$-3 = x$

Check:	$-5 = \frac{1}{4}(8x + 4)$	
Let $x = -3$.	$-5 \overset{?}{=} \frac{1}{2}(4(-3) + 2)$	
	$-5 \overset{?}{=} \frac{1}{2}(-12 + 2)$	
	$-5 \overset{?}{=} \frac{1}{2}(-10)$	
	$-5 = -5$ ✓	

Note:
Exercise:

TRY IT :: 2.101

Problem: Solve: $-11 = \frac{1}{2}(6p + 2)$.

Solution:

$p = -4$

Note:
Exercise:

TRY IT :: 2.102

Problem: Solve: $8=\frac{1}{3}(9q+6)$.

Solution:

$q=2$

In the next example, even after distributing, we still have fractions to clear.

Example:
Exercise:

Problem: **Solve: $\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$.

Solution:
Solution

	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2}\cdot y-\frac{1}{2}\cdot 5=\frac{1}{4}\cdot y-\frac{1}{4}\cdot 1$
Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$

Multiply by the LCD, 4.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$
Check:	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Let $y=9$.	$\frac{1}{2}(9-5)\stackrel{?}{=}\frac{1}{4}(9-1)$
Finish the check on your own.	

Note:
Exercise:

TRY IT :: 2.103

Problem: Solve: $\frac{1}{5}(n+3)=\frac{1}{4}(n+2)$.

Solution:

$n=2$

Note:
Exercise:

TRY IT :: 2.104

Problem: Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$.

Solution:

$m=-1$

Example:
Exercise:

Problem: **Solve: $\frac{5x-3}{4}=\frac{x}{2}$.

Solution:
Solution

		$\frac{5x-3}{4}=\frac{x}{2}$
Multiply by the LCD, 4.		$4\left(\frac{5x-3}{4}\right)=4\left(\frac{x}{2}\right)$
Simplify.		$5x-3=2x$
Collect the variables to the right.		$5x-5x-3=2x-5x$
Simplify.		$-3=-3x$
Divide.		$\frac{-3}{-3}=\frac{-3x}{-3}$
Simplify.		$1=x$
Check:	$\frac{5x-3}{4}=\frac{x}{2}$	
Let $x=1$.	$\frac{5(1)-3}{4}\overset{?}{=}\frac{1}{2}$	
	$\frac{2}{4}\overset{?}{=}\frac{1}{2}$	

	$\frac{1}{2}=\frac{1}{2}$ ✓	

Note:
Exercise:

TRY IT :: 2.105

Problem: Solve: $\frac{4y-7}{3}=\frac{y}{6}$.

Solution:

$y=2$

Note:
Exercise:

TRY IT :: 2.106

Problem: Solve: $\frac{-2z-5}{4}=\frac{z}{8}$.

Solution:

$z=-2$

Example:
Exercise:

Problem: **Solve: $\frac{a}{6} + 2 = \frac{a}{4} + 3$.

Solution:
Solution

	$\frac{a}{6} + 2 = \frac{a}{4} + 3$
Multiply by the LCD, 12.	$12\left(\frac{a}{6} + 2\right) = 12\left(\frac{a}{4} + 3\right)$
Distribute.	$12 \cdot \frac{a}{6} + 12 \cdot 2 = 12 \cdot \frac{a}{4} + 12 \cdot 3$
Simplify.	$2a + 24 = 3a + 36$
Collect the variables to the right.	$2a - 2a + 24 = 3a - 2a + 36$
Simplify.	$24 = a + 36$
Collect the constants to the left.	$24 - 36 = a + 36 - 36$
Simplify.	$a = -12$

Check:	$\frac{a}{6}+2=\frac{a}{4}+3$	
Let $a=-12$.	$\frac{-12}{6}+2\overset{?}{=}\frac{-12}{4}+3$	
	$-2+2\overset{?}{=}-3+3$	
	$0=0$ ✓	

Note:
Exercise:

TRY IT :: 2.107

Problem: Solve: $\frac{b}{10}+2=\frac{b}{4}+5$.

Solution:

$b=-20$

Note:
Exercise:

TRY IT :: 2.108

Problem: Solve: $\frac{c}{6}+3=\frac{c}{3}+4$.

Solution:

$c=-6$

Example:
Exercise:

Problem: **Solve: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$.

Solution:
Solution

	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Multiply by the LCD, 4.	$4\left(\frac{4q+3}{2}+6\right)=4\left(\frac{3q+5}{4}\right)$
Distribute.	$4\left(\frac{4q+3}{2}\right)+4\cdot 6=4\cdot\left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3)+24=3q+5$ $8q+6+24=3q+5$

		$8q+30=3q+5$
Collect the variables to the left.		$8q-3q+30=3q-3q+5$
Simplify.		$5q+30=5$
Collect the constants to the right.		$5q+30-30=5-30$
Simplify.		$5q=-25$
Divide by 5.		$\frac{5q}{5}=\frac{-25}{5}$
Simplify.		$q=-5$
Check:	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$	
Let $q=-5$.	$\frac{4(-5)+3}{2}+6\overset{?}{=}\frac{3(-5)+5}{4}$	
Finish the check on your own.		

Note:
Exercise:

TRY IT :: 2.109

Problem: Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$.

Solution:

$r=1$

Note:
Exercise:

TRY IT :: 2.110

Problem: Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$.

Solution:

$s=-8$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.3=\frac{3}{10}$ and $0.17=\frac{17}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the least common denominator.

Example:

Exercise:

Problem: **Solve: $0.06x + 0.02 = 0.25x - 1.5$.

Solution:
Solution

Look at the decimals and think of the equivalent fractions.
Equation:

$$0.06 = \frac{6}{100} \qquad 0.02 = \frac{2}{100} \qquad 0.25 = \frac{25}{100} \qquad 1.5 = 1\frac{5}{10}$$

Notice, the LCD is 100.

By multiplying by the LCD, we will clear the decimals from the equation.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both sides by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now we have no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the	$6x - 6x + 2 = 25x - 6x - 150$

right.	
Simplify.	$2 = 19x - 150$
Collect the constants to the left.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check: Let $x = 8$. $0.06(8) + 0.02 \overset{?}{=} 0.25(8) - 1.5$ $0.48 + 0.02 \overset{?}{=} 2.00 - 1.5$ $0.50 = 0.50 \checkmark$	

Note:
Exercise:

TRY IT :: 2.111

Problem: Solve: $0.14h + 0.12 = 0.35h - 2.4$.

Solution:

$h = 12$

Note:
Exercise:

TRY IT :: 2.112

Problem: Solve: $0.65k - 0.1 = 0.4k - 0.35$.

Solution:

$k = -1$

The next example uses an equation that is typical of the money applications in the next chapter. Notice that we distribute the decimal before we clear all the decimals.

Example:
Exercise:

Problem: **Solve: $0.25x + 0.05(x + 3) = 2.85$.

Solution:
Solution

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$

Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting $x = 9$ into the original equation.	

Note:
Exercise:

TRY IT :: 2.113

Problem: Solve: $0.25n + 0.05(n + 5) = 2.95$.

Solution:

$n = 9$

Note:
Exercise:

TRY IT :: 2.114

Problem: Solve: $0.10d + 0.05(d - 5) = 2.15$.

Solution:

$d = 16$

Key Concepts

- **Strategy to Solve an Equation with Fraction Coefficients**

  Find the least common denominator of all the fractions in the equation.
  Multiply both sides of the equation by that LCD. This clears the fractions.
  Solve using the General Strategy for Solving Linear Equations.

Practice Makes Perfect

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

Exercise:

Problem: $\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$

Exercise:

Problem: $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

Solution:

$x = 1$

Exercise:

Problem: $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

Exercise:

Problem: $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

Solution:

$y = -1$

Exercise:

Problem: $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

Exercise:

Problem: $\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

Solution:

$b = -2$

Exercise:

Problem: $2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$

Exercise:

Problem: $2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$

Solution:

$x = 3$

Exercise:

Problem: $\frac{1}{4}m - \frac{4}{5}m + \frac{1}{2}m = -1$

Exercise:

Problem: $\frac{5}{6}n - \frac{1}{4}n - \frac{1}{2}n = -2$

Solution:

$n = -24$

Exercise:

Problem: $x + \frac{1}{2} = \frac{2}{3}x - \frac{1}{2}$

Exercise:

Problem: $x + \frac{3}{4} = \frac{1}{2}x - \frac{5}{4}$

Solution:

$x = -4$

Exercise:

Problem: $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

Exercise:

Problem: $\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$

Solution:

$z = -2$

Exercise:

Problem: $\frac{1}{2}x - \frac{1}{4} = \frac{1}{12}x + \frac{1}{6}$

Exercise:

Problem: $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

Solution:

$a = 1$

Exercise:

Problem: $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

Exercise:

Problem: $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

Solution:

$x = -6$

Exercise:

Problem: $1 = \frac{1}{6}(12x - 6)$

Exercise:

Problem: $1 = \frac{1}{5}(15x - 10)$

Solution:

$x = 1$

Exercise:

Problem: $\frac{1}{4}(p - 7) = \frac{1}{3}(p + 5)$

Exercise:

Problem: $\frac{1}{5}(q + 3) = \frac{1}{2}(q - 3)$

Solution:

$q = 7$

Exercise:

Problem: $\frac{1}{2}(x + 4) = \frac{3}{4}$

Exercise:

Problem: $\frac{1}{3}(x + 5) = \frac{5}{6}$

Solution:

$x = -\frac{5}{2}$

Exercise:

Problem: $\frac{5q-8}{5} = \frac{2q}{10}$

Exercise:

Problem: $\frac{4m+2}{6} = \frac{m}{3}$

Solution:

$m = -1$

Exercise:

Problem: $\frac{4n+8}{4} = \frac{n}{3}$

Exercise:

Problem: $\frac{3p+6}{3} = \frac{p}{2}$

Solution:

$p = -4$

Exercise:

Problem: $\frac{u}{3} - 4 = \frac{u}{2} - 3$

Exercise:

Problem: $\frac{v}{10} + 1 = \frac{v}{4} - 2$

Solution:

$v = 20$

Exercise:

Problem: $\frac{c}{15} + 1 = \frac{c}{10} - 1$

Exercise:

Problem: $\frac{d}{6} + 3 = \frac{d}{8} + 2$

Solution:

$d = -24$

Exercise:

Problem: $\frac{3x+4}{2} + 1 = \frac{5x+10}{8}$

Exercise:

Problem: $\frac{10y-2}{3} + 3 = \frac{10y+1}{9}$

Solution:

$y = -1$

Exercise:

Problem: $\frac{7u-1}{4} - 1 = \frac{4u+8}{5}$

Exercise:

Problem: $\frac{3v-6}{2} + 5 = \frac{11v-4}{5}$

Solution:

$v = 4$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

Exercise:

Problem: $0.6y + 3 = 9$

Exercise:

Problem: $0.4y - 4 = 2$

Solution:

$y = 15$

Exercise:

Problem: $3.6j - 2 = 5.2$

Exercise:

Problem: $2.1k + 3 = 7.2$

Solution:

$k = 2$

Exercise:

Problem: $0.4x + 0.6 = 0.5x - 1.2$

Exercise:

Problem: $0.7x + 0.4 = 0.6x + 2.4$

Solution:

$x = 20$

Exercise:

Problem: $0.23x + 1.47 = 0.37x - 1.05$

Exercise:

Problem: $0.48x + 1.56 = 0.58x - 0.64$

Solution:

$x = 22$

Exercise:

Problem: $0.9x - 1.25 = 0.75x + 1.75$

Exercise:

Problem: $1.2x - 0.91 = 0.8x + 2.29$

Solution:

$x = 8$

Exercise:

Problem: $0.05n + 0.10\left(n + 8\right) = 2.15$

Exercise:

Problem: $0.05n + 0.10\left(n + 7\right) = 3.55$

Solution:

$n = 19$

Exercise:

Problem: $0.10d + 0.25\left(d + 5\right) = 4.05$

Exercise:

Problem: $0.10d + 0.25\left(d + 7\right) = 5.25$

Solution:

$d=10$

Exercise:

Problem: $0.05\left(q-5\right)+0.25q=3.05$

Exercise:

Problem: $0.05\left(q-8\right)+0.25q=4.10$

Solution:

$q=15$

Everyday Math

Exercise:

Problem:

Coins Taylor has $2.00 in dimes and pennies. The number of pennies is 2 more than the number of dimes. Solve the equation $0.10d+0.01(d+2)=2$ for d, the number of dimes.

Exercise:

Problem:

Stamps Paula bought $22.82 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 8 less than the number of 49-cent stamps. Solve the equation $0.49s+0.21(s-8)=22.82$ for s, to find the number of 49-cent stamps Paula bought.

Solution:

$s = 35$

Writing Exercises

Exercise:

Problem:

Explain how you find the least common denominator of $\frac{3}{8}$, $\frac{1}{6}$, and $\frac{2}{3}$.

Exercise:

Problem:

If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

Solution:

Answers will vary.

Exercise:

Problem:

If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

Exercise:

Problem:

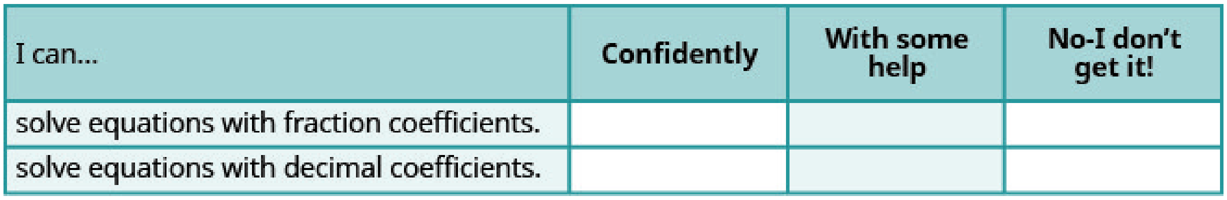
In the equation $0.35x + 2.1 = 3.85$ what is the LCD? How do you know?

Solution:

100. Justifications will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
solve equations with fraction coefficients.			
solve equations with decimal coefficients.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

Solve a Formula for a Specific Variable
By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

Note:
Before you get started, take this readiness quiz.

1. Solve: $15t = 120$.
   If you missed this problem, review [link].
2. Solve: $6x + 24 = 96$.
   If you missed this problem, review [link].

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

Note:
Distance, Rate, and Time
For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:
Equation:

$$d = rt \qquad \text{where} \qquad \begin{array}{rcl} d & = & \text{distance} \\ r & = & \text{rate} \\ t & = & \text{time} \end{array}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

Note:
Solve an application (with a formula).

Readthe problem. Make sure all the words and ideas are understood.
Identifywhat we are looking for.
Namewhat we are looking for. Choose a variable to represent that quantity.
Translateinto an equation. Write the appropriate formula for the situation.
Substitute in the given information.
Solvethe equation using good algebra techniques.
Checkthe answer in the problem and make sure it makes sense.
Answerthe question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

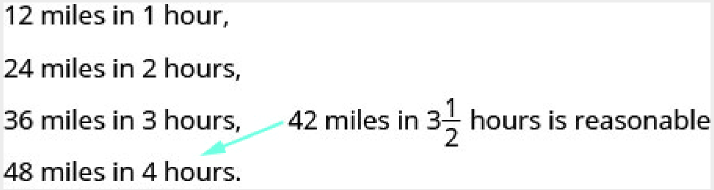
Example:
Exercise:

Problem:

**Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let *d* = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$
	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	

Does 42 miles make sense?	
Jamal rides:	
 12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 48 miles in 4 hours. 42 miles in $3\frac{1}{2}$ hours is reasonable	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Note:
Exercise:

Problem:

TRY IT :: 2.115
Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Solution:

330 miles

Note:
Exercise:

Problem:

TRY IT :: 2.116
Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Solution:

7 miles

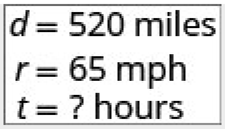
Example:
Exercise:

Problem:

**Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.

	 $d = 520$ miles $r = 65$ mph $t = ?$ hours
Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$\begin{array}{rcl} d & = & rt \\ 520 & \overset{?}{=} & 65 \cdot 8 \\ 520 & = & 520\checkmark \end{array}$	
Step 7. Answer the question with a complete sentence. Rey’s trip will take 8 hours.	

Note:
Exercise:

Problem:

TRY IT :: 2.117
Lee wants to drive from Phoenix to his brother’s apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Solution:

11 hours

Note:
Exercise:

Problem:

TRY IT :: 2.118
Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Solution:

56 mph

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In [link] and [link], we used the formula $d=rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in [link], we had to find the value of t. We substituted in values of d and r and then used algebra to solve for t. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d=rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

Example:
Exercise:

Problem: **Solve the formula $d=rt$ for t:

ⓐ when $d=520$ and $r=65$
ⓑ in general

Solution:
Solution

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

ⓐ when $d=520$ and			ⓑ in general

$r = 65$					
Write the formula.	$d = rt$			Write the formula.	$d = rt$
Substitute.	$520 = 65t$				
Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$			Divide, to isolate t.	$\frac{d}{r} = \frac{rt}{r}$
Simplify.	$8 = t$			Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

Note:
Exercise:

TRY IT :: 2.119

Problem: Solve the formula $d = rt$ for r:

ⓐ when $d = 180$ and $t = 4$ ⓑ in general

Solution:

ⓐ $r = 45$ ⓑ $r = \frac{d}{t}$

Note:
Exercise:

TRY IT :: 2.120

Problem: Solve the formula $d = rt$ for r:

ⓐ when $d = 780$ and $t = 12$ ⓑ in general

Solution:

ⓐ $r = 65$ ⓑ $r = \frac{d}{t}$

Example:
Exercise:

Problem: **Solve the formula $A = \frac{1}{2}bh$ for h:

ⓐ when $A = 90$ and $b = 15$ ⓑ in general

Solution:
Solution

ⓐ when $A = 90$ and $b = 15$				ⓑ in general	
Write the formula.	$A = \frac{1}{2}bh$			Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$				

Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2} 15h$			Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2} bh$
Simplify.	$180 = 15h$			Simplify.	$2A = bh$
Solve for h.	$12 = h$			Solve for h.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$.

Note:
Exercise:

TRY IT :: 2.121

Problem: Use the formula $A = \frac{1}{2}bh$ to solve for h:

ⓐ when $A = 170$ and $b = 17$ ⓑ in general

Solution:

ⓐ $h = 20$ ⓑ $h = \frac{2A}{b}$

Note:
Exercise:

TRY IT :: 2.122

Problem: Use the formula $A = \frac{1}{2}bh$ to solve for b:

ⓐ when $A = 62$ and $h = 31$ ⓑ in general

Solution:

ⓐ $b = 4$ ⓑ $b = \frac{2A}{h}$

The formula $I = Prt$ is used to calculate simple interest, *I*, for a principal, *P*, invested at rate, *r*, for *t* years.

Example:
Exercise:

Problem: **Solve the formula $I = Prt$ to find the principal, P:

ⓐ when $I = \$5{,}600, r = 4\%, t = 7$ years ⓑ in general

Solution:
Solution

ⓐ $I = \$5{,}600$, $r = 4\%$, $t = 7$ years				ⓑ in general	
Write the formula.	$I = Prt$			Write the formula.	$I = Prt$
Substitute.	$5600 = P(0.04)(7)$				

Simplify.	$5600 = P(0.28)$			Simplify.	$I = P(rt)$
Divide, to isolate P.	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$			Divide, to isolate P.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20,000 = P$			Simplify.	$\frac{I}{rt} = P$
The principal is	$20,000				$P = \frac{I}{rt}$

Note:
Exercise:

TRY IT :: 2.123

Problem: Use the formula $I = Prt$ to find the principal, P:

ⓐ when $I = \$2{,}160, r = 6\%, t = 3$ years ⓑ in general

Solution:

ⓐ $12,000 ⓑ $P = \frac{I}{rt}$

Note:
Exercise:

TRY IT :: 2.124

Problem: Use the formula $I = Prt$ to find the principal,P:

ⓐ when $I = \$5{,}400, r = 12\%, t = 5$ years ⓑ in general

Solution:

ⓐ \$9,000 ⓑ $P = \frac{I}{rt}$

Later in this class, and in future algebra classes, you’ll encounter equations that relate two variables, usually *x* and *y*. You might be given an equation that is solved for *y* and need to solve it for *x*, or vice versa. In the following example, we’re given an equation with both *x* and *y* on the same side and we’ll solve it for *y*.

Example:
Exercise:

Problem: **Solve the formula $3x + 2y = 18$ for *y*:

ⓐ when $x = 4$ ⓑ in general

Solution:
Solution

ⓐ when $x = 4$				ⓑ in general	

	$3x+2y=18$				$3x+2y=18$
Substitute.	$3(4)+2y=18$				
Subtract to isolate the y-term.	$12-12+2y=18-12$			Subtract to isolate the y-term.	$3x-3x+2y=18-3x$
Divide.	$\frac{2y}{2}=\frac{6}{2}$			Divide.	$\frac{2y}{2}=\frac{18}{2}-\frac{3x}{2}$
Simplify.	$y=3$			Simplify.	$y=-\frac{3x}{2}+9$

Note:
Exercise:

TRY IT :: 2.125

Problem: Solve the formula $3x+4y=10$ for y:

ⓐ when $x=\frac{14}{3}$ ⓑ in general

Solution:

ⓐ $y=-1$ ⓑ $y=\frac{10-3x}{4}$

Note:
Exercise:

TRY IT :: 2.126

Problem: Solve the formula $5x + 2y = 18$ for *y:*

ⓐ when $x = 4$ ⓑ in general

Solution:

ⓐ $y = -1$ ⓑ $y = \frac{18-5x}{2}$

In Examples 1.60 through 1.64 we used the numbers in part ⓐ as a guide to solving in general in part ⓑ. Now we will solve a formula in general without using numbers as a guide.

Example:
Exercise:

Problem: **Solve the formula $P = a + b + c$ for a.

Solution:
Solution

We will isolate a on one side of the equation.	$P = a + b + c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P - b - c = a + b + c - b - c$

Simplify.	$P - b - c = a$ $a = P - b - c$

Note:
Exercise:

TRY IT :: 2.127

Problem: Solve the formula $P = a + b + c$ for b.

Solution:

$b = P - a - c$

Note:
Exercise:

TRY IT :: 2.128

Problem: Solve the formula $P = a + b + c$ for c.

Solution:

$c = P - a - b$

Example:
Exercise:

Problem: **Solve the formula $6x + 5y = 13$ for y.

Solution:
Solution

	$6x + 5y = 13$
Subtract $6x$ from both sides to isolate the term with y.	$6x - 6x + 5y = 13 - 6x$
Simplify.	$5y = 13 - 6x$
Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13 - 6x}{5}$
Simplify.	$y = \frac{13 - 6x}{5}$

The fraction is simplified. We cannot divide $13 - 6x$ by 5.

Note:
Exercise:

TRY IT :: 2.129

Problem: Solve the formula $4x + 7y = 9$ for y.

Solution:

$y = \frac{9-4x}{7}$

Note:
Exercise:

TRY IT :: 2.130

Problem: Solve the formula $5x + 8y = 1$ for y.

Solution:

$y = \frac{1-5x}{8}$

Key Concepts

- **To Solve an Application (with a formula)**

  Read the problem. Make sure all the words and ideas are understood.
  Identify what we are looking for.
  Name what we are looking for. Choose a variable to represent that quantity.
  Translate into an equation. Write the appropriate formula for the situation. Substitute in the given information.
  Solve the equation using good algebra techniques.
  Check the answer in the problem and make sure it makes sense.
  Answer the question with a complete sentence.

- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: $d = rt$ where d = distance, r = rate, t = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

Practice Makes Perfect

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

Exercise:

Problem:

Steve drove for $8\frac{1}{2}$ hours at 72 miles per hour. How much distance did he travel?

Exercise:

Problem:

Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?

Solution:

290 miles

Exercise:

Problem:

Yuki walked for $1\frac{3}{4}$ hours at 4 miles per hour. How far did she walk?

Exercise:

Problem:

Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?

Solution:

30 miles

Exercise:

Problem:

Connor wants to drive from Tucson to the Grand Canyon, a distance of 338 miles. If he drives at a steady rate of 52 miles per hour, how many hours will the trip take?

Exercise:

Problem:

Megan is taking the bus from New York City to Montreal. The distance is 380 miles and the bus travels at a steady rate of 76 miles per hour. How long will the bus ride be?

Solution:

5 hours

Exercise:

Problem:

Aurelia is driving from Miami to Orlando at a rate of 65 miles per hour. The distance is 235 miles. To the nearest tenth of an hour, how long will the trip take?

Exercise:

Problem:

Kareem wants to ride his bike from St. Louis to Champaign, Illinois. The distance is 180 miles. If he rides at a steady rate of 16 miles per hour, how many hours will the trip take?

Solution:

11.25 hours

Exercise:

Problem:

Javier is driving to Bangor, 240 miles away. If he needs to be in Bangor in 4 hours, at what rate does he need to drive?

Exercise:

Problem:

Alejandra is driving to Cincinnati, 450 miles away. If she wants to be there in 6 hours, at what rate does she need to drive?

Solution:

75 mph

Exercise:

Problem:

Aisha took the train from Spokane to Seattle. The distance is 280 miles and the trip took 3.5 hours. What was the speed of the train?

Exercise:

Problem:

Philip got a ride with a friend from Denver to Las Vegas, a distance of 750 miles. If the trip took 10 hours, how fast was the friend driving?

Solution:

75 mph

Solve a Formula for a Specific Variable

In the following exercises, use the formula $d = rt$.

Exercise:

Problem: Solve for t
ⓐ when $d = 350$ and $r = 70$
ⓑ in general

Exercise:

Problem: Solve for t
ⓐ when $d = 240$ and $r = 60$
ⓑ in general

Solution:

ⓐ $t = 4$ ⓑ $t = \frac{d}{r}$

Exercise:

Problem: Solve for t
ⓐ when $d = 510$ and $r = 60$
ⓑ in general

Exercise:

Problem: Solve for t
ⓐ when $d = 175$ and $r = 50$
ⓑ in general

Solution:

ⓐ $t = 3.5$ ⓑ $t = \frac{d}{r}$

Exercise:

Problem: Solve for r
ⓐ when $d = 204$ and $t = 3$
ⓑ in general

Exercise:

Problem: Solve for r
ⓐ when $d = 420$ and $t = 6$
ⓑ in general

Solution:

ⓐ $r = 70$ ⓑ $r = \frac{d}{t}$

Exercise:

Problem: Solve for r
ⓐ when $d = 160$ and $t = 2.5$
ⓑ in general

Exercise:

Problem: Solve for r
ⓐ when $d = 180$ and $t = 4.5$
ⓑ in general

Solution:

ⓐ $r = 40$ ⓑ $r = \frac{d}{t}$

In the following exercises, use the formula $A = \frac{1}{2}bh$.

Exercise:

Solve for b
ⓐ when $A = 126$ and $h = 18$
Problem: ⓑ in general

Exercise:

Solve for h
ⓐ when $A = 176$ and $b = 22$
Problem: ⓑ in general

Solution:

ⓐ $h = 16$ ⓑ $h = \frac{2A}{b}$

Exercise:

Solve for h
ⓐ when $A = 375$ and $b = 25$
Problem: ⓑ in general

Exercise:

Solve for b
ⓐ when $A = 65$ and $h = 13$
Problem: ⓑ in general

Solution:

ⓐ $b = 10$ ⓑ $b = \frac{2A}{h}$

In the following exercises, use the formula $I = Prt$.

Exercise:

Problem: Solve for the principal, P for
ⓐ $I = \$5{,}480, r = 4\%,$
$t = 7$ years
ⓑ in general

Exercise:

Problem: Solve for the principal, P for
ⓐ $I = \$3{,}950, r = 6\%,$
$t = 5$ years
ⓑ in general

Solution:

ⓐ $P = \$\ 13,166.67$ ⓑ $P = \frac{I}{rt}$

Exercise:

Problem: Solve for the time, t for
ⓐ $I = \$2{,}376, P = \$9{,}000,$
$r = 4.4\%$
ⓑ in general

Exercise:

Problem: Solve for the time, t for
ⓐ $I = \$624, P = \$6{,}000,$
$r = 5.2\%$
ⓑ in general

Solution:

ⓐ $t = 2$ years ⓑ $t = \frac{I}{\text{Pr}}$

In the following exercises, solve.

Exercise:

Problem: Solve the formula $2x + 3y = 12$ for *y*
ⓐ when $x = 3$
ⓑ in general

Exercise:

Problem: Solve the formula $5x + 2y = 10$ for *y*
ⓐ when $x = 4$
ⓑ in general

Solution:

ⓐ $y = -5$ ⓑ $y = \frac{10-5x}{2}$

Exercise:

Problem: Solve the formula $3x - y = 7$ for *y*
ⓐ when $x = -2$
ⓑ in general

Exercise:

Problem: Solve the formula $4x + y = 5$ for *y*
ⓐ when $x = -3$
ⓑ in general

Solution:

ⓐ $y = 17$ ⓑ $y = 5 - 4x$

Exercise:

Problem: Solve $a + b = 90$ for b.

Exercise:

Problem: Solve $a + b = 90$ for a.

Solution:

$a = 90 - b$

Exercise:

Problem: Solve $180 = a + b + c$ for a.

Exercise:

Problem: Solve $180 = a + b + c$ for c.

Solution:

$c = 180 - a - b$

Exercise:

Problem: Solve the formula $8x + y = 15$ for y.

Exercise:

Problem: Solve the formula $9x + y = 13$ for y.

Solution:

$y = 13 - 9x$

Exercise:

Problem: Solve the formula $-4x + y = -6$ for y.

Exercise:

Problem: Solve the formula $-5x + y = -1$ for y.

Solution:

$y = -1 + 5x$

Exercise:

Problem: Solve the formula $4x + 3y = 7$ for y.

Exercise:

Problem: Solve the formula $3x + 2y = 11$ for y.

Solution:

$y = \frac{11 - 3x}{4}$

Exercise:

Problem: Solve the formula $x - y = -4$ for y.

Exercise:

Problem: Solve the formula $x - y = -3$ for y.

Solution:

$y = 3 + x$

Exercise:

Problem: Solve the formula $P = 2L + 2W$ for L.

Exercise:

Problem: Solve the formula $P = 2L + 2W$ for W.

Solution:

$W = \frac{P - 2L}{2}$

Exercise:

Problem: Solve the formula $C = \pi d$ for d.

Exercise:

Problem: Solve the formula $C = \pi d$ for π.

Solution:

$\pi = \frac{C}{d}$

Exercise:

Problem: Solve the formula $V = LWH$ for L.

Exercise:

Problem: Solve the formula $V = LWH$ for H.

Solution:

$H = \frac{V}{LW}$

Everyday Math

Exercise:

Problem:

Converting temperature While on a tour in Greece, Tatyana saw that the temperature was 40^{o} Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the Fahrenheit temperature.

Exercise:

Problem:

Converting temperature Yon was visiting the United States and he saw that the temperature in Seattle one day was 50^{o} Fahrenheit. Solve for C in the formula $F=\frac{9}{5}C+32$ to find the Celsius temperature.

Solution:

10°C

Writing Exercises

Exercise:

Problem:
Solve the equation $2x+3y=6$ for y
ⓐ when $x=-3$
ⓑ in general
ⓒ Which solution is easier for you, ⓐ or ⓑ? Why?

Exercise:

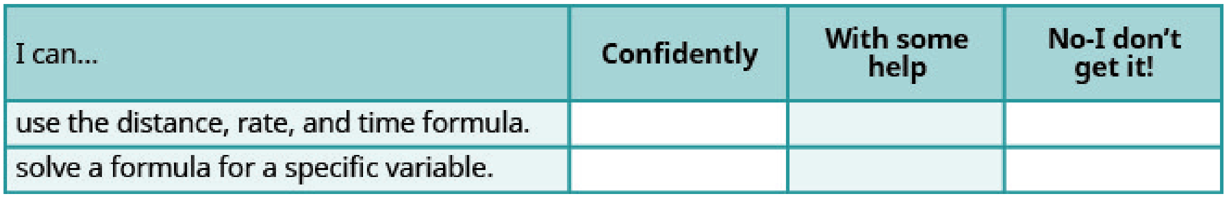
Problem:
Solve the equation $5x-2y=10$ for x
ⓐ when $y=10$
ⓑ in general
ⓒ Which solution is easier for you, ⓐ or ⓑ? Why?

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
use the distance, rate, and time formula.			
solve a formula for a specific variable.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Solve Linear Inequalities
By the end of this section, you will be able to:

- Graph inequalities on the number line
- Solve inequalities using the Subtraction and Addition Properties of inequality
- Solve inequalities using the Division and Multiplication Properties of inequality
- Solve inequalities that require simplification
- Translate to an inequality and solve

Note:
Before you get started, take this readiness quiz.

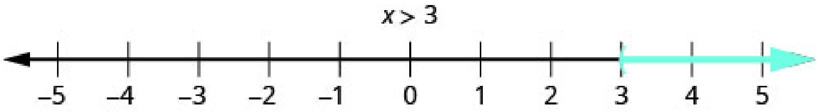
1. Translate from algebra to English: $15 > x$.
   If you missed this problem, review [link].
2. Solve: $n - 9 = -42$.
   If you missed this problem, review [link].
3. Solve: $-5p = -23$.
   If you missed this problem, review [link].
4. Solve: $3a - 12 = 7a - 20$.
   If you missed this problem, review [link].

Graph Inequalities on the Number Line

Do you remember what it means for a number to be a solution to an equation? A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

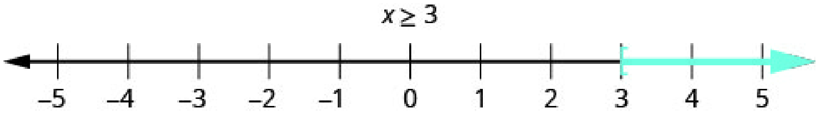
What about the solution of an inequality? What number would make the inequality $x > 3$ true? Are you thinking, '*x* could be 4'? That's correct, but *x* could be 5 too, or 20, or even 3.001. Any number greater than 3 is a solution to the inequality $x > 3$.

We show the solutions to the inequality $x > 3$ on the number line by shading in all the numbers to the right of 3, to show that all numbers greater than 3 are solutions. Because the number 3 itself is not a solution, we put an open parenthesis at 3. The graph of $x > 3$ is shown in [link]. Please note that the following convention is used: light blue arrows point in the positive direction and dark blue arrows point in the negative direction.



The inequality $x > 3$ is graphed on this number line.

The graph of the inequality $x \geq 3$ is very much like the graph of $x > 3$, but now we need to show that 3 is a solution, too. We do that by putting a bracket at $x = 3$, as shown in [link].



The inequality $x \geq 3$ is graphed on this number line.

Notice that the open parentheses symbol, (, shows that the endpoint of the inequality is not included. The open bracket symbol, [, shows that the endpoint is included.

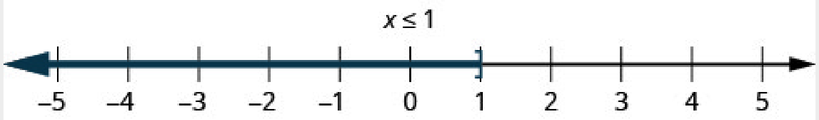
Example:
Exercise:

Problem: **Graph on the number line:

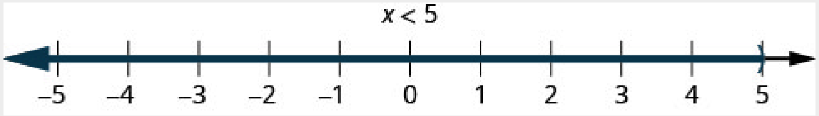
ⓐ $x \leq 1$ ⓑ $x < 5$ ⓒ $x > -1$

Solution:
Solution

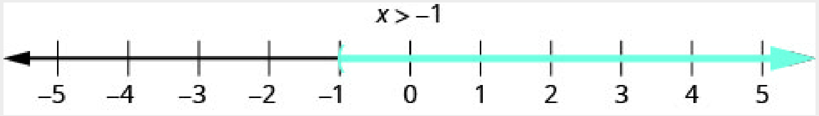
ⓐ $x \leq 1$
This means all numbers less than or equal to 1. We shade in all the numbers on the number line to the left of 1 and put a bracket at $x = 1$ to show that it is included.



ⓑ $x < 5$
This means all numbers less than 5, but not including 5. We shade in all the numbers on the number line to the left of 5 and put a parenthesis at $x = 5$ to show it is not included.



ⓒ $x > -1$
This means all numbers greater than -1, but not including -1. We shade in all the numbers on the number line to the right of -1, then put a parenthesis at $x = -1$ to show it is not included.



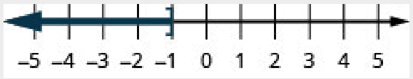
Note:
Exercise:

TRY IT :: 2.131

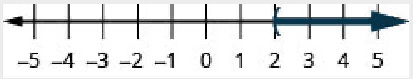
Problem: Graph on the number line: ⓐ $x \leq -1$ ⓑ $x > 2$ ⓒ $x < 3$

Solution:

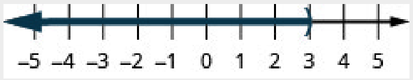
ⓐ



ⓑ



ⓒ



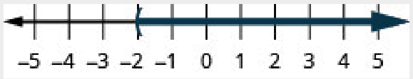
Note:
Exercise:

TRY IT :: 2.132

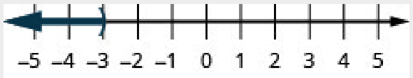
Problem: Graph on the number line: ⓐ $x > -2$ ⓑ $x < -3$ ⓒ $x \geq -1$

Solution:

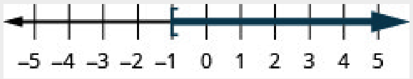
ⓐ



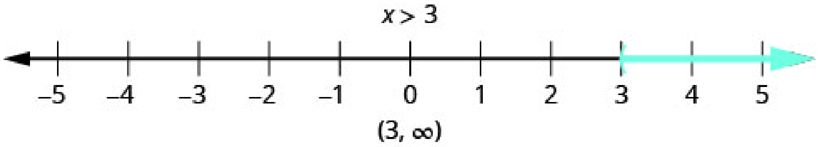
ⓑ



ⓒ

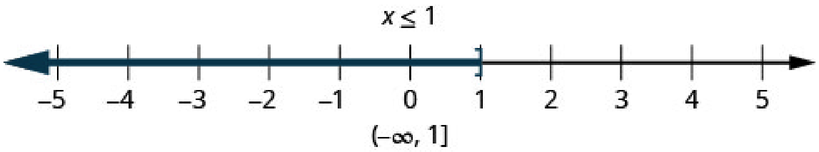


We can also represent inequalities using *interval notation.* As we saw above, the inequality $x > 3$ means all numbers greater than 3. There is no upper end to the solution to this inequality. In interval notation, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as ‘infinity’. It is not an actual number. [link] shows both the number line and the interval notation.



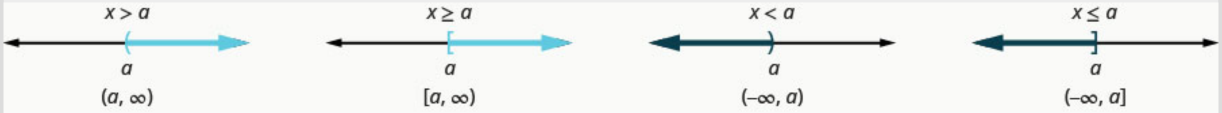
The inequality $x > 3$ is graphed on this number line and written in interval notation.

The inequality $x \leq 1$ means all numbers less than or equal to 1. There is no lower end to those numbers. We write $x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as 'negative infinity'. [link] shows both the number line and interval notation.

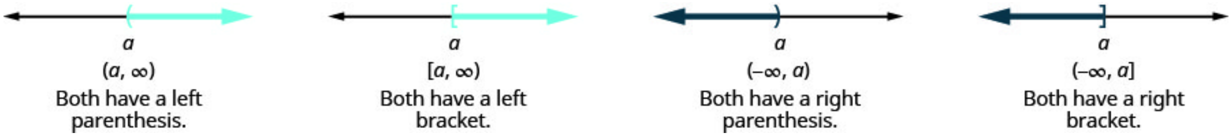


The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

Note:
Inequalities, Number Lines, and Interval Notation



Did you notice how the parenthesis or bracket in the interval notation matches the symbol at the endpoint of the arrow? These relationships are shown in [link].


a
(a, ∞)
Both have a left parenthesis.
a
[a, ∞)
Both have a left bracket.
a
(–∞, a)
Both have a right parenthesis.
a
(–∞, a]
Both have a right bracket.

The notation for inequalities on a number line and in interval notation use similar symbols to express the endpoints of intervals.

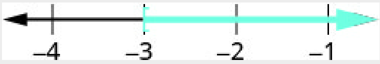
Example:
Exercise:

Problem: **Graph on the number line and write in interval notation.

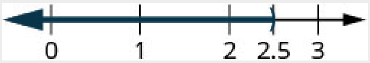
ⓐ $x \geq -3$ ⓑ $x < 2.5$ ⓒ $x \leq -\frac{3}{5}$

Solution:
Solution

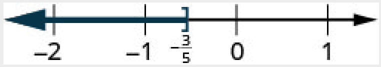
ⓐ

	$x \geq -3$
Shade to the right of -3, and put a bracket at -3.	
Write in interval notation.	$[-3, \infty)$

ⓑ

	$x < 2.5$
Shade to the left of 2.5, and put a parenthesis at 2.5.	
Write in interval notation.	$(-\infty, 2.5)$

ⓒ

	$x \le -\frac{3}{5}$
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, -\frac{3}{5}\right]$

Note:
Exercise:

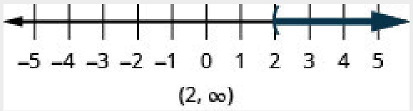
TRY IT :: 2.133

Problem: Graph on the number line and write in interval notation:

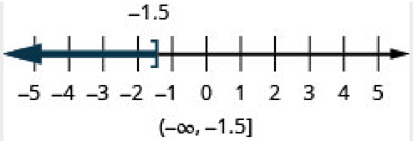
ⓐ $x > 2$ ⓑ $x \le -1.5$ ⓒ $x \ge \frac{3}{4}$

Solution:

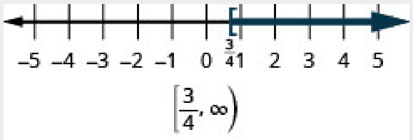
ⓐ



ⓑ



ⓒ



Note:
Exercise:

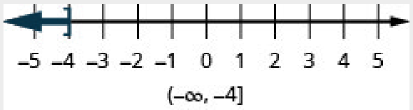
TRY IT :: 2.134

Problem: Graph on the number line and write in interval notation:

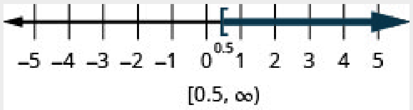
ⓐ $x \leq -4$ ⓑ $x \geq 0.5$ ⓒ $x < -\frac{2}{3}$

Solution:

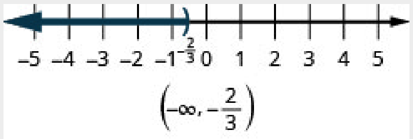
ⓐ



ⓑ



ⓒ



Solve Inequalities using the Subtraction and Addition Properties of Inequality

The Subtraction and Addition Properties of Equality state that if two quantities are equal, when we add or subtract the same amount from both quantities, the results will be equal.

Note:
Properties of Equality
Equation:

Subtraction Property of Equality

For any numbers a, b, and c,

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a-c & = & b-c. \end{array}$$

Addition Property of Equality

For any numbers a, b, and c,

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a+c & = & b+c. \end{array}$$

Similar properties hold true for inequalities.

For example, we know that −4 is less than 2.	$-4<2$
If we subtract 5 from both quantities, is the left side still less than the right side?	$-4-5 \; ? \; 2-5$
We get −9 on the left and −3 on the right.	$-9 \; ? \; -3$
And we know −9 is less than −3.	$-9<-3$
	The inequality sign stayed the same.

Similarly we could show that the inequality also stays the same for addition.

This leads us to the Subtraction and Addition Properties of Inequality.

Note:
Properties of Inequality
Equation:

Subtraction Property of Inequality

For any numbers $a, b,$ and $c,$

$$\begin{array}{lrcl} \text{if} & a & < & b \\ \text{then} & a-c & < & b-c. \end{array}$$

$$\begin{array}{lrcl} \text{if} & a & > & b \\ \text{then} & a-c & > & b-c. \end{array}$$

Addition Property of Inequality

For any numbers $a, b,$ and $c,$

$$\begin{array}{lrcl} \text{if} & a & < & b \\ \text{then} & a+c & < & b+c. \end{array}$$

$$\begin{array}{lrcl} \text{if} & a & > & b \\ \text{then} & a+c & > & b+c. \end{array}$$

We use these properties to solve inequalities, taking the same steps we used to solve equations. Solving the inequality $x+5>9$, the steps would look like this:

	$x+5>9$
Subtract 5 from both sides to isolate x.	$x+5-5>9-5$
Simplify.	$x>4$

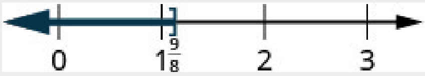
Any number greater than 4 is a solution to this inequality.

Example:
Exercise:

Problem:

**Solve the inequality $n-\frac{1}{2}\leq\frac{5}{8}$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$n - \frac{1}{2} \leq \frac{5}{8}$
Add $\frac{1}{2}$ to both sides of the inequality.	$n - \frac{1}{2} + \frac{1}{2} \leq \frac{5}{8} + \frac{1}{2}$
Simplify.	$n \leq \frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	[missing_resource: CNX_ElemAlg_Figure_02_07_026e_img_new.jpg]

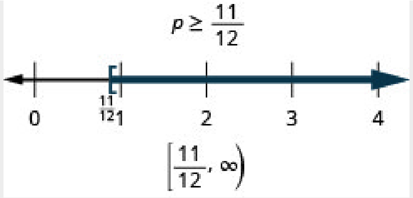
Note:
Exercise:

Problem:

TRY IT :: 2.135
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$p - \frac{3}{4} \geq \frac{1}{6}$

Solution:



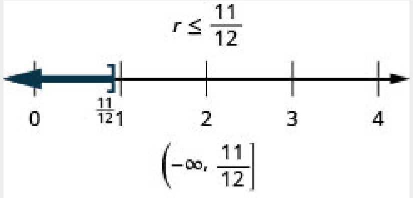
Note:
Exercise:

Problem:

TRY IT :: 2.136
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$r - \frac{1}{3} \leq \frac{7}{12}$

Solution:



Solve Inequalities using the Division and Multiplication Properties of Inequality

The Division and Multiplication Properties of Equality state that if two quantities are equal, when we divide or multiply both quantities by the same amount, the results will also be equal (provided we don’t divide by 0).

Note:
Properties of Equality
Equation:

Division Property of Equality

For any numbers a, b, c, and $c \neq 0$,

$$\begin{array}{llcl} \text{if} & a & = & b, \\ \text{then} & \frac{a}{c} & = & \frac{b}{c}. \end{array}$$

Multiplication Property of Equality

For any real numbers a, b, c,

$$\begin{array}{llcl} \text{if} & a & = & b, \\ \text{then} & ac & = & bc. \end{array}$$

Are there similar properties for inequalities? What happens to an inequality when we divide or multiply both sides by a constant?

Consider some numerical examples.

	$10 < 15$		$10 < 15$
Divide both sides by 5.	$\frac{10}{5} ? \frac{15}{5}$	Multiply both sides by 5.	$10(5) ? 15(5)$
Simplify.	$2 ? 3$		$50 ? 75$
Fill in the inequality signs.	$2 < 3$		$50 < 75$

Equation:

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

	$10 < 15$		$10 < 15$
Divide both sides by −5.	$\frac{10}{-5} \; ? \; \frac{15}{-5}$	Multiply both sides by −5.	$10(-5) \; ? \; 15(-5)$
Simplify.	$-2 \; ? \; -3$		$-50 \; ? \; -75$
Fill in the inequality signs.	$-2 > -3$		$-50 > -75$

Equation:

$$\textbf{The inequality signs reversed their direction.}$$

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

Here are the Division and Multiplication Properties of Inequality for easy reference.

Note:
Division and Multiplication Properties of Inequality
Equation:

$$\begin{array}{l}\text{For any real numbers } a, b, c\\ \\ \begin{array}{ll}\text{if } a < b \text{ and } c > 0\text{, then} & \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc.\\ \text{if } a > b \text{ and } c > 0\text{, then} & \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc.\\ \text{if } a < b \text{ and } c < 0\text{, then} & \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc.\\ \text{if } a > b \text{ and } c < 0\text{, then} & \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc.\end{array}\end{array}$$

When we **divide or multiply** an inequality by a:

- **positive** number, the inequality stays the **same**.

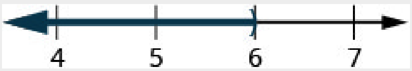
- **negative** number, the inequality **reverses**.

Example:
Exercise:

Problem:

**Solve the inequality $7y < 42$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$7y < 42$
Divide both sides of the inequality by 7. Since $7 > 0$, the inequality stays the same.	$\frac{7y}{7} < \frac{42}{7}$
Simplify.	$y < 6$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

Note:
Exercise:

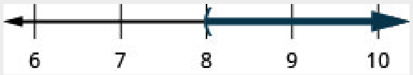
Problem:

TRY IT :: 2.137
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$(8, \infty)$

Solution:

$c > 8$



$(8, \infty)$

Note:
Exercise:

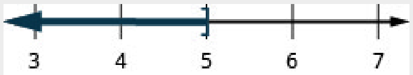
Problem:

TRY IT :: 2.138
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$12d \leq 60$

Solution:

$d \leq 5$



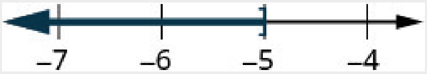
$(-\infty, 5]$

Example:
Exercise:

Problem:

**Solve the inequality $-10a \geq 50$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$-10a \geq 50$
Divide both sides of the inequality by −10. Since $-10 < 0$, the inequality reverses.	$\frac{-10a}{-10} \leq \frac{50}{-10}$
Simplify.	$a \leq -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

Note:
Exercise:

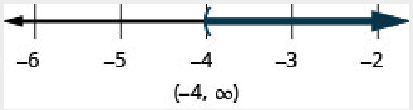
Problem:

TRY IT :: 2.139
Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-8q < 32$

Solution:

$q > -4$



Note:
Exercise:

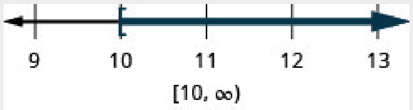
Problem:

TRY IT :: 2.140
Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-7r \leq \ -70$

Solution:

$r \geq 10$



Note:
Solving Inequalities
Sometimes when solving an inequality, the variable ends up on the right. We can rewrite the inequality in reverse to get the variable to the left.
Equation:

$$x > a \text{ has the same meaning as } a < x$$

Think about it as “If Xavier is taller than Alex, then Alex is shorter than Xavier.”

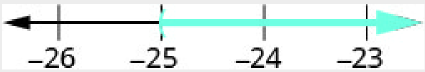
Example:

Exercise:

Problem:

**Solve the inequality $-20 < \frac{4}{5}u$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$-20 < \frac{4}{5}u$
Multiply both sides of the inequality by $\frac{5}{4}$. Since $\frac{5}{4} > 0$, the inequality stays the same.	$\frac{5}{4}(-20) < \frac{5}{4}\left(\frac{4}{5}u\right)$
Simplify.	$-25 < u$
Rewrite the variable on the left.	$u > -25$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

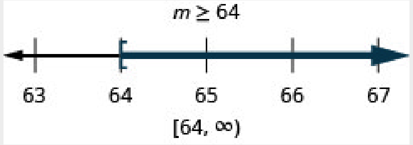
Note:
Exercise:

Problem:

TRY IT :: 2.141
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$24 \leq \frac{3}{8}m$

Solution:



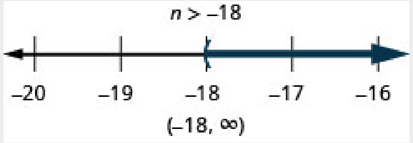
Note:
Exercise:

Problem:

TRY IT :: 2.142
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$-24 < \frac{4}{3}n$

Solution:

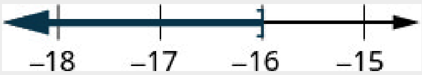


Example:
Exercise:

Problem:

**Solve the inequality $\frac{t}{-2} \geq 8$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$\frac{t}{-2} \geq 8$
Multiply both sides of the inequality by -2. Since $-2 < 0$, the inequality reverses.	$-2\left(\frac{t}{-2}\right) \leq -2(8)$
Simplify.	$t \leq -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

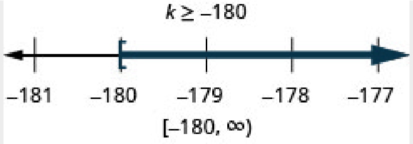
Note:
Exercise:

Problem:

TRY IT :: 2.143
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{k}{-12} \leq 15$

Solution:



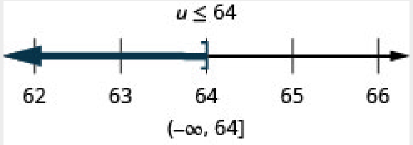
Note:
Exercise:

Problem:

TRY IT :: 2.144
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{u}{-4} \geq -16$

Solution:



Solve Inequalities That Require Simplification

Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but be sure to pay close attention during multiplication or division.

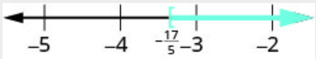
Example:
Exercise:

Problem:

**Solve the inequality $4m \leq 9m + 17$, graph the solution on the number line, and write the solution in interval notation.

Solution:

Solution

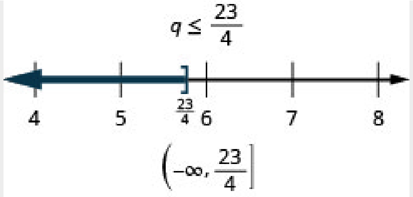
	$4m \leq 9m + 17$
Subtract $9m$ from both sides to collect the variables on the left.	$4m - 9m \leq 9m - 9m + 17$
Simplify.	$-5m \leq 17$
Divide both sides of the inequality by -5, and reverse the inequality.	$\frac{-5m}{-5} \geq \frac{17}{-5}$
Simplify.	$m \geq -\frac{17}{5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

Note:
Exercise:

Problem:

TRY IT :: 2.145
Solve the inequality $3q \geq 7q - 23$, graph the solution on the number line, and write the solution in interval notation.

Solution:

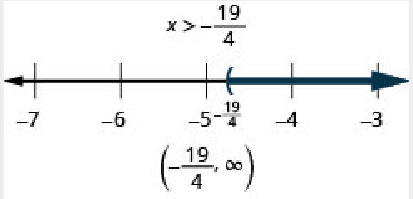


Note:
Exercise:

Problem:

TRY IT :: 2.146
Solve the inequality $6x < 10x + 19$, graph the solution on the number line, and write the solution in interval notation.

Solution:



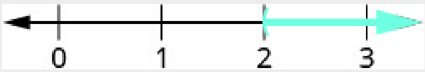
Example:
Exercise:

Problem:

**Solve the inequality $8p + 3(p - 12) > 7p - 28$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

Simplify each side as much as	$8p + 3(p - 12) > 7p - 28$

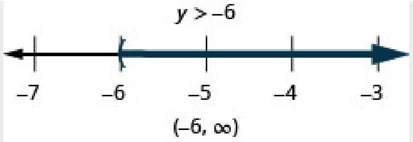
possible.	
Distribute.	$8p+3p-36>7p-28$
Combine like terms.	$11p-36>7p-28$
Subtract $7p$ from both sides to collect the variables on the left.	$11p-36-7p>7p-28-7p$
Simplify.	$4p-36>-28$
Add 36 to both sides to collect the constants on the right.	$4p-36+36>-28+36$
Simplify.	$4p>8$
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4}>\frac{8}{4}$
Simplify.	$p>2$
Graph the solution on the number line.	
Write the solution in interal notation.	$(2,\infty)$

Note:
Exercise:

Problem:

TRY IT :: 2.147
Solve the inequality $9y+2(y+6)>5y-24$, graph the solution on the number line, and write the solution in interval notation.

Solution:

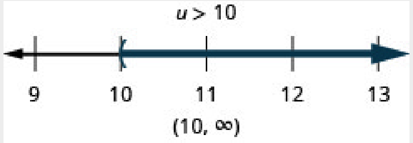


Note:
Exercise:

Problem:

TRY IT :: 2.148
Solve the inequality $6u + 8(u - 1) > 10u + 32$, graph the solution on the number line, and write the solution in interval notation.

Solution:



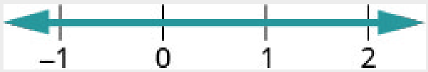
Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction.

Example:
Exercise:

Problem:

**Solve the inequality $8x - 2\left(5 - x\right) < 4\left(x + 9\right) + 6x$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

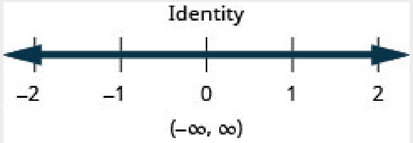
Simplify each side as much as possible.	$8x - 2(5 - x) < 4(x + 9) + 6x$
Distribute.	$8x - 10 + 2x < 4x + 36 + 6x$
Combine like terms.	$10x - 10 < 10x + 36$
Subtract $10x$ from both sides to collect the variables on the left.	$10x - 10 - 10x < 10x + 36 - 10x$
Simplify.	$-10 < 36$
The x's are gone, and we have a true statement.	The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, \infty)$

Note:
Exercise:

Problem:

TRY IT :: 2.149
Solve the inequality $4b - 3(3 - b) > 5(b - 6) + 2b$, graph the solution on the number line, and write the solution in interval notation.

Solution:

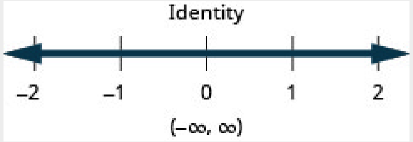


Note:
Exercise:

Problem:

TRY IT :: 2.150
Solve the inequality $9h-7\left(2-h\right)<8\left(h+11\right)+8h$, graph the solution on the number line, and write the solution in interval notation.

Solution:



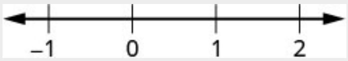
Example:
Exercise:

Problem:

**Solve the inequality $\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

Solution:
Solution

	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$

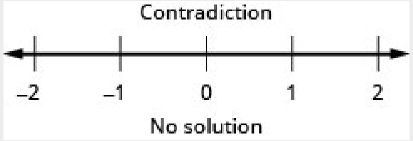
Combine like terms.	$5a > 5a + 18$
Subtract $5a$ from both sides to collect the variables on the left.	$5a - 5a > 5a - 5a + 18$
Simplify.	$0 > 18$
The statement is false!	The inequality is a contradiction.
	There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Note:
Exercise:

Problem:

TRY IT :: 2.151
Solve the inequality $\frac{1}{4}x - \frac{1}{12}x > \frac{1}{6}x + \frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

Solution:

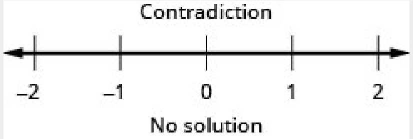


Note:
Exercise:

Problem:

TRY IT :: 2.152
Solve the inequality $\frac{2}{5}z - \frac{1}{3}z < \frac{1}{15}z - \frac{3}{5}$, graph the solution on the number line, and write the solution in interval notation.

Solution:



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like ‘more than’ and ‘less than’. But others are not as obvious.

Think about the phrase ‘at least’ – what does it mean to be ‘at least 21 years old’? It means 21 or more. The phrase ‘at least’ is the same as ‘greater than or equal to’.

[link] shows some common phrases that indicate inequalities.

$>$	$\geq$	$<$	$\leq$
is greater than	is greater than or equal to	is less than	is less than or equal to
is more than	is at least	is smaller than	is at most
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

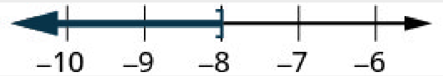
Example:
Exercise:

Problem:

**Translate and solve. Then write the solution in interval notation and graph on the number line.

Twelve times *c* is no more than 96.

Solution:
Solution

Translate.	Twelve times c $\underbrace{\text{is no more than}}_{}$ 96 $12c \le 96$
Solve—divide both sides by 12.	$\frac{12c}{12} \le \frac{96}{12}$
Simplify.	$c \le 8$
Write in interval notation.	$(-\infty, -8]$
Graph on the number line.	

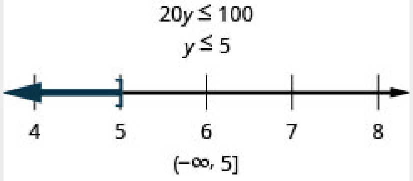
Note:
Exercise:

Problem:

TRY IT :: 2.153
Translate and solve. Then write the solution in interval notation and graph on the number line.

Twenty times *y* is at most 100

Solution:



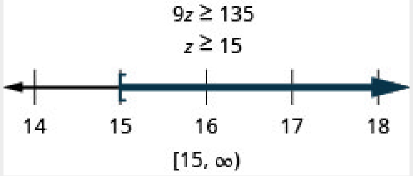
Note:
Exercise:

Problem:

TRY IT :: 2.154
Translate and solve. Then write the solution in interval notation and graph on the number line.

Nine times *z* is no less than 135

Solution:



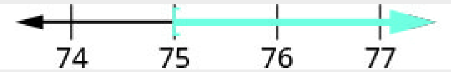
Example:
Exercise:

Problem:

**Translate and solve. Then write the solution in interval notation and graph on the number line.

Thirty less than *x* is at least 45.

Solution:
Solution

Translate.	$\text{Thirty less than } x \underbrace{\text{ is at least }} 45.$ $x-30\geq 45$
Solve—add 30 to both sides.	$x-30+30\geq 45+30$
Simplify.	$x\geq 75$
Write in interval notation.	$[75,\infty)$
Graph on the number line.	

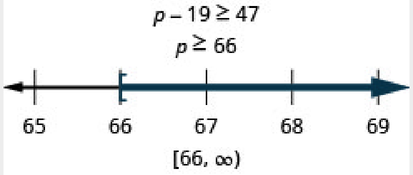
Note:
Exercise:

Problem:

TRY IT :: 2.155
Translate and solve. Then write the solution in interval notation and graph on the number line.

Nineteen less than *p* is no less than 47

Solution:



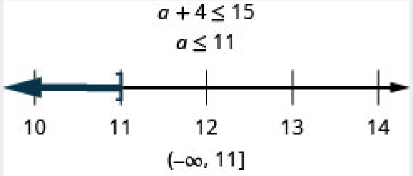
Note:
Exercise:

Problem:

TRY IT :: 2.156
Translate and solve. Then write the solution in interval notation and graph on the number line.

Four more than *a* is at most 15.

Solution:



Key Concepts

- **Subtraction Property of Inequality**
  For any numbers a, b, and c,

if $a < b$ then $a - c < b - c$ and
if $a > b$ then $a - c > b - c$.

- **Addition Property of Inequality**
  For any numbers a, b, and c,
  if $a < b$ then $a + c < b + c$ and
  if $a > b$ then $a + c > b + c$.
- **Division and Multiplication Properties of Inequality**
  For any numbers a, b, and c,
  if $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$ and $ac > bc$.
  if $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$.
  if $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$.
  if $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.
- When we **divide or multiply** an inequality by a:
  - **positive** number, the inequality stays the **same**.
  - **negative** number, the inequality **reverses**.

Section Exercises

Practice Makes Perfect

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line.

Exercise:

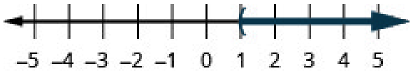
Problem:
ⓐ $x \leq 2$
ⓑ $x > -1$
ⓒ $x < 0$

Exercise:

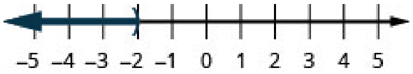
Problem:
ⓐ $x > 1$
ⓑ $x < -2$
ⓒ $x \geq -3$

Solution:

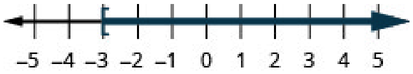
ⓐ



ⓑ



ⓒ



Exercise:

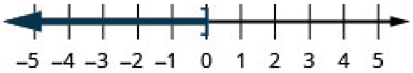
Problem:
ⓐ $x \geq -3$
ⓑ $x < 4$
ⓒ $x \leq -2$

Exercise:

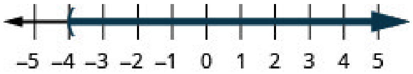
Problem:
ⓐ $x \leq 0$
ⓑ $x > -4$
ⓒ $x \geq -1$

Solution:

ⓐ



ⓑ



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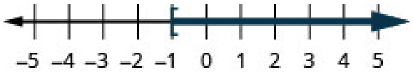


In the following exercises, graph each inequality on the number line and write in interval notation.

Exercise:

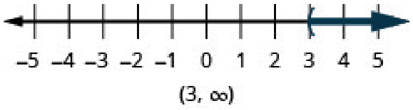
Problem: ⓐ $x < -2$
ⓑ $x \geq -3.5$
ⓒ $x \leq \frac{2}{3}$

Exercise:

Problem: ⓐ $x > 3$
ⓑ $x \leq -0.5$
ⓒ $x \geq \frac{1}{3}$

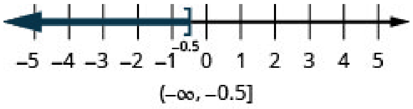
Solution:

ⓐ



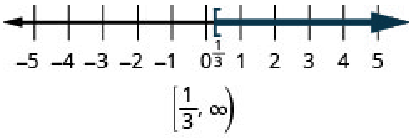
$(3, \infty)$

ⓑ



$(-\infty, -0.5]$

ⓒ



$\left[\frac{1}{3}, \infty\right)$

Exercise:

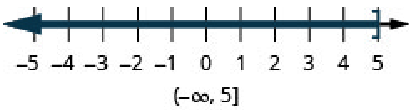
Problem: ⓐ $x \geq -4$
ⓑ $x < 2.5$
ⓒ $x > -\frac{3}{2}$

Exercise:

Problem: ⓐ $x \leq 5$
ⓑ $x \geq -1.5$
ⓒ $x < -\frac{7}{3}$

Solution:

ⓐ



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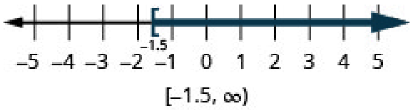

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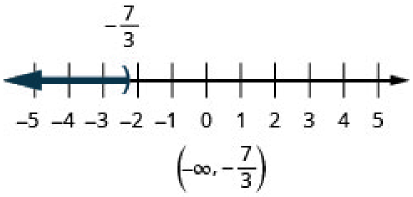


Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

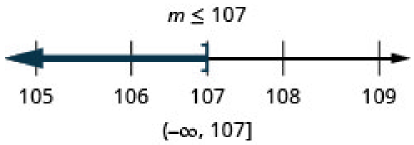
Exercise:

Problem: $n - 11 < 33$

Exercise:

Problem: $m - 45 \leq 62$

Solution:



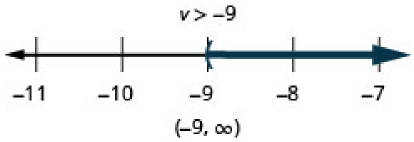
Exercise:

Problem: $u + 25 > 21$

Exercise:

Problem: $v + 12 > 3$

Solution:



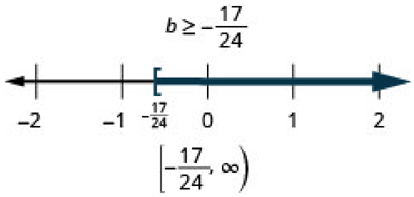
Exercise:

Problem: $a + \frac{3}{4} \geq \frac{7}{10}$

Exercise:

Problem: $b + \frac{7}{8} \geq \frac{1}{6}$

Solution:



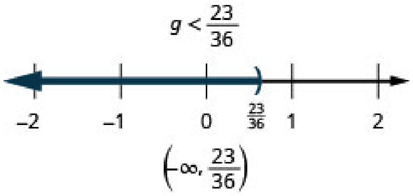
Exercise:

Problem: $f - \frac{13}{20} < -\frac{5}{12}$

Exercise:

Problem: $g - \frac{11}{12} < -\frac{5}{18}$

Solution:



Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

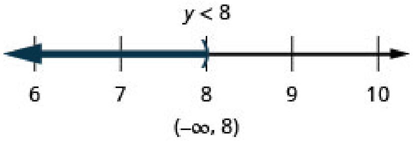
Exercise:

Problem: $8x > 72$

Exercise:

Problem: $6y < 48$

Solution:



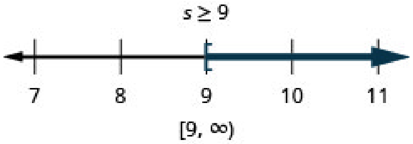
Exercise:

Problem: $7r \leq 56$

Exercise:

Problem: $9s \geq 81$

Solution:



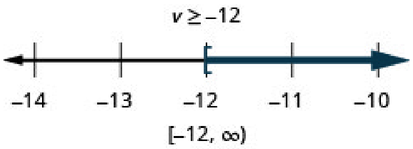
Exercise:

Problem: $-5u \geq 65$

Exercise:

Problem: $-8v \leq 96$

Solution:



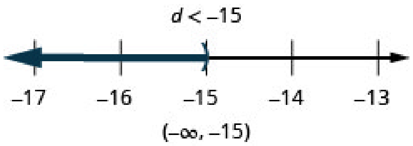
Exercise:

Problem: $-9c < 126$

Exercise:

Problem: $-7d > 105$

Solution:



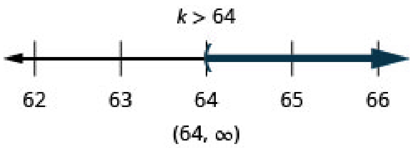
Exercise:

Problem: $20 > \frac{2}{5}h$

Exercise:

Problem: $40 < \frac{5}{8}k$

Solution:



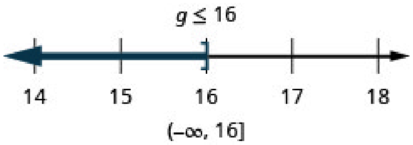
Exercise:

Problem: $\frac{7}{6}j \geq 42$

Exercise:

Problem: $\frac{9}{4}g \leq 36$

Solution:



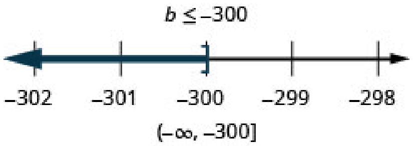
Exercise:

Problem: $\frac{a}{-3} \leq 9$

Exercise:

Problem: $\frac{b}{-10} \geq 30$

Solution:



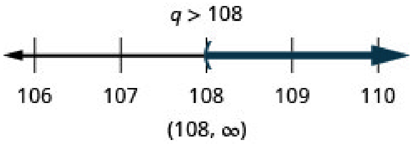
Exercise:

Problem: $-25 < \frac{p}{-5}$

Exercise:

Problem: $-18 > \frac{q}{-6}$

Solution:



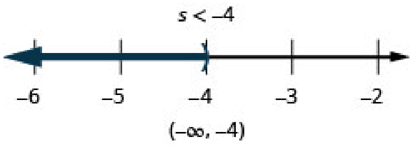
Exercise:

Problem: $9t \geq -27$

Exercise:

Problem: $7s < -28$

Solution:



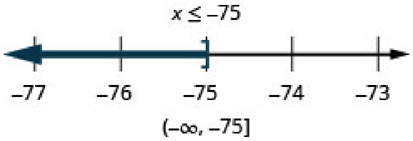
Exercise:

Problem: $\frac{2}{3}y > -36$

Exercise:

Problem: $\frac{3}{5}x \leq -45$

Solution:



Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

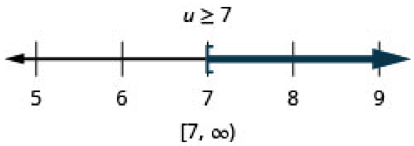
Exercise:

Problem: $4v \geq 9v - 40$

Exercise:

Problem: $5u \leq 8u - 21$

Solution:



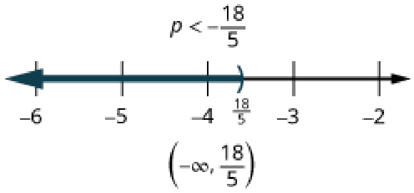
Exercise:

Problem: $13q < 7q - 29$

Exercise:

Problem: $9p > 14p - 18$

Solution:



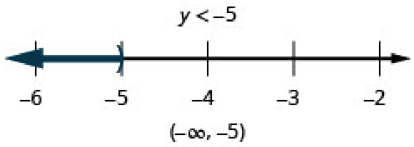
Exercise:

Problem: $12x + 3(x + 7) > 10x - 24$

Exercise:

Problem: $9y + 5(y + 3) < 4y - 35$

Solution:



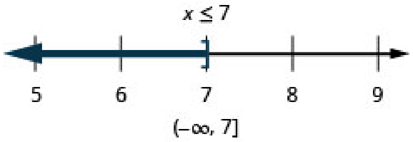
Exercise:

Problem: $6h - 4(h - 1) \leq 7h - 11$

Exercise:

Problem: $4k - (k - 2) \geq 7k - 26$

Solution:



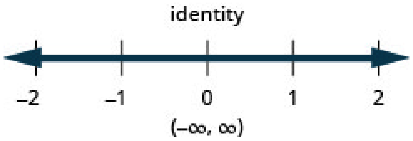
Exercise:

Problem: $8m - 2\left(14 - m\right) \geq 7\left(m - 4\right) + 3m$

Exercise:

Problem: $6n - 12\left(3 - n\right) \leq 9\left(n - 4\right) + 9n$

Solution:



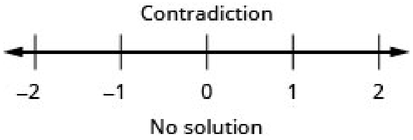
Exercise:

Problem: $\frac{3}{4}b - \frac{1}{3}b < \frac{5}{12}b - \frac{1}{2}$

Exercise:

Problem: $9u + 5(2u - 5) \geq 12(u - 1) + 7u$

Solution:



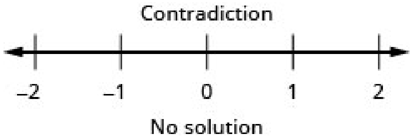
Exercise:

Problem: $\frac{2}{3}g - \frac{1}{2}\left(g - 14\right) \leq \frac{1}{6}\left(g + 42\right)$

Exercise:

Problem: $\frac{5}{6}a - \frac{1}{4}a > \frac{7}{12}a + \frac{2}{3}$

Solution:



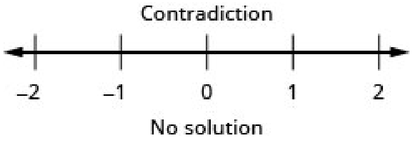
Exercise:

Problem: $\frac{4}{5}h - \frac{2}{3}(h - 9) \geq \frac{1}{15}(2h + 90)$

Exercise:

Problem: $12v + 3(4v - 1) \leq 19(v - 2) + 5v$

Solution:



Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

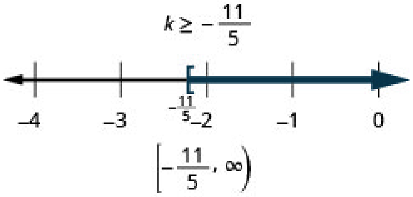
Exercise:

Problem: $15k \leq -40$

Exercise:

Problem: $35k \geq -77$

Solution:



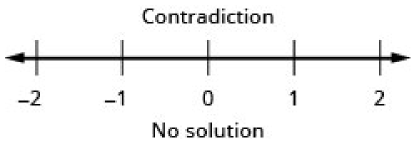
Exercise:

Problem: $23p-2(6-5p)>3(11p-4)$

Exercise:

Problem: $18q-4(10-3q)<5(6q-8)$

Solution:



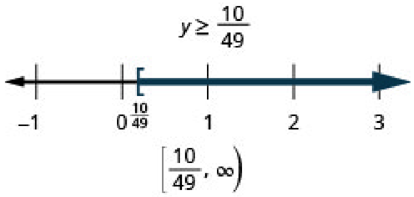
Exercise:

Problem: $-\frac{9}{4}x\geq-\frac{5}{12}$

Exercise:

Problem: $-\frac{21}{8}y\leq-\frac{15}{28}$

Solution:



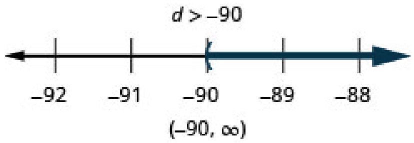
Exercise:

Problem: $c+34<-99$

Exercise:

Problem: $d+29>-61$

Solution:



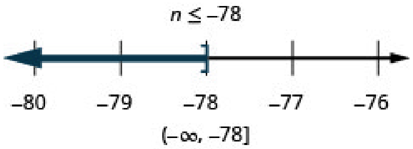
Exercise:

Problem: $\frac{m}{18} \geq -4$

Exercise:

Problem: $\frac{n}{13} \leq -6$

Solution:



Translate to an Inequality and Solve

In the following exercises, translate and solve .Then write the solution in interval notation and graph on the number line.

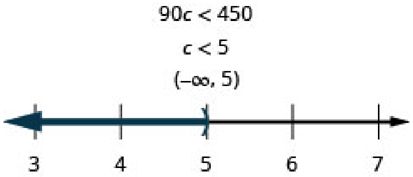
Exercise:

Problem: Fourteen times *d* is greater than 56.

Exercise:

Problem: Ninety times *c* is less than 450.

Solution:



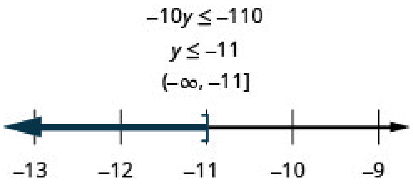
Exercise:

Problem: Eight times *z* is smaller than -40.

Exercise:

Problem: Ten times *y* is at most -110.

Solution:



$-10y \leq -110$

$y \leq -11$

$(-\infty, -11]$

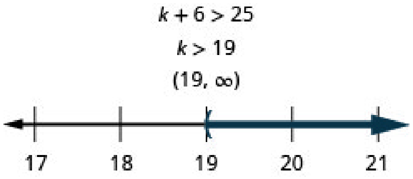
Exercise:

Problem: Three more than *h* is no less than 25.

Exercise:

Problem: Six more than *k* exceeds 25.

Solution:



$k + 6 > 25$

$k > 19$

$(19, \infty)$

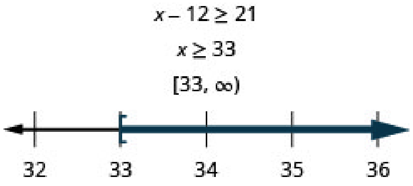
Exercise:

Problem: Ten less than *w* is at least 39.

Exercise:

Problem: Twelve less than *x* is no less than 21.

Solution:



$x - 12 \geq 21$

$x \geq 33$

$[33, \infty)$

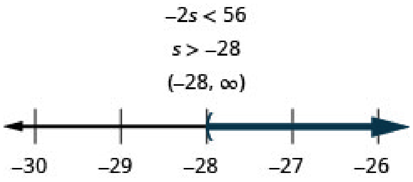
Exercise:

Problem: Negative five times *r* is no more than 95.

Exercise:

Problem: Negative two times *s* is lower than 56.

Solution:



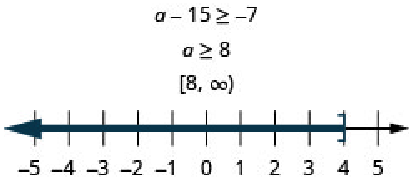
Exercise:

Problem: Nineteen less than *b* is at most -22.

Exercise:

Problem: Fifteen less than *a* is at least -7.

Solution:



Everyday Math

Exercise:

Problem:

Safety A child’s height, *h*, must be at least 57 inches for the child to safely ride in the front seat of a car. Write this as an inequality.

Exercise:

Problem:

Fighter pilots The maximum height, *h*, of a fighter pilot is 77 inches. Write this as an inequality.

Solution:

$h \leq 77$

Exercise:

Problem:

Elevators The total weight, *w*, of an elevator’s passengers can be no more than 1,200 pounds. Write this as an inequality.

Exercise:

Problem:

Shopping The number of items, *n*, a shopper can have in the express check-out lane is at most 8. Write this as an inequality.

Solution:

$n \leq 8$

Writing Exercises

Exercise:

Problem: Give an example from your life using the phrase ‘at least’.

Exercise:

Problem: Give an example from your life using the phrase ‘at most’.

Solution:

Answers will vary.

Exercise:

Problem:

Explain why it is necessary to reverse the inequality when solving $-5x > 10$.

Exercise:

Problem:

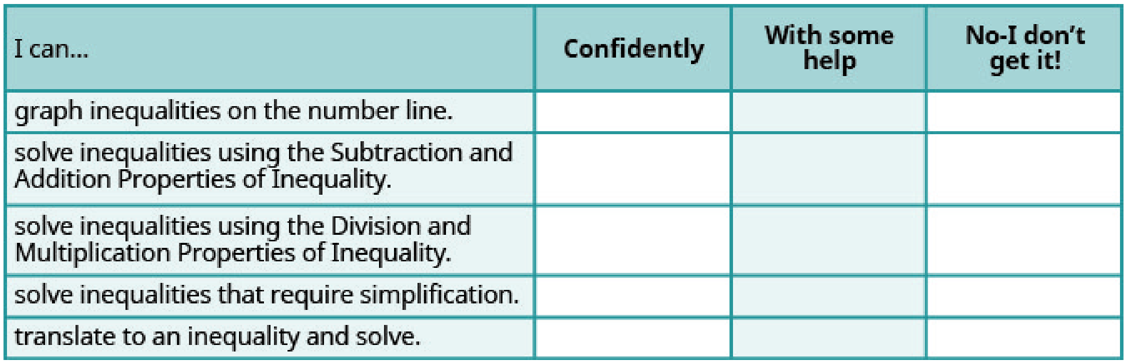
Explain why it is necessary to reverse the inequality when solving $\frac{n}{-3} < 12$.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
graph inequalities on the number line.			
solve inequalities using the Subtraction and Addition Properties of Inequality.			
solve inequalities using the Division and Multiplication Properties of Inequality.			
solve inequalities that require simplification.			
translate to an inequality and solve.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Review & Practice Test
By the end of this section, you will be prepared to take your Chapter 2 Assessment.

Chapter 2 Review Exercises

Solve Equations using the Subtraction and Addition Properties of Equality

Verify a Solution of an Equation

In the following exercises, determine whether each number is a solution to the equation.

Exercise:

Problem: $10x-1=5x; x=\frac{1}{5}$

Exercise:

Problem: $w+2=\frac{5}{8}; w=\frac{3}{8}$

Solution:

no

Exercise:

Problem: $-12n+5=8n; n=-\frac{5}{4}$

Exercise:

Problem: $6a-3=-7a, a=\frac{3}{13}$

Solution:

yes

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction Property of Equality.

Exercise:

Problem: $x + 7 = 19$

Exercise:

Problem: $y + 2 = -6$

Solution:

$y = -8$

Exercise:

Problem: $a + \frac{1}{3} = \frac{5}{3}$

Exercise:

Problem: $n + 3.6 = 5.1$

Solution:

$n = 1.5$

In the following exercises, solve each equation using the Addition Property of Equality.

Exercise:

Problem: $u - 7 = 10$

Exercise:

Problem: $x-9=-4$

Solution:

$x=5$

Exercise:

Problem: $c-\frac{3}{11}=\frac{9}{11}$

Exercise:

Problem: $p-4.8=14$

Solution:

$p=18.8$

In the following exercises, solve each equation.

Exercise:

Problem: $n-12=32$

Exercise:

Problem: $y+16=-9$

Solution:

$y=-25$

Exercise:

Problem: $f+\frac{2}{3}=4$

Exercise:

Problem: $d - 3.9 = 8.2$

Solution:

$d = 12.1$

Solve Equations That Require Simplification

In the following exercises, solve each equation.

Exercise:

Problem: $y + 8 - 15 = -3$

Exercise:

Problem: $7x + 10 - 6x + 3 = 5$

Solution:

$x = -8$

Exercise:

Problem: $6(n - 1) - 5n = -14$

Exercise:

Problem: $8(3p + 5) - 23(p - 1) = 35$

Solution:

$p = -28$

Translate to an Equation and Solve

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

Exercise:

Problem: The sum of -6 and m is 25.

Exercise:

Problem: Four less than n is 13.

Solution:

$n-4=13; n=17$

Translate and Solve Applications

In the following exercises, translate into an algebraic equation and solve.

Exercise:

Problem:

Rochelle’s daughter is 11 years old. Her son is 3 years younger. How old is her son?

Exercise:

Problem:

Tan weighs 146 pounds. Minh weighs 15 pounds more than Tan. How much does Minh weigh?

Solution:

161 pounds

Exercise:

Problem:

Peter paid \$9.75 to go to the movies, which was \$46.25 less than he paid to go to a concert. How much did he pay for the concert?

Exercise:

Problem:

Elissa earned \$152.84 this week, which was \$21.65 more than she earned last week. How much did she earn last week?

Solution:

\$131.19

Solve Equations using the Division and Multiplication Properties of Equality

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the division and multiplication properties of equality and check the solution.

Exercise:

Problem: $8x = 72$

Exercise:

Problem: $13a = -65$

Solution:

$a = -5$

Exercise:

Problem: $0.25p = 5.25$

Exercise:

Problem: $-y = 4$

Solution:

$y = -4$

Exercise:

Problem: $\frac{n}{6} = 18$

Exercise:

Problem: $\frac{y}{-10} = 30$

Solution:

$y = -300$

Exercise:

Problem: $36 = \frac{3}{4}x$

Exercise:

Problem: $\frac{5}{8}u = \frac{15}{16}$

Solution:

$u = \frac{3}{2}$

Exercise:

Problem: $-18m = -72$

Exercise:

Problem: $\frac{c}{9} = 36$

Solution:

$c = 324$

Exercise:

Problem: $0.45x = 6.75$

Exercise:

Problem: $\frac{11}{12} = \frac{2}{3}y$

Solution:

$y = \frac{11}{8}$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

Exercise:

Problem: $5r - 3r + 9r = 35 - 2$

Exercise:

Problem: $24x + 8x - 11x = -7 - 14$

Solution:

$x = -1$

Exercise:

Problem: $\frac{11}{12}n - \frac{5}{6}n = 9 - 5$

Exercise:

Problem: $-9(d-2) - 15 = -24$

Solution:

$d = 3$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

Exercise:

Problem: 143 is the product of -11 and *y*.

Exercise:

Problem: The quotient of *b* and and 9 is -27.

Solution:

$\frac{b}{9} = -27; b = -243$

Exercise:

Problem: The sum of *q* and one-fourth is one.

Exercise:

Problem: The difference of *s* and one-twelfth is one fourth.

Solution:

$s - \frac{1}{12} = \frac{1}{4}; s = \frac{1}{3}$

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise:

Problem:

Ray paid \$21 for 12 tickets at the county fair. What was the price of each ticket?

Exercise:

Problem:

Janet gets paid \$24 per hour. She heard that this is $\frac{3}{4}$ of what Adam is paid. How much is Adam paid per hour?

Solution:

\$32

Solve Equations with Variables and Constants on Both Sides

Solve an Equation with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

Exercise:

Problem: $8p + 7 = 47$

Exercise:

Problem: $10w - 5 = 65$

Solution:

$w = 7$

Exercise:

Problem: $3x + 19 = -47$

Exercise:

Problem: $32 = -4 - 9n$

Solution:

$n = -4$

Solve an Equation with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

Exercise:

Problem: $7y = 6y - 13$

Exercise:

Problem: $5a + 21 = 2a$

Solution:

$a = -7$

Exercise:

Problem: $k = -6k - 35$

Exercise:

Problem: $4x - \frac{3}{8} = 3x$

Solution:

$x = \frac{3}{8}$

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

Exercise:

Problem: $12x - 9 = 3x + 45$

Exercise:

Problem: $5n - 20 = -7n - 80$

Solution:

$n = -5$

Exercise:

Problem: $4u + 16 = -19 - u$

Exercise:

Problem: $\frac{5}{8}c - 4 = \frac{3}{8}c + 4$

Solution:

$c = 32$

Use a General Strategy for Solving Linear Equations

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

Exercise:

Problem: $6(x+6)=24$

Exercise:

Problem: $9(2p-5)=72$

Solution:

$p=\frac{13}{2}$

Exercise:

Problem: $-(s+4)=18$

Exercise:

Problem: $8+3(n-9)=17$

Solution:

$n=12$

Exercise:

Problem: $23-3(y-7)=8$

Exercise:

Problem: $\frac{1}{3}(6m+21)=m-7$

Solution:

$m = -14$

Exercise:

Problem: $4(3.5y + 0.25) = 365$

Exercise:

Problem: $0.25(q - 8) = 0.1(q + 7)$

Solution:

$q = 18$

Exercise:

Problem: $8(r - 2) = 6(r + 10)$

Exercise:

Problem: $5 + 7(2 - 5x) = 2(9x + 1) - (13x - 57)$

Solution:

$x = -1$

Exercise:

Problem: $(9n + 5) - (3n - 7) = 20 - (4n - 2)$

Exercise:

Problem: $2[-16 + 5(8k - 6)] = 8(3 - 4k) - 32$

Solution:

$k = \frac{3}{4}$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

Exercise:

Problem: $17y - 3(4 - 2y) = 11(y - 1) + 12y - 1$

Exercise:

Problem: $9u + 32 = 15(u - 4) - 3(2u + 21)$

Solution:

contradiction; no solution

Exercise:

Problem: $-8(7m + 4) = -6(8m + 9)$

Exercise:

Problem: $21(c - 1) - 19(c + 1) = 2(c - 20)$

Solution:

identity; all real numbers

Solve Equations with Fractions and Decimals

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

Exercise:

Problem: $\frac{2}{5}n-\frac{1}{10}=\frac{7}{10}$

Exercise:

Problem: $\frac{1}{3}x+\frac{1}{5}x=8$

Solution:

$x=15$

Exercise:

Problem: $\frac{3}{4}a-\frac{1}{3}=\frac{1}{2}a-\frac{5}{6}$

Exercise:

Problem: $\frac{1}{2}(k-3)=\frac{1}{3}(k+16)$

Solution:

$k=41$

Exercise:

Problem: $\frac{3x-2}{5}=\frac{3x+4}{8}$

Exercise:

Problem: $\frac{5y-1}{3}+4=\frac{-8y+4}{6}$

Solution:

$y = -1$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

Exercise:

Problem: $0.8x - 0.3 = 0.7x + 0.2$

Exercise:

Problem: $0.36u + 2.55 = 0.41u + 6.8$

Solution:

$u = -85$

Exercise:

Problem: $0.6p - 1.9 = 0.78p + 1.7$

Exercise:

Problem: $0.6p - 1.9 = 0.78p + 1.7$

Solution:

$d = -20$

Solve a Formula for a Specific Variable

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

Exercise:

Problem:

Natalie drove for $7\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Exercise:

Problem:

Mallory is taking the bus from St. Louis to Chicago. The distance is 300 miles and the bus travels at a steady rate of 60 miles per hour. How long will the bus ride be?

Solution:

5 hours

Exercise:

Problem:

Aaron’s friend drove him from Buffalo to Cleveland. The distance is 187 miles and the trip took 2.75 hours. How fast was Aaron’s friend driving?

Exercise:

Problem:

Link rode his bike at a steady rate of 15 miles per hour for $2\frac{1}{2}$ hours. How much distance did he travel?

Solution:

37.5 miles

Solve a Formula for a Specific Variable

In the following exercises, solve.

Exercise:

Use the formula. $d = rt$ to solve for t
ⓐ when $d = 510$ and $r = 60$
Problem: ⓑ in general

Exercise:

Use the formula. $d = rt$ to solve for r
ⓐ when when $d = 451$ and $t = 5.5$
Problem: ⓑ in general

Solution:

ⓐ $r = 82\,\text{mph}$; ⓑ $r = \frac{D}{t}$

Exercise:

Use the formula $A = \frac{1}{2}bh$ to solve for b
ⓐ when $A = 390$ and $h = 26$
Problem: ⓑ in general

Exercise:

Use the formula $A = \frac{1}{2}bh$ to solve for h
ⓐ when $A = 153$ and $b = 18$
Problem: ⓑ in general

Solution:

ⓐ $h = 17$ ⓑ $h = \frac{2A}{b}$

Exercise:

Use the formula $I = Prt$ to solve for the principal, P for
ⓐ $I = \$2,501, r = 4.1\%$,
$t = 5$ years
Problem: ⓑ in general

Exercise:

Problem: Solve the formula $4x + 3y = 6$ for y
ⓐ when $x = -2$
ⓑ in general

Solution:

ⓐ $y = \frac{14}{3}$ ⓑ $y = \frac{6-4x}{3}$

Exercise:

Problem: Solve $180 = a + b + c$ for c.

Exercise:

Problem: Solve the formula $V = LWH$ for H.

Solution:

$H = \frac{V}{LW}$

Solve Linear Inequalities

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line.

Exercise:

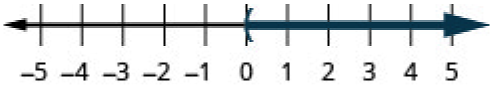
Problem:
ⓐ $x \leq 4$
ⓑ $x > -2$
ⓒ $x < 1$

Exercise:

ⓐ $x > 0$
ⓑ $x < -3$
Problem: ⓒ $x \geq -1$

Solution:

ⓐ



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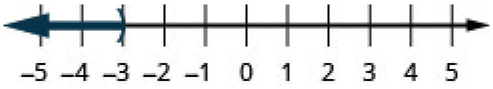

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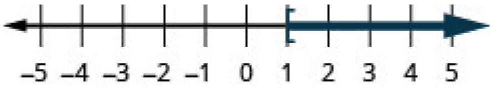


In the following exercises, graph each inequality on the number line and write in interval notation.

Exercise:

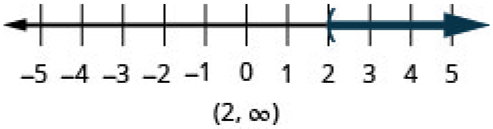
ⓐ $x < -1$
ⓑ $x \geq -2.5$
Problem: ⓒ $x \leq \frac{5}{4}$

Exercise:

Problem:
ⓐ $x > 2$
ⓑ $x \leq -1.5$
ⓒ $x \geq \frac{5}{3}$

Solution:

ⓐ



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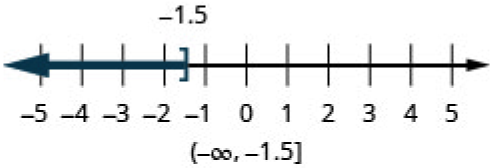

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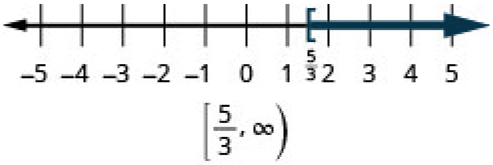


Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

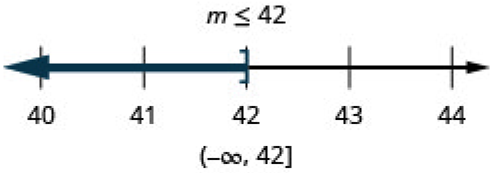
Exercise:

Problem: $n - 12 \leq 23$

Exercise:

Problem: $m + 14 \leq 56$

Solution:



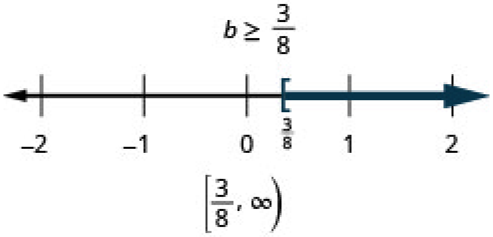
Exercise:

Problem: $a + \frac{2}{3} \geq \frac{7}{12}$

Exercise:

Problem: $b - \frac{7}{8} \geq -\frac{1}{2}$

Solution:



Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

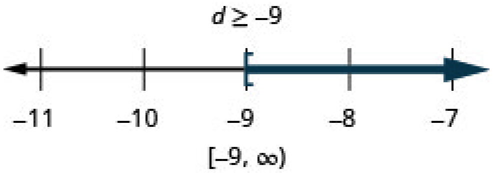
Exercise:

Problem: $9x > 54$

Exercise:

Problem: $-12d \leq 108$

Solution:



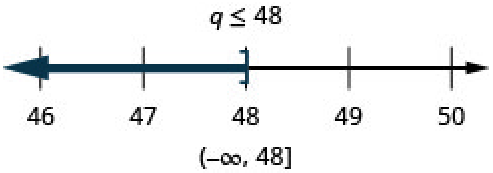
Exercise:

Problem: $\frac{5}{2}j < -60$

Exercise:

Problem: $\frac{q}{-2} \geq -24$

Solution:



Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

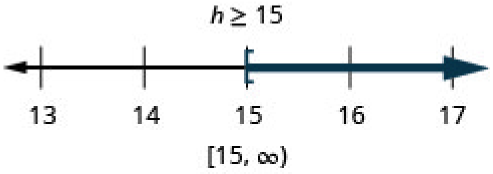
Exercise:

Problem: $6p > 15p - 30$

Exercise:

Problem: $9h-7(h-1)\le 4h-23$

Solution:



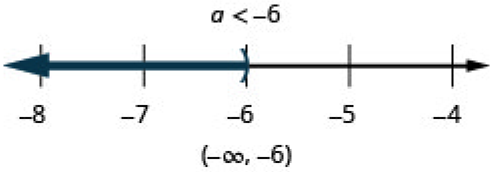
Exercise:

Problem: $5n-15(4-n)<10(n-6)+10n$

Exercise:

Problem: $\frac{3}{8}a-\frac{1}{12}a>\frac{5}{12}a+\frac{3}{4}$

Solution:



Translate to an Inequality and Solve

In the following exercises, translate and solve. Then write the solution in interval notation and graph on the number line.

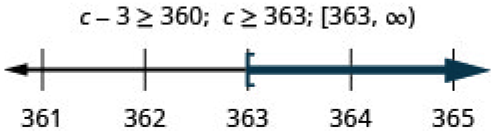
Exercise:

Problem: Five more than *z* is at most 19.

Exercise:

Problem: Three less than *c* is at least 360.

Solution:



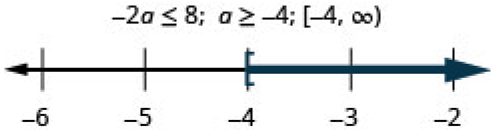
Exercise:

Problem: Nine times *n* exceeds 42.

Exercise:

Problem: Negative two times *a* is no more than 8.

Solution:



Everyday Math

Exercise:

Problem:

Describe how you have used two topics from this chapter in your life outside of your math class during the past month.

Chapter 2 Practice Test

Exercise:

Problem:

Determine whether each number is a solution to the equation $6x - 3 = x + 20$.

ⓐ 5
ⓑ $\frac{23}{5}$

Solution:

ⓐ no ⓑ yes

In the following exercises, solve each equation.

Exercise:

Problem: $n - \frac{2}{3} = \frac{1}{4}$

Exercise:

Problem: $\frac{9}{2}c = 144$

Solution:

$c = 32$

Exercise:

Problem: $4y - 8 = 16$

Exercise:

Problem: $-8x - 15 + 9x - 1 = -21$

Solution:

$x = -5$

Exercise:

Problem: $-15a = 120$

Exercise:

Problem: $\frac{2}{3}x = 6$

Solution:

$x = 9$

Exercise:

Problem: $x - 3.8 = 8.2$

Exercise:

Problem: $10y = -5y - 60$

Solution:

$y = -4$

Exercise:

Problem: $8n - 2 = 6n - 12$

Exercise:

Problem: $9m - 2 - 4m - m = 42 - 8$

Solution:

$m = 9$

Exercise:

Problem: $-5(2x-1)=45$

Exercise:

Problem: $-(d-9)=23$

Solution:

$d=-14$

Exercise:

Problem: $\frac{1}{4}(12m-28)=6-2(3m-1)$

Exercise:

Problem: $2(6x-5)-8=-22$

Solution:

$x=-\frac{1}{3}$

Exercise:

Problem: $8(3a-5)-7(4a-3)=20-3a$

Exercise:

Problem: $\frac{1}{4}p-\frac{1}{3}=\frac{1}{2}$

Solution:

$p=\frac{10}{3}$

Exercise:

Problem: $0.1d+0.25(d+8)=4.1$

Exercise:

Problem: $14n - 3(4n + 5) = -9 + 2(n - 8)$

Solution:

contradiction; no solution

Exercise:

Problem: $9(3u - 2) - 4[6 - 8(u - 1)] = 3(u - 2)$

Exercise:

Problem: Solve the formula $x - 2y = 5$ for y
ⓐ when $x = -3$
ⓑ in general

Solution:

ⓐ $y = 4$ ⓑ $y = \frac{5-x}{2}$

In the following exercises, graph on the number line and write in interval notation.

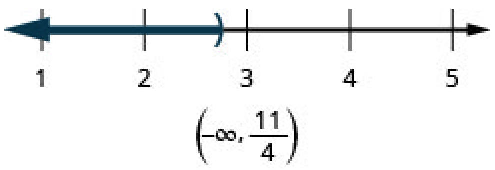
Exercise:

Problem: $x \geq -3.5$

Exercise:

Problem: $x < \frac{11}{4}$

Solution:



In the following exercises,, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

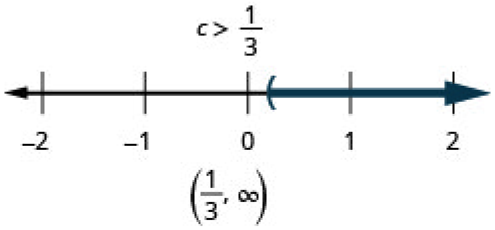
Exercise:

Problem: $8k \geq 5k - 120$

Exercise:

Problem: $3c - 10(c - 2) < 5c + 16$

Solution:



In the following exercises, translate to an equation or inequality and solve.

Exercise:

Problem: 4 less than twice *x* is 16.

Exercise:

Problem: Fifteen more than *n* is at least 48.

Solution:

$n + 15 \geq 48; n \geq 33$

Exercise:

Problem:

Samuel paid \$25.82 for gas this week, which was \$3.47 less than he paid last week. How much had he paid last week?

Exercise:

Problem:

Jenna bought a coat on sale for \$120, which was $\frac{2}{3}$ of the original price. What was the original price of the coat?

Solution:

$120=\frac{2}{3}p$; The original price was \$180.

Exercise:

Problem:

Sean took the bus from Seattle to Boise, a distance of 506 miles. If the trip took $7\frac{2}{3}$ hours, what was the speed of the bus?

Introduction class="introduction"

Sophisticated mathematical models are used to predict traffic patterns on our nation’s highways.

Mathematical formulas model phenomena in every facet of our lives. They are used to explain events and predict outcomes in fields such as transportation, business, economics, medicine, chemistry, engineering, and many more. In this chapter, we will apply our skills in solving equations to solve problems in a variety of situations.

Use a Problem-Solving Strategy
By the end of this section, you will be able to:

- Approach word problems with a positive attitude
- Use a problem-solving strategy for word problems
- Solve number problems

Note:
Before you get started, take this readiness quiz.

1. Translate “6 less than twice *x*” into an algebraic expression.
   If you missed this problem, review [link].
2. Solve: $\frac{2}{3}x = 24$.
   If you missed this problem, review [link].
3. Solve: $3x + 8 = 14$.
   If you missed this problem, review [link].

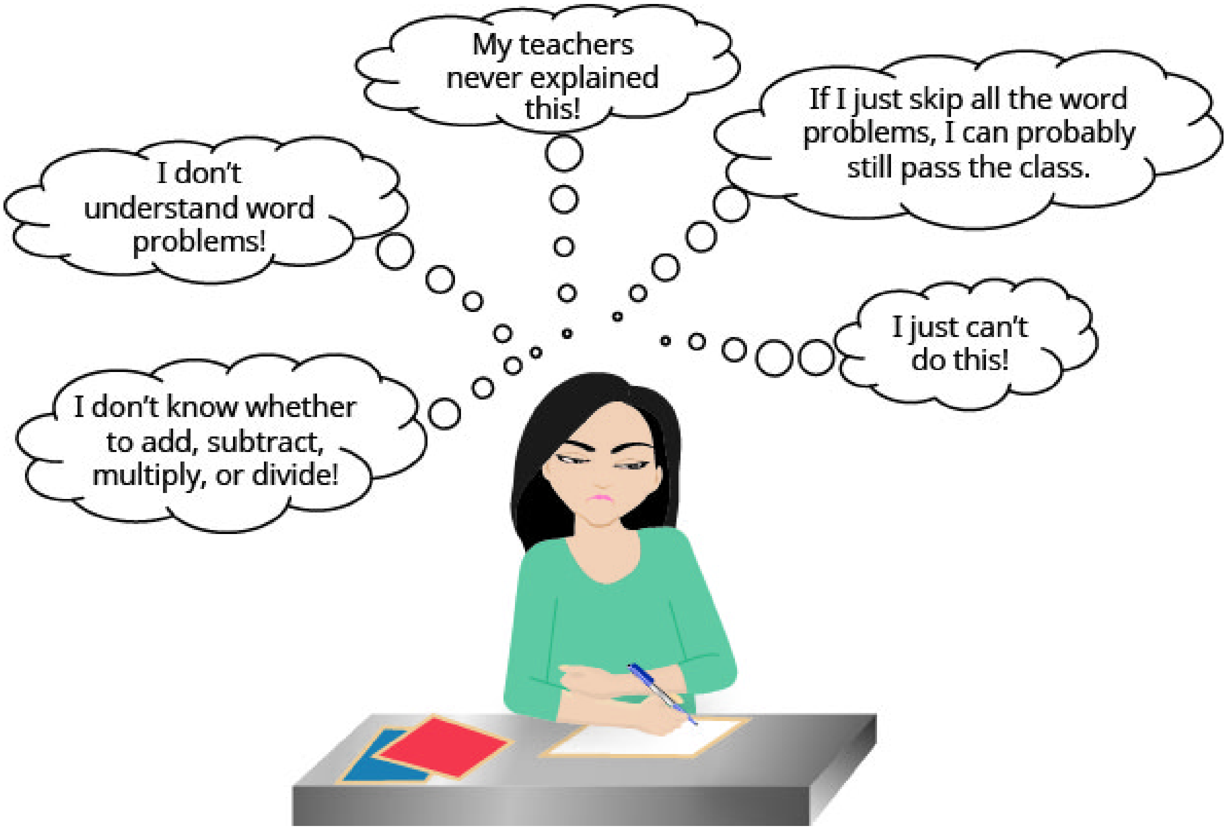
Approach Word Problems with a Positive Attitude

“If you think you can… or think you can’t… you’re right.”—Henry Ford

The world is full of word problems! Will my income qualify me to rent that apartment? How much punch do I need to make for the party? What size diamond can I afford to buy my girlfriend? Should I fly or drive to my family reunion?

How much money do I need to fill the car with gas? How much tip should I leave at a restaurant? How many socks should I pack for vacation? What size turkey do I need to buy for Thanksgiving dinner, and then what time do I need to put it in the oven? If my sister and I buy our mother a present, how much does each of us pay?

Now that we can solve equations, we are ready to apply our new skills to word problems. Do you know anyone who has had negative experiences in the past with word problems? Have you ever had thoughts like the student below?


My teachers never explained this!
If I just skip all the word problems, I can probably still pass the class.
I don't understand word problems!
I just can't do this!
I don't know whether to add, subtract, multiply, or divide!

Negative thoughts can be barriers to success.

When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. We need to calm our fears and change our negative feelings.

Start with a fresh slate and begin to think positive thoughts. If we take control and believe we can be successful, we will be able to master word problems! Read the positive thoughts in [link] and say them out loud.

I am better prepared now. I think I will begin to understand word problems.
I think I can! I think I can!
While word problems were hard in the past, I think I can try them now.
It may take time, but I can begin to solve word problems.

Thinking positive thoughts is a first step towards success.

Think of something, outside of school, that you can do now but couldn't do 3 years ago. Is it driving a car? Snowboarding? Cooking a gourmet meal? Speaking a new language? Your past experiences with word problems happened when you were younger—now you're older and ready to succeed!

Use a Problem-Solving Strategy for Word Problems

We have reviewed translating English phrases into algebraic expressions, using some basic mathematical vocabulary and symbols. We have also translated English sentences into algebraic equations and solved some word problems. The word problems applied math to everyday situations. We restated the situation in one sentence, assigned a variable, and then wrote an equation to solve the problem. This method works as long as the situation is familiar and the math is not too complicated.

Now, we’ll expand our strategy so we can use it to successfully solve any word problem. We’ll list the strategy here, and then we’ll use it to solve some problems. We summarize below an effective strategy for problem solving.

Note:
Use a Problem-Solving Strategy to Solve Word Problems.

Readthe problem. Make sure all the words and ideas are understood.
Identifywhat we are looking for.
Namewhat we are looking for. Choose a variable to represent that quantity.
Translateinto an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
Solvethe equation using good algebra techniques.
Checkthe answer in the problem and make sure it makes sense.
Answerthe question with a complete sentence.

Example:
Exercise:

Problem:

**Pilar bought a purse on sale for $18, which is one-half of the original price. What was the original price of the purse?

Solution:
Solution

Step 1. Read the problem. Read the problem two or more times if necessary. Look up any unfamiliar words in a dictionary or on the internet.

- *In this problem, is it clear what is being discussed? Is every word familiar?*

Step 2. Identify what you are looking for. Did you ever go into your bedroom to get something and then forget what you were looking for? It’s hard to find something if you are not sure what it is! Read the problem again and look for words that tell you what you are looking for!

- *In this problem, the words “what was the original price of the purse” tell us what we need to find.*

Step 3. Name what we are looking for. Choose a variable to represent that quantity. We can use any letter for the variable, but choose one that makes it easy to remember what it represents.

- Let $p =$ the original price of the purse.

Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Translate the English sentence into an algebraic equation.

Reread the problem carefully to see how the given information is related. Often, there is one sentence that gives this information, or it may help to write one sentence with all the important information. Look for clue words to help translate the sentence into algebra. Translate the sentence into an equation.

Restate the problem in one sentence with all the important information.	$\underbrace{18}\quad\underbrace{\text{is}}\quad\underbrace{\text{one-half the original price.}}$

Translate into an equation.	$18 \quad = \quad \frac{1}{2} \cdot p$

Step 5. Solve the equation using good algebraic techniques. Even if you know the solution right away, using good algebraic techniques here will better prepare you to solve problems that do not have obvious answers.

Solve the equation.	$18 = \frac{1}{2}p$
Multiply both sides by 2.	$2 \cdot 18 = 2 \cdot \frac{1}{2}p$
Simplify.	$36 = p$

Step 6. Check the answer in the problem to make sure it makes sense. We solved the equation and found that $p = 36$, which means "the original price" was $36.

- *Does $36 make sense in the problem? Yes, because 18 is one-half of 36, and the purse was on sale at half the original price.*

Step 7. Answer the question with a complete sentence. The problem asked "What was the original price of the purse?"

- *The answer to the question is: "The original price of the purse was $36."*

If this were a homework exercise, our work might look like this:

Pilar bought a purse on sale for $18, which is one-half the original price. What was the original price of the purse?

	Let $p =$ the original price.
	18 is one-half the original price.
	$18 = \frac{1}{2}p$
Multiply both sides by 2.	$2 \cdot 18 = 2 \cdot \frac{1}{2}p$
Simplify.	$36 = p$
Check. Is $36 a reasonable price for a purse?	
Yes.	
Is 18 one half of 36?	
$18 \overset{?}{=} \frac{1}{2} \cdot 36$	
$18 = 18\checkmark$	

	The original price of the purse was $36.

Note:
Exercise:

Problem:

TRY IT :: 3.1
Joaquin bought a bookcase on sale for $120, which was two-thirds of the original price. What was the original price of the bookcase?

Solution:

$180

Note:
Exercise:

Problem:

TRY IT :: 3.2
Two-fifths of the songs in Mariel’s playlist are country. If there are 16 country songs, what is the total number of songs in the playlist?

Solution:

40

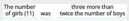
Let’s try this approach with another example.

Example:
Exercise:

Problem:

**Ginny and her classmates formed a study group. The number of girls in the study group was three more than twice the number of boys. There were 11 girls in the study group. How many boys were in the study group?

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	How many boys were in the study group?
Step 3. Name. Choose a variable to represent the number of boys.	Let $n =$ the number of boys.
Step 4. Translate. Restate the problem in one sentence with all the important information.	 The number of girls (11) was three more than twice the number of boys

Translate into an equation.	$11 = 2b + 3$
Step 5. Solve the equation.	$11 = 2b + 3$
Subtract 3 from each side.	$11 - 3 = 2b + 3 - 3$
Simplify.	$8 = 2b$
Divide each side by 2.	$\frac{8}{2} = \frac{2b}{2}$
Simplify.	$4 = b$
Step 6. Check. First, is our answer reasonable? Yes, having 4 boys in a study group seems OK. The problem says the number of girls was 3 more than twice the number of boys. If there are four boys, does that make eleven girls? Twice 4 boys is 8. Three more than 8 is 11.	
Step 7. Answer the question.	There were 4 boys in the study group.

Note:
Exercise:

Problem:

TRY IT :: 3.3
Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was 3 more than twice the number of notebooks. He bought 7 textbooks. How many notebooks did he buy?

Solution:

2

Note:
Exercise:

Problem:

TRY IT :: 3.4
Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. How many crossword puzzles did he do?

Solution:

7

Solve Number Problems

Now that we have a problem solving strategy, we will use it on several different types of word problems. The first type we will work on is “number problems.” Number problems give some clues about one or more numbers. We use these clues to write an equation. Number problems don’t usually arise on an everyday basis, but they provide a good introduction to practicing the problem solving strategy outlined above.

Example:
Exercise:

Problem:

**The difference of a number and six is 13. Find the number.

Solution:
Solution

Step 1. Read the problem. Are all the words familiar?	
Step 2. Identify what we are looking for.	the number
Step 3. Name. Choose a variable to represent the number.	Let $n =$ the number.
Step 4. Translate. Remember to look for clue words like "difference... of... and..."	
Restate the problem as one sentence.	$\underbrace{\text{The difference of the number and 6}}\ \underbrace{\text{is}}\ \underbrace{13}$
Translate into an equation.	$n - 6 \quad = \quad 13$

Step 5. Solve the equation.	$n - 6 = 13$
Simplify.	$n = 19$
Step 6. Check.	
The difference of 19 and 6 is 13. It checks!	
Step 7. Answer the question.	The number is 19.

Note:
Exercise:

Problem:

TRY IT :: 3.5
The difference of a number and eight is 17. Find the number.

Solution:

25

Note:
Exercise:

Problem:

TRY IT :: 3.6
The difference of a number and eleven is -7. Find the number.

Solution:

4

Example:
Exercise:

Problem:

**The sum of twice a number and seven is 15. Find the number.

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number
Step 3. Name. Choose a variable to represent the number.	Let $n =$ the number.
Step 4. Translate.	
Restate the problem as one sentence.	$\underbrace{\text{The sum of twice a number and 7}}\ \underbrace{\text{is}}\ \underbrace{15}$
Translate into an equation.	$2n + 7 \quad = \quad 15$

Step 5. Solve the equation.	$2n + 7 = 15$
Subtract 7 from each side and simplify.	$2n = 8$
Divide each side by 2 and simplify.	$n = 4$
Step 6. Check.	
Is the sum of twice 4 and 7 equal to 15?	
$2 \cdot 4 + 7 \overset{?}{=} 15$ $15 = 15\checkmark$	
Step 7. Answer the question.	The number is 4.

Did you notice that we left out some of the steps as we solved this equation? If you’re not yet ready to leave out these steps, write down as many as you need.

Note:
Exercise:

Problem:

TRY IT :: 3.7
The sum of four times a number and two is 14. Find the number.

Solution:

3

Note:
Exercise:

Problem:

TRY IT :: 3.8
The sum of three times a number and seven is 25. Find the number.

Solution:

6

Some number word problems ask us to find two or more numbers. It may be tempting to name them all with different variables, but so far we have only solved equations with one variable. In order to avoid using more than one variable, we will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

Example:
Exercise:

Problem:

**One number is five more than another. The sum of the numbers is 21. Find the numbers.

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		We are looking for two numbers.
Step 3. Name. We have two numbers to name and need a name for each.		
Choose a variable to represent the first number.		Let $n = 1^{\text{st}}$ number.
What do we know about the second number?		One number is five more than another.
		$n + 5 = 2^{\text{nd}}$ number
Step 4. Translate. Restate the problem as one sentence with all the important information.		The sum of the 1^{st} number and the 2^{nd} number is 21.
Translate into an equation.		$\underbrace{1^{\text{st}}\text{ number}} + \underbrace{2^{\text{nd}}\text{ number}} \; \underbrace{=} \; \underbrace{21}$
Substitute the variable expressions.		$n + n + 5 = 21$

Step 5. Solve the equation.		$n + n + 5 = 21$
Combine like terms.		$2n + 5 = 21$
Subtract 5 from both sides and simplify.		$2n = 16$
Divide by 2 and simplify.		$n = 8$ 1st number
Find the second number, too.		$n + 5$ 2nd number
		$8 + 5$
		13
Step 6. Check.		
Do these numbers check in the problem?		
Is one number 5 more than the other?	$13 \overset{?}{=} 8 + 5$	
Is thirteen 5 more than 8? Yes.	$13 = 13\checkmark$	
Is the sum of the two numbers 21?	$8 + 13 \overset{?}{=} 21$	
	$21 = 21\checkmark$	

Step 7. Answer the question.		The numbers are 8 and 13.

Note:
Exercise:

Problem:

TRY IT :: 3.9
One number is six more than another. The sum of the numbers is twenty-four. Find the numbers.

Solution:

9, 15

Note:
Exercise:

Problem:

TRY IT :: 3.10
The sum of two numbers is fifty-eight. One number is four more than the other. Find the numbers.

Solution:

27, 31

Example:
Exercise:

Problem:

**The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		We are looking for two numbers.
Step 3. Name.		
Choose a variable.		Let $n = 1^{\text{st}}$ number.
One number is 4 less than the other.		$n - 4 = 2^{\text{nd}}$ number
Step 4. Translate.		
Write as one sentence.		The sum of the 2 numbers is negative 14.

Translate into an equation.		$\underbrace{1^{\text{st}}\text{ number}} + \underbrace{2^{\text{nd}}\text{ number}}\quad\underbrace{\text{is}}\quad\underbrace{\text{negative fourteen}}$
Step 5. Solve the equation.		$n \quad + \quad n-4 \quad = \quad -14$
Combine like terms.		$n+n-4=-14$
Add 4 to each side and simplify.		$2n-4=-14$
Simplify.		$2n=-10$
		$n=-5 \quad 1^{\text{st}}$ number
		$n-4 \quad 2^{\text{nd}}$ number
		$-5-4$
		-9
Step 6. Check.		
Is −9 four less than −5?	$-5-4 \overset{?}{=} -9$	
	$-9=-9\checkmark$	
Is their sum −14?	$-5+(-9) \overset{?}{=} -14$	
	$-14=-14\checkmark$	

Step 7. Answer the question.		The numbers are −5 and −9.

Note:
Exercise:

Problem:

TRY IT :: 3.11
The sum of two numbers is negative twenty-three. One number is seven less than the other. Find the numbers.

Solution:

$-15, -8$

Note:
Exercise:

Problem:

TRY IT :: 3.12
The sum of two numbers is -18. One number is 40 more than the other. Find the numbers.

Solution:

$-29, 11$

Example:
Exercise:

Problem:

**One number is ten more than twice another. Their sum is one. Find the numbers.

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what you are looking for.		We are looking for two numbers.
Step 3. Name.		
Choose a variable.		Let $x = 1^{\text{st}}$ number.
One number is 10 more than twice another.		$2x + 10 = 2^{\text{nd}}$ number
Step 4. Translate.		
Restate as one sentence.		Their sum is one.
		The sum of the

		two numbers is 1.
Translate into an equation.		$x + 2x + 10 = 1$
Step 5. Solve the equation.		
Combine like terms.		$x + 2x + 10 = 1$
Subtract 10 from each side.		$3x + 10 = 1$
Divide each side by 3.		$3x = -9$
		$x = -3$ 1st number
		$2x + 10$ 2nd number
		$2(-3) + 10$
		4
Step 6. Check.		
Is ten more than twice −3 equal to 4?	$2(-3) + 10 \stackrel{?}{=} 4$	
	$-6 + 10 \stackrel{?}{=} 4$	

	$4 = 4\checkmark$	
Is their sum 1?	$-3 + 4 \overset{?}{=} 1$	
	$1 = 1\checkmark$	
Step 7. Answer the question.		The numbers are −3 and −4.

Note:
Exercise:

Problem:

TRY IT :: 3.13
One number is eight more than twice another. Their sum is negative four. Find the numbers.

Solution:

$-4, 0$

Note:
Exercise:

Problem:

TRY IT :: 3.14
One number is three more than three times another. Their sum is -5. Find the numbers.

Solution:

$-3, -2$

Some number problems involve consecutive integers. *Consecutive integers* are integers that immediately follow each other. Examples of consecutive integers are:

Equation:

$$\begin{array}{l} 1, 2, 3, 4 \\ -10, -9, -8, -7 \\ 150, 151, 152, 153 \end{array}$$

Notice that each number is one more than the number preceding it. So if we define the first integer as n, the next consecutive integer is $n+1$. The one after that is one more than $n+1$, so it is $n+1+1$, which is $n+2$.

Equation:

$$\begin{array}{cl} n & 1^{\text{st}} \text{ integer} \\ n+1 & 2^{\text{nd}} \text{ consecutive integer} \\ n+2 & 3^{\text{rd}} \text{ consecutive integer} \ldots \text{etc.} \end{array}$$

Example:

Exercise:

Problem:

**The sum of two consecutive integers is 47. Find the numbers.

Solution:

Solution

Step 1. Read the problem.		
Step 2. Identify what you are looking for.		two consecutive integers
Step 3. Name each number.		Let $n = 1^{\text{st}}$ integer.
		$n + 1 =$ next consecutive integer
Step 4. Translate.		
Restate as one sentence.		The sum of the integers is 47.
Translate into an equation.		$n + n + 1 = 47$
Step 5. Solve the equation.		$n + n + 1 = 47$
Combine like terms.		$2n + 1 = 47$
Subtract 1 from each side.		$2n = 46$
Divide each side by 2.		$n = 23$ 1^{st} integer
		$n + 1$ next consecutive integer
		$23 + 1$

		24
Step 6. Check.		
$23 + 24 \overset{?}{=} 47$ $47 = 47\checkmark$		
Step 7. Answer the question.		The two consecutive integers are 23 and 24.

Note:
Exercise:

Problem:

TRY IT :: 3.15
The sum of two consecutive integers is 95. Find the numbers.

Solution:

47, 48

Note:
Exercise:

Problem:

TRY IT :: 3.16
The sum of two consecutive integers is -31. Find the numbers.

Solution:

$-16, -15$

Example:
Exercise:

Problem: **Find three consecutive integers whose sum is -42.

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		three consecutive integers
Step 3. Name each of the three numbers.		Let $n = 1^{\text{st}}$ integer.
		$n + 1 = 2^{\text{nd}}$ consecutive integer
		$n + 2 = 3^{\text{rd}}$ consecutive integer

Step 4. Translate.		
Restate as one sentence.		The sum of the three integers is −42.
Translate into an equation.		$n + n + 1 + n + 2 = -42$
Step 5. Solve the equation.		$n + n + 1 + n + 2 = -42$
Combine like terms.		$3n + 3 = -42$
Subtract 3 from each side.		$3n = -45$
Divide each side by 3.		$n = -15$ 1st integer
		$n + 1$ 2nd integer
		$-15 + 1$
		-14
		$n + 2$ 3rd integer
		$-15 + 2$

		−13
Step 6. Check.		
$-13+(-14)+(-15) \stackrel{?}{=} -42$ $-42 = -42\checkmark$		
Step 7. Answer the question.		The three consecutive integers are −15, −14, and −13.

Note:
Exercise:

TRY IT :: 3.17

Problem: Find three consecutive integers whose sum is -96.

Solution:

$-33, -32, -31$

Note:
Exercise:

TRY IT :: 3.18

Problem: Find three consecutive integers whose sum is -36.

Solution:

$-13, -12, -11$

Now that we have worked with consecutive integers, we will expand our work to include consecutive even integers and consecutive odd integers. *Consecutive even integers* are even integers that immediately follow one another. Examples of consecutive even integers are:

Equation:

$$\begin{array}{l}18, 20, 22\\64, 66, 68\\-12, -10, -8\end{array}$$

Notice each integer is 2 more than the number preceding it. If we call the first one *n*, then the next one is $n + 2$. The next one would be $n + 2 + 2$ or $n + 4$.

Equation:

$$\begin{array}{cl}n & 1^{\text{st}}\text{ even integer}\\n+2 & 2^{\text{nd}}\text{ consecutive even integer}\\n+4 & 3^{\text{rd}}\text{ consecutive even integer . . . etc.}\end{array}$$

Consecutive odd integers are odd integers that immediately follow one another. Consider the consecutive odd integers 77, 79, and 81.

Equation:

$$\begin{array}{l}77, 79, 81\\n, n+2, n+4\end{array}$$

Equation:

n	1^{st} odd integer
$n+2$	2^{nd} consecutive odd integer
$n+4$	3^{rd} consecutive odd integer . . . etc.

Does it seem strange to add 2 (an even number) to get from one odd integer to the next? Do you get an odd number or an even number when we add 2 to 3? to 11? to 47?

Whether the problem asks for consecutive even numbers or odd numbers, you don’t have to do anything different. The pattern is still the same—to get from one odd or one even integer to the next, add 2.

Example:
Exercise:

Problem: **Find three consecutive even integers whose sum is 84.

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	three consecutive even integers
Step 3. Name the integers.	Let $n = 1^{\text{st}}$ even integer. $n+2 = 2^{\text{nd}}$ consecutive even integer

	$n+4=3^{\text{rd}}$ consecutive even integer
Step 4. Translate.	
Restate as one sentence.	The sume of the three even integers is 84.
Translate into an equation.	$n+n+2+n+4\ =\ 84$
Step 5. Solve the equation.	
Combine like terms.	$n+n+2+n+4\ =\ 84$
Subtract 6 from each side.	$3n+6\ =\ 84$
Divide each side by 3.	$3n\ =\ 78$
	$n=26\quad 1^{\text{st}}$ integer $n+2\quad 2^{\text{nd}}$ integer $26+2$ 28 $n+4\quad 3^{\text{rd}}$ integer $26+4$ 30
Step 6. Check.	
$26+28+30\overset{?}{=}84$ $84=84\checkmark$	

Step 7. Answer the question.	The three consecutive integers are 26, 28, and 30.

Note:
Exercise:

TRY IT :: 3.19

Problem: Find three consecutive even integers whose sum is 102.

Solution:

32, 34, 36

Note:
Exercise:

TRY IT :: 3.20

Problem: Find three consecutive even integers whose sum is -24.

Solution:

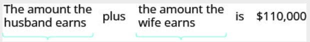
$-10, -8, -6$

Example:
Exercise:

Problem:

**A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		How much does the husband earn?
Step 3. Name.		
Choose a variable to represent the amount the husband earns.		Let $h =$ the amount the husband earns.
The wife earns $16,000 less than twice that.		$2h - 16{,}000$ the amount the wife earns.
Step 4. Translate.		Together the husband and wife earn $110,000.
Restate the problem in one sentence with all the important information.		 The amount the husband earns plus the amount the wife earns is $110,000

Translate into an equation.		$h \quad + \quad 2h - 16{,}000 \quad = \quad 110{,}000$
Step 5. Solve the equation.		h + 2h − 16,000 = 110,000
Combine like terms.		3h − 16,000 = 110,000
Add 16,000 to both sides and simplify.		3h = 126,000
Divide each side by 3.		h = 42,000
		$42,000 amount husband earns
		2h − 16,000 amount wife earns
		2(42,000) − 16,000
		84,000 − 16,000
		68,000
Step 6. Check.		
If the wife earns $68,000 and the husband earns $42,000 is the total $110,000? Yes!		

Step 7. Answer the question.		The husband earns $42,000 a year.

Note:
Exercise:

Problem:

TRY IT :: 3.21
According to the National Automobile Dealers Association, the average cost of a car in 2014 was $28,500. This was $1,500 less than 6 times the cost in 1975. What was the average cost of a car in 1975?

Solution:

$5,000

Note:
Exercise:

Problem:

TRY IT :: 3.22
U.S. Census data shows that the median price of new home in the United States in November 2014 was $280,900. This was $10,700 more than 14 times the price in November 1964. What was the median price of a new home in November 1964?

Solution:

$19,300

Key Concepts

- **Problem-Solving Strategy**

  Read the problem. Make sure all the words and ideas are understood.
  Identify what we are looking for.
  Name what we are looking for. Choose a variable to represent that quantity.
  Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  Solve the equation using good algebra techniques.
  Check the answer in the problem and make sure it makes sense.
  Answer the question with a complete sentence.

- **Consecutive Integers**
  Consecutive integers are integers that immediately follow each other.
  Equation:

n	1^{st} integer
$n+1$	2^{nd} integer consecutive integer
$n+2$	3^{rd} consecutive integer . . . etc.

  Consecutive even integers are even integers that immediately follow one another.
  Equation:

n	1^{st} integer
$n+2$	2^{nd} integer consecutive integer
$n+4$	3^{rd} consecutive integer . . . etc.

  Consecutive odd integers are odd integers that immediately follow one another.
  Equation:

n	1st integer
$n+2$	2nd integer consecutive integer
$n+4$	3rd consecutive integer . . . etc.

Practice Makes Perfect

Use the Approach Word Problems with a Positive Attitude

In the following exercises, prepare the lists described.

Exercise:

Problem:

List five positive thoughts you can say to yourself that will help you approach word problems with a positive attitude. You may want to copy them on a sheet of paper and put it in the front of your notebook, where you can read them often.

Solution:

Answers will vary

Exercise:

Problem:

List five negative thoughts that you have said to yourself in the past that will hinder your progress on word problems. You may want to write each one on a small piece of paper and rip it up to symbolically destroy the negative thoughts.

Use a Problem-Solving Strategy for Word Problems

In the following exercises, solve using the problem solving strategy for word problems. Remember to write a complete sentence to answer each question.

Exercise:

Problem:

Two-thirds of the children in the fourth-grade class are girls. If there are 20 girls, what is the total number of children in the class?

Solution:

30

Exercise:

Problem:

Three-fifths of the members of the school choir are women. If there are 24 women, what is the total number of choir members?

Exercise:

Problem:

Zachary has 25 country music CDs, which is one-fifth of his CD collection. How many CDs does Zachary have?

Solution:

125

Exercise:

Problem:

One-fourth of the candies in a bag of M&M’s are red. If there are 23 red candies, how many candies are in the bag?

Exercise:

Problem:

There are 16 girls in a school club. The number of girls is four more than twice the number of boys. Find the number of boys.

Solution:

6

Exercise:

Problem:

There are 18 Cub Scouts in Pack 645. The number of scouts is three more than five times the number of adult leaders. Find the number of adult leaders.

Exercise:

Problem:

Huong is organizing paperback and hardback books for her club’s used book sale. The number of paperbacks is 12 less than three times the number of hardbacks. Huong had 162 paperbacks. How many hardback books were there?

Solution:

58

Exercise:

Problem:

Jeff is lining up children’s and adult bicycles at the bike shop where he works. The number of children’s bicycles is nine less than three times the number of adult bicycles. There are 42 adult bicycles. How many children’s bicycles are there?

Exercise:

Problem:

Philip pays $1,620 in rent every month. This amount is $120 more than twice what his brother Paul pays for rent. How much does Paul pay for rent?

Solution:

$750

Exercise:

Problem:

Marc just bought an SUV for $54,000. This is $7,400 less than twice what his wife paid for her car last year. How much did his wife pay for her car?

Exercise:

Problem:

Laurie has $46,000 invested in stocks and bonds. The amount invested in stocks is $8,000 less than three times the amount invested in bonds. How much does Laurie have invested in bonds?

Solution:

$13,500

Exercise:

Problem:

Erica earned a total of $50,450 last year from her two jobs. The amount she earned from her job at the store was $1,250 more than three times the amount she earned from her job at the college. How much did she earn from her job at the college?

Solve Number Problems

In the following exercises, solve each number word problem.

Exercise:

Problem: The sum of a number and eight is 12. Find the number.

Solution:

4

Exercise:

Problem: The sum of a number and nine is 17. Find the number.

Exercise:

Problem:

The difference of a number and 12 is three. Find the number.

Solution:

15

Exercise:

Problem:

The difference of a number and eight is four. Find the number.

Exercise:

Problem:

The sum of three times a number and eight is 23. Find the number.

Solution:

5

Exercise:

Problem: The sum of twice a number and six is 14. Find the number.

Exercise:

Problem:

The difference of twice a number and seven is 17. Find the number.

Solution:

12

Exercise:

Problem:

The difference of four times a number and seven is 21. Find the number.

Exercise:

Problem:

Three times the sum of a number and nine is 12. Find the number.

Solution:

-5

Exercise:

Problem:

Six times the sum of a number and eight is 30. Find the number.

Exercise:

Problem:

One number is six more than the other. Their sum is 42. Find the numbers.

Solution:

18, 24

Exercise:

Problem:

One number is five more than the other. Their sum is 33. Find the numbers.

Exercise:

Problem:

The sum of two numbers is 20. One number is four less than the other. Find the numbers.

Solution:

8, 12

Exercise:

Problem:

The sum of two numbers is 27. One number is seven less than the other. Find the numbers.

Exercise:

Problem:

The sum of two numbers is -45. One number is nine more than the other. Find the numbers.

Solution:

-18, -27

Exercise:

Problem:

The sum of two numbers is -61. One number is 35 more than the other. Find the numbers.

Exercise:

Problem:

The sum of two numbers is $-316.$ One number is 94 less than the other. Find the numbers.

Solution:

$-111, -205$

Exercise:

Problem:

The sum of two numbers is $-284.$ One number is 62 less than the other. Find the numbers.

Exercise:

Problem:

One number is 14 less than another. If their sum is increased by seven, the result is 85. Find the numbers.

Solution:

32, 46

Exercise:

Problem:

One number is 11 less than another. If their sum is increased by eight, the result is 71. Find the numbers.

Exercise:

Problem:

One number is five more than another. If their sum is increased by nine, the result is 60. Find the numbers.

Solution:

23, 28

Exercise:

Problem:

One number is eight more than another. If their sum is increased by 17, the result is 95. Find the numbers.

Exercise:

Problem:

One number is one more than twice another. Their sum is -5. Find the numbers.

Solution:

$-2, -3$

Exercise:

Problem:

One number is six more than five times another. Their sum is six. Find the numbers.

Exercise:

Problem:

The sum of two numbers is 14. One number is two less than three times the other. Find the numbers.

Solution:

4, 10

Exercise:

Problem:

The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.

Exercise:

Problem:

The sum of two consecutive integers is 77. Find the integers.

Solution:

38, 39

Exercise:

Problem:

The sum of two consecutive integers is 89. Find the integers.

Exercise:

Problem:

The sum of two consecutive integers is -23. Find the integers.

Solution:

$-11, -12$

Exercise:

Problem:

The sum of two consecutive integers is -37. Find the integers.

Exercise:

Problem:

The sum of three consecutive integers is 78. Find the integers.

Solution:

25, 26, 27

Exercise:

Problem:

The sum of three consecutive integers is 60. Find the integers.

Exercise:

Problem: Find three consecutive integers whose sum is -36.

Solution:

$-11, -12, -13$

Exercise:

Problem: Find three consecutive integers whose sum is -3.

Exercise:

Problem: Find three consecutive even integers whose sum is 258.

Solution:

84, 86, 88

Exercise:

Problem: Find three consecutive even integers whose sum is 222.

Exercise:

Problem: Find three consecutive odd integers whose sum is 171.

Solution:

55, 57, 59

Exercise:

Problem: Find three consecutive odd integers whose sum is 291.

Exercise:

Problem: Find three consecutive even integers whose sum is -36.

Solution:

-10, -12, -14

Exercise:

Problem: Find three consecutive even integers whose sum is -84.

Exercise:

Problem: Find three consecutive odd integers whose sum is -213.

Solution:

-69, -71, -73

Exercise:

Problem: Find three consecutive odd integers whose sum is -267.

Everyday Math

Exercise:

Problem:

Sale Price Patty paid $35 for a purse on sale for $10 off the original price. What was the original price of the purse?

Solution:

$45

Exercise:

Problem:

Sale Price Travis bought a pair of boots on sale for $25 off the original price. He paid $60 for the boots. What was the original price of the boots?

Exercise:

Problem:

Buying in Bulk Minh spent $6.25 on five sticker books to give his nephews. Find the cost of each sticker book.

Solution:

$1.25

Exercise:

Problem:

Buying in Bulk Alicia bought a package of eight peaches for $3.20. Find the cost of each peach.

Exercise:

Problem:

Price before Sales Tax Tom paid $1,166.40 for a new refrigerator, including $86.40 tax. What was the price of the refrigerator?

Solution:

$1080

Exercise:

Problem:

Price before Sales Tax Kenji paid $2,279 for a new living room set, including $129 tax. What was the price of the living room set?

Writing Exercises

Exercise:

Problem:

What has been your past experience solving word problems?

Solution:

answers will vary

Exercise:

Problem:

When you start to solve a word problem, how do you decide what to let the variable represent?

Exercise:

Problem:

What are consecutive odd integers? Name three consecutive odd integers between 50 and 60.

Solution:

Consecutive odd integers are odd numbers that immediately follow each other. An example of three consecutive odd integers between 50 and 60 would be 51, 53, and 55.

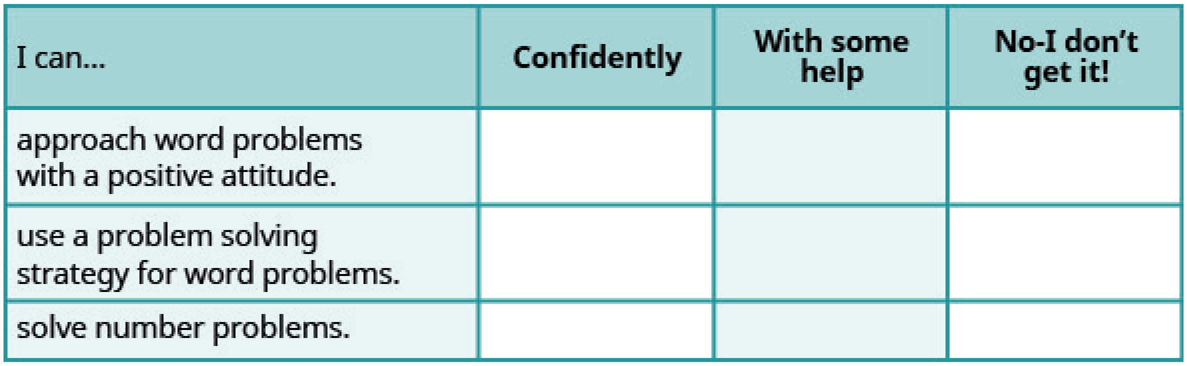
Exercise:

Problem:

What are consecutive even integers? Name three consecutive even integers between -50 and -40.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
approach word problems with a positive attitude.			
use a problem solving strategy for word problems.			
solve number problems.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved your goals in this section! Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific!

…with some help. This must be addressed quickly as topics you do not master become potholes in your road to success. Math is sequential—every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is critical and you must not ignore it. You need to get help immediately or you will quickly be overwhelmed. See your instructor as soon as possible to discuss your situation. Together you can come up with a plan to get you the help you need.

Solve Percent Applications
By the end of this section, you will be able to:

- Translate and solve basic percent equations
- Solve percent applications
- Find percent increase and percent decrease
- Solve simple interest applications
- Solve applications with discount or mark-up

Note:
Before you get started, take this readiness quiz.

1. Convert 4.5% to a decimal.
   If you missed this problem, review [link].
2. Convert 0.6 to a percent.
   If you missed this problem, review [link].
3. Round 0.875 to the nearest hundredth.
   If you missed this problem, review [link].
4. Multiply (4.5)(2.38).
   If you missed this problem, review [link].
5. Solve $3.5=0.7n$.
   If you missed this problem, review [link].
6. Subtract $50-37.45$.
   If you missed this problem, review [link].

Translate and Solve Basic Percent Equations

We will solve percent equations using the methods we used to solve equations with fractions or decimals. Without the tools of algebra, the best method available to solve percent problems was by setting them up as proportions. Now as an algebra student, you can just translate English sentences into algebraic equations and then solve the equations.

We can use any letter you like as a variable, but it is a good idea to choose a letter that will remind us of what you are looking for. We must be sure to change the given percent to a decimal when we put it in the equation.

Example:
Exercise:

Problem: **Translate and solve: What number is 35% of 90?

Solution:
Solution

	$\underbrace{\text{What number}}\ \underbrace{\text{is}}\ \underbrace{35\%}\ \underbrace{\text{of}}\ \underbrace{90?}$
Translate into algebra. Let n= the number.	$n = 0.35 \cdot 90$
Remember "of" means multiply, "is" means equals.	
Multiply.	$n = 31.5$
	31.5 is 35% of 90

Note:
Exercise:

TRY IT :: 3.23

Problem: Translate and solve:

What number is 45% of 80?

Solution:

36

Note:
Exercise:

TRY IT :: 3.24
Problem: Translate and solve:

What number is 55% of 60?

Solution:

33

We must be very careful when we translate the words in the next example. The unknown quantity will not be isolated at first, like it was in [link]. We will again use direct translation to write the equation.

Example:
Exercise:

Problem: **Translate and solve: 6.5% of what number is $1.17?

Solution:
Solution

	$\underbrace{6.5\%}\ \underbrace{\text{of}}\ \underbrace{\text{what number}}\ \underbrace{\text{is}}\ \underbrace{\$1.17?}$

Translate. Let $n =$ the number.	$0.065 \cdot n = 1.17$
Multiply.	$0.065n = 1.17$
Divide both sides by 0.065 and simplify.	$n = 18$
	6.5% of$18 is $1.17

Note:
Exercise:

TRY IT :: 3.25
Problem: Translate and solve:

7.5% of what number is $1.95?

Solution:

$26

Note:
Exercise:

TRY IT :: 3.26
Problem: Translate and solve:

8.5% of what number is $3.06?

Solution:

$36

In the next example, we are looking for the percent.

Example:
Exercise:

Problem: **Translate and solve: 144 is what percent of 96?

Solution:
Solution

	$\underbrace{144}\quad\underbrace{\text{is}}\quad\underbrace{\text{what percent}}\quad\underbrace{\text{of}}\quad\underbrace{96?}$
Translate into algebra. Let $p =$ the percent.	$144 \quad = \quad p \quad \cdot \quad 96$
Multiply.	$144 = 96p$
Divide by 96 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	144 is 150% of 96

Note that we are asked to find percent, so we must have our final result in percent form.

Note:
Exercise:

TRY IT :: 3.27

Problem: Translate and solve:

110 is what percent of 88?

Solution:

125%

Note:
Exercise:

TRY IT :: 3.28

Problem: Translate and solve:

126 is what percent of 72?

Solution:

175%

Solve Applications of Percent

Many applications of percent—such as tips, sales tax, discounts, and interest—occur in our daily lives. To solve these applications we'll translate to a basic percent equation, just like those we solved in previous examples. Once we translate the sentence into a percent equation, we know how to solve it.

We will restate the problem solving strategy we used earlier for easy reference.

Note:
Use a Problem-Solving Strategy to Solve an Application.

Read the problem. Make sure all the words and ideas are understood.
Identify what we are looking for.
Name what we are looking for. Choose a variable to represent that quantity.
Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
Solve the equation using good algebra techniques.
Check the answer in the problem and make sure it makes sense.
Answer the question with a complete sentence.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications will involve everyday situations, you can rely on your own experience.

Example:
Exercise:

Problem:

**Dezohn and his girlfriend enjoyed a nice dinner at a restaurant and his bill was $68.50. He wants to leave an 18% tip. If the tip will be 18% of the total bill, how much tip should he leave?

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the amount of tip should Dezohn leave

Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let t = amount of tip.
Step 4. Translate into an equation.		The tip is 18% of the total bill.
Write a sentence that gives the information to find it.		$\underbrace{\text{The tip}}\ \underbrace{\text{is}}\ \underbrace{18\%}\ \underbrace{\text{of}}\ \underbrace{\$68.50}$
Translate the sentence into an equation.		$t = 0.18 \cdot 68.50$
Step 5. Solve the equation. Multiply.		$t = 12.33$
Step 6. Check. Does this make sense?		
Yes, 20% of \$70 is \$14.		
Step 7. Answer the question with a complete sentence.		Dezohn should leave a tip of \$12.33.

Notice that we used t to represent the unknown tip.

Note:
Exercise:

Problem:

TRY IT :: 3.29
Cierra and her sister enjoyed a dinner in a restaurant and the bill was \$81.50. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Solution:

$14.67

Note:
Exercise:

Problem:

TRY IT :: 3.30
Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was $14.40, how much will she leave for the tip?

Solution:

$2.16

Example:
Exercise:

Problem:

**The label on Masao’s breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the total amount of potassium that is recommended

Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $a =$ total amount of potassium.
Step 4. Translate. Write a sentence that gives the information to find it.		$\underbrace{85\text{ mg}}\ \underbrace{\text{is}}\ \underbrace{2\%}\ \underbrace{\text{of the}}\ \underbrace{\text{total amount}}$
Translate into an equation.		$85 = 0.02 \cdot a$
Step 5. Solve the equation.		$4{,}250 = a$
Step 6. Check. Does this make sense?		
Yes, 2% is a small percent and 85 is a small part of 4,250.		
Step 7. Answer the question with a complete sentence.		The amount of potassium that is recommended is 4,250 mg.

Note:
Exercise:

Problem:

TRY IT :: 3.31
One serving of wheat square cereal has seven grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

Solution:

25 grams

Note:
Exercise:

Problem:

TRY IT :: 3.32
One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution:

2,375 mg

Example:
Exercise:

Problem:

**Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the percent of the total calories from fat

Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $p =$ percent of fat.
Step 4. Translate. Write a sentence that gives the information to find it.		$\underbrace{\text{What percent}}_{}\ \underbrace{\text{of}}_{}\ \underbrace{480}_{}\ \underbrace{\text{is}}_{}\ \underbrace{240}_{}?$
Translate into an equation.		$p \cdot 480 = 240$
Step 5. Solve the equation.		$480p = 240$
Divide by 480.		$p = 0.5$
Put in a percent form.		$p = 50\%$
Step 6. Check. Does this make sense?		
Yes, 240 is half of 480, so 50% makes sense.		
Step 7. Answer the question with a complete sentence.		Of the total calories in each brownie, 50% is fat.

Note:
Exercise:

TRY IT :: 3.33

Problem: Solve. Round to the nearest whole percent.

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat?

Solution:

26%

Note:
Exercise:

TRY IT :: 3.34
Problem: Solve. Round to the nearest whole percent.

The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat?

Solution:

40%

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the percent increase, first we find the amount of increase, the difference of the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

Note:
Find the Percent Increase.

Find the amount of increase. $\text{new amount} - \text{original amount} = \text{increase}$
Find the percent increase. The increase is what percent of the original amount?

Example:
Exercise:

Problem:

**In 2011, the California governor proposed raising community college fees from $26 a unit to $36 a unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the percent increase
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $p =$ the percent.
Step 4. Translate. Write a sentence that gives the information to find it.		
First find the amount of increase.		new amount − original amount = increase
		$36 - 26 = 10$
Find the percent.		Increase is what percent

		of the original amount?
		$\underbrace{10}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{26?}$
Translate into an equation.		$10 = p \cdot 26$
Step 5. Solve the equation.		$10 = 26p$
Divide by 26.		$0.385 = p$
Change to percent form; round to the nearest tenth.		$38.5\% = p$
Step 6. Check. Does this make sense?		
Yes, 38.4% is close to $\frac{1}{3}$, and 10 is close to $\frac{1}{3}$ of 26.		
Step 7. Answer the question with a complete sentence.		The new fees represent a 38.5% increase over the old fees.

Notice that we rounded the division to the nearest thousandth in order to round the percent to the nearest tenth.

Note:
Exercise:

TRY IT :: 3.35

Problem: Find the percent increase. (Round to the nearest tenth of a percent.)

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents.

Solution:

8.8%

Note:
Exercise:

TRY IT :: 3.36

Problem: Find the percent increase.

In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was $2.25.

Solution:

50%

Finding the percent decrease is very similar to finding the percent increase, but now the amount of decrease is the difference of the original amount and the new amount. Then we find what percent the amount of decrease is of the original amount.

Note:
Find the Percent Decrease.

Find the amount of decrease. $\text{original amount} - \text{new amount} = \text{decrease}$
Find the percent decrease. Decrease is what percent of the original amount?

Example:
Exercise:

Problem:

**The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solution:
Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		the percent decrease
Step 3. Name what we are looking for.		
Choose a variable to represent that quantity.		Let $p =$ the percent decrease.
Step 4. Translate. Write a sentence that gives the information to find it.		
First find the amount of decrease.		$3.71 - 3.64 = 0.07$
Find the percent.		Decrease is what percent of the original amaount?
		$\underbrace{0.07}\quad\underbrace{\text{is}}\quad\underbrace{\text{what percent}}\quad\underbrace{\text{of}}\quad\underbrace{3.71}?$
Translate into an equation.		$0.07 \quad = \quad p \quad \cdot \quad 3.71$

Step 5. Solve the equation.		$0.07 = 3.71p$
Divide by 3.71.		$0.019 = p$
Change to percent form; round to the nearest tenth.		$1.9\% = p$
Step 6. Check. Does this make sense?		
Yes, if the original price was $4, a 2% decrease would be 8 cents.		
Step 7. Answer the question with a complete sentence.		The price of gas decreased 1.9%.

Note:
Exercise:

TRY IT :: 3.37

Problem: Find the percent decrease. (Round to the nearest tenth of a percent.)

The population of North Dakota was about 672,000 in 2010. The population is projected to be about 630,000 in 2020.

Solution:

6.3%

Note:
Exercise:

TRY IT :: 3.38

Problem: Find the percent decrease.

Last year, Sheila’s salary was $42,000. Because of furlough days, this year, her salary was $37,800.

Solution:

10%

Solve Simple Interest Applications

Do you know that banks pay you to keep your money? The money a customer puts in the bank is called the **principal**, *P*, and the money the bank pays the customer is called the **interest**. The interest is computed as a certain percent of the principal; called the **rate of interest**, *r*. We usually express rate of interest as a percent per year, and we calculate it by using the decimal equivalent of the percent. The variable *t*, (for *time*) represents the number of years the money is in the account.

To find the interest we use the simple interest formula, $I = Prt$.

Note:

Simple Interest

If an amount of money, *P*, called the principal, is invested for a period of *t* years at an annual interest rate *r*, the amount of interest, *I*, earned is

Equation:

$$I = Prt \qquad \text{where} \qquad \begin{aligned} I &= \text{interest} \\ P &= \text{principal} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned}$$

Interest earned according to this formula is called **simple interest**.

Interest may also be calculated another way, called compound interest. This type of interest will be covered in later math classes.

The formula we use to calculate simple interest is $I = Prt$. To use the formula, we substitute in the values the problem gives us for the variables, and then solve for the unknown variable. It may be helpful to organize the information in a chart.

Example:
Exercise:

Problem:

**Nathaly deposited $12,500 in her bank account where it will earn 4% interest. How much interest will Nathaly earn in 5 years?

Equation:

$$\begin{aligned} I &= ? \\ P &= \$12{,}500 \\ r &= 4\% \\ t &= 5 \text{ years} \end{aligned}$$

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of interest earned
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let $I =$ the amount of interest.
Step 4. Translate into an equation.	

Write the formula. Substitute in the given information.	$I = Prt$ $I = (12{,}500)(.04)(5)$
Step 5. Solve the equation.	$I = 2{,}500$
Step 6. Check: Does this make sense? Is $2,500 a reasonable interest on $12,500? Yes.	
Step 7. Answer the question with a complete sentence.	The interest is $2,500.

Note:
Exercise:

Problem:

TRY IT :: 3.39
Areli invested a principal of $950 in her bank account with interest rate 3%. How much interest did she earn in 5 years?

Solution:

$142.50

Note:
Exercise:

Problem:

TRY IT :: 3.40
Susana invested a principal of $36,000 in her bank account with interest rate 6.5%. How much interest did she earn in 3 years?

Solution:

$7,020

There may be times when we know the amount of interest earned on a given principal over a certain length of time, but we don’t know the rate. To find the rate, we use the simple interest formula, substitute in the given values for the principal and time, and then solve for the rate.

Example:
Exercise:

Problem:

**Loren loaned his brother $3,000 to help him buy a car. In 4 years his brother paid him back the $3,000 plus $660 in interest. What was the rate of interest?

Equation:

$$\begin{aligned} I &= \$660 \\ P &= \$3{,}000 \\ r &= ? \\ t &= 4\,\text{years} \end{aligned}$$

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the rate of interest
Step 3. Name what we are looking for. Choose a variable to represent that	Let $r =$ the rate of interest.

quantity.	
Step 4. Translate into an equation. Write the formula. Substitute in the given information.	$I = Prt$ $660 = (3{,}000)r(4)$
Step 5. Solve the equation. Divide. Change to percent form.	$660 = (12{,}000)r$ $0.055 = r$ $5.5\% = r$
Step 6. Check: Does this make sense? $I = Prt$ $660 \overset{?}{=} (3{,}000)(0.055)(4)$ $660 = 660\checkmark$	
Step 7. Answer the question with a complete sentence.	The rate of interest was 5.5%.

Notice that in this example, Loren's brother paid Loren interest, just like a bank would have paid interest if Loren invested his money there.

Note:
Exercise:

Problem:

TRY IT :: 3.41
Jim loaned his sister \$5,000 to help her buy a house. In 3 years, she paid him the \$5,000, plus \$900 interest. What was the rate of interest?

Solution:

6%

Note:
Exercise:

Problem:

TRY IT :: 3.42
Hang borrowed $7,500 from her parents to pay her tuition. In 5 years, she paid them $1,500 interest in addition to the $7,500 she borrowed. What was the rate of interest?

Solution:

4%

Example:
Exercise:

Problem:

**Eduardo noticed that his new car loan papers stated that with a 7.5% interest rate, he would pay $6,596.25 in interest over 5 years. How much did he borrow to pay for his car?

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount borrowed (the principal)
Step 3. Name what we are looking for.	Let $P =$ principal

Choose a variable to represent that quantity.	borrowed.
Step 4. Translate into an equation. Write the formula. Substitute in the given information.	$$\begin{aligned} I &= Prt \\ 6{,}596.25 &= P(0.075)(5) \end{aligned}$$
Step 5. Solve the equation. Divide.	$$\begin{aligned} 6{,}596.25 &= 0.375P \\ 17{,}590 &= P \end{aligned}$$
Step 6. Check: Does this make sense? $$\begin{aligned} I &= Prt \\ 6{,}596.25 &\overset{?}{=} (17{,}590)(0.075)(5) \\ 6{,}596.25 &= 6{,}596.25\checkmark \end{aligned}$$	
Step 7. Answer the question with a complete sentence.	The principal was $17,590.

Note:
Exercise:

Problem:

TRY IT :: 3.43
Sean’s new car loan statement said he would pay $4,866.25 in interest from an interest rate of 8.5% over 5 years. How much did he borrow to buy his new car?

Solution:

$11,450

Note:
Exercise:

Problem:

TRY IT :: 3.44
In 5 years, Gloria’s bank account earned $2,400 interest at 5%. How much had she deposited in the account?

Solution:

$9,600

Solve Applications with Discount or Mark-up

Applications of discount are very common in retail settings. When you buy an item on sale, the original price has been discounted by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the amount of the discount. To determine the **amount of discount**, we multiply the discount rate by the original price.

We summarize the discount model in the box below.

Note:
Discount
Equation:

$$\begin{aligned}&\text{amount of discount} = \text{discount rate} \times \text{original price}\\&\text{sale price} = \text{original price} - \text{amount of discount}\end{aligned}$$

Keep in mind that the sale price should always be less than the original price.

Example:
Exercise:

Problem:

**Elise bought a dress that was discounted 35% off of the original price of $140. What was ⓐ the amount of discount and ⓑ the sale price of the dress?

Solution:
Solution

ⓐ $$\begin{aligned}\text{Original price} &= \$140\\ \text{Discount rate} &= 35\%\\ \text{Discount} &= ?\end{aligned}$$	
Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of discount
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let $d =$ the amount of discount.
Step 4. Translate into an equation. Write a sentence that gives the information to find it. Translate into an equation.	The discount is 35% of \$140. $d = 0.35(140)$
Step 5. Solve the equation.	$d = 49$
Step 6. Check: Does this make sense? Is a \$49 discount reasonable for a \$140 dress? Yes.	
Step 7. Write a complete sentence to answer the question.	The amount of discount was \$49.

ⓑ
Read the problem again.

Step 1. Identify what we are looking for.		the sale price of the dress
Step 2. Name what we are looking for.		
Choose a variable to represent that quantity.		Let $s =$ the sale price.
Step 3. Translate into an equation.		
Write a sentence that gives the information to find it.		$\underbrace{\text{The sale price}}\ \underbrace{\text{is}}\ \underbrace{\text{the \$140}}\ \underbrace{\text{minus}}\ \underbrace{\text{the \$49 discount}}$
Translate into an equation.		$s \quad = \quad 140 \quad - \quad 49$
Step 4. Solve the equation.		$s = 91$
Step 5. Check. Does this make sense?		
Is the sale price less than the original price?		
Yes, $91 is less than $140.		
Step 6. Answer the question with a complete sentence.		The sale price of the dress was $91.

Note:
Exercise:

TRY IT :: 3.45

Problem: Find ⓐ the amount of discount and ⓑ the sale price:

Sergio bought a belt that was discounted 40% from an original price of $29.

Solution:

ⓐ $11.60 ⓑ $17.40

Note:
Exercise:

TRY IT :: 3.46

Problem: Find ⓐ the amount of discount and ⓑ the sale price:

Oscar bought a barbecue that was discounted 65% from an original price of $395.

Solution:

ⓐ $256.75 ⓑ $138.25

There may be times when we know the original price and the sale price, and we want to know the discount rate. To find the discount rate, first we will find the amount of discount and then use it to compute the rate as a percent of the original price. [link] will show this case.

Example:
Exercise:

Problem:

**Jeannette bought a swimsuit at a sale price of $13.95. The original price of the swimsuit was $31. Find the ⓐ amount of discount and ⓑ discount rate.

Solution:
Solution

ⓐ $$\begin{aligned}\text{Original price} &= \$31\\ \text{Discount} &= ?\\ \text{Sale Price} &= \$13.95\end{aligned}$$	
Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of discount
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let $d =$ the amount of discount.
Step 4. Translate into an equation. Write a sentence that gives the information to find it. Translate into an equation.	The discount is the difference between the original price and the sale price. $d = 31 - 13.95$
Step 5. Solve the equation.	$d = 17.05$
Step 6. Check: Does this make sense? Is 17.05 less than 31? Yes.	
Step 7. Answer the question with a complete sentence.	The amount of discount was $17.05.

ⓑ
Read the problem again.

Step 1. Identify what we are looking for.		the discount rate
Step 2. Name what we are looking for.		
Choose a variable to represent it.		Let $r =$ the discount rate.
Step 3. Translate into an equation.		
Write a sentence that gives the information to find it.		$\underbrace{\text{The discount of \$17.05}}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{\text{\$31}}\text{?}$
Translate into an equation.		$17.05 \quad = \quad r \quad \cdot \quad 31$
Step 4. Solve the equation.		$17.05 = 31r$
Divide both sides by 31.		$0.55 = r$
Change to percent form.		$r = 55\%$
Step 5. Check. Does this make sense?		
Is $17.05 equal to 55% of $31?		
$17.05 \overset{?}{=} 0.55(31)$		

$17.05 = 17.05\checkmark$		
Step 6. Answer the question with a complete sentence.		The rate of discount was 55%.

Note:
Exercise:

TRY IT :: 3.47

Problem: Find ⓐ the amount of discount and ⓑ the discount rate.

Lena bought a kitchen table at the sale price of $375.20. The original price of the table was $560.

Solution:

ⓐ $184.80 ⓑ 33%

Note:
Exercise:

TRY IT :: 3.48

Problem: Find ⓐ the amount of discount and ⓑ the discount rate.

Nick bought a multi-room air conditioner at a sale price of $340. The original price of the air conditioner was $400.

Solution:

ⓐ $60 ⓑ 15%

Applications of mark-up are very common in retail settings. The price a retailer pays for an item is called the **original cost**. The retailer then adds a **mark-up** to the original cost to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the original cost. To determine the amount of mark-up, multiply the mark-up rate by the original cost.

We summarize the mark-up model in the box below.

Note:
Mark-Up
Equation:

$$\begin{aligned}&\text{amount of mark-up} = \text{mark-up rate } \times \text{ original cost}\\&\text{list price} = \text{original cost} + \text{amount of mark up}\end{aligned}$$

Keep in mind that the list price should always be more than the original cost.

Example:
Exercise:

Problem:

**Adam’s art gallery bought a photograph at original cost $250. Adam marked the price up 40%. Find the ⓐ amount of mark-up and ⓑ the list price of the photograph.

Solution:
Solution

ⓐ

Step 1. Read the problem.		

Step 2. Identify what we are looking for.		the amount of mark-up
Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $m =$ the amount of markup.
Step 4. Translate into an equation.		
Write a sentence that gives the information to find it.		$\underbrace{\text{The mark-up}}\ \underbrace{\text{is}}\ \underbrace{40\%}\ \underbrace{\text{of}}\ \underbrace{\text{the \$250 original cost}}$
Translate into an equation.		$m = 0.40 \cdot 250$
Step 5. Solve the equation.		$m = 100$
Step 6. Check. Does this make sense?		
Yes, 40% is less than one-half and 100 is less than half of 250.		
Step 7. Answer the question with a complete sentence.		The mark-up on the phtograph was $100.

ⓑ

Step 1. Read the problem again.		
Step 2. Identify what we are looking for.		the list price

Step 3. Name what we are looking for.		
Choose a variable to represent it.		Let $p =$ the list price.
Step 4. Translate into an equation.		
Write a sentence that gives the information to find it.		$\underbrace{\text{The list price}}\ \underbrace{\text{is}}\ \underbrace{\text{original cost}}\ \underbrace{\text{plus}}\ \underbrace{\text{the mark-up}}$
Translate into an equation.		$p \quad = \quad 250 \quad + \quad 100$
Step 5. Solve the equation.		$p = 350$
Step 6. Check. Does this make sense?		
Is the list price more than the net price? Is $350 more than $250? Yes.		
Step 7. Answer the question with a complete sentence.		The list price of the photograph was $350.

Note:
Exercise:

TRY IT :: 3.49

Problem: Find ⓐ the amount of mark-up and ⓑ the list price.

Jim’s music store bought a guitar at original cost $1,200. Jim marked the price up 50%.

Solution:

ⓐ \$600 ⓑ \$1,800

Note:
Exercise:

TRY IT :: 3.50

Problem: Find ⓐ the amount of mark-up and ⓑ the list price.

The Auto Resale Store bought Pablo’s Toyota for \$8,500. They marked the price up 35%.

Solution:

ⓐ \$2,975 ⓑ \$11,475

Key Concepts

- **Percent Increase** To find the percent increase:

  Find the amount of increase. $\text{increase} = \text{new amount} - \text{original amount}$
  Find the percent increase. Increase is what percent of the original amount?

- **Percent Decrease** To find the percent decrease:

  Find the amount of decrease. $\text{decrease} = \text{original amount} - \text{new amount}$
  Find the percent decrease. Decrease is what percent of the original amount?

- **Simple Interest** If an amount of money, P, called the principal, is invested for a period of t years at an annual interest rate r, the amount of interest, I, earned is
  Equation:

$$\begin{array}{rrcl} & I & = & Prt \\ \text{where} & I & = & \text{interest} \\ & P & = & \text{principal} \\ & r & = & \text{rate} \\ & t & = & \text{time} \end{array}$$

- **Discount**
    - amount of discount is discount rate · original price
    - sale price is original price – discount
- **Mark-up**
    - amount of mark-up is mark-up rate · original cost
    - list price is original cost + mark up

Practice Makes Perfect

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

Exercise:

Problem: What number is 45% of 120?

Solution:

54

Exercise:

Problem: What number is 65% of 100?

Exercise:

Problem: What number is 24% of 112?

Solution:

26.88

Exercise:

Problem: What number is 36% of 124?

Exercise:

Problem: 250% of 65 is what number?

Solution:

162.5

Exercise:

Problem: 150% of 90 is what number?

Exercise:

Problem: 800% of 2250 is what number?

Solution:

18,000

Exercise:

Problem: 600% of 1740 is what number?

Exercise:

Problem: 28 is 25% of what number?

Solution:

112

Exercise:

Problem: 36 is 25% of what number?

Exercise:

Problem: 81 is 75% of what number?

Solution:

108

Exercise:

Problem: 93 is 75% of what number?

Exercise:

Problem: 8.2% of what number is \$2.87?

Solution:

\$35

Exercise:

Problem: 6.4% of what number is \$2.88?

Exercise:

Problem: 11.5% of what number is \$108.10?

Solution:

\$940

Exercise:

Problem: 12.3% of what number is \$92.25?

Exercise:

Problem: What percent of 260 is 78?

Solution:

30%

Exercise:

Problem: What percent of 215 is 86?

Exercise:

Problem: What percent of 1500 is 540?

Solution:

36%

Exercise:

Problem: What percent of 1800 is 846?

Exercise:

Problem: 30 is what percent of 20?

Solution:

150%

Exercise:

Problem: 50 is what percent of 40?

Exercise:

Problem: 840 is what percent of 480?

Solution:

175%

Exercise:

Problem: 790 is what percent of 395?

Solve Percent Applications

In the following exercises, solve.

Exercise:

Problem:

Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?

Solution:

$11.88

Exercise:

Problem:

When Hiro and his co-workers had lunch at a restaurant near their work, the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?

Exercise:

Problem:

Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $2165. How much money was deposited to Trong’s savings account?

Solution:

$259.80

Exercise:

Problem:

Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $1,485. How much did Cherise deposit into her retirement account?

Exercise:

Problem:

One serving of oatmeal has eight grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Solution:

24.2 g

Exercise:

Problem:

One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

Exercise:

Problem:

A bacon cheeseburger at a popular fast food restaurant contains 2070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Solution:

2407 mg

Exercise:

Problem:

A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

Exercise:

Problem:

After 3 months on a diet, Lisa had lost 12% of her original weight. She lost 21 pounds. What was Lisa’s original weight?

Solution:

175 lb.

Exercise:

Problem:

Tricia got a 6% raise on her weekly salary. The raise was $30 per week. What was her original salary?

Exercise:

Problem:

Yuki bought a dress on sale for $72. The sale price was 60% of the original price. What was the original price of the dress?

Solution:

$120

Exercise:

Problem:

Kim bought a pair of shoes on sale for $40.50. The sale price was 45% of the original price. What was the original price of the shoes?

Exercise:

Problem:

Tim left a $9 tip for a $50 restaurant bill. What percent tip did he leave?

Solution:

18%

Exercise:

Problem:

Rashid left a $15 tip for a $75 restaurant bill. What percent tip did he leave?

Exercise:

Problem:

The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Solution:

45%

Exercise:

Problem:

The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

Exercise:

Problem:

Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

Solution:

25%

Exercise:

Problem:

Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, solve.

Exercise:

Problem:

Tamanika got a raise in her hourly pay, from $15.50 to $17.36. Find the percent increase.

Solution:

12%

Exercise:

Problem:

Ayodele got a raise in her hourly pay, from \$24.50 to \$25.48. Find the percent increase.

Exercise:

Problem:

Annual student fees at the University of California rose from about \$4,000 in 2000 to about \$12,000 in 2010. Find the percent increase.

Solution:

200%

Exercise:

Problem:

The price of a share of one stock rose from \$12.50 to \$50. Find the percent increase.

Exercise:

Problem:

According to *Time* magazine annual global seafood consumption rose from 22 pounds per person in the 1960s to 38 pounds per person in 2011. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution:

72.7%

Exercise:

Problem:

In one month, the median home price in the Northeast rose from \$225,400 to \$241,500. Find the percent increase. (Round to the nearest tenth of a percent.)

Exercise:

Problem:

A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent decrease.

Solution:

2.5%

Exercise:

Problem:

The price of a share of one stock fell from $8.75 to $8.54. Find the percent decrease.

Exercise:

Problem:

Hernando’s salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent decrease.

Solution:

11%

Exercise:

Problem:

In 10 years, the population of Detroit fell from 950,000 to about 712,500. Find the percent decrease.

Exercise:

Problem:

In 1 month, the median home price in the West fell from $203,400 to $192,300. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solution:

5.5%

Exercise:

Problem:

Sales of video games and consoles fell from $1,150 million to $1,030 million in 1 year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solve Simple Interest Applications

In the following exercises, solve.

Exercise:

Problem:

Casey deposited $1,450 in a bank account with interest rate 4%. How much interest was earned in two years?

Solution:

$116

Exercise:

Problem:

Terrence deposited $5,720 in a bank account with interest rate 6%. How much interest was earned in 4 years?

Exercise:

Problem:

Robin deposited $31,000 in a bank account with interest rate 5.2%. How much interest was earned in 3 years?

Solution:

$4,836

Exercise:

Problem:

Carleen deposited $16,400 in a bank account with interest rate 3.9%. How much interest was earned in 8 years?

Exercise:

Problem:

Hilaria borrowed $8,000 from her grandfather to pay for college. Five years later, she paid him back the $8,000, plus $1,200 interest. What was the rate of interest?

Solution:

3%

Exercise:

Problem:

Kenneth loaned his niece $1,200 to buy a computer. Two years later, she paid him back the $1,200, plus $96 interest. What was the rate of interest?

Exercise:

Problem:

Lebron loaned his daughter $20,000 to help her buy a condominium. When she sold the condominium four years later, she paid him the $20,000, plus $3,000 interest. What was the rate of interest?

Solution:

3.75%

Exercise:

Problem:

Pablo borrowed $50,000 to start a business. Three years later, he repaid the $50,000, plus $9,375 interest. What was the rate of interest?

Exercise:

Problem:

In 10 years, a bank account that paid 5.25% earned $18,375 interest. What was the principal of the account?

Solution:

$35,000

Exercise:

Problem:

In 25 years, a bond that paid 4.75% earned $2,375 interest. What was the principal of the bond?

Exercise:

Problem:

Joshua's computer loan statement said he would pay $1,244.34 in interest for a 3-year loan at 12.4%. How much did Joshua borrow to buy the computer?

Solution:

$3,345

Exercise:

Problem:

Margaret's car loan statement said she would pay $7,683.20 in interest for a 5-year loan at 9.8%. How much did Margaret borrow to buy the car?

Solve Applications with Discount or Mark-up

In the following exercises, find the sale price.

Exercise:

Problem:

Perla bought a cell phone that was on sale for $50 off. The original price of the cell phone was $189.

Solution:

$139

Exercise:

Problem:

Sophie saw a dress she liked on sale for $15 off. The original price of the dress was $96.

Exercise:

Problem:

Rick wants to buy a tool set with original price $165. Next week the tool set will be on sale for $40 off.

Solution:

$125

Exercise:

Problem:

Angelo’s store is having a sale on televisions. One television, with original price $859, is selling for $125 off.

In the following exercises, find ⓐ the amount of discount and ⓑ the sale price.

Exercise:

Problem:

Janelle bought a beach chair on sale at 60% off. The original price was $44.95.

Solution:

ⓐ $26.97 ⓑ $17.98

Exercise:

Problem:

Errol bought a skateboard helmet on sale at 40% off. The original price was $49.95.

Exercise:

Problem:

Kathy wants to buy a camera that lists for $389. The camera is on sale with a 33% discount.

Solution:

ⓐ $128.37 ⓑ $260.63

Exercise:

Problem:

Colleen bought a suit that was discounted 25% from an original price of $245.

Exercise:

Problem:

Erys bought a treadmill on sale at 35% off. The original price was $949.95 (round to the nearest cent.)

Solution:

ⓐ $332.48 ⓑ $617.47

Exercise:

Problem:

Jay bought a guitar on sale at 45% off. The original price was $514.75 (round to the nearest cent.)

In the following exercises, find ⓐ the amount of discount and ⓑ the discount rate. (Round to the nearest tenth of a percent if needed.)

Exercise:

Problem:

Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.

Solution:

ⓐ $576 ⓑ 30%

Exercise:

Problem:

Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.

Exercise:

Problem:

Patty bought a baby stroller on sale for $301.75. The original price of the stroller was $355.

Solution:

ⓐ $53.25 ⓑ 15%

Exercise:

Problem:

Bill found a book he wanted on sale for $20.80. The original price of the book was $32.

Exercise:

Problem:

Nikki bought a patio set on sale for $480. The original price was $850. To the nearest tenth of a percent, what was the rate of discount?

Solution:

ⓐ $370 ⓑ 43.5%

Exercise:

Problem:

Stella bought a dinette set on sale for $725. The original price was $1,299. To the nearest tenth of a percent, what was the rate of discount?

In the following exercises, find ⓐ the amount of the mark-up and ⓑ the list price.

Exercise:

Problem:

Daria bought a bracelet at original cost $16 to sell in her handicraft store. She marked the price up 45%.

Solution:

ⓐ $7.20 ⓑ $23.20

Exercise:

Problem:

Regina bought a handmade quilt at original cost $120 to sell in her quilt store. She marked the price up 55%.

Exercise:

Problem:

Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up.

Solution:

ⓐ $0.20 ⓑ $0.80

Exercise:

Problem:

Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up.

Exercise:

Problem:

Alan bought a used bicycle for $115. After re-conditioning it, he added 225% mark-up and then advertised it for sale.

Solution:

ⓐ $258.75 ⓑ $373.75

Exercise:

Problem:

Michael bought a classic car for $8,500. He restored it, then added 150% mark-up before advertising it for sale.

Everyday Math

Exercise:

Problem:

Leaving a Tip At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

Solution:

21.2%

Exercise:

Problem:

Splitting a Bill Four friends went out to lunch and the bill came to $53.75. They decided to add enough tip to make a total of $64, so that they could easily split the bill evenly among themselves. What percent tip did they leave?

Writing Exercises

Exercise:

Problem:

Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

Solution:

The number should be greater than 44. Since 80% equals 0.8 in decimal form, 0.8 is less than one, and we must multiply the number by 0.8 to get 44, the number must be greater than 44.

Exercise:

Problem:

Without solving the problem “What is 20% of 300?” think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.

Exercise:

Problem:

After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

Solution:

He meant that he should have packed half the shorts and twice the shirts.

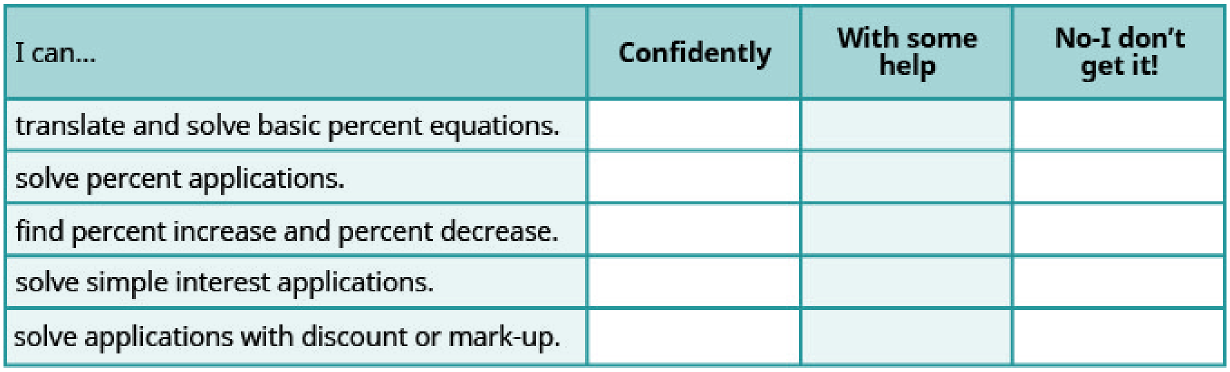
Exercise:

Problem:

Because of road construction in one city, commuters were advised to plan that their Monday morning commute would take 150% of their usual commuting time. Explain what this means.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
translate and solve basic percent equations.			
solve percent applications.			
find percent increase and percent decrease.			
solve simple interest applications.			
solve applications with discount or mark-up.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

Glossary

amount of discount

The amount of discount is the amount resulting when a discount rate is multiplied by the original price of an item.

discount rate

The discount rate is the percent used to determine the amount of a discount, common in retail settings.

interest

Interest is the money that a bank pays its customers for keeping their money in the bank.

list price

The list price is the price a retailer sells an item for.

mark-up

A mark-up is a percentage of the original cost used to increase the price of an item.

original cost

The original cost in a retail setting, is the price that a retailer pays for an item.

principal

The principal is the original amount of money invested or borrowed for a period of time at a specific interest rate.

rate of interest

The rate of interest is a percent of the principal, usually expressed as a percent per year.

simple interest

Simple interest is the interest earned according to the formula $I = Prt$.

Solve Geometry Applications: Triangles, Rectangles, and the Pythagorean Theorem

By the end of this section, you will be able to:

- Solve applications using properties of triangles
- Use the Pythagorean Theorem
- Solve applications using rectangle properties

Note:
Before you get started, take this readiness quiz.

1. Simplify: $\frac{1}{2}(6h)$.
   If you missed this problem, review [link].
2. The length of a rectangle is three less than the width. Let *w* represent the width. Write an expression for the length of the rectangle.
   If you missed this problem, review [link].
3. Solve: $A = \frac{1}{2}bh$ for *b* when $A = 260$ and $h = 52$.
   If you missed this problem, review [link].
4. Simplify: $\sqrt{144}$.
   If you missed this problem, review [link].

Solve Applications Using Properties of Triangles

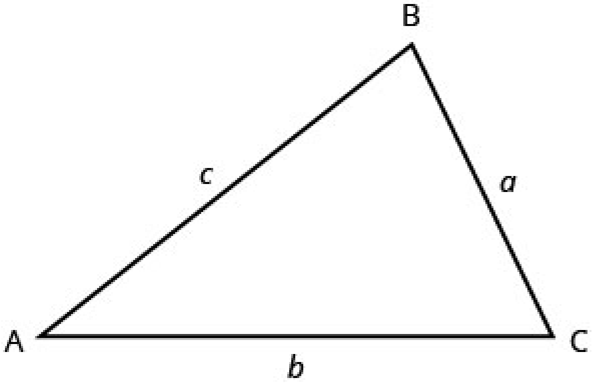
In this section we will use some common geometry formulas. We will adapt our problem-solving strategy so that we can solve geometry applications. The geometry formula will name the variables and give us the equation to solve. In addition, since these applications will all involve shapes of some sort, most people find it helpful to draw a figure and label it with the given information. We will include this in the first step of the problem solving strategy for geometry applications.

Note:
Solve Geometry Applications.

Read the problem and make sure all the words and ideas are understood. Draw the figure and label it with the given information.
Identify what we are looking for.
Label what we are looking for by choosing a variable to represent it.
Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
Solve the equation using good algebra techniques.
Check the answer by substituting it back into the equation solved in step 5 and by making sure it makes sense in the context of the problem.
Answer the question with a complete sentence.

We will start geometry applications by looking at the properties of triangles. Let’s review some basic facts about triangles. Triangles have three sides and three interior angles. Usually each side is labeled with a lowercase letter to match the uppercase letter of the opposite vertex.

The plural of the word *vertex* is *vertices*. All triangles have three vertices. Triangles are named by their vertices: The triangle in [link] is called $\triangle ABC$.



Triangle ABC has vertices A,

B, and C. The lengths of the sides are a, b, and c.

The three angles of a triangle are related in a special way. The sum of their measures is 180°. Note that we read $m\angle A$ as “the measure of angle A.” So in $\triangle ABC$ in [link],

Equation:

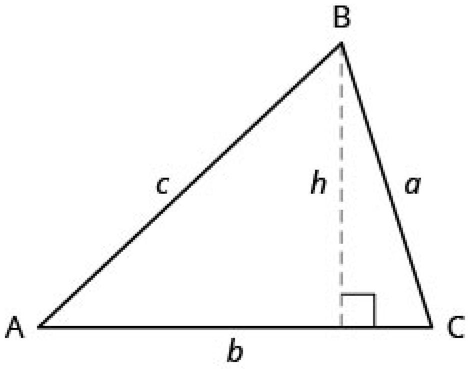
$$m\angle A+m\angle B+m\angle C=180^{\circ}$$

Because the perimeter of a figure is the length of its boundary, the perimeter of $\triangle ABC$ is the sum of the lengths of its three sides.

Equation:

$$P=a+b+c$$

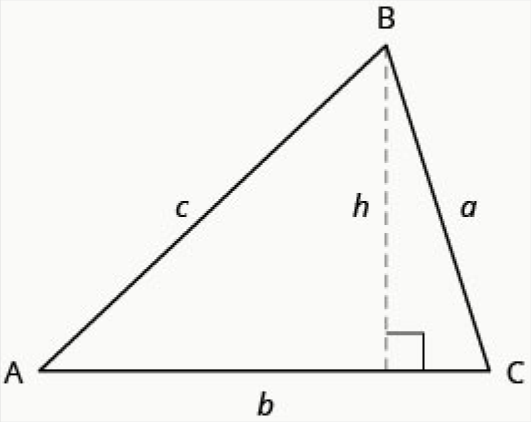
To find the area of a triangle, we need to know its base and height. The height is a line that connects the base to the opposite vertex and makes a 90° angle with the base. We will draw $\triangle ABC$ again, and now show the height, *h*. See [link].



The formula for the area of $\triangle ABC$ is

$A = \frac{1}{2}bh$, where b is the base and h is the height.

Note:
Triangle Properties



For $\triangle ABC$
Angle measures:
Equation:

$$m\angle A + m\angle B + m\angle C = 180$$

- The sum of the measures of the angles of a triangle is 180°.

Perimeter:
Equation:

$$P = a + b + c$$

- The perimeter is the sum of the lengths of the sides of the triangle.

Area:

Equation:

$$A = \frac{1}{2}bh, b = \text{base}, h = \text{height}$$

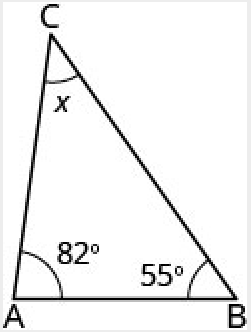
- The area of a triangle is one-half the base times the height.

Example:
Exercise:

Problem:

**The measures of two angles of a triangle are 55 and 82 degrees. Find the measure of the third angle.

Solution:
Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	

Step 2. Identify what you are looking for.	the measure of the third angle in a triangle
Step 3. Name. Choose a variable to represent it.	Let $x =$ the measure of the angle.
Step 4. Translate.	
Write the appropriate formula and substitute.	$m\angle A + m\angle B + m\angle C = 180$
Step 5. Solve the equation.	$55 + 82 + x = 180$ $137 + x = 180$ $x = 43$
Step 6. Check. $55 + 82 + 43 \overset{?}{=} 180$ $180 = 180 \checkmark$	
Step 7. Answer the question.	The measure of the third angle is 43 degrees.

Note:
Exercise:

Problem:

TRY IT :: 3.51
The measures of two angles of a triangle are 31 and 128 degrees. Find the measure of the third angle.

Solution:

21 degrees

Note:
Exercise:

Problem:

TRY IT :: 3.52
The measures of two angles of a triangle are 49 and 75 degrees. Find the measure of the third angle.

Solution:

56 degrees

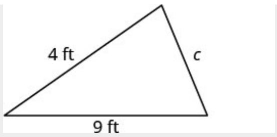
Example:
Exercise:

Problem:

**The perimeter of a triangular garden is 24 feet. The lengths of two sides are four feet and nine feet. How long is the third side?

Solution:
Solution

Step 1. Read the problem. Draw the	

figure and label it with the given information.	 $P = 24$ ft
Step 2. Identify what you are looking for.	length of the third side of a triangle
Step 3. Name. Choose a variable to represent it.	Let $c =$ the third side.
Step 4. Translate.	
Write the appropriate formula and substitute.	$\underbrace{P}_{} = \underbrace{a}_{} + \underbrace{b}_{} + \underbrace{c}_{}$
Substitute in the given information.	$24 \text{ ft} = 4 \text{ ft} + 9 \text{ ft} + c$
Step 5. Solve the equation.	$24 = 13 + c$ $11 = c$
Step 6. Check. $\text{P} = a + b + c$ $24 \overset{?}{=} 4 + 9 + 11$ $24 = 24 ✓$	
Step 7. Answer the question.	The third side is

	11 feet long.

Note:
Exercise:

Problem:

TRY IT :: 3.53
The perimeter of a triangular garden is 48 feet. The lengths of two sides are 18 feet and 22 feet. How long is the third side?

Solution:

8 feet

Note:
Exercise:

Problem:

TRY IT :: 3.54
The lengths of two sides of a triangular window are seven feet and five feet. The perimeter is 18 feet. How long is the third side?

Solution:

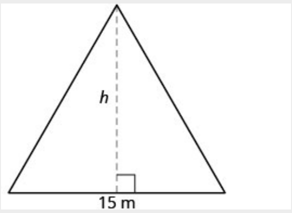
6 feet

Example:
Exercise:

Problem:

**The area of a triangular church window is 90 square meters. The base of the window is 15 meters. What is the window’s height?

Solution:
Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	 Area $= 90m^2$
Step 2. Identify what you are looking for.	height of a triangle
Step 3. Name. Choose a variable to represent it.	Let $h =$ the height.
Step 4. Translate.	
Write the appropriate formula.	$\underbrace{A}\ \underbrace{=}\ \underbrace{\frac{1}{2}}\ \underbrace{\cdot}\ \underbrace{b}\ \underbrace{\cdot}\ \underbrace{h}$
Substitute in the given information.	$90\text{ m}^2 = \frac{1}{2} \cdot 15\text{ m} \cdot h$

Step 5. Solve the equation.	$90 = \frac{15}{2}h$ $12 = h$
Step 6. Check. $A = \frac{1}{2}bh$ $90 \stackrel{?}{=} \frac{1}{2} \cdot 15 \cdot 12$ $90 = 90\checkmark$	
Step 7. Answer the question.	The height of the triangle is 12 meters.

Note:
Exercise:

Problem:

TRY IT :: 3.55
The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Solution:

14 inches

Note:
Exercise:

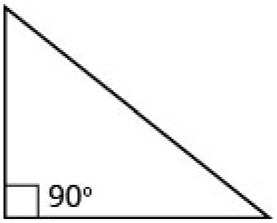
Problem:

TRY IT :: 3.56
A triangular tent door has area 15 square feet. The height is five feet. What is the base?

Solution:

6 feet

The triangle properties we used so far apply to all triangles. Now we will look at one specific type of triangle—a right triangle. A right triangle has one 90° angle, which we usually mark with a small square in the corner.



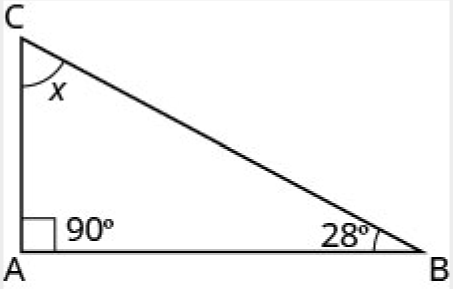
Note:
Right Triangle
A **right triangle** has one 90° angle, which is often marked with a square at the vertex.

Example:
Exercise:

Problem:

**One angle of a right triangle measures 28°. What is the measure of the third angle?

Solution:
Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the measure of an angle
Step 3. Name. Choose a variable to represent it.	Let $x =$ the measure of an angle.
Step 4. Translate.	$m\angle A + m\angle B + m\angle C = 180$
Write the appropriate formula and substitute.	$x + 90 + 28 = 180$
Step 5. Solve the equation.	$\begin{aligned} x + 118 &= 180 \\ x &= 62 \end{aligned}$
Step 6. Check. $\begin{aligned} 180 &\overset{?}{=} 90 + 28 + 62 \\ 180 &= 180\checkmark \end{aligned}$	
Step 7. Answer the	The measure of the third angle

question.	is 62°.

Note:
Exercise:

Problem:

TRY IT :: 3.57
One angle of a right triangle measures 56°. What is the measure of the other small angle?

Solution:

34°

Note:
Exercise:

Problem:

TRY IT :: 3.58
One angle of a right triangle measures 45°. What is the measure of the other small angle?

Solution:

45°

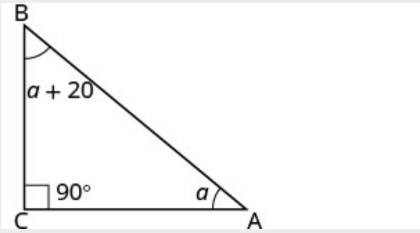
In the examples we have seen so far, we could draw a figure and label it directly after reading the problem. In the next example, we will have to define one angle in terms of another. We will wait to draw the figure until we write expressions for all the angles we are looking for.

Example:
Exercise:

Problem:

**The measure of one angle of a right triangle is 20 degrees more than the measure of the smallest angle. Find the measures of all three angles.

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the measures of all three angles
Step 3. Name. Choose a variable to represent it.	Let $a = 1^{\text{st}}$ angle. $a + 20 = 2^{\text{nd}}$ angle $90 = 3^{\text{rd}}$ angle (the right angle)
Draw the figure and label it with the given information	

Step 4. Translate	$m\angle A + m\angle B + m\angle C = 180$
Write the appropriate formula. Substitute into the formula.	$a + (a + 20) + 90 = 180$
Step 5. Solve the equation.	$2a + 110 = 180$ $2a = 70$ $a = 35$ first angle $a + 20$ second angle $35 + 20$ 55 90 third angle
Step 6. Check. $35 + 55 + 90 \overset{?}{=} 180$ $180 = 180✓$	
Step 7. Answer the question.	The three angles measure 35°, 55°, and 90°.

Note:
Exercise:

Problem:

TRY IT :: 3.59
The measure of one angle of a right triangle is 50° more than the measure of the smallest angle. Find the measures of all three angles.

Solution:

$20^\circ, 70^\circ, 90^\circ$

Note:
Exercise:

Problem:

TRY IT :: 3.60
The measure of one angle of a right triangle is 30° more than the measure of the smallest angle. Find the measures of all three angles.

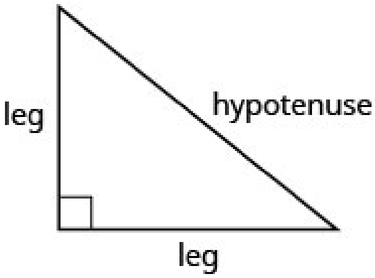
Solution:

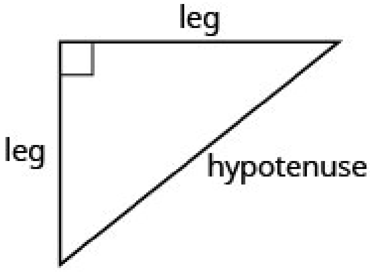
$30^\circ, 60^\circ, 90^\circ$

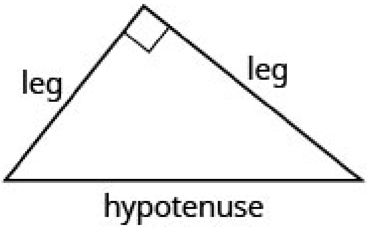
Use the Pythagorean Theorem

We have learned how the measures of the angles of a triangle relate to each other. Now, we will learn how the lengths of the sides relate to each other. An important property that describes the relationship among the lengths of the three sides of a right triangle is called the Pythagorean Theorem. This theorem has been used around the world since ancient times. It is named after the Greek philosopher and mathematician, Pythagoras, who lived around 500 BC.

Before we state the Pythagorean Theorem, we need to introduce some terms for the sides of a triangle. Remember that a right triangle has a 90° angle, marked with a small square in the corner. The side of the triangle opposite the 90° angle is called the *hypotenuse* and each of the other sides are called *legs*.



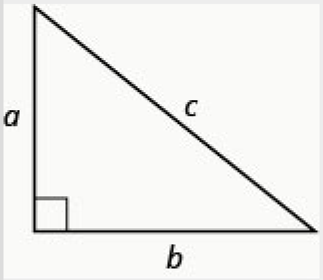




The Pythagorean Theorem tells how the lengths of the three sides of a right triangle relate to each other. It states that in any right triangle, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse. In symbols we say: in any right triangle, $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse.

Writing the formula in every exercise and saying it aloud as you write it, may help you remember the Pythagorean Theorem.

Note:
The Pythagorean Theorem
In any right triangle, $a^2 + b^2 = c^2$.



where a and b are the lengths of the legs, c is the length of the hypotenuse.

To solve exercises that use the Pythagorean Theorem, we will need to find square roots. We have used the notation $\sqrt{m}$ and the definition:

If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.

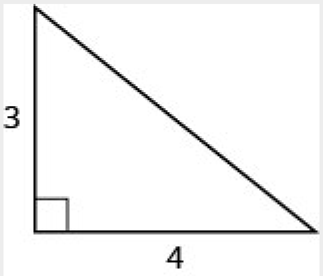
For example, we found that $\sqrt{25}$ is 5 because $25 = 5^2$.

Because the Pythagorean Theorem contains variables that are squared, to solve for the length of a side in a right triangle, we will have to use square roots.

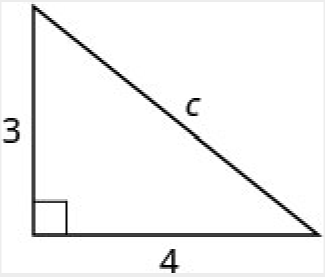
Example:
Exercise:

Problem:

**Use the Pythagorean Theorem to find the length of the hypotenuse shown below.



Solution:
Solution

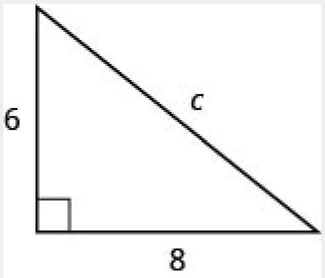
Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the hypotenuse of the triangle
Step 3. Name. Choose a variable to represent it. Label side *c* on the figure.	Let *c* = the length of the hypotenuse. 
Step 4. Translate.	
Write the appropriate formula.	$a^2 + b^2 = c^2$
Substitute.	$3^2 + 4^2 = c^2$
Step 5. Solve the equation.	$9 + 16 = c^2$
Simplify.	$25 = c^2$
Use the definition of square root.	$\sqrt{25} = c$
Simplify.	$5 = c$
Step 6. Check.	

$3^2 + 4^2 \overset{?}{=} 5^2$ $9 + 16 \overset{?}{=} 25$ $25 = 25\ \checkmark$	
Step 7. Answer the question.	The length of the hypotenuse is 5.

Note:
Exercise:

Problem:

TRY IT :: 3.61
Use the Pythagorean Theorem to find the length of the hypotenuse in the triangle shown below.



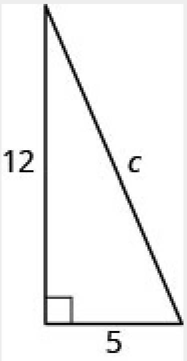
Solution:

$c = 10$

Note:
Exercise:

Problem:

TRY IT :: 3.62
Use the Pythagorean Theorem to find the length of the hypotenuse in the triangle shown below.



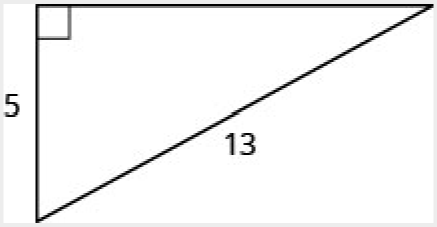
Solution:

$c = 13$

Example:
Exercise:

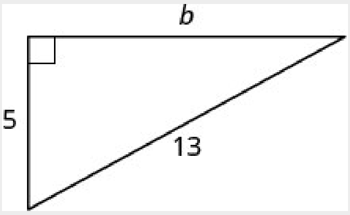
Problem:

**Use the Pythagorean Theorem to find the length of the leg shown below.



Solution:

Solution

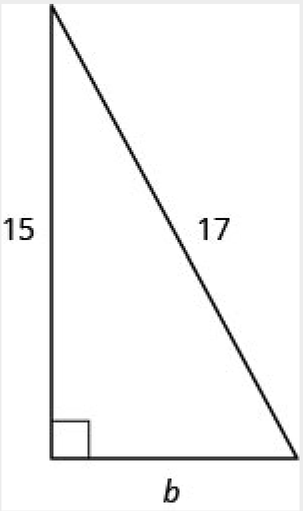
Step 1. Read the problem.		
Step 2. Identify what you are looking for.		the length of the leg of the triangle
Step 3. Name. Choose a variable to represent it.		Let *b* = the leg of the triangle.
Lable side *b*.		
Step 4. Translate		
Write the appropriate formula.		$a^2+b^2=c^2$
Substitute.		$5^2+b^2=13^2$
Step 5. Solve the equation.		$25+b^2=169$
Isolate the variable term.		$b^2=144$
Use the definition of square root.		$b^2=\sqrt{144}$
Simplify.		$b=12$

Step 6. Check. $5^2 + {\color{red}12}^2 \overset{?}{=} 13^2$ $25 + 144 \overset{?}{=} 169$ $169 = 169 \checkmark$		
Step 7. Answer the question.		The length of the leg is 12.

Note:
Exercise:

Problem:

TRY IT :: 3.63
Use the Pythagorean Theorem to find the length of the leg in the triangle shown below.



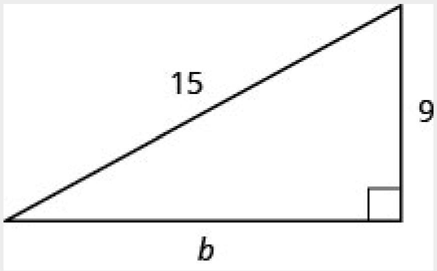
Solution:

8

Note:
Exercise:

Problem:

TRY IT :: 3.64
Use the Pythagorean Theorem to find the length of the leg in the triangle shown below.

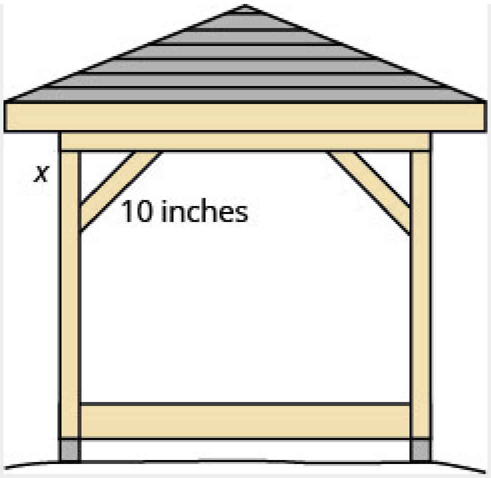


Solution:

12

Example:
Exercise:

Problem:



**Kelvin is building a gazebo and wants to brace each corner by placing a 10″ piece of wood diagonally as shown above.

If he fastens the wood so that the ends of the brace are the same distance from the corner, what is the length of the legs of the right triangle formed? Approximate to the nearest tenth of an inch.

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the distance from the corner that the bracket should be attached
Step 3. Name. Choose a variable to represent it.	Let $x =$ the distance from the corner.

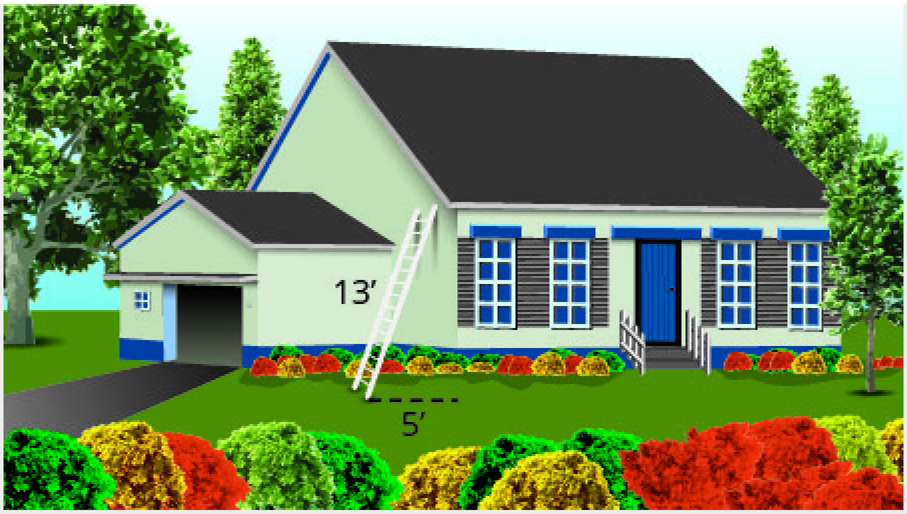
Step 4. Translate Write the appropriate formula and substitute.	$a^2 + b^2 = c^2$ $x^2 + x^2 = 10^2$
Step 5. Solve the equation. Isolate the variable. Use the definition of square root. Simplify. Approximate to the nearest tenth.	$2x^2 = 100$ $x^2 = 50$ $x = \sqrt{50}$ $x \approx 7.1$
Step 6. Check. $a^2 + b^2 = c^2$ $(7.1)^2 + (7.1)^2 \approx 10^2$ Yes.	
Step 7. Answer the question.	Kelvin should fasten each piece of wood approximately 7.1" from the corner.

Note:
Exercise:

Problem:

TRY IT :: 3.65

John puts the base of a 13-foot ladder five feet from the wall of his house as shown below. How far up the wall does the ladder reach?



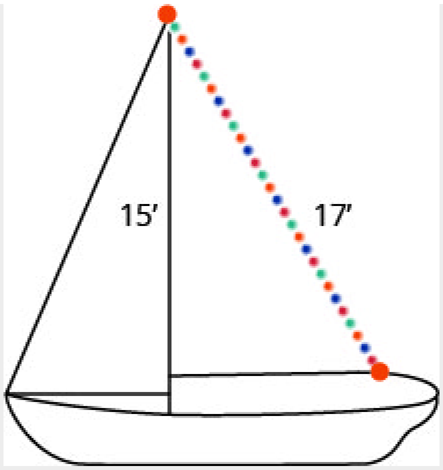
Solution:

12 feet

Note:
Exercise:

Problem:

TRY IT :: 3.66
Randy wants to attach a 17 foot string of lights to the top of the 15 foot mast of his sailboat, as shown below. How far from the base of the mast should he attach the end of the light string?

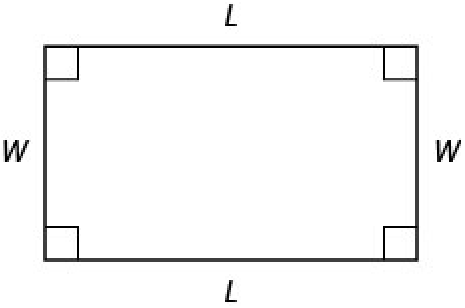


Solution:

8 feet

Solve Applications Using Rectangle Properties

You may already be familiar with the properties of rectangles. Rectangles have four sides and four right (90°) angles. The opposite sides of a rectangle are the same length. We refer to one side of the rectangle as the length, L, and its adjacent side as the width, W.

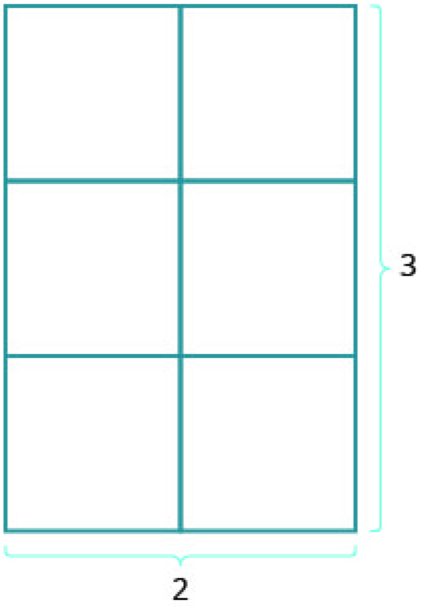


The distance around this rectangle is $L + W + L + W$, or $2L + 2W$. This is the perimeter, P, of the rectangle.

Equation:

$$P = 2L + 2W$$

What about the area of a rectangle? Imagine a rectangular rug that is 2-feet long by 3-feet wide. Its area is 6 square feet. There are six squares in the figure.



Equation:

$$\begin{aligned} A &= 6 \\ A &= 2 \cdot 3 \\ A &= L \cdot W \end{aligned}$$

The area is the length times the width.

The formula for the area of a rectangle is $A = LW$.

Note:
Properties of Rectangles
Rectangles have four sides and four right (90°) angles.
The lengths of opposite sides are equal.

The perimeter of a rectangle is the sum of twice the length and twice the width.
Equation:

$$P = 2L + 2W$$

The area of a rectangle is the product of the length and the width.
Equation:

$$A = L \cdot W$$

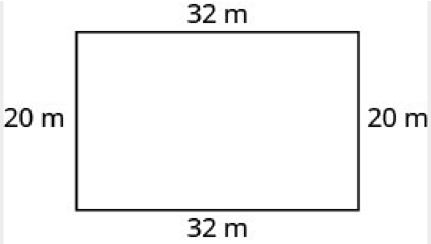
Example:
Exercise:

Problem:

**The length of a rectangle is 32 meters and the width is 20 meters. What is the perimeter?

Solution:
Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	

	
Step 2. Identify what you are looking for.	the perimeter of a rectangle
Step 3. Name. Choose a variable to represent it.	Let P = the perimeter.
Step 4. Translate.	
Write the appropriate formula.	$\underbrace{P}\ \underbrace{=}\ \underbrace{2L}\ \underbrace{+}\ \underbrace{2W}$
Substitute.	$P = 2(32\text{ m}) + 2(20\text{ m})$
Step 5. Solve the equation.	$P = 64 + 40$ $P = 104$
Step 6. Check. $P \stackrel{?}{=} 104$ $20 + 32 + 20 + 32 \stackrel{?}{=} 104$ $104 = 104\checkmark$	
Step 7. Answer the question.	The perimeter of the rectangle is 104 meters.

Note:
Exercise:

Problem:

TRY IT :: 3.67
The length of a rectangle is 120 yards and the width is 50 yards. What is the perimeter?

Solution:

340 yards

Note:
Exercise:

Problem:

TRY IT :: 3.68
The length of a rectangle is 62 feet and the width is 48 feet. What is the perimeter?

Solution:

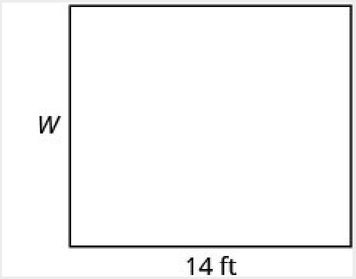
220 feet

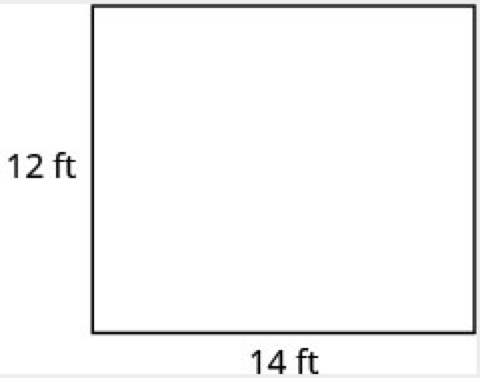
Example:
Exercise:

Problem:

**The area of a rectangular room is 168 square feet. The length is 14 feet. What is the width?

Solution:
Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the width of a rectangular room
Step 3. Name. Choose a variable to represent it.	Let W = the width.
Step 4. Translate.	
Write the appropriate formula.	$A = LW$
Substitute.	$168 = 14W$
Step 5. Solve the equation.	$\frac{168}{14} = \frac{14W}{14}$ $12 = W$

Step 6. Check.  $A = LW$ $168 \overset{?}{=} 14 \cdot 12$ $168 = 168\checkmark$	
Step 7. Answer the question.	The width of the room is 12 feet.

Note:
Exercise:

Problem:

TRY IT :: 3.69
The area of a rectangle is 598 square feet. The length is 23 feet. What is the width?

Solution:

26 feet

Note:
Exercise:

Problem:

TRY IT :: 3.70
The width of a rectangle is 21 meters. The area is 609 square meters. What is the length?

Solution:

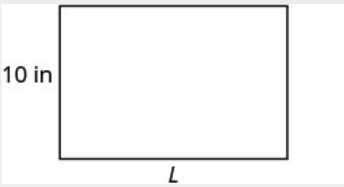
29 meters

Example:
Exercise:

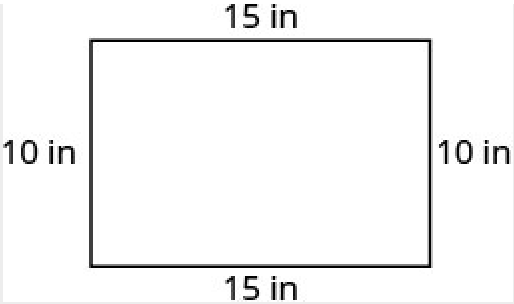
Problem:

**Find the length of a rectangle with perimeter 50 inches and width 10 inches.

Solution:
Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	

	$P = 50$ in
Step 2. Identify what you are looking for.	the length of the rectangle
Step 3. Name. Choose a variable to represent it.	Let L = the length.
Step 4. Translate.	
Write the appropriate formula.	$P = 2L + 2W$
Substitute.	$50 = 2L + 2(10)$
Step 5. Solve the equation.	$50 - 20 = 2L + 20 - 20$ $30 = 2L$ $\frac{30}{2} = \frac{2L}{2}$ $15 = L$
Step 6. Check.	

 $\begin{aligned} P &= 50 \\ 15+10+15+10 &\stackrel{?}{=} 50 \\ 50 &= 50\checkmark \end{aligned}$	
Step 7. Answer the question.	The length is 15 inches.

Note:
Exercise:

Problem:

TRY IT :: 3.71
Find the length of a rectangle with: perimeter 80 and width 25.

Solution:

15

Note:
Exercise:

Problem:

TRY IT :: 3.72
Find the length of a rectangle with: perimeter 30 and width 6.

Solution:

9

We have solved problems where either the length or width was given, along with the perimeter or area; now we will learn how to solve problems in which the width is defined in terms of the length. We will wait to draw the figure until we write an expression for the width so that we can label one side with that expression.

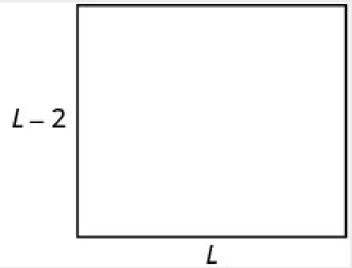
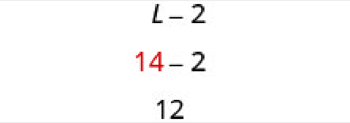
Example:
Exercise:

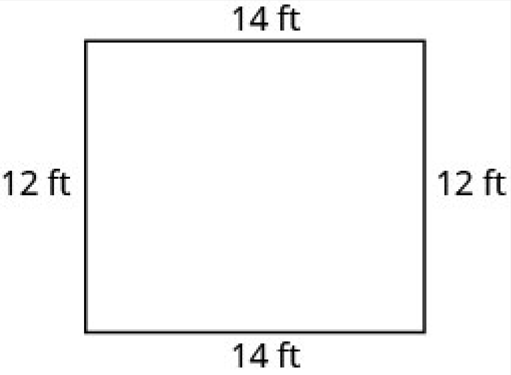
Problem:

**The width of a rectangle is two feet less than the length. The perimeter is 52 feet. Find the length and width.

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what you are	the length and width

looking for.	of a rectangle
Step 3. Name. Choose a variable to represent it. Since the width is defined in terms of the length, we let L = length. The width is two feet less than the length, so we let $L - 2$ = width.	 $P = 52\text{ ft}$
Step 4. Translate.	
Write the appropriate formula. The formula for the perimeter of a rectangle relates all the information.	$P = 2L + 2W$
Substitute in the given information.	$52 = 2L + 2(L - 2)$
Step 5. Solve the equation.	$52 = 2L + 2L - 4$
Combine like terms.	$52 = 4L - 4$
Add 4 to each side.	$56 = 4L$
Divide by 4.	$\frac{56}{4} = \frac{4L}{4}$ $14 = L$ The length is 14 feet.
Now we need to find the width.	The width is $L - 2$.  $L - 2$ $14 - 2$ 12

	The width is 12 feet.
Step 6. Check. Since $14 + 12 + 14 + 12 = 52$, this works!  	
Step 7. Answer the question.	The length is 14 feet and the width is 12 feet.

Note:
Exercise:

Problem:

TRY IT :: 3.73
The width of a rectangle is seven meters less than the length. The perimeter is 58 meters. Find the length and width.

Solution:

18 meters, 11 meters

Note:
Exercise:

Problem:

TRY IT :: 3.74
The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

Solution:

19 feet, 11 feet

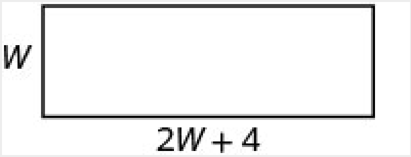
Example:
Exercise:

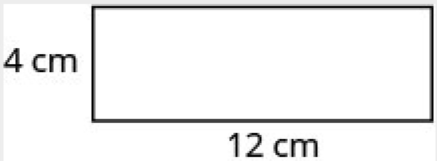
Problem:

**The length of a rectangle is four centimeters more than twice the width. The perimeter is 32 centimeters. Find the length and width.

Solution:
Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length and the width
Step 3. Name. Choose a variable to represent the width.	Let $W =$ width

The length is four more than twice the width.	$2W + 4 = \text{length}$  $P = 32\text{ cm}$
Step 4. Translate	
Write the appropriate formula.	$P = 2L + 2W$
Substitute in the given information.	$32 = 2(2W + 4) + 2W$
Step 5. Solve the equation.	$32 = 4W + 8 + 2W$ $32 = 6W + 8$ $24 = 6W$ $4 = W$ (width)

	$2W + 4$ (length) $2(4) + 4$ 12 The length is 12 cm.
Step 6. Check.  $P = 2L + 2W$ $32 \overset{?}{=} 2 \cdot 12 + 2 \cdot 4$ $32 = 32\checkmark$	
Step 7. Answer the question.	The length is 12 cm and the width is 4 cm.

Note:

Exercise:

Problem:

TRY IT :: 3.75

The length of a rectangle is eight more than twice the width. The perimeter is 64. Find the length and width.

Solution:

24, 8

Note:
Exercise:

Problem:

TRY IT :: 3.76
The width of a rectangle is six less than twice the length. The perimeter is 18. Find the length and width.

Solution:

5, 4

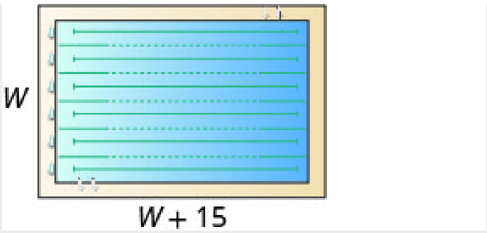
Example:
Exercise:

Problem:

**The perimeter of a rectangular swimming pool is 150 feet. The length is 15 feet more than the width. Find the length and width.

Solution:
Solution

Step 1. Read the problem.	

Draw the figure and label it with the given information.	 $P = 150$ ft
Step 2. Identify what you are looking for.	the length and the width of the pool
Step 3. Name. Choose a variable to represent the width. The length is 15 feet more than the width.	Let $W =$ width $W + 15 =$ length
Step 4. Translate	
Write the appropriate formula.	$P = 2L + 2W$
Substitute.	$150 = 2(W + 15) + 2W$
Step 5. Solve the equation.	$150 = 2W + 30 + 2W$ $150 = 4W + 30$ $120 = 4W$

	$30 = W$ (the width of the pool) $W + 15$ (the length of the pool) $30 + 15$ 45
Step 6. Check. $P = 2L + 2W$ $150 \stackrel{?}{=} 2(45) + 2(30)$ $150 = 150 \checkmark$	
Step 7. Answer the question.	The length of the pool is 45 feet and the width is 30 feet.

Note:
Exercise:

Problem:

TRY IT :: 3.77
The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Solution:

70 feet, 30 feet

Note:
Exercise:

Problem:

TRY IT :: 3.78
The length of a rectangular garden is 30 yards more than the width. The perimeter is 300 yards. Find the length and width.

Solution:

90 yards, 60 yards

Key Concepts

- **Problem-Solving Strategy for Geometry Applications**

  Read the problem and make all the words and ideas are understood. Draw the figure and label it with the given information.
  Identify what we are looking for.
  Name what we are looking for by choosing a variable to represent it.
  Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
  Solve the equation using good algebra techniques.
  Check the answer in the problem and make sure it makes sense.
  Answer the question with a complete sentence.

- **Triangle Properties For $\triangle ABC$**
  Angle measures:

  - $m\angle A + m\angle B + m\angle C = 180$

Perimeter:

  - $P = a + b + c$

Area:

  - $A = \frac{1}{2}bh, \text{ b} = \text{base}, \text{h} = \text{height}$

A right triangle has one 90° angle.

- **The Pythagorean Theorem** In any right triangle, $a^2 + b^2 = c^2$ where c is the length of the hypotenuse and a and b are the lengths of the legs.
- **Properties of Rectangles**
  - Rectangles have four sides and four right (90°) angles.
  - The lengths of opposite sides are equal.
  - The perimeter of a rectangle is the sum of twice the length and twice the width: $P = 2L + 2W$. The area of a rectangle is the length times the width: $A = LW$.

Practice Makes Perfect

Solving Applications Using Triangle Properties

In the following exercises, solve using triangle properties.

Exercise:

Problem:

The measures of two angles of a triangle are 26 and 98 degrees. Find the measure of the third angle.

Solution:

56 degrees

Exercise:

Problem:

The measures of two angles of a triangle are 61 and 84 degrees. Find the measure of the third angle.

Exercise:

Problem:

The measures of two angles of a triangle are 105 and 31 degrees. Find the measure of the third angle.

Solution:

44 degrees

Exercise:

Problem:

The measures of two angles of a triangle are 47 and 72 degrees. Find the measure of the third angle.

Exercise:

Problem:

The perimeter of a triangular pool is 36 yards. The lengths of two sides are 10 yards and 15 yards. How long is the third side?

Solution:

11 feet

Exercise:

Problem:

A triangular courtyard has perimeter 120 meters. The lengths of two sides are 30 meters and 50 meters. How long is the third side?

Exercise:

Problem:

If a triangle has sides 6 feet and 9 feet and the perimeter is 23 feet, how long is the third side?

Solution:

8 feet

Exercise:

Problem:

If a triangle has sides 14 centimeters and 18 centimeters and the perimeter is 49 centimeters, how long is the third side?

Exercise:

Problem:

A triangular flag has base one foot and height 1.5 foot. What is its area?

Solution:

0.75 sq. ft.

Exercise:

Problem:

A triangular window has base eight feet and height six feet. What is its area?

Exercise:

Problem:

What is the base of a triangle with area 207 square inches and height 18 inches?

Solution:

23 inches

Exercise:

Problem:

What is the height of a triangle with area 893 square inches and base 38 inches?

Exercise:

Problem:

One angle of a right triangle measures 33 degrees. What is the measure of the other small angle?

Solution:

57

Exercise:

Problem:

One angle of a right triangle measures 51 degrees. What is the measure of the other small angle?

Exercise:

Problem:

One angle of a right triangle measures 22.5 degrees. What is the measure of the other small angle?

Solution:

67.5

Exercise:

Problem:

One angle of a right triangle measures 36.5 degrees. What is the measure of the other small angle?

Exercise:

Problem:

The perimeter of a triangle is 39 feet. One side of the triangle is one foot longer than the second side. The third side is two feet longer than the second side. Find the length of each side.

Solution:

13 ft., 12 ft., 14 ft.

Exercise:

Problem:

The perimeter of a triangle is 35 feet. One side of the triangle is five feet longer than the second side. The third side is three feet longer than the second side. Find the length of each side.

Exercise:

Problem:

One side of a triangle is twice the shortest side. The third side is five feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.

Solution:

3 ft., 6 ft., 8 ft.

Exercise:

Problem:

One side of a triangle is three times the shortest side. The third side is three feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

Exercise:

Problem:

The two smaller angles of a right triangle have equal measures. Find the measures of all three angles.

Solution:

$45^\circ, 45^\circ, 90^\circ$

Exercise:

Problem:

The measure of the smallest angle of a right triangle is 20° less than the measure of the next larger angle. Find the measures of all three angles.

Exercise:

Problem:

The angles in a triangle are such that one angle is twice the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

Solution:

$30^\circ, 60^\circ, 90^\circ$

Exercise:

Problem:

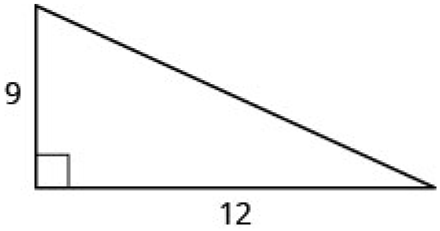
The angles in a triangle are such that one angle is 20° more than the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

Exercise:

Problem:

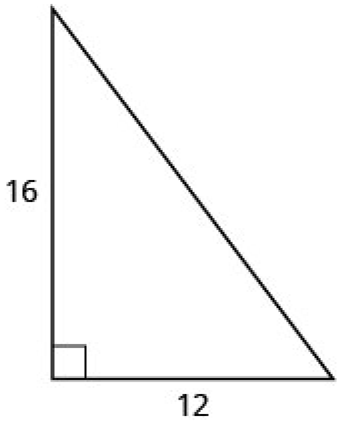


Solution:

15

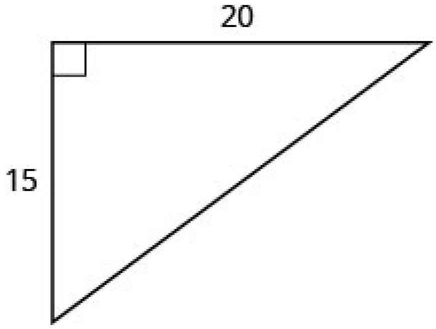
Exercise:

Problem:



Exercise:

Problem:

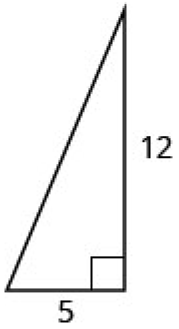


Solution:

25

Exercise:

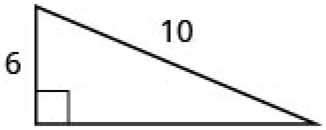
Problem:



In the following exercises, use the Pythagorean Theorem to find the length of the leg. Round to the nearest tenth, if necessary.

Exercise:

Problem:

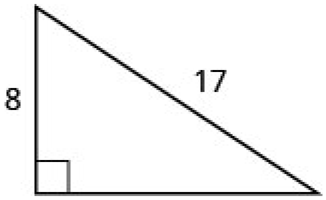


Solution:

8

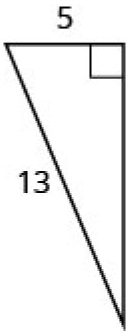
Exercise:

Problem:



Exercise:

Problem:

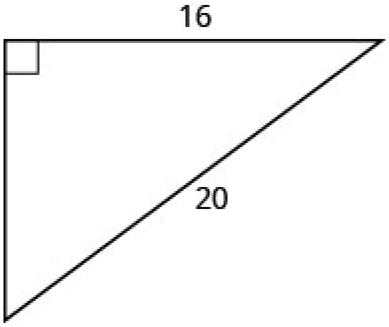


Solution:

12

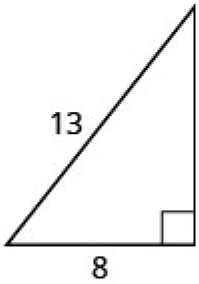
Exercise:

Problem:



Exercise:

Problem:

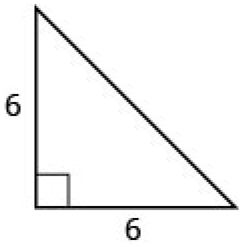


Solution:

10.2

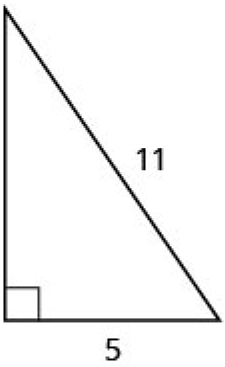
Exercise:

Problem:



Exercise:

Problem:

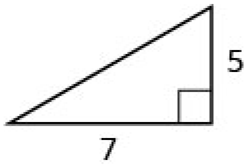


Solution:

9.8

Exercise:

Problem:

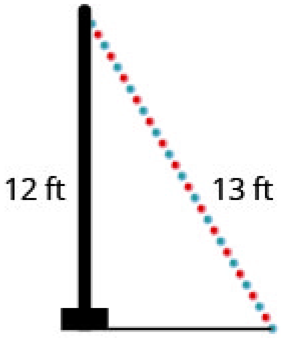


In the following exercises, solve using the Pythagorean Theorem. Approximate to the nearest tenth, if necessary.

Exercise:

Problem:

A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display, as shown below. How far from the base of the pole should the end of the string of lights be anchored?



Solution:

5 feet

Exercise:

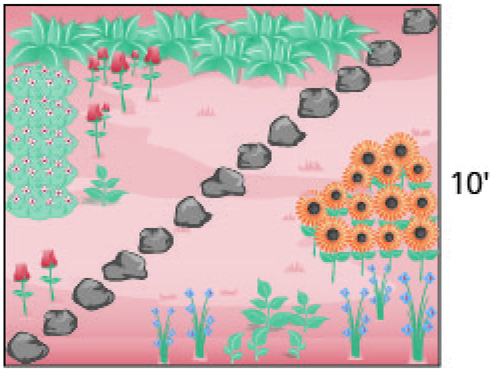
Problem:

Pam wants to put a banner across her garage door, as shown below, to congratulate her son for his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door?

Exercise:

Problem:

Chi is planning to put a path of paving stones through her flower garden, as shown below. The flower garden is a square with side 10 feet. What will the length of the path be?



Solution:

14.1 feet

Exercise:

Problem:

Brian borrowed a 20 foot extension ladder to use when he paints his house. If he sets the base of the ladder 6 feet from the house, as shown below, how far up will the top of the ladder reach?

Solve Applications Using Rectangle Properties

In the following exercises, solve using rectangle properties.

Exercise:

Problem:

The length of a rectangle is 85 feet and the width is 45 feet. What is the perimeter?

Solution:

260 feet

Exercise:

Problem:

The length of a rectangle is 26 inches and the width is 58 inches. What is the perimeter?

Exercise:

Problem:

A rectangular room is 15 feet wide by 14 feet long. What is its perimeter?

Solution:

58 feet

Exercise:

Problem:

A driveway is in the shape of a rectangle 20 feet wide by 35 feet long. What is its perimeter?

Exercise:

Problem:

The area of a rectangle is 414 square meters. The length is 18 meters. What is the width?

Solution:

23 meters

Exercise:

Problem:

The area of a rectangle is 782 square centimeters. The width is 17 centimeters. What is the length?

Exercise:

Problem:

The width of a rectangular window is 24 inches. The area is 624 square inches. What is the length?

Solution:

26 inches

Exercise:

Problem:

The length of a rectangular poster is 28 inches. The area is 1316 square inches. What is the width?

Exercise:

Problem:

Find the length of a rectangle with perimeter 124 and width 38.

Solution:

24

Exercise:

Problem:

Find the width of a rectangle with perimeter 92 and length 19.

Exercise:

Problem:

Find the width of a rectangle with perimeter 16.2 and length 3.2.

Solution:

4.9

Exercise:

Problem:

Find the length of a rectangle with perimeter 20.2 and width 7.8.

Exercise:

Problem:

The length of a rectangle is nine inches more than the width. The perimeter is 46 inches. Find the length and the width.

Solution:

16 in., 7 in.

Exercise:

Problem:

The width of a rectangle is eight inches more than the length. The perimeter is 52 inches. Find the length and the width.

Exercise:

Problem:

The perimeter of a rectangle is 58 meters. The width of the rectangle is five meters less than the length. Find the length and the width of the rectangle.

Solution:

17 m, 12 m

Exercise:

Problem:

The perimeter of a rectangle is 62 feet. The width is seven feet less than the length. Find the length and the width.

Exercise:

Problem:

The width of the rectangle is 0.7 meters less than the length. The perimeter of a rectangle is 52.6 meters. Find the dimensions of the rectangle.

Solution:

13.5 m length, 12.8 m width

Exercise:

Problem:

The length of the rectangle is 1.1 meters less than the width. The perimeter of a rectangle is 49.4 meters. Find the dimensions of the rectangle.

Exercise:

Problem:

The perimeter of a rectangle is 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.

Solution:

50 ft., 25 ft.

Exercise:

Problem:

The length of a rectangle is three times the width. The perimeter of the rectangle is 72 feet. Find the length and width of the rectangle.

Exercise:

Problem:

The length of a rectangle is three meters less than twice the width. The perimeter of the rectangle is 36 meters. Find the dimensions of the rectangle.

Solution:

7 m width, 11 m length

Exercise:

Problem:

The length of a rectangle is five inches more than twice the width. The perimeter is 34 inches. Find the length and width.

Exercise:

Problem:

The perimeter of a rectangular field is 560 yards. The length is 40 yards more than the width. Find the length and width of the field.

Solution:

160 yd., 120 yd.

Exercise:

Problem:

The perimeter of a rectangular atrium is 160 feet. The length is 16 feet more than the width. Find the length and width of the atrium.

Exercise:

Problem:

A rectangular parking lot has perimeter 250 feet. The length is five feet more than twice the width. Find the length and width of the parking lot.

Solution:

85 ft., 40 ft.

Exercise:

Problem:

A rectangular rug has perimeter 240 inches. The length is 12 inches more than twice the width. Find the length and width of the rug.

Everyday Math

Exercise:

Problem:

Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are six feet, eight feet and 10 feet. How many feet of fencing will she need to enclose her flowerbed?

Solution:

24 feet

Exercise:

Problem:

Jose just removed the children’s playset from his back yard to make room for a rectangular garden. He wants to put a fence around the garden to keep out the dog. He has a 50 foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other length?

Writing Exercises

Exercise:

Problem:

If you need to put tile on your kitchen floor, do you need to know the perimeter or the area of the kitchen? Explain your reasoning.

Solution:

area; answers will vary

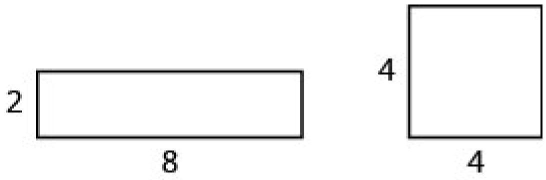
Exercise:

Problem:

If you need to put a fence around your backyard, do you need to know the perimeter or the area of the backyard? Explain your reasoning.

Exercise:

Problem: Look at the two figures below.



ⓐ Which figure looks like it has the larger area?
ⓑ Which looks like it has the larger perimeter?
ⓒ Now calculate the area and perimeter of each figure.
ⓓ Which has the larger area?
ⓔ Which has the larger perimeter?

Solution:

ⓐ Answers will vary.
ⓑ Answers will vary.
ⓒ Answers will vary.
ⓓ The areas are the same.
ⓔ The 2x8 rectangle has a larger perimeter than the 4x4 square.

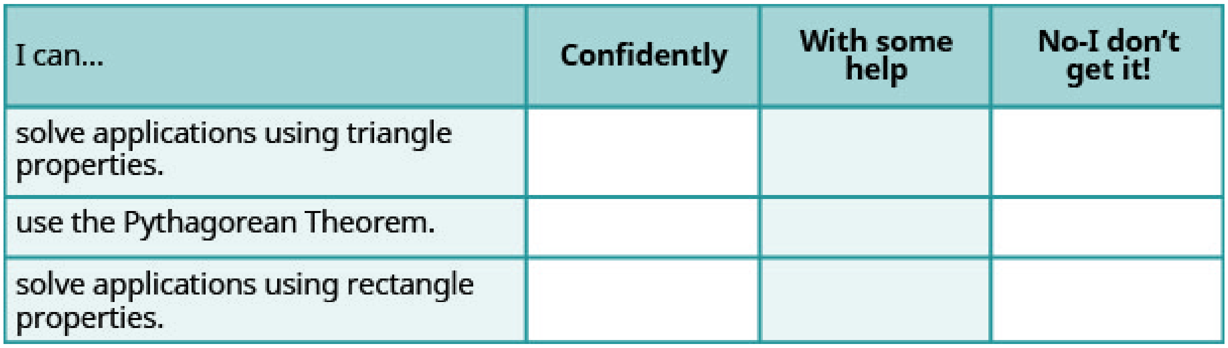
Exercise:

Problem:

Write a geometry word problem that relates to your life experience, then solve it and explain all your steps.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
solve applications using triangle properties.			
use the Pythagorean Theorem.			
solve applications using rectangle properties.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Solve Uniform Motion Applications
By the end of this section, you will be able to:

- Solve uniform motion applications

Note:
Before you get started, take this readiness quiz.

1. Find the distance travelled by a car going 70 miles per hour for 3 hours.
   If you missed this problem, review [link].
2. Solve $x+1.2(x-10)=98$.
   If you missed this problem, review [link].
3. Convert 90 minutes to hours.
   If you missed this problem, review [link].

Solve Uniform Motion Applications

When planning a road trip, it often helps to know how long it will take to reach the destination or how far to travel each day. We would use the distance, rate, and time formula, $D=rt$, which we have already seen.

In this section, we will use this formula in situations that require a little more algebra to solve than the ones we saw earlier. Generally, we will be looking at comparing two scenarios, such as two vehicles travelling at different rates or in opposite directions. When the speed of each vehicle is constant, we call applications like this *uniform motion problems*.

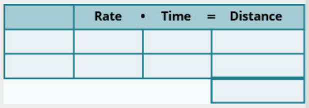
Our problem-solving strategies will still apply here, but we will add to the first step. The first step will include drawing a diagram that shows what is happening in the example. Drawing the diagram helps us understand what is happening so that we will write an appropriate equation. Then we will make a table to organize the information, like we did for the money applications.

The steps are listed here for easy reference:

Note:
Use a Problem-Solving Strategy in Distance, Rate, and Time Applications.

Read the problem. Make sure all the words and ideas are understood.

- Draw a diagram to illustrate what it happening.
- Create a table to organize the information.
- Label the columns rate, time, distance.
- List the two scenarios.
- Write in the information you know.



	Rate	• Time	= Distance

Identify what we are looking for.

Name what we are looking for. Choose a variable to represent that quantity.

- Complete the chart.
- Use variable expressions to represent that quantity in each row.
- Multiply the rate times the time to get the distance.

Translate into an equation.

- Restate the problem in one sentence with all the important information.
- Then, translate the sentence into an equation.

Solvethe equation using good algebra techniques.
Checkthe answer in the problem and make sure it makes sense.
Answerthe question with a complete sentence.

Example:
Exercise:

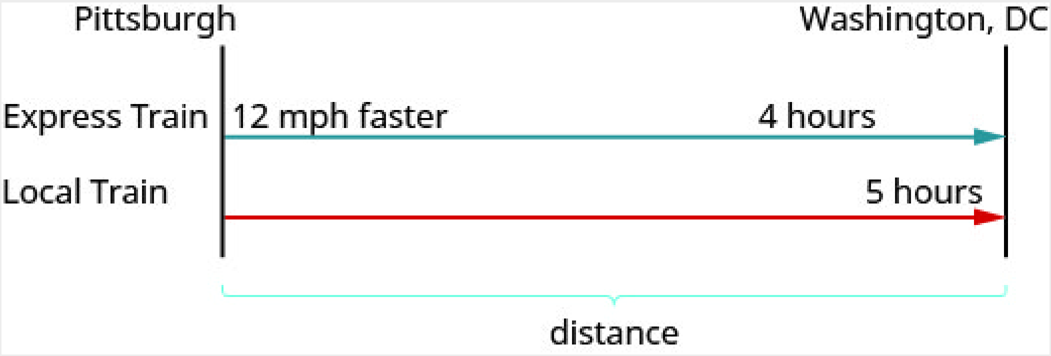
Problem:

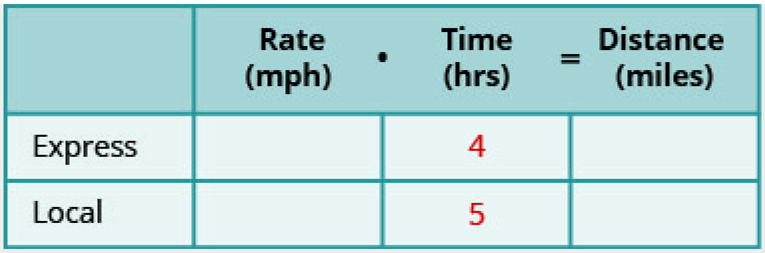
**An express train and a local train leave Pittsburgh to travel to Washington, D.C. The express train can make the trip in 4 hours and the local train takes 5 hours for the trip. The speed of the express train is 12 miles per hour faster than the speed of the local train. Find the speed of both trains.

Solution:
Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

- Draw a diagram to illustrate what it happening. Shown below is a sketch of what is happening in the example.





	Rate (mph) •	Time (hrs) =	Distance (miles)
Express		4	
Local		5	

- Create a table to organize the information.
- Label the columns “Rate,” “Time,” and “Distance.”
- List the two scenarios.
- Write in the information you know.

Step 2. Identify what we are looking for.

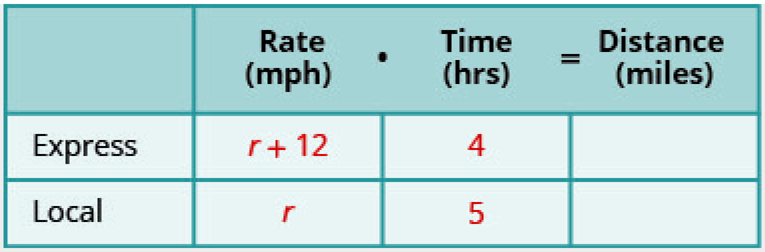
- We are asked to find the speed of both trains.
- Notice that the distance formula uses the word “rate,” but it is more common to use “speed” when we talk about vehicles in everyday English.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

- Complete the chart
- Use variable expressions to represent that quantity in each row.
- We are looking for the speed of the trains. Let’s let *r* represent the speed of the local train. Since the speed of the express train is 12 mph faster, we represent that as $r + 12$.

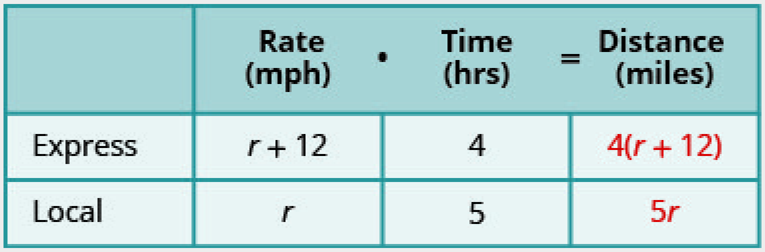
$$\begin{aligned} r &= \text{speed of the local train} \\ r+12 &= \text{speed of the express train} \end{aligned}$$

Fill in the speeds into the chart.



	Rate (mph) •	Time (hrs) =	Distance (miles)
Express	$r + 12$	4	
Local	r	5	

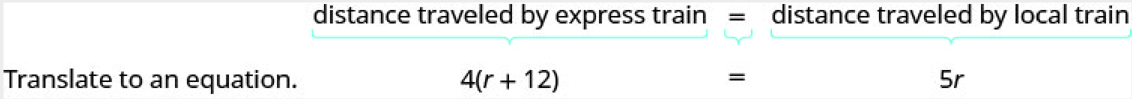
Multiply the rate times the time to get the distance.



	Rate (mph) •	Time (hrs) =	Distance (miles)
Express	$r + 12$	4	$4(r + 12)$
Local	r	5	$5r$

Step 4. Translate into an equation.

- Restate the problem in one sentence with all the important information.
- Then, translate the sentence into an equation.
- The equation to model this situation will come from the relation between the distances. Look at the diagram we drew above. How is the distance travelled by the express train related to the distance travelled by the local train?
- Since both trains leave from Pittsburgh and travel to Washington, D.C. they travel the same distance. So we write:



$$\underbrace{\text{distance traveled by express train}}_{4(r+12)} \underbrace{=}_{=} \underbrace{\text{distance traveled by local train}}_{5r}$$

Translate to an equation. $4(r + 12) = 5r$

Step 5. Solve the equation using good algebra techniques.

Now solve this equation.	$4(r+12)=5r$ $4r+48=5r$ $48=r$ So the speed of the local train is 48 mph.
Find the speed of the express train.	$r+12$ $48+12$ 60 The speed of the express train is 60 mph.

Step 6. Check the answer in the problem and make sure it makes sense.

express train	$60 \text{ mph } (4 \text{ hours}) = 240 \text{ miles}$
local train	$48 \text{ mph } (5 \text{ hours}) = 240 \text{ miles}$ ✓

Step 7. Answer the question with a complete sentence.

- The speed of the local train is 48 mph and the speed of the express train is 60 mph.

Note:
Exercise:

Problem:

TRY IT :: 3.79
Wayne and Dennis like to ride the bike path from Riverside Park to the beach. Dennis's speed is seven miles per hour faster than Wayne's speed, so it takes Wayne 2 hours to ride to the beach while it takes Dennis 1.5 hours for the ride. Find the speed of both bikers.

Solution:

Wayne 21 mph, Dennis 28 mph

Note:
Exercise:

Problem:

TRY IT :: 3.80
Jeromy can drive from his house in Cleveland to his college in Chicago in 4.5 hours. It takes his mother 6 hours to make the same drive. Jeromy drives 20 miles per hour faster than his mother. Find Jeromy's speed and his mother's speed.

Solution:

Jeromy 80 mph, mother 60 mph

In [link], the last example, we had two trains traveling the same distance. The diagram and the chart helped us write the equation we solved. Let’s see how this works in another case.

Example:
Exercise:

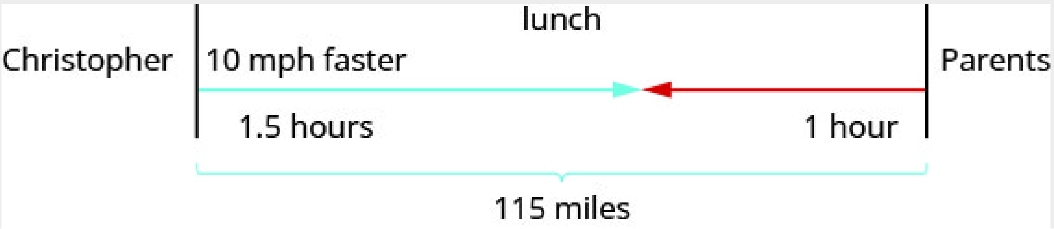
Problem:

**Christopher and his parents live 115 miles apart. They met at a restaurant between their homes to celebrate his mother’s birthday. Christopher drove 1.5 hours while his parents drove 1 hour to get to the restaurant. Christopher’s average speed was 10 miles per hour faster than his parents’ average speed. What were the average speeds of Christopher and of his parents as they drove to the restaurant?

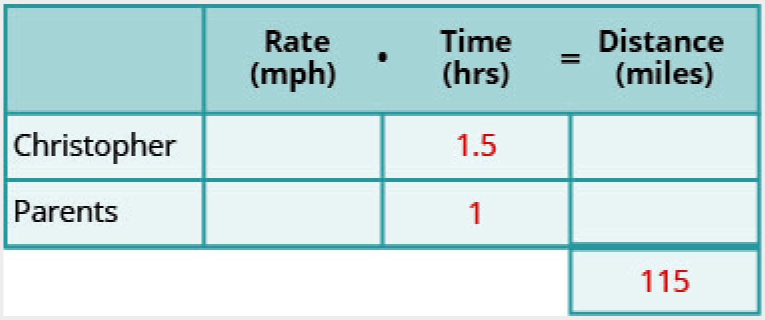
Solution:
Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

- Draw a diagram to illustrate what it happening. Below shows a sketch of what is happening in the example.



- Create a table to organize the information.
- Label the columns rate, time, distance.
- List the two scenarios.
- Write in the information you know.



	Rate (mph) •	Time (hrs) =	Distance (miles)
Christopher		1.5	
Parents		1	
			115

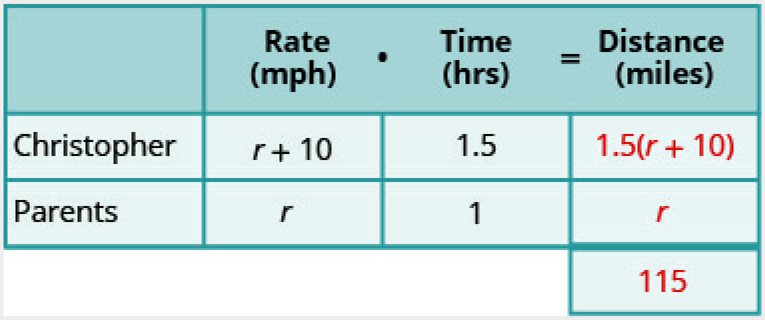
Step 2. Identify what we are looking for.

- We are asked to find the average speeds of Christopher and his parents.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

- Complete the chart.
- Use variable expressions to represent that quantity in each row.
- We are looking for their average speeds. Let's let *r* represent the average speed of the parents. Since the Christopher's speed is 10 mph faster, we represent that as $r + 10$.

Fill in the speeds into the chart.



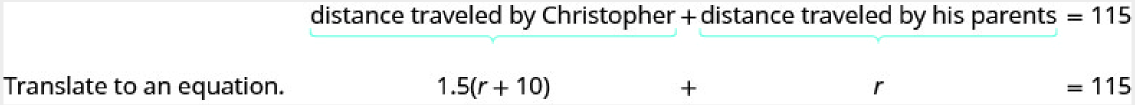
	Rate (mph) •	Time (hrs) =	Distance (miles)
Christopher	$r + 10$	1.5	$1.5(r + 10)$
Parents	r	1	r
			115

Multiply the rate times the time to get the distance.

Step 4. Translate into an equation.

- Restate the problem in one sentence with all the important information.
- Then, translate the sentence into an equation.
- Again, we need to identify a relationship between the distances in order to write an equation. Look at the diagram we created above and notice the relationship between the distance Christopher traveled and the distance his parents traveled.

The distance Christopher travelled plus the distance his parents travel must add up to 115 miles. So we write:



	distance traveled by Christopher	+	distance traveled by his parents	= 115
Translate to an equation.	$1.5(r + 10)$	+	r	= 115

Step 5. Solve the equation using good algebra techniques.

Now solve this equation.	$1.5(r+10)+r = 115$ $1.5r+15+r = 115$ $2.5r+15 = 115$ $2.5r = 100$ $r = 40$
	So the parents' speed was 40 mph.
Christopher's speed is $r+10$.	

	$r+10$ $40+10$ 50
	Christopher's speed was 50 mph.

Step 6. Check the answer in the problem and make sure it makes sense.

Christopher drove	$50 \text{ mph } (1.5 \text{ hours}) \quad = \quad 75 \text{ miles}$
His parents drove	$40 \text{ mph } (1 \text{ hours}) \quad = \quad \underline{40 \text{ miles}}$ 115 miles

Step 7. Answer the question with a complete sentence.	Christopher's speed was 50 mph. His parents' speed was 40 mph.

Note:
Exercise:

Problem:

TRY IT :: 3.81
Carina is driving from her home in Anaheim to Berkeley on the same day her brother is driving from Berkeley to Anaheim, so they decide to meet for lunch along the way in Buttonwillow. The distance from Anaheim to Berkeley is 410 miles. It takes Carina 3 hours to get to Buttonwillow, while her brother drives 4 hours to get there. The average speed Carina's brother drove was 15 miles per hour faster than Carina's average speed. Find Carina's and her brother's average speeds.

Solution:

Carina 50 mph, brother 65 mph

Note:
Exercise:

Problem:

TRY IT :: 3.82
Ashley goes to college in Minneapolis, 234 miles from her home in Sioux Falls. She wants her parents to bring her more winter clothes, so they decide to meet at a restaurant on the road between Minneapolis and Sioux Falls. Ashley and her parents both drove 2 hours to the restaurant. Ashley's average speed was seven miles per hour faster than her parents' average speed. Find Ashley's and her parents' average speed.

Solution:

parents 55 mph, Ashley 62 mph

As you read the next example, think about the relationship of the distances traveled. Which of the previous two examples is more similar to this situation?

Example:
Exercise:

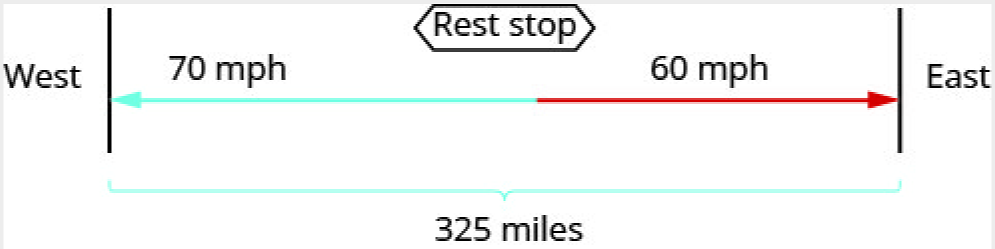
Problem:

**Two truck drivers leave a rest area on the interstate at the same time. One truck travels east and the other one travels west. The truck traveling west travels at 70 mph and the truck traveling east has an average speed of 60 mph. How long will they travel before they are 325 miles apart?

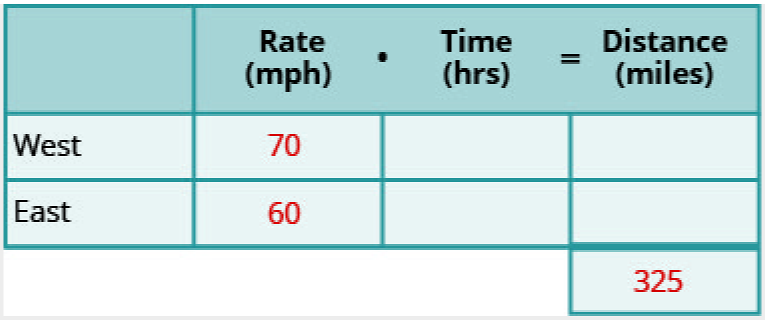
Solution:
Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

- Draw a diagram to illustrate what it happening.



- Create a table to organize the information.



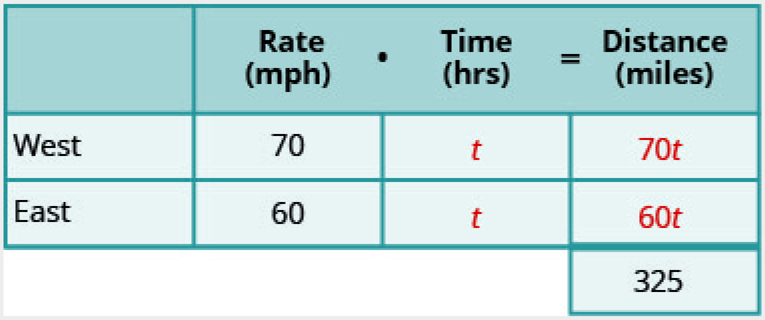
	Rate (mph)	•	Time (hrs)	=	Distance (miles)
West	70				
East	60				
					325

Step 2. Identify what we are looking for.

- We are asked to find the amount of time the trucks will travel until they are 325 miles apart.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

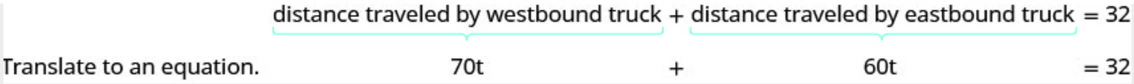
- We are looking for the time travelled. Both trucks will travel the same amount of time. Let's call the time *t*. Since their speeds are different, they will travel different distances.
- Complete the chart.



	Rate (mph)	•	Time (hrs)	=	Distance (miles)
West	70		t		$70t$
East	60		t		$60t$
					325

Step 4. Translate into an equation.

- We need to find a relation between the distances in order to write an equation. Looking at the diagram, what is the relationship between the distance each of the trucks will travel?
- The distance traveled by the truck going west plus the distance travelled by the truck going east must add up to 325 miles. So we write:



	distance traveled by westbound truck	+	distance traveled by eastbound truck	= 32
Translate to an equation.	70t	+	60t	= 32

Step 5. Solve the equation using good algebra techniques.

Now solve this equation.	$\begin{aligned} 70t + 60t &= 325 \\ 130t &= 325 \\ t &= 2.5 \end{aligned}$

So it will take the trucks 2.5 hours to be 325 miles apart.

Step 6. Check the answer in the problem and make sure it makes sense.

Truck going West	70 mph (2.5 hours)	=	175 miles
Truck going East	60 mph (2.5 hours)	=	$\underline{\text{150 miles}}$ 325 miles

Step 7. Answer the question with a complete sentence.	It will take the trucks 2.5 hours to be 325 miles apart.

Note:
Exercise:

Problem:

TRY IT :: 3.83
Pierre and Monique leave their home in Portland at the same time. Pierre drives north on the turnpike at a speed of 75 miles per hour while Monique drives south at a speed of 68 miles per hour. How long will it take them to be 429 miles apart?

Solution:

3 hours

Note:
Exercise:

Problem:

TRY IT :: 3.84
Thanh and Nhat leave their office in Sacramento at the same time. Thanh drives north on I-5 at a speed of 72 miles per hour. Nhat drives south on I-5 at a speed of 76 miles per hour. How long will it take them to be 330 miles apart? (Round to the nearest tenth of an hour.)

Solution:

2.2 hours

Note:

Matching Units in Problems

It is important to make sure the units match when we use the distance rate and time formula. For instance, if the rate is in miles per hour, then the time must be in hours.

Example:

Exercise:

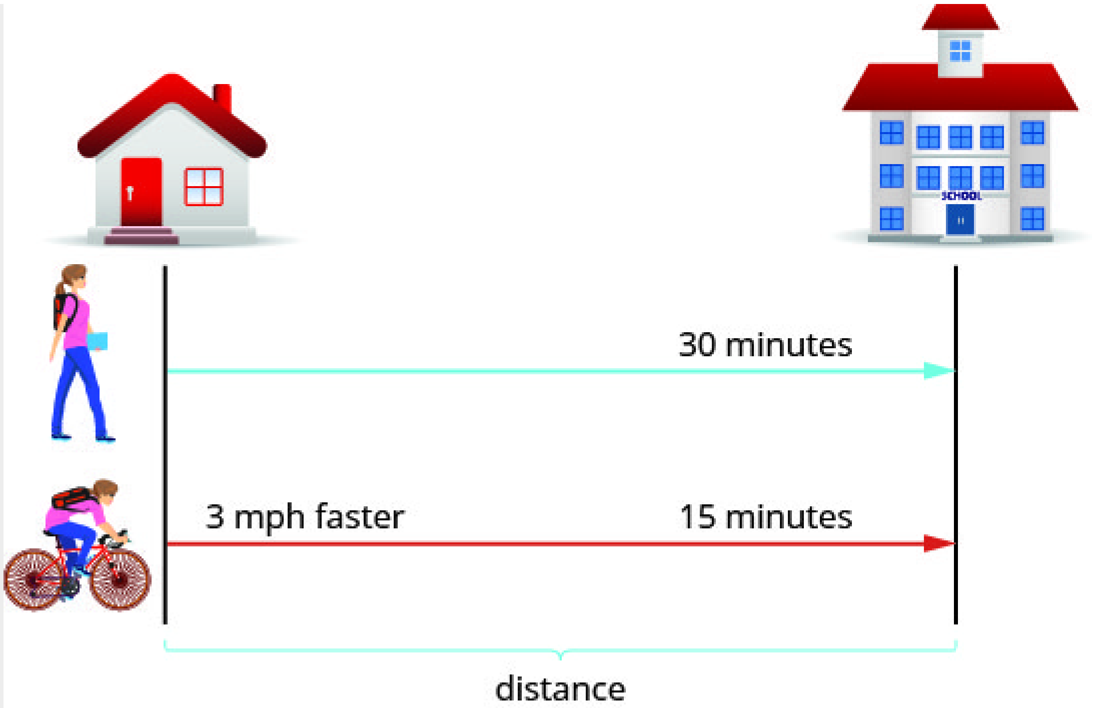
Problem:

**When Katie Mae walks to school, it takes her 30 minutes. If she rides her bike, it takes her 15 minutes. Her speed is three miles per hour faster when she rides her bike than when she walks. What are her walking speed and her speed riding her bike?

Solution:

Solution

First, we draw a diagram that represents the situation to help us see what is happening.



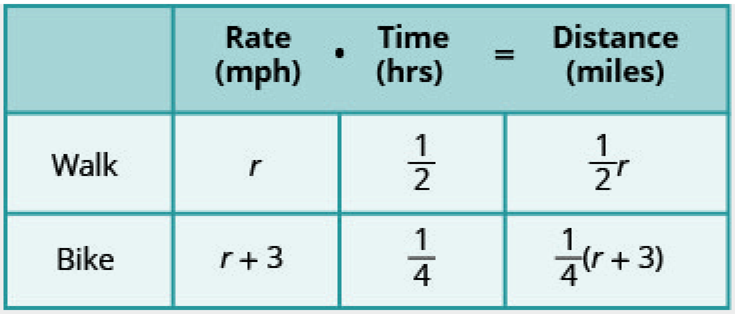
We are asked to find her speed walking and riding her bike. Let’s call her walking speed r. Since her biking speed is three miles per hour faster, we will call that speed $r + 3$. We write the speeds in the chart.

The speed is in miles per hour, so we need to express the times in hours, too, in order for the units to be the same. Remember, one hour is 60 minutes. So:

Equation:

$$\begin{array}{l} \text{30 minutes is } \frac{30}{60} \text{ or } \frac{1}{2} \text{ hour} \\ \text{15 minutes is } \frac{15}{60} \text{ or } \frac{1}{4} \text{ hour} \end{array}$$

Next, we multiply rate times time to fill in the distance column.



	Rate (mph)	• Time (hrs)	= Distance (miles)
Walk	r	$\frac{1}{2}$	$\frac{1}{2}r$
Bike	$r+3$	$\frac{1}{4}$	$\frac{1}{4}(r+3)$

The equation will come from the fact that the distance from Katie Mae's home to her school is the same whether she is walking or riding her bike.

So we say:

	$\underbrace{\text{distance walked}} = \underbrace{\text{distance covered by bike}}$
Translate into an equation.	$\frac{1}{2}r \quad = \quad \frac{1}{4}(r+3)$
Solve this equation.	$\frac{1}{2}r = \frac{1}{4}(r+3)$
Clear the fractions by multiplying by the LCD of all the fractions in the equation.	$8 \cdot \frac{1}{2}r = 8 \cdot \frac{1}{4}(r+3)$
Simplify.	$4r = 2(r+3)$ $4r = 2r + 6$

	$2r = 6$ $r = 3$ mph (Katie Mae’s walking speed) $r + 3$ biking speed $3 + 3$ 6 mph (Katie Mae's biking speed)
Let's check if this works. Walk 3 mph (0.5 hour) = 1.5 miles Bike 6 mph (0.25 hour) = 1.5 miles	
Yes, either way Katie Mae travels 1.5 miles to school.	Katie Mae’s walking speed is 3 mph. Her speed riding her bike is 6 mph.

Note:
Exercise:

Problem:

TRY IT :: 3.85
Suzy takes 50 minutes to hike uphill from the parking lot to the lookout tower. It takes her 30 minutes to hike back down to the parking lot. Her speed going downhill is 1.2 miles per hour faster than her speed going uphill. Find Suzy's uphill and downhill speeds.

Solution:

uphill 1.8 mph, downhill three mph

Note:
Exercise:

Problem:

TRY IT :: 3.86
Llewyn takes 45 minutes to drive his boat upstream from the dock to his favorite fishing spot. It takes him 30 minutes to drive the boat back downstream to the dock. The boat's speed going downstream is four miles per hour faster than its speed going upstream. Find the boat's upstream and downstream speeds.

Solution:

upstream 8 mph, downstream 12 mph

In the distance, rate, and time formula, time represents the actual amount of elapsed time (in hours, minutes, etc.). If a problem gives us starting and ending times as clock times, we must find the elapsed time in order to use the formula.

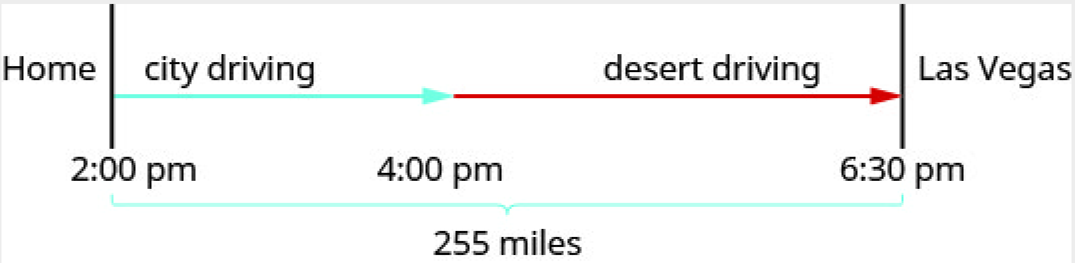
Example:
Exercise:

Problem:

**Hamilton loves to travel to Las Vegas, 255 miles from his home in Orange County. On his last trip, he left his house at 2:00 pm. The first part of his trip was on congested city freeways. At 4:00 pm, the traffic cleared and he was able to drive through the desert at a speed 1.75 times faster than when he drove in the congested area. He arrived in Las Vegas at 6:30 pm. How fast was he driving during each part of his trip?

Solution:
Solution

A diagram will help us model this trip.

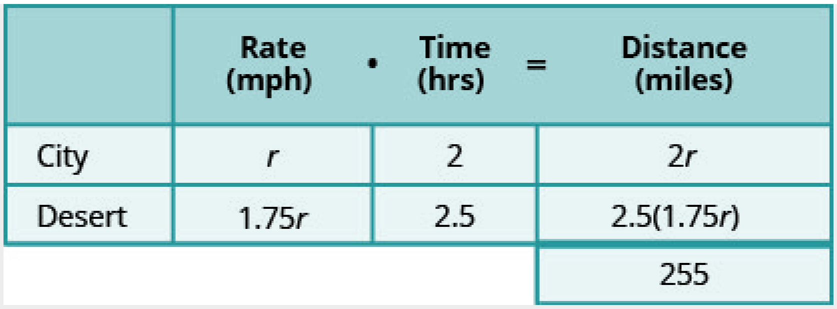


Next, we create a table to organize the information.

We know the total distance is 255 miles. We are looking for the rate of speed for each part of the trip. The rate in the desert is 1.75 times the rate in the city. If we let $r =$ the rate in the city, then the rate in the desert is $1.75r$.

The times here are given as clock times. Hamilton started from home at 2:00 pm and entered the desert at 4:30 pm. So he spent two hours driving the congested freeways in the city. Then he drove faster from 4:00 pm until 6:30 pm in the desert. So he drove 2.5 hours in the desert.

Now, we multiply the rates by the times.



	Rate (mph) •	Time (hrs) =	Distance (miles)
City	r	2	$2r$
Desert	$1.75r$	2.5	$2.5(1.75r)$
			255

By looking at the diagram below, we can see that the sum of the distance driven in the city and the distance driven in the desert is 255 miles.

	$\underbrace{\text{distance driven in the city}} + \underbrace{\text{distance driven in desert}} = 255$
Translate into an equation.	$2r + 2.5(1.75r) = 255$
Solve this equation.	$2r + 2.5(1.75r) = 255$ $2r + 4.375r = 255$ $6.375r = 255$ $r = 40$ mph city $1.75r$ desert speed

	$1.75(40)$ 70 mph
Check. City 40 mph (2 hours) = 80 miles Desert 70 mph (2.5 hours) = 175 miles 255 miles	
	Hamilton drove 40 mph in the city and 70 mph in the desert.

Note:

Exercise:

Problem:

TRY IT :: 3.87

Cruz is training to compete in a triathlon. He left his house at 6:00 and ran until 7:30. Then he rode his bike until 9:45. He covered a total distance of 51 miles. His speed when biking was 1.6 times his speed when running. Find Cruz’s biking and running speeds.

Solution:

biking 16 mph, running 10 mph

Note:
Exercise:

Problem:

TRY IT :: 3.88
Phuong left home on his bicycle at 10:00. He rode on the flat street until 11:15, then rode uphill until 11:45. He rode a total of 31 miles. His speed riding uphill was 0.6 times his speed on the flat street. Find his speed biking uphill and on the flat street.

Solution:

uphill 12 mph, flat street 20 mph

Key Concepts

- **Distance, Rate, and Time**
  - $D = rt$ where D = distance, r = rate, t = time
- **Problem-Solving Strategy—Distance, Rate, and Time Applications**

Read	the problem. Make sure all the words and ideas are understood.	Draw a diagram to illustrate what it happening.	Create a table to organize the information: Label the columns rate, time, distance. List the two scenarios. Write in the information you know.	
Identify	what we are looking for.			
Name	what we are looking for. Choose a variable to represent that quantity.	Complete the chart.	Use variable expressions to represent that quantity in each row.	Multiply the rate times the time to get the distance.
Translate	into an equation.	Restate the problem in one sentence with all the important	Then, translate the sentence into an	

information. equation.

Solvethe equation using good algebra techniques.
Checkthe answer in the problem and make sure it makes sense.
Answerthe question with a complete sentence.

Practice Makes Perfect

Solve Uniform Motion Applications

In the following exercises, solve.

Exercise:

Problem:

Lilah is moving from Portland to Seattle. It takes her three hours to go by train. Mason leaves the train station in Portland and drives to the train station in Seattle with all Lilah’s boxes in his car. It takes him 2.4 hours to get to Seattle, driving at 15 miles per hour faster than the speed of the train. Find Mason’s speed and the speed of the train.

Solution:

Mason 75 mph, train 60 mph

Exercise:

Problem:

Kathy and Cheryl are walking in a fundraiser. Kathy completes the course in 4.8 hours and Cheryl completes the course in 8 hours. Kathy walks two miles per hour faster than Cheryl. Find Kathy’s speed and Cheryl’s speed.

Exercise:

Problem:

Two busses go from Sacramento for San Diego. The express bus makes the trip in 6.8 hours and the local bus takes 10.2 hours for the trip. The speed of the express bus is 25 mph faster than the speed of the local bus. Find the speed of both busses.

Solution:

express bus 75mph, local 50 mph

Exercise:

Problem:

A commercial jet and a private airplane fly from Denver to Phoenix. It takes the commercial jet 1.1 hours for the flight, and it takes the private airplane 1.8 hours. The speed of the commercial jet is 210 miles per hour faster than the speed of the private airplane. Find the speed of both airplanes.

Exercise:

Problem:

Saul drove his truck 3 hours from Dallas towards Kansas City and stopped at a truck stop to get dinner. At the truck stop he met Erwin, who had driven 4 hours from Kansas City towards Dallas. The distance between Dallas and Kansas City is 542 miles, and Erwin’s speed was eight miles per hour slower than Saul’s speed. Find the speed of the two truckers.

Solution:

Saul 82 mph, Erwin 74 mph

Exercise:

Problem:

Charlie and Violet met for lunch at a restaurant between Memphis and New Orleans. Charlie had left Memphis and drove 4.8 hours towards New Orleans. Violet had left New Orleans and drove 2 hours towards Memphis, at a speed 10 miles per hour faster than Charlie's speed. The distance between Memphis and New Orleans is 394 miles. Find the speed of the two drivers.

Exercise:

Problem:

Sisters Helen and Anne live 332 miles apart. For Thanksgiving, they met at their other sister's house partway between their homes. Helen drove 3.2 hours and Anne drove 2.8 hours. Helen's average speed was four miles per hour faster than Anne's. Find Helen's average speed and Anne's average speed.

Solution:

Helen 60 mph, Anne 56 mph

Exercise:

Problem:

Ethan and Leo start riding their bikes at the opposite ends of a 65-mile bike path. After Ethan has ridden 1.5 hours and Leo has ridden 2 hours, they meet on the path. Ethan's speed is six miles per hour faster than Leo's speed. Find the speed of the two bikers.

Exercise:

Problem:

Elvira and Aletheia live 3.1 miles apart on the same street. They are in a study group that meets at a coffee shop between their houses. It took Elvira half an hour and Aletheia two-thirds of an hour to walk to the coffee shop. Aletheia's speed is 0.6 miles per hour slower than Elvira's speed. Find both women's walking speeds.

Solution:

Aletheia 2.4 mph, Elvira 3 mph

Exercise:

Problem:

DaMarcus and Fabian live 23 miles apart and play soccer at a park between their homes. DaMarcus rode his bike for three-quarters of an hour and Fabian rode his bike for half an hour to get to the park. Fabian’s speed was six miles per hour faster than DaMarcus’ speed. Find the speed of both soccer players.

Exercise:

Problem:

Cindy and Richard leave their dorm in Charleston at the same time. Cindy rides her bicycle north at a speed of 18 miles per hour. Richard rides his bicycle south at a speed of 14 miles per hour. How long will it take them to be 96 miles apart?

Solution:

3 hours

Exercise:

Problem:

Matt and Chris leave their uncle’s house in Phoenix at the same time. Matt drives west on I-60 at a speed of 76 miles per hour. Chris drives east on I-60 at a speed of 82 miles per hour. How many hours will it take them to be 632 miles apart?

Exercise:

Problem:

Two busses leave Billings at the same time. The Seattle bus heads west on I-90 at a speed of 73 miles per hour while the Chicago bus heads east at a speed of 79 miles an hour. How many hours will it take them to be 532 miles apart?

Solution:

3.5 hours

Exercise:

Problem:

Two boats leave the same dock in Cairo at the same time. One heads north on the Mississippi River while the other heads south. The northbound boat travels four miles per hour. The southbound boat goes eight miles per hour. How long will it take them to be 54 miles apart?

Exercise:

Problem:

Lorena walks the path around the park in 30 minutes. If she jogs, it takes her 20 minutes. Her jogging speed is 1.5 miles per hour faster than her walking speed. Find Lorena’s walking speed and jogging speed.

Solution:

walking 3 mph, jogging 4.5 mph

Exercise:

Problem:

Julian rides his bike uphill for 45 minutes, then turns around and rides back downhill. It takes him 15 minutes to get back to where he started. His uphill speed is 3.2 miles per hour slower than his downhill speed. Find Julian’s uphill and downhill speed.

Exercise:

Problem:

Cassius drives his boat upstream for 45 minutes. It takes him 30 minutes to return downstream. His speed going upstream is three miles per hour slower than his speed going downstream. Find his upstream and downstream speeds.

Solution:

upstream 6 mph, downstream 9 mph

Exercise:

Problem:

It takes Darline 20 minutes to drive to work in light traffic. To come home, when there is heavy traffic, it takes her 36 minutes. Her speed in light traffic is 24 miles per hour faster than her speed in heavy traffic. Find her speed in light traffic and in heavy traffic.

Exercise:

Problem:

At 1:30 Marlon left his house to go to the beach, a distance of 7.6 miles. He rode his skateboard until 2:15, then walked the rest of the way. He arrived at the beach at 3:00. Marlon’s speed on his skateboard is 2.5 times his walking speed. Find his speed when skateboarding and when walking.

Solution:

skateboarding 8 mph, walking 3.2 mph

Exercise:

Problem:

Aaron left at 9:15 to drive to his mountain cabin 108 miles away. He drove on the freeway until 10:45, and then he drove on the mountain road. He arrived at 11:05. His speed on the freeway was three times his speed on the mountain road. Find Aaron's speed on the freeway and on the mountain road.

Exercise:

Problem:

Marisol left Los Angeles at 2:30 to drive to Santa Barbara, a distance of 95 miles. The traffic was heavy until 3:20. She drove the rest of the way in very light traffic and arrived at 4:20. Her speed in heavy traffic was 40 miles per hour slower than her speed in light traffic. Find her speed in heavy traffic and in light traffic.

Solution:

heavy traffic 30 mph, light traffic 70 mph

Exercise:

Problem:

Lizette is training for a marathon. At 7:00 she left her house and ran until 8:15, then she walked until 11:15. She covered a total distance of 19 miles. Her running speed was five miles per hour faster than her walking speed. Find her running and walking speeds.

Everyday Math

Exercise:

Problem:

John left his house in Irvine at 8:35 am to drive to a meeting in Los Angeles, 45 miles away. He arrived at the meeting at 9:50. At 3:30 pm, he left the meeting and drove home. He arrived home at 5:18.

ⓐ What was his average speed on the drive from Irvine to Los Angeles?
ⓑ What was his average speed on the drive from Los Angeles to Irvine?
ⓒ What was the total time he spent driving to and from this meeting?
ⓓ John drove a total of 90 miles roundtrip. Find his average speed. (Round to the nearest tenth.)

Solution:

ⓐ 36 mph ⓑ 25 mph ⓒ 3.05 hours ⓓ 29.5 mph

Exercise:

Problem:

Sarah wants to arrive at her friend’s wedding at 3:00. The distance from Sarah’s house to the wedding is 95 miles. Based on usual traffic patterns, Sarah predicts she can drive the first 15 miles at 60 miles per hour, the next 10 miles at 30 miles per hour, and the remainder of the drive at 70 miles per hour.

ⓐ How long will it take Sarah to drive the first 15 miles?
ⓑ How long will it take Sarah to drive the next 10 miles?
ⓒ How long will it take Sarah to drive the rest of the trip?
ⓓ What time should Sarah leave her house?

Writing Exercises

Exercise:

Problem:

When solving a uniform motion problem, how does drawing a diagram of the situation help you?

Solution:

Answers will vary.

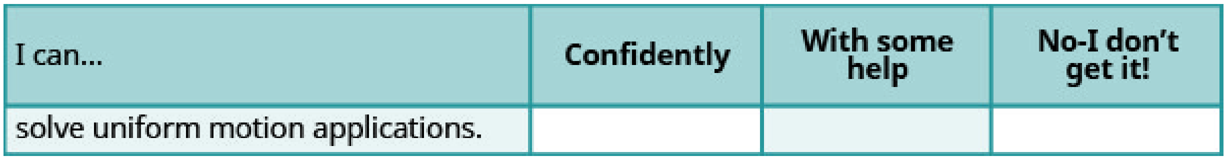
Exercise:

Problem:

When solving a uniform motion problem, how does creating a table help you?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
solve uniform motion applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Solve Proportion and Similar Figure Applications
By the end of this section, you will be able to:

- Solve proportions
- Solve similar figure applications

Note:
Before you get started, take this readiness quiz.
If you miss a problem, go back to the section listed and review the material.

1. Solve $\frac{n}{3}=30$.
   If you missed this problem, review [link].
2. The perimeter of a triangular window is 23 feet. The lengths of two sides are ten feet and six feet. How long is the third side?
   If you missed this problem, review [link].

Solve Proportions

When two rational expressions are equal, the equation relating them is called a *proportion*.

Note:
Proportion
A **proportion** is an equation of the form $\frac{a}{b}=\frac{c}{d}$, where $b\neq 0, d\neq 0$.
The proportion is read "a is to b, as c is to d."

The equation $\frac{1}{2}=\frac{4}{8}$ is a proportion because the two fractions are equal. The proportion $\frac{1}{2}=\frac{4}{8}$ is read "1 is to 2 as 4 is to 8."

Proportions are used in many applications to 'scale up' quantities. We'll start with a very simple example so you can see how proportions work. Even if you can figure out the answer to the example right away, make sure you also learn to solve it using proportions.

Suppose a school principal wants to have 1 teacher for 20 students. She could use proportions to find the number of teachers for 60 students. We let x be the number of teachers for 60 students and then set up the proportion:

Equation:

$$\frac{1\text{ teacher}}{20\text{ students}} = \frac{x\text{ teachers}}{60\text{ students}}$$

We are careful to match the units of the numerators and the units of the denominators—teachers in the numerators, students in the denominators.

Since a proportion is an equation with rational expressions, we will solve proportions the same way we solved equations in Solve Rational Equations. We'll multiply both sides of the equation by the LCD to clear the fractions and then solve the resulting equation.

So let's finish solving the principal's problem now. We will omit writing the units until the last step.

	$\frac{1}{20} = \frac{x}{60}$
Multiply both sides by the LCD, 60.	$\frac{1}{20} \cdot 60 = \frac{x}{60} \cdot 60$
Simplify.	$3 = x$
	The principal needs 3 teachers for 60 students.

Now we'll do a few examples of solving numerical proportions without any units. Then we will solve applications using proportions.

Example:

Exercise:

Problem: **Solve the proportion: $\frac{x}{63} = \frac{4}{7}$.

Solution:
Solution

		$\frac{x}{63} = \frac{4}{7}$
To isolate x, multiply both sides by the LCD, 63.		$63\left(\frac{x}{63}\right) = 63\left(\frac{4}{7}\right)$
Simplify.		$x = \frac{9 \cdot \cancel{7} \cdot 4}{\cancel{7}}$
Divide the common factors.		$x = 36$
Check. To check our answer, we substitute into the original proportion.		
	$\frac{x}{63} = \frac{4}{7}$	
Substitute $x = 36$.	$\frac{36}{63} \stackrel{?}{=} \frac{4}{7}$	
Show common factors.	$\frac{4 \cdot 9}{7 \cdot 9} \stackrel{?}{=} \frac{4}{7}$	
Simplify.	$\frac{4}{7} = \frac{4}{7}$✓	

Note:
Exercise:

TRY IT :: 3.89

Problem: Solve the proportion: $\frac{n}{84} = \frac{11}{12}$.

Solution:

77

Note:
Exercise:

TRY IT :: 3.90

Problem: Solve the proportion: $\frac{y}{96} = \frac{13}{12}$.

Solution:

104

When we work with proportions, we exclude values that would make either denominator zero, just like we do for all rational expressions. What value(s) should be excluded for the proportion in the next example?

Example:
Exercise:

Problem: **Solve the proportion: $\frac{144}{a} = \frac{9}{4}$.

Solution:
Solution

		$\frac{144}{a}=\frac{9}{4}$
Multiply both sides by the LCD.		$\frac{144}{a}\cdot 4a=\frac{9}{4}\cdot 4a$
Remove common factors on each side.		$4\cdot 144=a\cdot 9$
Simplify.		$576=9a$
Divide both sides by 9.		$\frac{576}{9}=\frac{9a}{9}$
Simplify.		$64=a$
Check.		
	$\frac{144}{a}=\frac{9}{4}$	
Substitute $a=64$.	$\frac{144}{64}\overset{?}{=}\frac{9}{4}$	
Show common factors.	$\frac{9\cdot 16}{4\cdot 16}\overset{?}{=}\frac{9}{4}$	
Simplify.	$\frac{9}{4}=\frac{9}{4}$ ✓	

Note:
Exercise:

TRY IT :: 3.91

Problem: Solve the proportion: $\frac{91}{b} = \frac{7}{5}$.

Solution:

65

Note:
Exercise:

TRY IT :: 3.92

Problem: Solve the proportion: $\frac{39}{c} = \frac{13}{8}$.

Solution:

24

Example:
Exercise:

Problem: **Solve the proportion: $\frac{n}{n+14} = \frac{5}{7}$.

Solution:
Solution

		$\frac{n}{n+14} = \frac{5}{7}$
Multiply both sides by the		

LCD.		$7(n+14)\left(\frac{n}{n+14}\right)=7(n+14)\left(\frac{5}{7}\right)$
Remove common factors on each side.		$7n=5(n+14)$
Simplify.		$7n=5n+70$
Solve for n.		$2n=70$
		$n=35$
Check.		
	$\frac{n}{n+14}=\frac{5}{7}$	
Substitute $n=35$.	$\frac{35}{35+14}\overset{?}{=}\frac{5}{7}$	
Simplify.	$\frac{35}{49}\overset{?}{=}\frac{5}{7}$	
Show common factors.	$\frac{5\cdot7}{7\cdot7}\overset{?}{=}\frac{5}{7}$	
Simplify.	$\frac{5}{7}=\frac{5}{7}$ ✓	

Note:
Exercise:

TRY IT :: 3.93

Problem: Solve the proportion: $\frac{y}{y+55} = \frac{3}{8}$.

Solution:

33

Note:
Exercise:

TRY IT :: 3.94

Problem: Solve the proportion: $\frac{z}{z-84} = -\frac{1}{5}$.

Solution:

14

Example:
Exercise:

Problem: **Solve: $\frac{p+12}{9} = \frac{p-12}{6}$.

Solution:
Solution

		$\frac{p+12}{9} = \frac{p-12}{6}$
Multiply both sides by the LCD, 18.		$18\left(\frac{p+12}{9}\right) = 18\left(\frac{p-12}{6}\right)$

Simplify.		$2(p+12)=3(p-12)$
Distribute.		$2p+24=3p-36$
Solve for p.		$60=p$
Check.		
	$\frac{p+12}{9}=\frac{p-12}{6}$	
Substitute $p=60$.	$\frac{60+12}{9}\stackrel{?}{=}\frac{60-12}{6}$	
Simplify.	$\frac{72}{9}\stackrel{?}{=}\frac{48}{6}$	
Divide.	$8=8$ ✓	

Note:
Exercise:

TRY IT :: 3.95

Problem: Solve: $\frac{v+30}{8}=\frac{v+66}{12}$.

Solution:

42

Note:
Exercise:

TRY IT :: 3.96

Problem: Solve: $\frac{2x+15}{9} = \frac{7x+3}{15}$.

Solution:

6

To solve applications with proportions, we will follow our usual strategy for solving applications. But when we set up the proportion, we must make sure to have the units correct—the units in the numerators must match and the units in the denominators must match.

Example:
Exercise:

Problem:

**When pediatricians prescribe acetaminophen to children, they prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of the child's weight. If Zoe weighs 80 pounds, how many milliliters of acetaminophen will her doctor prescribe?

Solution:
Solution

Identify what we are asked to find, and choose a variable to represent it.		How many ml of acetaminophen will the doctor prescribe?
		Let $a =$ ml of acetaminophen.

Write a sentence that gives the information to find it.		If 5 ml is prescribed for every 25 pounds, how much will be prescribed for 80 pounds?
Translate into a proportion–be careful of the units. $\frac{\text{ml}}{\text{pounds}} = \frac{\text{ml}}{\text{pounds}}$		$\frac{5}{25} = \frac{a}{80}$
Multiply both sides by the LCD, 400.		$400\left(\frac{5}{25}\right) = 400\left(\frac{a}{80}\right)$
Remove common factors on each side.		$\cancel{25} \cdot 16\left(\frac{5}{\cancel{25}}\right) = \cancel{80} \cdot 5\left(\frac{a}{\cancel{80}}\right)$
Simplify, but don't multiply on the left. Notice what the next step will be.		$16 \cdot 5 = 5a$
Solve for *a*.		$\frac{16 \cdot 5}{5} = \frac{5a}{5}$
Check.		$16 = a$
Is the answer reasonable?		
Yes, since 80 is about 3 times 25, the medicine should be about 3 times 5. So 16 ml makes sense.		
Substitute $a = 16$ in the original proportion. $\frac{5}{25} = \frac{a}{80}$ $\frac{5}{25} \stackrel{?}{=} \frac{16}{80}$ $\frac{1}{5} = \frac{1}{5}$ ✓		

Write a complete sentence.		The pediatrician would prescribe 16 ml of acetaminophen to Zoe.

Note:
Exercise:

Problem:

TRY IT :: 3.97
Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Emilia, who weighs 60 pounds?

Solution:

12 ml

Note:
Exercise:

Problem:

TRY IT :: 3.98
For every 1 kilogram (kg) of a child's weight, pediatricians prescribe 15 milligrams (mg) of a fever reducer. If Isabella weighs 12 kg, how many milligrams of the fever reducer will the pediatrician prescribe?

Solution:

180 ml

Example:
Exercise:

Problem:

**A 16-ounce iced caramel macchiato has 230 calories. How many calories are there in a 24-ounce iced caramel macchiato?

Solution:
Solution

Identify what we are asked to find, and choose a variable to represent it.		How many calories are in a 24 ounce iced caramel macchiato?
		Let $c =$ calories in 24 ounces.
Write a sentence that gives the information to find it.		If there are 230 calories in 16 ounces, then how many calories are in 24 ounces?
Translate into a proportion–be careful of the units. $\frac{\text{calories}}{\text{ounce}} = \frac{\text{calories}}{\text{ounce}}$		$\frac{230}{16} = \frac{c}{24}$
Multiply both sides by the LCD, 48.		$48\left(\frac{230}{16}\right) = 48\left(\frac{c}{24}\right)$
Remove common factors on each side.		$\cancel{16}\cdot 3\left(\frac{230}{\cancel{16}}\right) = \cancel{24}\cdot 2\left(\frac{c}{\cancel{24}}\right)$
Simplify.		$690 = 2c$
Solve for c.		

		$\frac{690}{2} = \frac{2c}{2}$
		$345 = c$
Check.		
Is the answer reasonable?		
Yes, 345 calories for 24 ounces is more than 290 calories for 16 ounces, but not too much more.		
Substitute $c = 345$ in the original proportion. $\frac{230}{16} = \frac{c}{24}$ $\frac{230}{16} \overset{?}{=} \frac{345}{24}$ $\frac{115}{8} = \frac{115}{8}$ ✓		
Write a complete sentence.		There are 345 calories in a 24-ounce iced caramel macchiato.

Note:
Exercise:

Problem:

TRY IT :: 3.99
At a fast-food restaurant, a 22-ounce chocolate shake has 850 calories. How many calories are in their 12-ounce chocolate shake? Round your answer to nearest whole number.

Solution:

464 calories

Note:
Exercise:

Problem:

TRY IT :: 3.100
Yaneli loves Starburst candies, but wants to keep her snacks to 100 calories. If the candies have 160 calories for 8 pieces, how many pieces can she have in her snack?

Solution:

5 pieces

Example:
Exercise:

Problem:

**Josiah went to Mexico for spring break and changed $325 dollars into Mexican pesos. At that time, the exchange rate had $1 US is equal to 12.54 Mexican pesos. How many Mexican pesos did he get for his trip?

Solution:
Solution

What are you asked to find?		How many Mexican pesos did Josiah get?
Assign a variable.		Let $p =$ the number of Mexican pesos.
Write a sentence that gives the information to find it.		If $1 US is equal to 12.54 Mexican pesos, then $325 is how many pesos?
Translate into a		

proportion–be careful of the units.		
$\frac{\$}{\text{pesos}} = \frac{\$}{\text{pesos}}$		$\frac{1}{12.54} = \frac{325}{p}$
Multiply both sides by the LCD, $12.54p$.		$12.54p\left(\frac{1}{12.54}\right) = 12.54p\left(\frac{325}{p}\right)$
Remove common factors on each side.		$\cancel{12.54}p\left(\frac{1}{\cancel{12.54}}\right) = 12.54\cancel{p}\left(\frac{325}{\cancel{p}}\right)$
Simplify.		$p = 4075.5$
Check.		
Is the answer reasonable?		
Yes, \$100 would be 1,254 pesos. \$325 is a little more than 3 times this amount, so our answer of 4075.5 pesos makes sense.		
Substitute $p = 4075.5$ in the original proportion. Use a calculator. $\frac{1}{12.54} = \frac{325}{p}$ $\frac{1}{12.54} \overset{?}{=} \frac{325}{4075.5}$ $0.07874\ldots = 0.07874\ldots$ ✓		
Write a complete sentence.		Josiah got 4075.5 pesos for his spring break trip.

Note:
Exercise:

Problem:

TRY IT :: 3.101
Yurianna is going to Europe and wants to change $800 dollars into Euros. At the current exchange rate, $1 US is equal to 0.738 Euro. How many Euros will she have for her trip?

Solution:

590.4 Euros

Note:
Exercise:

Problem:

TRY IT :: 3.102
Corey and Nicole are traveling to Japan and need to exchange $600 into Japanese yen. If each dollar is 94.1 yen, how many yen will they get?

Solution:

$56{,}460\text{ yen}$

In the example above, we related the number of pesos to the number of dollars by using a proportion. We could say the number of pesos *is proportional to* the number of dollars. If two quantities are related by a proportion, we say that they are proportional.

Solve Similar Figure Applications

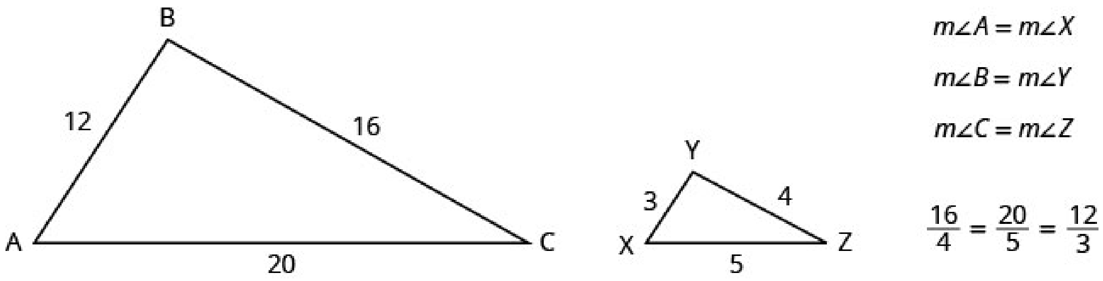
When you shrink or enlarge a photo on a phone or tablet, figure out a distance on a map, or use a pattern to build a bookcase or sew a dress, you are working with **similar figures**. If two figures have exactly the same shape, but different sizes, they are said to be *similar.* One is a scale model of the other. All their corresponding angles have the same measures and their corresponding sides are in the same ratio.

Note:

Similar Figures

Two figures are similar if the measures of their corresponding angles are equal and their corresponding sides are in the same ratio.

For example, the two triangles in [link] are similar. Each side of ΔABC is 4 times the length of the corresponding side of ΔXYZ.

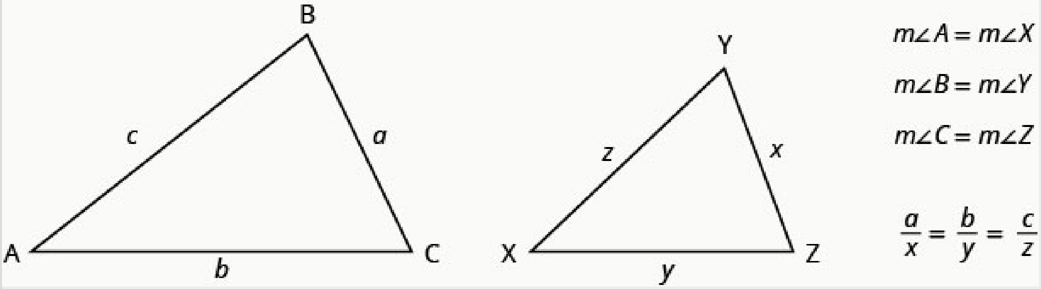


This is summed up in the Property of Similar Triangles.

Note:

Property of Similar Triangles

If ΔABC is similar to ΔXYZ, then their corresponding angle measure are equal and their corresponding sides are in the same ratio.



To solve applications with similar figures we will follow the Problem-Solving Strategy for Geometry Applications we used earlier.

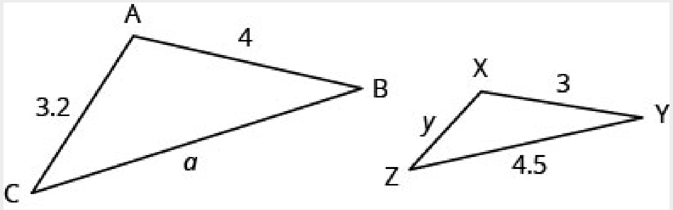
Note:
Solve geometry applications.

Readthe problem and make all the words and ideas are understood. Draw the figure and label it with the given information.
Identifywhat we are looking for.
Namewhat we are looking for by choosing a variable to represent it.
Translateinto an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
Solve the equationusing good algebra techniques.
Checkthe answer in the problem and make sure it makes sense.
Answerthe question with a complete sentence.

Example:
Exercise:

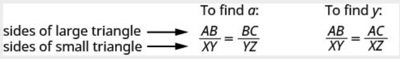
Problem:

**ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given. Find the lengths of the third sides.



Solution:
Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	Figure is given.
Step 2. Identify what we are looking for.	the length of the sides of similar triangles

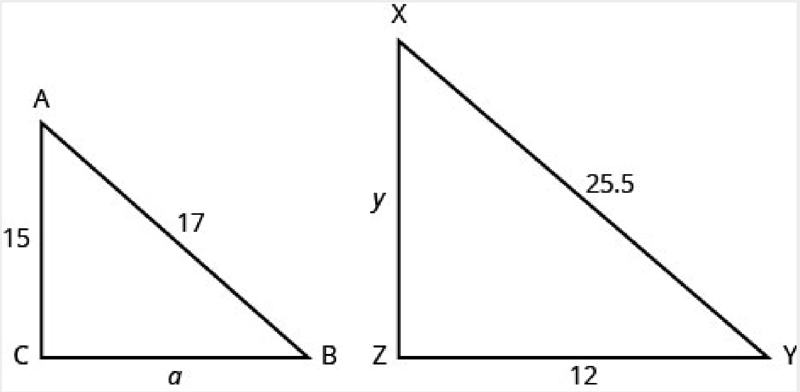
Step 3. Name the variables.	Let $a=$ length of the third side of ΔABC. $y=$ length of the third side of ΔXYZ
Step 4. Translate.	Since the triangles are similar, the corresponding sides are proportional.
We need to write an equation that compares the side we are looking for to a known ratio. Since the side *AB* = 4 corresponds to the side *XY* = 3 we know $\frac{AB}{XY}=\frac{4}{3}$. So we write equations with $\frac{AB}{XY}$ to find the sides we are looking for. Be careful to match up corresponding sides correctly.	$\frac{AB}{XY}=\frac{BC}{YZ}=\frac{AC}{XZ}$.  To find *a*: To find *y*: sides of large triangle sides of small triangle $\frac{AB}{XY}=\frac{BC}{YZ}$ $\frac{AB}{XY}=\frac{AC}{XZ}$
Substitute.	$\frac{4}{3}=\frac{a}{4.5}$ $\frac{4}{3}=\frac{3.2}{y}$
Step 5. Solve the equation.	$3a=4(4.5)$ $4y=3(3.2)$
	$a=6$ $y=2.4$
Step 6. Check.	

$\begin{array}{rcl} \frac{4}{3} & \overset{?}{=} & \frac{6}{4.5} \\ 4(4.5) & \overset{?}{=} & 6(3) \\ 18 & = & 18 \end{array}$ $\quad$ $\begin{array}{rcl} \frac{4}{3} & \overset{?}{=} & \frac{3.2}{2.4} \\ 4(2.4) & \overset{?}{=} & 3.2(3) \\ 9.6 & = & 9.6 \end{array}$	
Step 7. Answer the question.	The third side of ΔABC is 6 and the third side of ΔXYZ is 2.4.

Note:
Exercise:

Problem:

TRY IT :: 3.103
ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given in the figure.



Find the length of side a.

Solution:

8

Note:
Exercise:

Problem:

TRY IT :: 3.104
ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given in the figure.

Find the length of side y.

Solution:

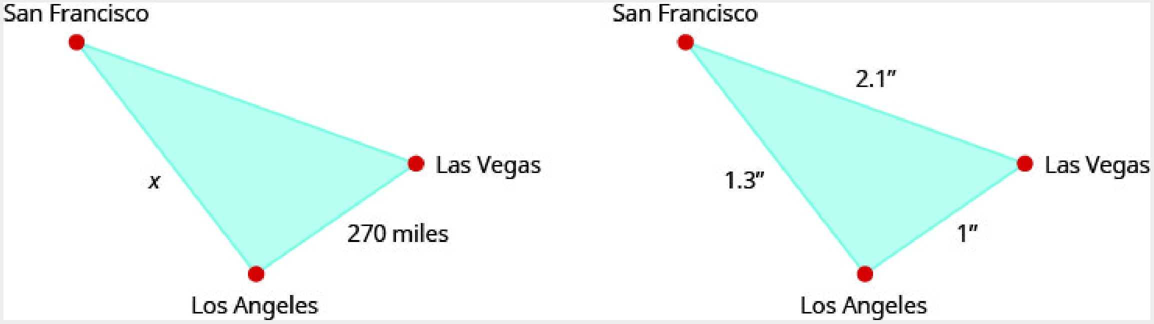
22.5

The next example shows how similar triangles are used with maps.

Example:
Exercise:

Problem:

**On a map, San Francisco, Las Vegas, and Los Angeles form a triangle whose sides are shown in the figure below. If the actual distance from Los Angeles to Las Vegas is 270 miles find the distance from Los Angeles to San Francisco.



Solution:
Solution

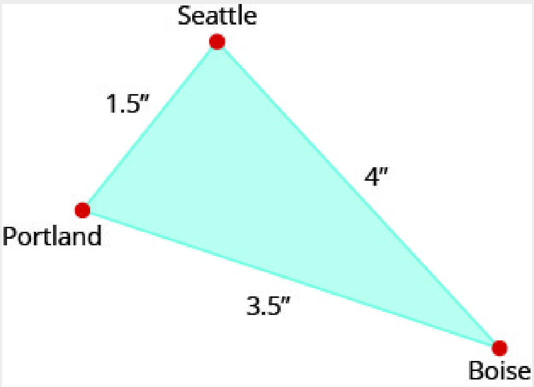
Read the problem. Draw the figures and label with the given information.	The figures are shown above.
Identify what we are looking for.	The actual distance from Los Angeles to San Francisco.
Name the variables.	Let $x =$ distance from Los Angeles to San Francisco.
Translate into an equation. Since the triangles are similar, the corresponding sides are proportional. We'll make the numerators "miles" and the denominators "inches."	$\frac{x \text{ miles}}{1.3 \text{ inches}} = \frac{270 \text{ miles}}{1 \text{ inch}}$
Solve the equation.	$1.3\left(\frac{x}{1.3}\right) = 1.3\left(\frac{270}{1}\right)$
	$x = 351$
Check.	
On the map, the distance from Los Angeles to San Francisco is more than the distance from Los Angeles to Las Vegas. Since 351 is more than 270 the answer makes sense.	
Check $x = 351$ in the original proportion. Use a calculator. $\frac{x \text{ miles}}{1.3 \text{ inches}} = \frac{270 \text{ miles}}{1 \text{ inch}}$ $\frac{351 \text{ miles}}{1.3 \text{ inches}} \stackrel{?}{=} \frac{270 \text{ miles}}{1 \text{ inch}}$ $\frac{270 \text{ miles}}{1 \text{ inch}} = \frac{270 \text{ miles}}{1 \text{ inch}}$ ✓	

Answer the question.	The distance from Los Angeles to San Francisco is 351 miles.

Note:
Exercise:

Problem:

TRY IT :: 3.105
On the map, Seattle, Portland, and Boise form a triangle whose sides are shown in the figure below. If the actual distance from Seattle to Boise is 400 miles, find the distance from Seattle to Portland.



Solution:

150 miles

Note:
Exercise:

TRY IT :: 3.106
Problem: Using the map above, find the distance from Portland to Boise.

Solution:

350 miles

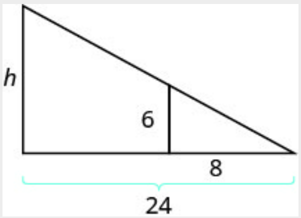
We can use similar figures to find heights that we cannot directly measure.

Example:
Exercise:

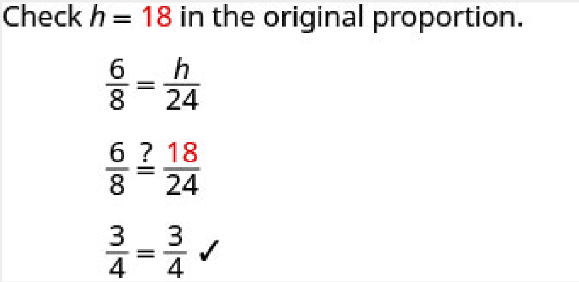
Problem:

**Tyler is 6 feet tall. Late one afternoon, his shadow was 8 feet long. At the same time, the shadow of a tree was 24 feet long. Find the height of the tree.

Solution:
Solution

Read the problem and draw a figure.	
We are looking for *h*, the height of the tree.	
We will use similar triangles to write an equation.	
The small triangle is similar to the large triangle.	$\frac{h}{24} = \frac{6}{8}$
Solve the proportion.	$24\left(\frac{6}{8}\right) = 24\left(\frac{h}{24}\right)$
Simplify.	$18 = h$
Check.	

Tyler's height is less than his shadow's length so it makes
sense that the tree's height is less than the length of its shadow.



Check $h = 18$ in the original proportion.

$$\frac{6}{8} = \frac{h}{24}$$

$$\frac{6}{8} \overset{?}{=} \frac{18}{24}$$

$$\frac{3}{4} = \frac{3}{4} \checkmark$$

Note:
Exercise:

Problem:

TRY IT :: 3.107
A telephone pole casts a shadow that is 50 feet long. Nearby, an 8 foot tall traffic sign casts a shadow that is 10 feet long. How tall is the telephone pole?

Solution:

40 feet

Note:
Exercise:

Problem:

TRY IT :: 3.108
A pine tree casts a shadow of 80 feet next to a 30-foot tall building which casts a 40 feet shadow. How tall is the pine tree?

Solution:

60 feet

Key Concepts

- **Property of Similar Triangles**
  - If ΔABC is similar to ΔXYZ, then their corresponding angle measures are equal and their corresponding sides are in the same ratio.
- **Problem Solving Strategy for Geometry Applications**

  Read the problem and make sure all the words and ideas are understood. Draw the figure and label it with the given information.
  Identify what we are looking for.
  Name what we are looking for by choosing a variable to represent it.
  Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
  Solve the equation using good algebra techniques.
  Check the answer in the problem and make sure it makes sense.
  Answer the question with a complete sentence.

Practice Makes Perfect

Solve Proportions

In the following exercises, solve.

Exercise:

Problem: $\frac{x}{56} = \frac{7}{8}$

Solution:

49

Exercise:

Problem: $\frac{n}{91} = \frac{8}{13}$

Exercise:

Problem: $\frac{49}{63} = \frac{z}{9}$

Solution:

7

Exercise:

Problem: $\frac{56}{72} = \frac{y}{9}$

Exercise:

Problem: $\frac{5}{a} = \frac{65}{117}$

Solution:

9

Exercise:

Problem: $\frac{4}{b} = \frac{64}{144}$

Exercise:

Problem: $\frac{98}{154} = \frac{-7}{p}$

Solution:

-11

Exercise:

Problem: $\frac{72}{156} = \frac{-6}{q}$

Exercise:

Problem: $\frac{a}{-8} = \frac{-42}{48}$

Solution:

7

Exercise:

Problem: $\frac{b}{-7} = \frac{-30}{42}$

Exercise:

Problem: $\frac{2.7}{j} = \frac{0.9}{0.2}$

Solution:

0.6

Exercise:

Problem: $\frac{2.8}{k} = \frac{2.1}{1.5}$

Exercise:

Problem: $\frac{a}{a+12} = \frac{4}{7}$

Solution:

16

Exercise:

Problem: $\frac{b}{b-16} = \frac{11}{9}$

Exercise:

Problem: $\frac{c}{c-104} = -\frac{5}{8}$

Solution:

$-\frac{5}{8}$

Exercise:

Problem: $\frac{d}{d-48} = -\frac{13}{3}$

Exercise:

Problem: $\frac{m+90}{25} = \frac{m+30}{15}$

Solution:

60

Exercise:

Problem: $\frac{n+10}{4} = \frac{40-n}{6}$

Exercise:

Problem: $\frac{2p+4}{8} = \frac{p+18}{6}$

Solution:

30

Exercise:

Problem: $\frac{q-2}{2} = \frac{2q-7}{18}$

Exercise:

Problem:

Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child’s weight. How many milliliters of acetaminophen will the doctor prescribe for Jocelyn, who weighs 45 pounds?

Solution:

9 ml

Exercise:

Problem:

Brianna, who weighs 6 kg, just received her shots and needs a pain killer. The pain killer is prescribed for children at 15 milligrams (mg) for every 1 kilogram (kg) of the child’s weight. How many milligrams will the doctor prescribe?

Exercise:

Problem:

A veterinarian prescribed Sunny, a 65 pound dog, an antibacterial medicine in case an infection emerges after her teeth were cleaned. If the dosage is 5 mg for every pound, how much medicine was Sunny given?

Solution:

325 mg

Exercise:

Problem:

Belle, a 13 pound cat, is suffering from joint pain. How much medicine should the veterinarian prescribe if the dosage is 1.8 mg per pound?

Exercise:

Problem:

A new energy drink advertises 106 calories for 8 ounces. How many calories are in 12 ounces of the drink?

Solution:

159 calories

Exercise:

Problem:

One 12 ounce can of soda has 150 calories. If Josiah drinks the big 32 ounce size from the local mini-mart, how many calories does he get?

Exercise:

Problem:

A new 7 ounce lemon ice drink is advertised for having only 140 calories. How many ounces could Sally drink if she wanted to drink just 100 calories?

Solution:

5 oz

Exercise:

Problem:

Reese loves to drink healthy green smoothies. A 16 ounce serving of smoothie has 170 calories. Reese drinks 24 ounces of these smoothies in one day. How many calories of smoothie is he consuming in one day?

Exercise:

Problem:

Janice is traveling to Canada and will change $250 US dollars into Canadian dollars. At the current exchange rate, $1 US is equal to $1.01 Canadian. How many Canadian dollars will she get for her trip?

Solution:

252.5 Canadian dollars

Exercise:

Problem:

Todd is traveling to Mexico and needs to exchange $450 into Mexican pesos. If each dollar is worth 12.29 pesos, how many pesos will he get for his trip?

Exercise:

Problem:

Steve changed $600 into 480 Euros. How many Euros did he receive for each US dollar?

Solution:

0.80 Euros

Exercise:

Problem:

Martha changed $350 US into 385 Australian dollars. How many Australian dollars did she receive for each US dollar?

Exercise:

Problem:

When traveling to Great Britain, Bethany exchanged her $900 into 570 British pounds. How many pounds did she receive for each American dollar?

Solution:

0.63 British pounds

Exercise:

Problem:

A missionary commissioned to South Africa had to exchange his $500 for the South African Rand which is worth 12.63 for every dollar. How many Rand did he have after the exchange?

Exercise:

Problem:

Ronald needs a morning breakfast drink that will give him at least 390 calories. Orange juice has 130 calories in one cup. How many cups does he need to drink to reach his calorie goal?

Solution:

3 cups

Exercise:

Problem:

Sarah drinks a 32-ounce energy drink containing 80 calories per 12 ounce. How many calories did she drink?

Exercise:

Problem:

Elizabeth is returning to the United States from Canada. She changes the remaining 300 Canadian dollars she has to $230.05 in American dollars. What was $1 worth in Canadian dollars?

Solution:

1.30 Canadian dollars

Exercise:

Problem:

Ben needs to convert $1000 to the Japanese Yen. One American dollar is worth 123.3 Yen. How much Yen will he have?

Exercise:

Problem:

A golden retriever weighing 85 pounds has diarrhea. His medicine is prescribed as 1 teaspoon per 5 pounds. How much medicine should he be given?

Solution:

17 tsp

Exercise:

Problem:

Five-year-old Lacy was stung by a bee. The dosage for the anti-itch liquid is 150 mg for her weight of 40 pounds. What is the dosage per pound?

Exercise:

Problem:

Karen eats $\frac{1}{2}$ cup of oatmeal that counts for 2 points on her weight loss program. Her husband, Joe, can have 3 points of oatmeal for breakfast. How much oatmeal can he have?

Solution:

$\frac{3}{4}$ cup

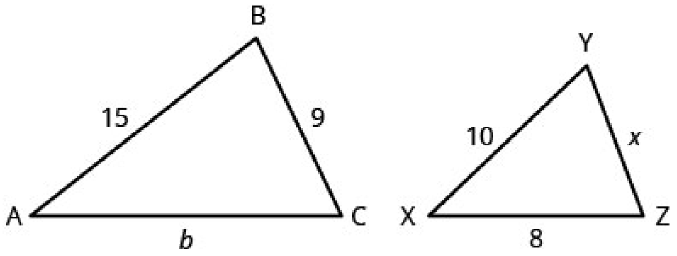
Exercise:

Problem:

An oatmeal cookie recipe calls for $\frac{1}{2}$ cup of butter to make 4 dozen cookies. Hilda needs to make 10 dozen cookies for the bake sale. How many cups of butter will she need?

Solve Similar Figure Applications

In the following exercises, ΔABC is similar to ΔXYZ. Find the length of the indicated side.



Exercise:

Problem: side *b*

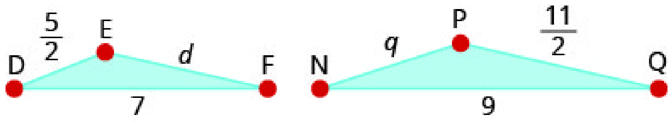
Solution:

12

Exercise:

Problem: side x

In the following exercises, ΔDEF is similar to ΔNPQ.



Exercise:

Problem: Find the length of side d.

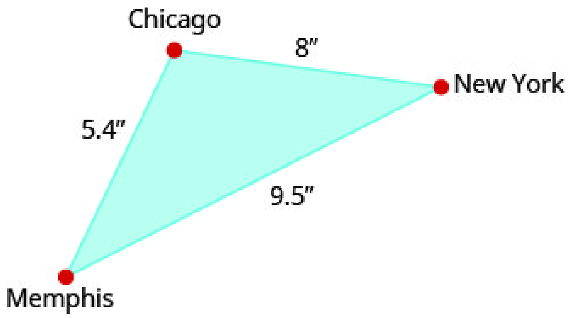
Solution:

$\frac{77}{18}$

Exercise:

Problem: Find the length of side q.

In the following two exercises, use the map shown. On the map, New York City, Chicago, and Memphis form a triangle whose sides are shown in the figure below. The actual distance from New York to Chicago is 800 miles.



Exercise:

Problem: Find the actual distance from New York to Memphis.

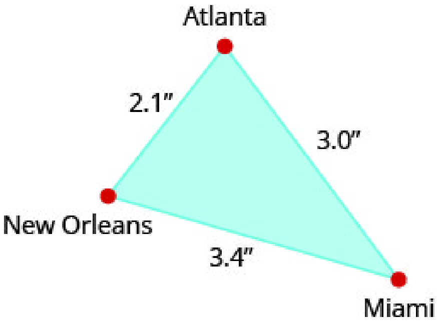
Solution:

950 miles

Exercise:

Problem: Find the actual distance from Chicago to Memphis.

In the following two exercises, use the map shown. On the map, Atlanta, Miami, and New Orleans form a triangle whose sides are shown in the figure below. The actual distance from Atlanta to New Orleans is 420 miles.



Exercise:

Problem: Find the actual distance from New Orleans to Miami.

Solution:

680 miles

Exercise:

Problem: Find the actual distance from Atlanta to Miami.

Exercise:

Problem:

A 2 foot tall dog casts a 3 foot shadow at the same time a cat casts a one foot shadow. How tall is the cat?

Solution:

$\frac{2}{3}$ foot (8 in)

Exercise:

Problem:

Larry and Tom were standing next to each other in the backyard when Tom challenged Larry to guess how tall he was. Larry knew his own height is 6.5 feet and when they measured their shadows, Larry's shadow was 8 feet and Tom's was 7.75 feet long. What is Tom's height?

Exercise:

Problem:

The tower portion of a windmill is 212 feet tall. A six foot tall person standing next to the tower casts a seven foot shadow. How long is the windmill's shadow?

Solution:

247.3 feet

Exercise:

Problem:

The height of the Statue of Liberty is 305 feet. Nicole, who is standing next to the statue, casts a 6 foot shadow and she is 5 feet tall. How long should the shadow of the statue be?

Everyday Math

Exercise:

Problem:

Heart Rate At the gym, Carol takes her pulse for 10 seconds and counts 19 beats.

ⓐ How many beats per minute is this?
ⓑ Has Carol met her target heart rate of 140 beats per minute?

Solution:

114 beats per minute
no

Exercise:

Problem:

Heart Rate Kevin wants to keep his heart rate at 160 beats per minute while training. During his workout he counts 27 beats in 10 seconds.

ⓐ How many beats per minute is this?
ⓑ Has Kevin met his target heart rate?

Exercise:

Problem: **Cost of a Road Trip** Jesse’s car gets 30 miles per gallon of gas.

ⓐ If Las Vegas is 285 miles away, how many gallons of gas are needed to get there and then home?
ⓑ If gas is $3.09 per gallon, what is the total cost of the gas for the trip?

Solution:

19 gallons
$58.71

Exercise:

Problem:

Cost of a Road Trip Danny wants to drive to Phoenix to see his grandfather. Phoenix is 370 miles from Danny’s home and his car gets 18.5 miles per gallon.

ⓐ How many gallons of gas will Danny need to get to and from Phoenix?
ⓑ If gas is $3.19 per gallon, what is the total cost for the gas to drive to see his grandfather?

Exercise:

Problem:

Lawn Fertilizer Phil wants to fertilize his lawn. Each bag of fertilizer covers about 4,000 square feet of lawn. Phil’s lawn is approximately 13,500 square feet. How many bags of fertilizer will he have to buy?

Solution:

4 bags

Exercise:

Problem:

House Paint April wants to paint the exterior of her house. One gallon of paint covers about 350 square feet, and the exterior of the house measures approximately 2000 square feet. How many gallons of paint will she have to buy?

Exercise:

Problem:

Cooking Natalia’s pasta recipe calls for 2 pounds of pasta for 1 quart of sauce. How many pounds of pasta should Natalia cook if she has 2.5 quarts of sauce?

Solution:

5

Exercise:

Problem:

Heating Oil A 275 gallon oil tank costs $400 to fill. How much would it cost to fill a 180 gallon oil tank?

Writing Exercises

Exercise:

Problem:

Marisol solves the proportion $\frac{144}{a} = \frac{9}{4}$ by ‘cross multiplying’, so her first step looks like $4 \cdot 144 = 9 \cdot a$. Explain how this differs from the method of solution shown in [link].

Solution:

Answers will vary.

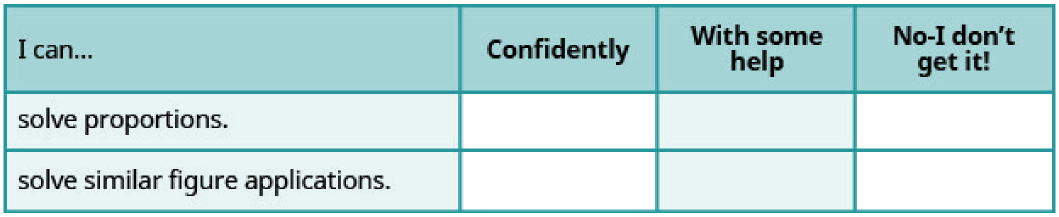
Exercise:

Problem:

Find a printed map and then write and solve an application problem similar to [link].

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
solve proportions.			
solve similar figure applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

proportion

A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$. The proportion is read "a is to b, as c is to d."

similar figures

Two figures are similar if the measures of their corresponding angles are equal and their corresponding sides are in the same ratio.

Review & Practice Test
By the end of this section, you will be prepared to take your Chapter 3 Assessment.

Chapter 3 Review Exercises

Using a Problem Solving Strategy

Approach Word Problems with a Positive Attitude

In the following exercises, reflect on your approach to word problems.

Exercise:

Problem:

How has your attitude towards solving word problems changed as a result of working through this chapter? Explain.

Solution:

answers will vary

Exercise:

Problem:

Did the problem-solving strategy help you solve word problems in this chapter? Explain.

Use a Problem-Solving Strategy for Word Problems

In the following exercises, solve using the problem-solving strategy for word problems. Remember to write a complete sentence to answer each question.

Exercise:

Problem:

Three-fourths of the people at a concert are children. If there are 87 children, what is the total number of people at the concert?

Solution:

116

Exercise:

Problem:

There are nine saxophone players in the band. The number of saxophone players is one less than twice the number of tuba players. Find the number of tuba players.

Solve Number Problems

In the following exercises, solve each number word problem.

Exercise:

Problem:

The sum of a number and three is forty-one. Find the number.

Solution:

38

Exercise:

Problem:

Twice the difference of a number and ten is fifty-four. Find the number.

Exercise:

Problem:

One number is nine less than another. Their sum is negative twenty-seven. Find the numbers.

Solution:

$-18, -9$

Exercise:

Problem:

One number is eleven more than another. If their sum is increased by seventeen, the result is 90. Find the numbers.

Exercise:

Problem:

One number is two more than four times another. Their sum is -13. Find the numbers.

Solution:

$-3, -10$

Exercise:

Problem:

The sum of two consecutive integers is -135. Find the numbers.

Exercise:

Problem: Find three consecutive integers whose sum is -141.

Solution:

$-48, -47, -46$

Exercise:

Problem: Find three consecutive even integers whose sum is 234.

Exercise:

Problem: Find three consecutive odd integers whose sum is 51.

Solution:

15, 17, 19

Exercise:

Problem:

Koji has $5,502 in his savings account. This is $30 less than six times the amount in his checking account. How much money does Koji have in his checking account?

Solve Percent Applications

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

Exercise:

Problem: What number is 67% of 250?

Solution:

167.5

Exercise:

Problem: 300% of 82 is what number?

Exercise:

Problem: 12.5% of what number is 20?

Solution:

160

Exercise:

Problem: 72 is 30% of what number?

Exercise:

Problem: What percent of 125 is 150?

Solution:

120%

Exercise:

Problem: 127.5 is what percent of 850?

Solve Percent Applications

In the following exercises, solve.

Exercise:

Problem:

The bill for Dino’s lunch was $19.45. He wanted to leave 20% of the total bill as a tip. How much should the tip be?

Solution:

$3.89

Exercise:

Problem:

Reza was very sick and lost 15% of his original weight. He lost 27 pounds. What was his original weight?

Exercise:

Problem:

Dolores bought a crib on sale for $350. The sale price was 40% of the original price. What was the original price of the crib?

Solution:

$875

Exercise:

Problem:

Jaden earns $2,680 per month. He pays $938 a month for rent. What percent of his monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, solve.

Exercise:

Problem:

Angel’s got a raise in his annual salary from $55,400 to $56,785. Find the percent increase.

Solution:

2.5%

Exercise:

Problem:

Rowena’s monthly gasoline bill dropped from $83.75 last month to $56.95 this month. Find the percent decrease.

Solve Simple Interest Applications

In the following exercises, solve.

Exercise:

Problem:

Winston deposited $3,294 in a bank account with interest rate 2.6%. How much interest was earned in 5 years?

Solution:

$428.22

Exercise:

Problem:

Moira borrowed $4,500 from her grandfather to pay for her first year of college. Three years later, she repaid the $4,500 plus $243 interest. What was the rate of interest?

Exercise:

Problem:

Jaime’s refrigerator loan statement said he would pay $1,026 in interest for a 4-year loan at 13.5%. How much did Jaime borrow to buy the refrigerator?

Solution:

$1,900

Exercise:

Problem:

In 12 years, a bond that paid 6.35% interest earned $7,620 interest. What was the principal of the bond?

Solve Applications with Discount or Mark-up

In the following exercises, find the sale price.

Exercise:

Problem:

The original price of a handbag was $84. Carole bought it on sale for $21 off.

Solution:

$63

Exercise:

Problem:

Marian wants to buy a coffee table that costs $495. Next week the coffee table will be on sale for $149 off.

In the following exercises, find ⓐ the amount of discount and ⓑ the sale price.

Exercise:

Problem:

Emmett bought a pair of shoes on sale at 40% off from an original price of $138.

Solution:

ⓐ $55.20 ⓑ $82.80

Exercise:

Problem:

Anastasia bought a dress on sale at 75% off from an original price of $280.

In the following exercises, find ⓐ the amount of discount and ⓑ the discount rate. (Round to the nearest tenth of a percent, if needed.)

Exercise:

Problem:

Zack bought a printer for his office that was on sale for $380. The original price of the printer was $450.

Solution:

ⓐ $70 ⓑ 15.6%

Exercise:

Problem:

Lacey bought a pair of boots on sale for $95. The original price of the boots was $200.

In the following exercises, find ⓐ the amount of the mark-up and ⓑ the list price.

Exercise:

Problem:

Nga and Lauren bought a chest at a flea market for $50. They re-finished it and then added a 350% mark-up.

Solution:

ⓐ $175 ⓑ $225

Exercise:

Problem:

Carly bought bottled water for $0.24 per bottle at the discount store. She added a 75% mark-up before selling them at the football game.

Solve Geometry Applications: Triangles, Rectangles and the Pythagorean Theorem

Solve Applications Using Triangle Properties

In the following exercises, solve using triangle properties.

Exercise:

Problem:

The measures of two angles of a triangle are 22 and 85 degrees. Find the measure of the third angle.

Solution:

73°

Exercise:

Problem:

The playground at a shopping mall is a triangle with perimeter 48 feet. The lengths of two sides are 19 feet and 14 feet. How long is the third side?

Exercise:

Problem:

A triangular road sign has base 30 inches and height 40 inches. What is its area?

Solution:

600 square inches

Exercise:

Problem:

What is the height of a triangle with area 67.5 square meters and base 9 meters?

Exercise:

Problem:

One angle of a triangle is 30° more than the smallest angle. The largest angle is the sum of the other angles. Find the measures of all three angles.

Solution:

$30^{\circ}, 60^{\circ}, 90^{\circ}$

Exercise:

Problem:

One angle of a right triangle measures 58°. What is the measure of the other angles of the triangle?

Exercise:

Problem:

The measure of the smallest angle in a right triangle is 45° less than the measure of the next larger angle. Find the measures of all three angles.

Solution:

$22.5^{\circ}, 67.5^{\circ}, 90^{\circ}$

Exercise:

Problem:

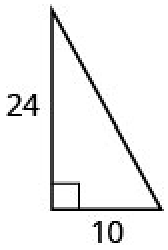
The perimeter of a triangle is 97 feet. One side of the triangle is eleven feet more than the smallest side. The third side is six feet more than twice the smallest side. Find the lengths of all sides.

Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

Exercise:

Problem:

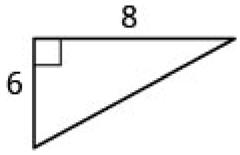


Solution:

26

Exercise:

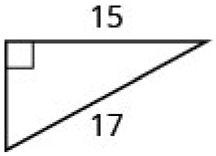
Problem:



In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

Exercise:

Problem:

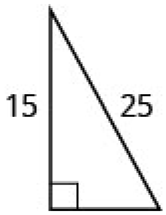


Solution:

8

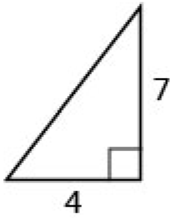
Exercise:

Problem:



Exercise:

Problem:

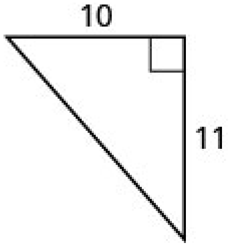


Solution:

8.1

Exercise:

Problem:

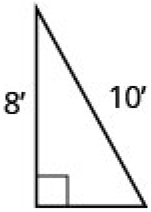


In the following exercises, solve. Approximate to the nearest tenth, if necessary.

Exercise:

Problem:

Sergio needs to attach a wire to hold the antenna to the roof of his house, as shown in the figure. The antenna is 8 feet tall and Sergio has 10 feet of wire. How far from the base of the antenna can he attach the wire?



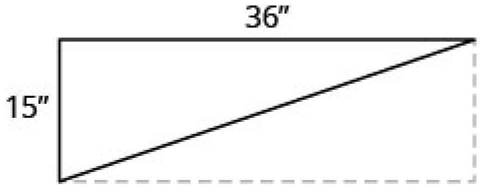
Solution:

6′

Exercise:

Problem:

Seong is building shelving in his garage. The shelves are 36 inches wide and 15 inches tall. He wants to put a diagonal brace across the back to stabilize the shelves, as shown. How long should the brace be?



Solve Applications Using Rectangle Properties

In the following exercises, solve using rectangle properties.

Exercise:

Problem:

The length of a rectangle is 36 feet and the width is 19 feet. Find the ⓐ perimeter ⓑ area.

Solution:

ⓐ 110 ft. ⓑ 684 sq. ft.

Exercise:

Problem:

A sidewalk in front of Kathy’s house is in the shape of a rectangle four feet wide by 45 feet long. Find the ⓐ perimeter ⓑ area.

Exercise:

Problem:

The area of a rectangle is 2356 square meters. The length is 38 meters. What is the width?

Solution:

62 m

Exercise:

Problem:

The width of a rectangle is 45 centimeters. The area is 2,700 square centimeters. What is the length?

Exercise:

Problem:

The length of a rectangle is 12 cm more than the width. The perimeter is 74 cm. Find the length and the width.

Solution:

24.5 cm, 12.5 cm

Exercise:

Problem:

The width of a rectangle is three more than twice the length. The perimeter is 96 inches. Find the length and the width.

Solve Uniform Motion Applications

Solve Uniform Motion Applications

In the following exercises, solve.

Exercise:

Problem:

When Gabe drives from Sacramento to Redding it takes him 2.2 hours. It takes Elsa 2 hours to drive the same distance. Elsa's speed is seven miles per hour faster than Gabe's speed. Find Gabe's speed and Elsa's speed.

Solution:

Gabe 70 mph, Elsa 77 mph

Exercise:

Problem:

Louellen and Tracy met at a restaurant on the road between Chicago and Nashville. Louellen had left Chicago and drove 3.2 hours towards Nashville. Tracy had left Nashville and drove 4 hours towards Chicago, at a speed one mile per hour faster than Louellen’s speed. The distance between Chicago and Nashville is 472 miles. Find Louellen’s speed and Tracy’s speed.

Exercise:

Problem:

Two busses leave Amarillo at the same time. The Albuquerque bus heads west on the I-40 at a speed of 72 miles per hour, and the Oklahoma City bus heads east on the I-40 at a speed of 78 miles per hour. How many hours will it take them to be 375 miles apart?

Solution:

2.5 hours

Exercise:

Problem:

Kyle rowed his boat upstream for 50 minutes. It took him 30 minutes to row back downstream. His speed going upstream is two miles per hour slower than his speed going downstream. Find Kyle’s upstream and downstream speeds.

Exercise:

Problem:

At 6:30, Devon left her house and rode her bike on the flat road until 7:30. Then she started riding uphill and rode until 8:00. She rode a total of 15 miles. Her speed on the flat road was three miles per hour faster than her speed going uphill. Find Devon’s speed on the flat road and riding uphill.

Solution:

flat road 11 mph, uphill 8 mph

Exercise:

Problem:

Anthony drove from New York City to Baltimore, a distance of 192 miles. He left at 3:45 and had heavy traffic until 5:30. Traffic was light for the rest of the drive, and he arrived at 7:30. His speed in light traffic was four miles per hour more than twice his speed in heavy traffic. Find Anthony’s driving speed in heavy traffic and light traffic.

Solve Proportion and Similar Figure Applications

Solve Proportions

In the following exercises, solve.

Exercise:

Problem: $\frac{}{4} = \frac{3}{5}$

Solution:

$\frac{12}{5}$

Exercise:

Problem: $\frac{3}{} = \frac{9}{5}$

Exercise:

Problem: $\frac{}{+20} = \frac{3}{7}$

Solution:

15

Exercise:

Problem: $\frac{-3}{5} = \frac{+2}{9}$

In the following exercises, solve using proportions.

Exercise:

Problem:

Rachael had a 21 ounce strawberry shake that has 739 calories. How many calories are there in a 32 ounce shake?

Solution:

1161 calories

Exercise:

Problem:

Leo went to Mexico over Christmas break and changed $525 dollars into Mexican pesos. At that time, the exchange rate had $1 US is equal to 16.25 Mexican pesos. How many Mexican pesos did he get for his trip?

Solve Similar Figure Applications

In the following exercises, solve.

Exercise:

Problem:

ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given in the figure. Find the lengths of the third sides.
[missing_resource: CNX_ElemAlg_Figure_08_09_204_img_new.jpg]

Solution:

$= 9; \quad = 2\frac{1}{3}$

Exercise:

Problem:

On a map of Europe, Paris, Rome, and Vienna form a triangle whose sides are shown in the figure below. If the actual distance from Rome to Vienna is 700 miles, find the distance from

ⓐ Paris to Rome
ⓑ Paris to Vienna

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Exercise:

Problem:

Tony is 5.75 feet tall. Late one afternoon, his shadow was 8 feet long. At the same time, the shadow of a nearby tree was 32 feet long. Find the height of the tree.

Solution:

23 feet

Exercise:

Problem:

The height of a lighthouse in Pensacola, Florida is 150 feet. Standing next to the statue, 5.5 foot tall Natalie cast a 1.1 foot shadow How long would the shadow of the lighthouse be?

Practice Test

Exercise:

Problem:

Four-fifths of the people on a hike are children. If there are 12 children, what is the total number of people on the hike?

Solution:

15

Exercise:

Problem:

One number is three more than twice another. Their sum is -63. Find the numbers.

Exercise:

Problem:

The sum of two consecutive odd integers is -96. Find the numbers.

Solution:

$-49, -47$

Exercise:

Problem:

Marla’s breakfast was 525 calories. This was 35% of her total calories for the day. How many calories did she have that day?

Exercise:

Problem:

Humberto’s hourly pay increased from $16.25 to $17.55. Find the percent increase.

Solution:

8%

Exercise:

Problem:

Melinda deposited $5,985 in a bank account with an interest rate of 1.9%. How much interest was earned in 2 years?

Exercise:

Problem:

Dotty bought a freezer on sale for $486.50. The original price of the freezer was $695. Find ⓐ the amount of discount and ⓑ the discount rate.

Solution:

ⓐ $208.50 ⓑ 30%

Exercise:

Problem:

Bonita has $2.95 in dimes and quarters in her pocket. If she has five more dimes than quarters, how many of each coin does she have?

Exercise:

Problem:

At a concert, $1,600 in tickets were sold. Adult tickets were $9 each and children's tickets were $4 each. If the number of adult tickets was 30 less than twice the number of children's tickets, how many of each kind were sold?

Solution:

140 adult, 85 children

Exercise:

Problem:

The measure of one angle of a triangle is twice the measure of the smallest angle. The measure of the third angle is 14 more than the measure of the smallest angle. Find the measures of all three angles.

Solution:

$41.5^\circ, 55.5^\circ, 83^\circ$

Exercise:

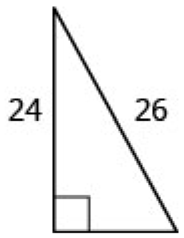
Problem:

What is the height of a triangle with area 277.2 square inches and base 44 inches?

In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

Exercise:

Problem:

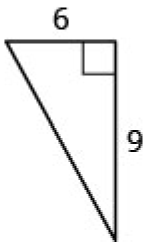


Solution:

10

Exercise:

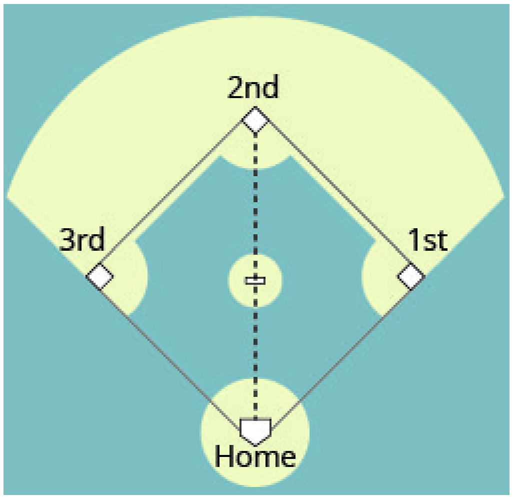
Problem:



Exercise:

Problem:

A baseball diamond is really a square with sides of 90 feet. How far is it from home plate to second base, as shown?



Solution:

127.3 ft.

Exercise:

Problem:

The length of a rectangle is two feet more than five times the width. The perimeter is 40 feet. Find the dimensions of the rectangle.

Exercise:

Problem:

Two planes leave Dallas at the same time. One heads east at a speed of 428 miles per hour. The other plane heads west at a speed of 382 miles per hour. How many hours will it take them to be 2,025 miles apart?

Solution:

2.5 hours

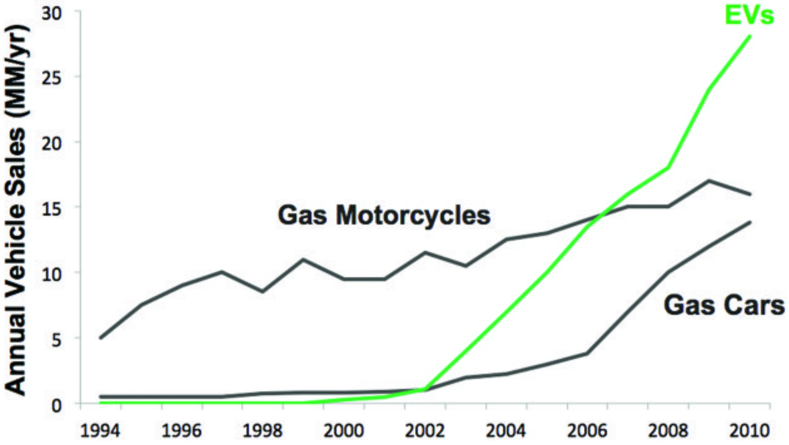
Exercise:

Problem:

Leon drove from his house in Cincinnati to his sister's house in Cleveland, a distance of 252 miles. It took him $4\frac{1}{2}$ hours. For the first half hour he had heavy traffic, and the rest of the time his speed was five miles per hour less than twice his speed in heavy traffic. What was his speed in heavy traffic?

Introduction class="introduction"

This graph illustrates the annual vehicle sales of gas motorcycles, gas cars, and electric vehicles from 1994 to 2010. It is a line graph with *x*- and *y*-axes, one of the most common types of graphs. (credit: Steve Jurvetson, Flickr)



Graphs are found in all areas of our lives—from commercials showing you which cell phone carrier provides the best coverage, to bank statements and news articles, to the boardroom of major corporations. In this chapter, we will study the rectangular coordinate system, which is the basis for most consumer graphs. We will look at linear graphs, slopes of lines, equations of lines, and linear inequalities.

Use the Rectangular Coordinate System
By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables

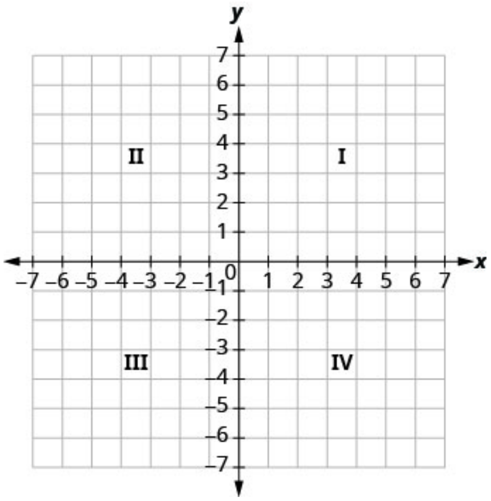
Note:
Before you get started, take this readiness quiz.

1. Evaluate $x+3$ when $x=-1$.
   If you missed this problem, review [link].
2. Evaluate $2x-5y$ when $x=3$ and $y=-2$.
   If you missed this problem, review [link].
3. Solve for y: $40-4y=20$.
   If you missed this problem, review [link].

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane or the 'coordinate plane'.

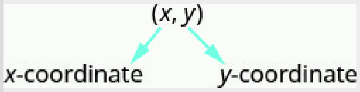
The horizontal number line is called the *x-axis*. The vertical number line is called the *y-axis*. The *x*-axis and the *y*-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See [link].



'Quadrant' has the root 'quad,'
which means 'four.'

In the rectangular coordinate system, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

Note:
Ordered Pair
An **ordered pair**, (x, y), gives the coordinates of a point in a rectangular coordinate system.

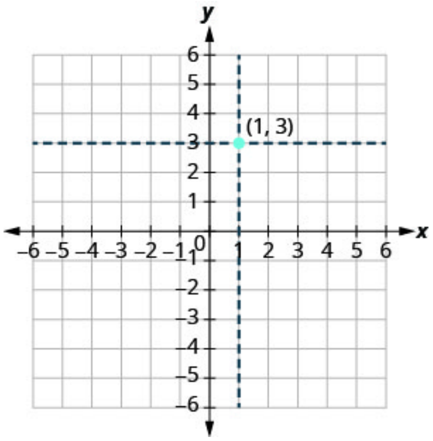


The first number is the *x*-coordinate.
The second number is the *y*-coordinate.

The phrase ‘ordered pair’ means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the origin.

Note:
The Origin
The point $(0, 0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let’s plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet—that is the point with coordinates $(1, 3)$.



Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example:
Exercise:

Problem:

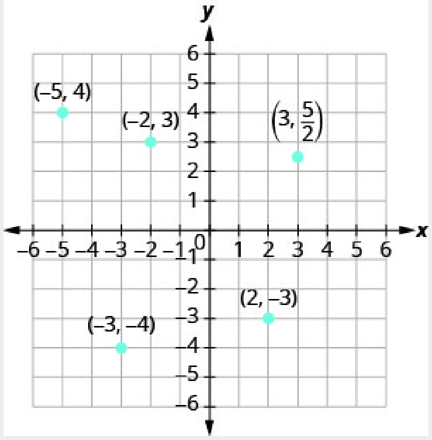
**Plot each point in the rectangular coordinate system and identify the quadrant in which the point is located:

ⓐ $(-5,4)$ ⓑ $(-3,-4)$ ⓒ $(2,-3)$ ⓓ $(-2,3)$ ⓔ $\left(3,\frac{5}{2}\right)$.

Solution:
Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

ⓐ Since $x=-5$, the point is to the left of the *y*-axis. Also, since $y=4$, the point is above the *x*-axis. The point $(-5,4)$ is in Quadrant II.
ⓑ Since $x=-3$, the point is to the left of the *y*-axis. Also, since $y=-4$, the point is below the *x*-axis. The point $(-3,-4)$ is in Quadrant III.
ⓒ Since $x=2$, the point is to the right of the *y*-axis. Since $y=-3$, the point is below the *x*-axis. The point $(2,-3)$ is in Quadrant lV.
ⓓ Since $x=-2$, the point is to the left of the *y*-axis. Since $y=3$, the point is above the *x*-axis. The point $(-2,3)$ is in Quadrant II.
ⓔ Since $x=3$, the point is to the right of the *y*-axis. Since $y=\frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $\left(3,\frac{5}{2}\right)$ is in Quadrant I.



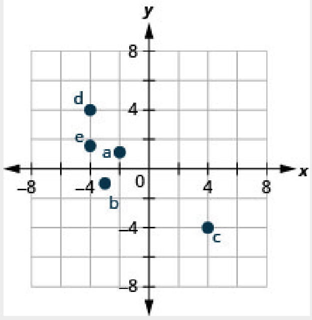
Note:
Exercise:

Problem:

TRY IT :: 4.1
Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

ⓐ $(-2,1)$ ⓑ $(-3,-1)$ ⓒ $(4,-4)$ ⓓ $(-4,4)$ ⓔ $\left(-4,\frac{3}{2}\right)$.

Solution:



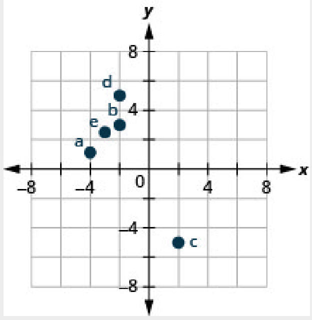
Note:
Exercise:

Problem:

TRY IT :: 4.2
Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

ⓐ $(-4, 1)$ ⓑ $(-2, 3)$ ⓒ $(2, -5)$ ⓓ $(-2, 5)$ ⓔ $\left(-3, \frac{5}{2}\right)$.

Solution:



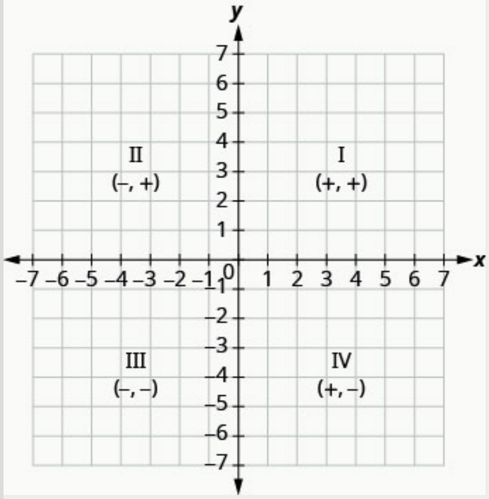
How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

For the point in [link] in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

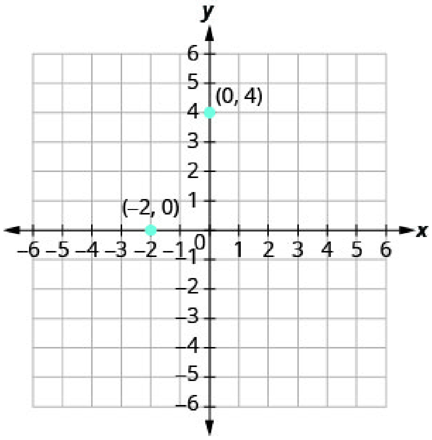
Can you tell just by looking at the coordinates in which quadrant the point $(-2, 5)$ is located? In which quadrant is $(2, -5)$ located?

Note:
Quadrants
We can summarize sign patterns of the quadrants in this way.
Equation:

$$\begin{array}{cccc} \text{Quadrant I} & \text{Quadrant II} & \text{Quadrant III} & \text{Quadrant IV} \\ (x,y) & (x,y) & (x,y) & (x,y) \\ (+,+) & (-,+) & (-,-) & (+,-) \end{array}$$



What if one coordinate is zero as shown in [link]? Where is the point $(0,4)$ located? Where is the point $(-2,0)$ located?



The point $(0,4)$ is on the *y*-axis and the point $(-2,0)$ is on the *x*-axis.

Note:

Points on the Axes
Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates $(a, 0)$.
Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates $(0, b)$.

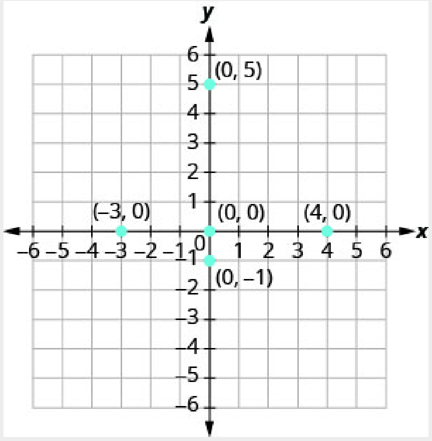
Example:
Exercise:

Problem: **Plot each point:

ⓐ $(0, 5)$ ⓑ $(4, 0)$ ⓒ $(-3, 0)$ ⓓ $(0, 0)$ ⓔ $(0, -1)$.

Solution:
Solution

ⓐ Since $x = 0$, the point whose coordinates are $(0, 5)$ is on the *y*-axis.
ⓑ Since $y = 0$, the point whose coordinates are $(4, 0)$ is on the *x*-axis.
ⓒ Since $y = 0$, the point whose coordinates are $(-3, 0)$ is on the *x*-axis.
ⓓ Since $x = 0$ and $y = 0$, the point whose coordinates are $(0, 0)$ is the origin.
ⓔ Since $x = 0$, the point whose coordinates are $(0, -1)$ is on the *y*-axis.

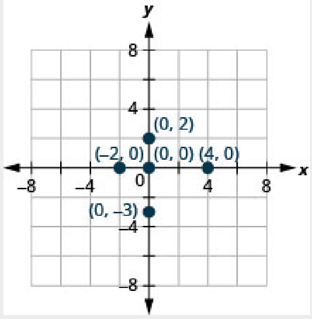


Note:
Exercise:

TRY IT :: 4.3
Problem: Plot each point:

ⓐ $(4, 0)$ ⓑ $(-2, 0)$ ⓒ $(0, 0)$ ⓓ $(0, 2)$ ⓔ $(0, -3)$.

Solution:



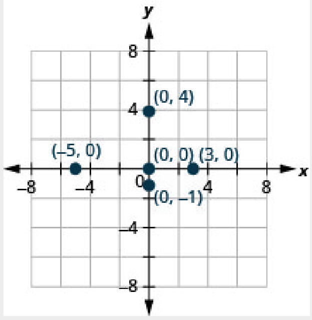
Note:
Exercise:

TRY IT :: 4.4

Problem: Plot each point:

ⓐ $(-5,0)$ ⓑ $(3,0)$ ⓒ $(0,0)$ ⓓ $(0,-1)$ ⓔ $(0,4)$.

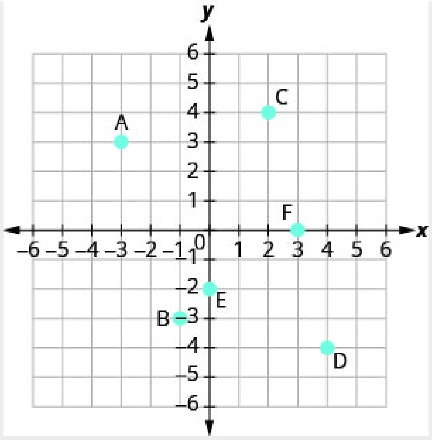
Solution:



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the ordered pair use the correct order, (x, y).

Example:
Exercise:

Problem: **Name the ordered pair of each point shown in the rectangular coordinate system.



Solution:
Solution

Point A is above -3 on the *x*-axis, so the *x*-coordinate of the point is -3.

- The point is to the left of 3 on the *y*-axis, so the *y*-coordinate of the point is 3.
- The coordinates of the point are $(-3, 3)$.

Point B is below -1 on the *x*-axis, so the *x*-coordinate of the point is -1.

- The point is to the left of -3 on the *y*-axis, so the *y*-coordinate of the point is -3.
- The coordinates of the point are $(-1, -3)$.

Point C is above 2 on the *x*-axis, so the *x*-coordinate of the point is 2.

- The point is to the right of 4 on the *y*-axis, so the *y*-coordinate of the point is 4.
- The coordinates of the point are $(2, 4)$.

Point D is below 4 on the *x*-axis, so the *x*-coordinate of the point is 4.

- The point is to the right of -4 on the *y*-axis, so the *y*-coordinate of the point is -4.
- The coordinates of the point are $(4, -4)$.

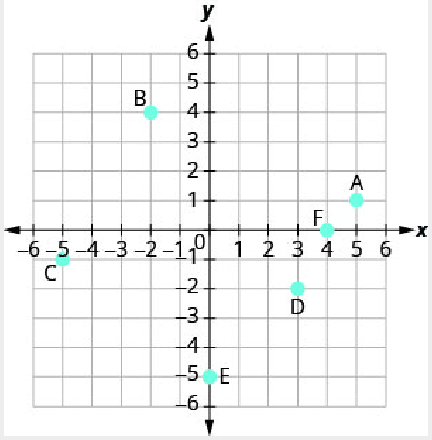
Point E is on the *y*-axis at $y = -2$. The coordinates of point E are $(0, -2)$.

Point F is on the *x*-axis at $x = 3$. The coordinates of point F are $(3, 0)$.

Note:
Exercise:

TRY IT :: 4.5

Problem: Name the ordered pair of each point shown in the rectangular coordinate system.



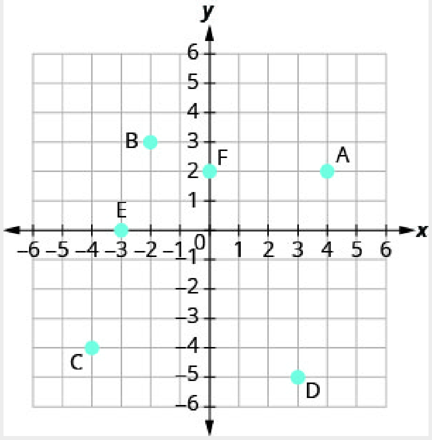
Solution:

A: $(5,1)$ B: $(-2,4)$ C: $(-5,-1)$ D: $(3,-2)$ E: $(0,-5)$ F: $(4,0)$

Note:
Exercise:

TRY IT :: 4.6

Problem: Name the ordered pair of each point shown in the rectangular coordinate system.



Solution:

A: $(4,2)$ B: $(-2,3)$ C: $(-4,-4)$ D: $(3,-5)$ E: $(-3,0)$ F: $(0,2)$

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like $x = 4$. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.
Equation:

$$\begin{aligned} 3x + 5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

But equations can have more than one variable. Equations with two variables may be of the form $Ax + By = C$. Equations of this form are called **linear equations in two variables**.

Note:
Linear Equation
An equation of the form $Ax + By = C$, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$\begin{aligned} Ax + By &= C \\ x + 4y &= 8 \end{aligned}$$

$A = 1,\ B = 4,\ C = 8$

The equation $y = -3x + 5$ is also a linear equation. But it does not appear to be in the form $Ax + By = C$. We can use the Addition Property of Equality and rewrite it in $Ax + By = C$ form.

	$y = -3x + 5$
Add to both sides.	$y + 3x = -3x + 5 + 3x$
Simplify.	$y + 3x = 5$
Use the Commutative Property to put it in $Ax + By = C$ form.	$3x + y = 5$

By rewriting $y = -3x + 5$ as $3x + y = 5$, we can easily see that it is a linear equation in two variables because it is of the form $Ax + By = C$. When an equation is in the form $Ax + By = C$, we say it is in *standard form*.

Note:
Standard Form of Linear Equation
A linear equation is in standard form when it is written $Ax + By = C$.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x, y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Note:
Solution of a Linear Equation in Two Variables
An **ordered pair** (x, y) is a **solution** of the linear equation $Ax + By = C$, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

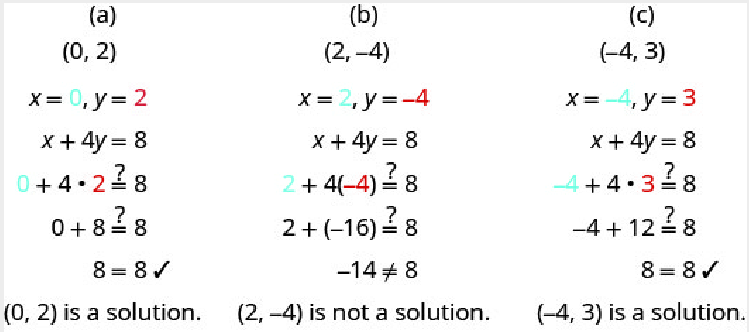
Example:
Exercise:

Problem: **Determine which ordered pairs are solutions to the equation $x + 4y = 8$.

ⓐ $(0, 2)$ ⓑ $(2, -4)$ ⓒ $(-4, 3)$

Solution:
Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if the result is a true statement.



(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \overset{?}{=} 8$	$2 + 4(-4) \overset{?}{=} 8$	$-4 + 4 \cdot 3 \overset{?}{=} 8$
$0 + 8 \overset{?}{=} 8$	$2 + (-16) \overset{?}{=} 8$	$-4 + 12 \overset{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Note:
Exercise:

TRY IT :: 4.7
Problem: Which of the following ordered pairs are solutions to $2x + 3y = 6$?

ⓐ $(3, 0)$ ⓑ $(2, 0)$ ⓒ $(6, -2)$

Solution:

a, c

Note:
Exercise:

TRY IT :: 4.8

Problem: Which of the following ordered pairs are solutions to the equation $4x - y = 8$?

ⓐ $(0, 8)$ ⓑ $(2, 0)$ ⓒ $(1, -4)$

Solution:

b, c

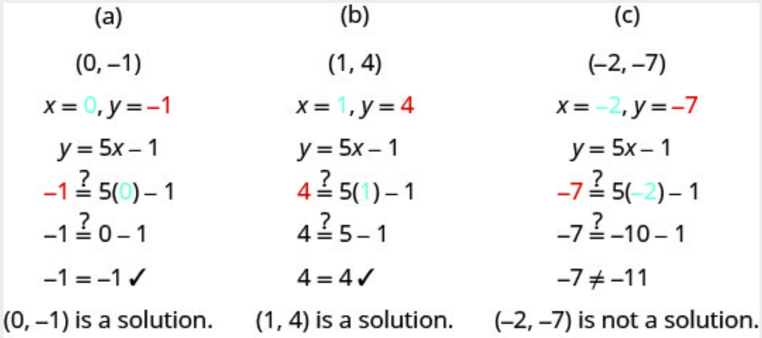
Example:
Exercise:

Problem: **Which of the following ordered pairs are solutions to the equation $y = 5x - 1$?

ⓐ $(0, -1)$ ⓑ $(1, 4)$ ⓒ $(-2, -7)$

Solution:
Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if it results in a true statement.



(a)	(b)	(c)
$(0, -1)$	$(1, 4)$	$(-2, -7)$
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \overset{?}{=} 5(0) - 1$	$4 \overset{?}{=} 5(1) - 1$	$-7 \overset{?}{=} 5(-2) - 1$
$-1 \overset{?}{=} 0 - 1$	$4 \overset{?}{=} 5 - 1$	$-7 \overset{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
$(0, -1)$ is a solution.	$(1, 4)$ is a solution.	$(-2, -7)$ is not a solution.

Note:
Exercise:

TRY IT :: 4.9

Problem: Which of the following ordered pairs are solutions to the equation $y = 4x - 3$?

ⓐ $(0,3)$ ⓑ $(1,1)$ ⓒ $(-1,-1)$

Solution:

b

Note:
Exercise:

TRY IT :: 4.10
Problem: Which of the following ordered pairs are solutions to the equation $y = -2x + 6$?

ⓐ $(0,6)$ ⓑ $(1,4)$ ⓒ $(-2,-2)$

Solution:

a, b

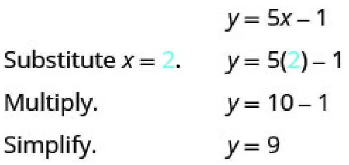
Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the *x*- and *y*-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It’s easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We’ll start by looking at the solutions to the equation $y = 5x - 1$ that we found in [link]. We can summarize this information in a table of solutions, as shown in [link].

$y = 5x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	4	$(1, 4)$

To find a third solution, we’ll let $x = 2$ and solve for y.



	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

The ordered pair $(2,9)$ is a solution to $y=5x-1$. We will add it to [link].

$y=5x-1$		
x	y	(x,y)
0	-1	$(0,-1)$
1	4	$(1,4)$
2	9	$(2,9)$

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

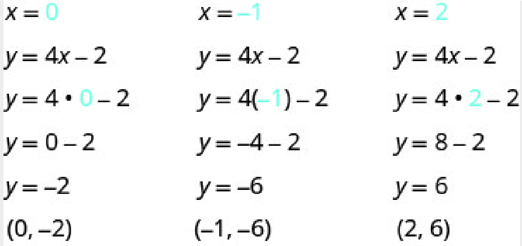
Example:
Exercise:

Problem: **Complete [link] to find three solutions to the equation $y=4x-2$.

$y=4x-2$		
x	y	(x,y)
0		
-1		
2		

Solution:
Solution

Substitute $x=0$, $x=-1$, and $x=2$ into $y=4x-2$.



$x = 0$	$x = -1$	$x = 2$
$y = 4x - 2$	$y = 4x - 2$	$y = 4x - 2$
$y = 4 \cdot 0 - 2$	$y = 4(-1) - 2$	$y = 4 \cdot 2 - 2$
$y = 0 - 2$	$y = -4 - 2$	$y = 8 - 2$
$y = -2$	$y = -6$	$y = 6$
$(0, -2)$	$(-1, -6)$	$(2, 6)$

The results are summarized in [link].

$y = 4x - 2$		
x	y	(x, y)
0	-2	$(0, -2)$
-1	-6	$(-1, -6)$
2	6	$(2, 6)$

Note:
Exercise:

TRY IT :: 4.11

Problem: Complete the table to find three solutions to this equation: $y = 3x - 1$.

$y = 3x - 1$		
x	y	(x, y)
0		
-1		
2		

Solution:

$y = 3x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
-1	-4	$(-1, -4)$
2	5	$(2, 5)$

Note:
Exercise:

TRY IT :: 4.12

Problem: Complete the table to find three solutions to this equation: $y = 6x + 1$.

$y = 6x + 1$		
x	y	(x, y)
0		
1		
-2		

Solution:

$y = 6x + 1$		
x	y	(x, y)
0	1	$(0, 1)$
1	7	$(1, 7)$
-2	-11	$(-2, -11)$

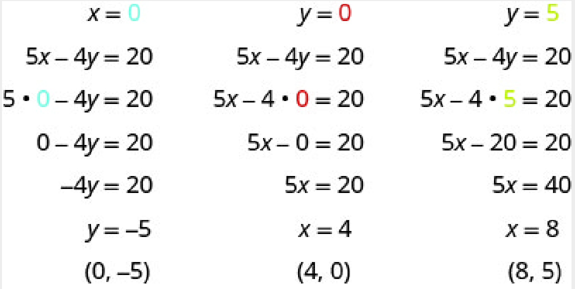
Example:
Exercise:

Problem: **Complete [link] to find three solutions to the equation $5x - 4y = 20$.

$5x - 4y = 20$		
x	y	(x, y)
0		
	0	
	5	

Solution:
Solution

Substitute the given value into the equation $5x - 4y = 20$ and solve for the other variable. Then, fill in the values in the table.



$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in [link].

$5x - 4y = 20$		
x	y	(x, y)

0	-5	$(0,-5)$
4	0	$(4,0)$
8	5	$(8,5)$

Note:
Exercise:

TRY IT :: 4.13

Problem: Complete the table to find three solutions to this equation: $2x-5y=20$.

$2x-5y=20$		
x	y	(x,y)
0		
	0	
-5		

Solution:

$2x-5y=20$		
x	y	(x,y)
0	-4	$(0,-4)$
10	0	$(10,0)$
-5	-6	$(-5,-6)$

Note:
Exercise:

TRY IT :: 4.14

Problem: Complete the table to find three solutions to this equation: $3x - 4y = 12$.

$3x - 4y = 12$		
x	y	(x, y)
0		
	0	
-4		

Solution:

$3x - 4y = 12$		
x	y	(x, y)
0	-3	$(0, -3)$
4	0	$(4, 0)$
-4	-6	$(-4, -6)$

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in *y*-form, with the *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example:
Exercise:

Problem: **Find three solutions to the equation $y = -3x + 2$.

Solution:
Solution

We can substitute any value we want for x or any value for y. Since the equation is in y-form, it will be easier to substitute in values of x. Let’s pick $x = 0$, $x = 1$, and $x = -1$.

			$x = 0$	$x = 1$	$x = -1$
			$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$
Substitute the value into the equation.			$y = -3 \cdot 0 + 2$	$y = -3 \cdot 1 + 2$	$y = -3(-1) + 2$
Simplify.			$y = 0 + 2$	$y = -3 + 2$	$y = 3 + 2$
Simplify.			$y = 2$	$y = -1$	$y = 5$
Write the ordered pair.			(0, 2)	(1, −1)	(−1, 5)
Check.					
$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$			
$2 \overset{?}{=} -3 \cdot 0 + 2$	$-1 \overset{?}{=} -3 \cdot 1 + 2$	$5 \overset{?}{=} -3(-1) + 2$			
$2 \overset{?}{=} 0 + 2$	$-1 \overset{?}{=} -3 + 2$	$5 \overset{?}{=} 3 + 2$			
$2 = 2 \checkmark$	$-1 = -1 \checkmark$	$5 = 5 \checkmark$			

So, $(0, 2)$, $(1, -1)$ and $(-1, 5)$ are all solutions to $y = -3x + 2$. We show them in [link].

$y = -3x + 2$

x	y	(x, y)
0	2	$(0, 2)$
1	-1	$(1, -1)$
-1	5	$(-1, 5)$

Note:
Exercise:

TRY IT :: 4.15
Problem: Find three solutions to this equation: $y = -2x + 3$.

Solution:

Answers will vary.

Note:
Exercise:

TRY IT :: 4.16
Problem: Find three solutions to this equation: $y = -4x + 1$.

Solution:

Answers will vary.

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when $x = 0$ find a second solution when $y = 0$, and then find a third solution.

Example:
Exercise:

Problem: **Find three solutions to the equation $3x + 2y = 6$.

Solution:
Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let’s pick first $x = 0$, then $y = 0$, and then find a third point.

	$x = 0$	$y = 0$	$x = 1$
	$3x + 2y = 6$	$3x + 2y = 6$	$3x + 2y = 6$
Substitute the value into the equation.	$3(0) + 2y = 6$	$3x + 2(0) = 6$	$3(1) + 2y = 6$
Simplify.	$0 + 2y = 6$	$3x + 0 = 6$	$3 + 2y = 6$
Solve.	$2y = 6$	$3x = 6$	$2y = 3$
	$y = 3$	$x = 2$	$y = \frac{3}{2}$
Write the ordered pair.	(0, 3)	(2, 0)	$\left(1, \frac{3}{2}\right)$
Check.			

$3x + 2y = 6$	$3x + 2y = 6$	$3x + 2y = 6$
$3 \cdot 0 + 2 \cdot 3 \overset{?}{=} 6$	$3 \cdot 2 + 2 \cdot 0 \overset{?}{=} 6$	$3 \cdot 1 + 2 \cdot \frac{3}{2} \overset{?}{=} 6$
$0 + 6 \overset{?}{=} 6$	$6 + 0 \overset{?}{=} 6$	$3 + 3 \overset{?}{=} 6$
$6 = 6\checkmark$	$6 = 6\checkmark$	$6 = 6\checkmark$

So $(0, 3)$, $(2, 0)$, and $\left(1, \frac{3}{2}\right)$ are all solutions to the equation $3x + 2y = 6$. We can list these three solutions in [link].

$3x + 2y = 6$		
x	y	(x, y)

0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$\left(1, \frac{3}{2}\right)$

Note:
Exercise:

TRY IT :: 4.17

Problem: Find three solutions to the equation $2x + 3y = 6$.

Solution:

Answers will vary.

Note:
Exercise:

TRY IT :: 4.18

Problem: Find three solutions to the equation $4x + 2y = 8$.

Solution:

Answers will vary.

Key Concepts

- **Sign Patterns of the Quadrants**

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- **Points on the Axes**
  - On the *x*-axis, $y = 0$. Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates $(a, 0)$.
  - On the *y*-axis, $x = 0$. Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates $(0, b)$.
- **Solution of a Linear Equation**
  - An ordered pair (x, y) is a solution of the linear equation $Ax + By = C$, if the equation is a true statement when the *x*- and *y*- values of the ordered pair are substituted into the equation.

Practice Makes Perfect

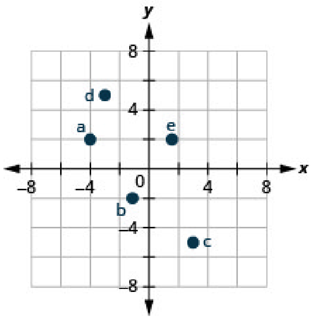
Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

Exercise:

ⓐ $(-4, 2)$
ⓑ $(-1, -2)$
ⓒ $(3, -5)$
ⓓ $(-3, 5)$
Problem: ⓔ $\left(\frac{5}{3}, 2\right)$

Solution:



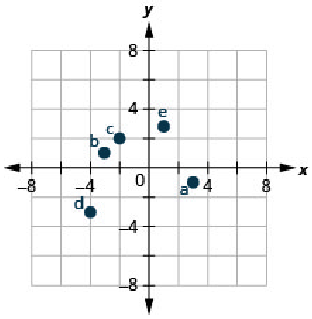
Exercise:

ⓐ $(-2, -3)$
ⓑ $(3, -3)$
ⓒ $(-4, 1)$
ⓓ $(4, -1)$
Problem: ⓔ $\left(\frac{3}{2}, 1\right)$

Exercise:

ⓐ $(3, -1)$
ⓑ $(-3, 1)$
ⓒ $(-2, 2)$
ⓓ $(-4, -3)$
Problem: ⓔ $\left(1, \frac{14}{5}\right)$

Solution:



Exercise:

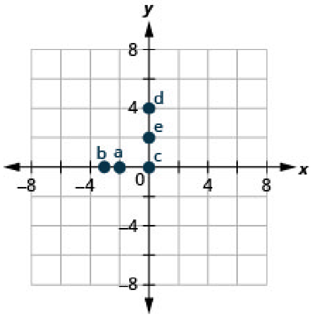
ⓐ $(-1, 1)$
ⓑ $(-2, -1)$
ⓒ $(2, 1)$
ⓓ $(1, -4)$
Problem: ⓔ $\left(3, \frac{7}{2}\right)$

In the following exercises, plot each point in a rectangular coordinate system.

Exercise:

ⓐ $(-2, 0)$
ⓑ $(-3, 0)$
ⓒ $(0, 0)$
ⓓ $(0, 4)$
Problem: ⓔ $(0, 2)$

Solution:



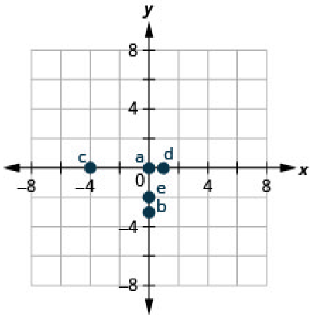
Exercise:

ⓐ $(0, 1)$
ⓑ $(0, -4)$
ⓒ $(-1, 0)$
ⓓ $(0, 0)$
Problem: ⓔ $(5, 0)$

Exercise:

ⓐ $(0,0)$
ⓑ $(0,-3)$
ⓒ $(-4,0)$
ⓓ $(1,0)$
Problem: ⓔ $(0,-2)$

Solution:



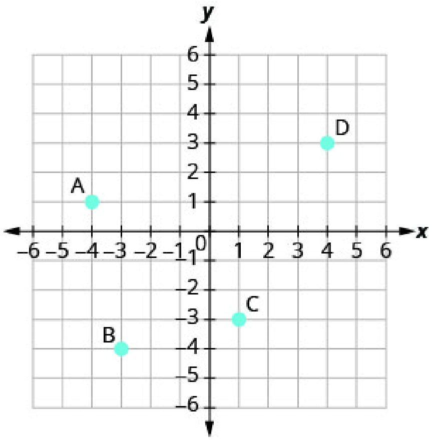
Exercise:

ⓐ $(-3,0)$
ⓑ $(0,5)$
ⓒ $(0,-2)$
ⓓ $(2,0)$
Problem: ⓔ $(0,0)$

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

Exercise:

Problem:

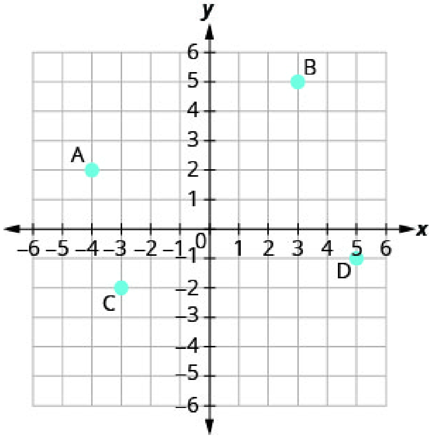


Solution:

A: $(-4, 1)$ B: $(-3, -4)$ C: $(1, -3)$ D: $(4, 3)$

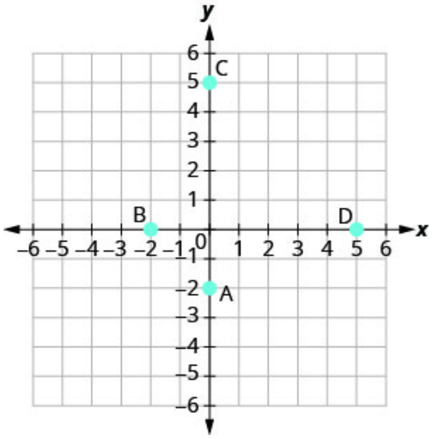
Exercise:

Problem:



Exercise:

Problem:

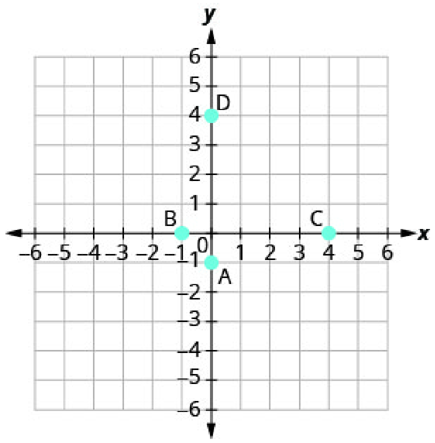


Solution:

A: $(0, -2)$ B: $(-2, 0)$ C: $(0, 5)$ D: $(5, 0)$

Exercise:

Problem:



Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise:

Problem: $2x + y = 6$

ⓐ $(1, 4)$
ⓑ $(3, 0)$
ⓒ $(2, 3)$

Solution:

a, b

Exercise:

Problem: $x + 3y = 9$

ⓐ $(0, 3)$
ⓑ $(6, 1)$
ⓒ $(-3, -3)$

Exercise:

Problem: $4x - 2y = 8$

ⓐ $(3, 2)$
ⓑ $(1, 4)$
ⓒ $(0, -4)$

Solution:

a, c

Exercise:

Problem: $3x - 2y = 12$

ⓐ $(4, 0)$
ⓑ $(2, -3)$
ⓒ $(1, 6)$

Exercise:

Problem: $y = 4x + 3$

ⓐ $(4, 3)$
ⓑ $(-1, -1)$
ⓒ $\left(\frac{1}{2}, 5\right)$

Solution:

b, c

Exercise:

Problem: $y = 2x - 5$

ⓐ $(0, -5)$
ⓑ $(2, 1)$
ⓒ $\left(\frac{1}{2}, -4\right)$

Exercise:

Problem: $y = \frac{1}{2}x - 1$

ⓐ $(2, 0)$
ⓑ $(-6, -4)$
ⓒ $(-4, -1)$

Solution:

a, b

Exercise:

Problem: $y = \frac{1}{3}x + 1$

ⓐ $(-3, 0)$
ⓑ $(9, 4)$
ⓒ $(-6, -1)$

Complete a Table of Solutions to a Linear Equation

In the following exercises, complete the table to find solutions to each linear equation.

Exercise:

Problem: $y = 2x - 4$

x	y	(x,y)
0		
2		
-1		

Solution:

x	y	(x,y)
0	-4	$(0,-4)$
2	0	$(2,0)$
-1	-6	$(-1,-6)$

Exercise:

Problem: $y=3x-1$

x	y	(x,y)
0		
2		
-1		

Exercise:

Problem: $y=-x+5$

x	y	(x, y)
0		
3		
-2		

Solution:

x	y	(x, y)
0	5	$(0, 5)$
3	2	$(3, 2)$
-2	7	$(-2, 7)$

Exercise:

Problem: $y = -x + 2$

x	y	(x, y)
0		
3		
-2		

Exercise:

Problem: $y = \frac{1}{3}x + 1$

x	y	(x,y)
0		
3		
6		

Solution:

x	y	(x,y)
0	1	$(0,1)$
3	2	$(3,2)$
6	3	$(6,3)$

Exercise:

Problem: $y=\frac{1}{2}x+4$

x	y	(x,y)
0		
2		
4		

Exercise:

Problem: $y=-\frac{3}{2}x-2$

x	y	(x, y)
0		
2		
-2		

Solution:

x	y	(x, y)
0	-2	$(0, -2)$
2	-5	$(2, -5)$
-2	1	$(-2, 1)$

Exercise:

Problem: $y = -\frac{2}{3}x - 1$

x	y	(x, y)
0		
3		
-3		

Exercise:

Problem: $x + 3y = 6$

x	y	(x, y)
0		
3		
	0	

Solution:

x	y	(x, y)
0	2	$(0, 2)$
3	4	$(3, 1)$
6	0	$(6, 0)$

Exercise:

Problem: $x + 2y = 8$

x	y	(x, y)
0		
4		
	0	

Exercise:

Problem: $2x - 5y = 10$

x	y	(x, y)
0		
10		
	0	

Solution:

x	y	(x, y)
0	-2	$(0, -2)$
10	2	$(10, 2)$
5	0	$(5, 0)$

Exercise:

Problem: $3x - 4y = 12$

x	y	(x, y)
0		
8		
	0	

Find Solutions to a Linear Equation

In the following exercises, find three solutions to each linear equation.

Exercise:

Problem: $y = 5x - 8$

Solution:

Answers will vary.

Exercise:

Problem: $y = 3x - 9$

Exercise:

Problem: $y = -4x + 5$

Solution:

Answers will vary.

Exercise:

Problem: $y = -2x + 7$

Exercise:

Problem: $x + y = 8$

Solution:

Answers will vary.

Exercise:

Problem: $x + y = 6$

Exercise:

Problem: $x + y = -2$

Solution:

Answers will vary.

Exercise:

Problem: $x + y = -1$

Exercise:

Problem: $3x + y = 5$

Solution:

Answers will vary.

Exercise:

Problem: $2x + y = 3$

Exercise:

Problem: $4x - y = 8$

Solution:

Answers will vary.

Exercise:

Problem: $5x - y = 10$

Exercise:

Problem: $2x + 4y = 8$

Solution:

Answers will vary.

Exercise:

Problem: $3x + 2y = 6$

Exercise:

Problem: $5x - 2y = 10$

Solution:

Answers will vary.

Exercise:

Problem: $4x - 3y = 12$

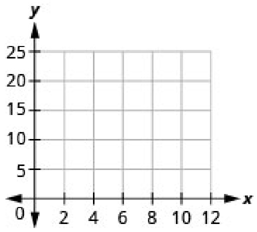
Everyday Math

Exercise:

Problem:

Weight of a baby. Mackenzie recorded her baby’s weight every two months. The baby’s age, in months, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

ⓐ Plot the points on a coordinate plane.

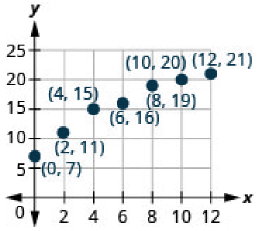


ⓑ Why is only Quadrant I needed?

Age x	**Weight** y	(x, y)
0	7	(0, 7)
2	11	(2, 11)
4	15	(4, 15)
6	16	(6, 16)
8	19	(8, 19)
10	20	(10, 20)
12	21	(12, 21)

Solution:

ⓐ



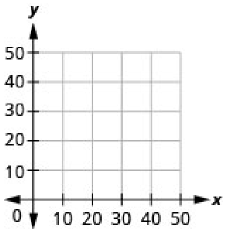
ⓑ Age and weight are only positive.

Exercise:

Problem:

Weight of a child. Latresha recorded her son’s height and weight every year. His height, in inches, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

ⓐ Plot the points on a coordinate plane.



ⓑ Why is only Quadrant I needed?

Height x	**Weight** y	(x, y)
28	22	(28, 22)
31	27	(31, 27)
33	33	(33, 33)
37	35	(37, 35)
40	41	(40, 41)
42	45	(42, 45)

Writing Exercises

Exercise:

Problem: Explain in words how you plot the point $(4, -2)$ in a rectangular coordinate system.

Solution:

Answers will vary.

Exercise:

Problem: How do you determine if an ordered pair is a solution to a given equation?

Exercise:

Problem: Is the point $(-3, 0)$ on the *x*-axis or *y*-axis? How do you know?

Solution:

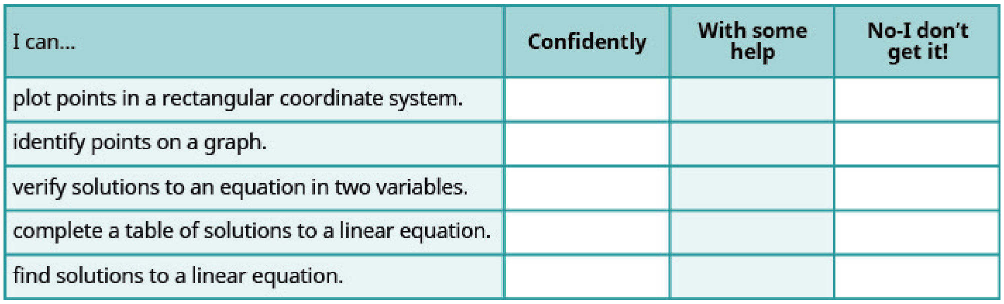
Answers will vary.

Exercise:

Problem: Is the point $(0,8)$ on the *x*-axis or *y*-axis? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
plot points in a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to a linear equation.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no, I don't get it. This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

linear equation

A linear equation is of the form $Ax+By=C$, where A and B are not both zero, is called a linear equation in two variables.

ordered pair

An ordered pair (x,y) gives the coordinates of a point in a rectangular coordinate system.

origin

The point $(0,0)$ is called the origin. It is the point where the *x*-axis and *y*-axis intersect.

quadrant

The *x*-axis and the *y*-axis divide a plane into four regions, called quadrants.

rectangular coordinate system

A grid system is used in algebra to show a relationship between two variables; also called the *xy*-plane or the ‘coordinate plane’.

x-coordinate
The first number in an ordered pair (x, y).

y-coordinate
The second number in an ordered pair (x, y).

Graph Linear Equations in Two Variables
By the end of this section, you will be able to:

- Recognize the relationship between the solutions of an equation and its graph.
- Graph a linear equation by plotting points.
- Graph vertical and horizontal lines.

Note:
Before you get started, take this readiness quiz.

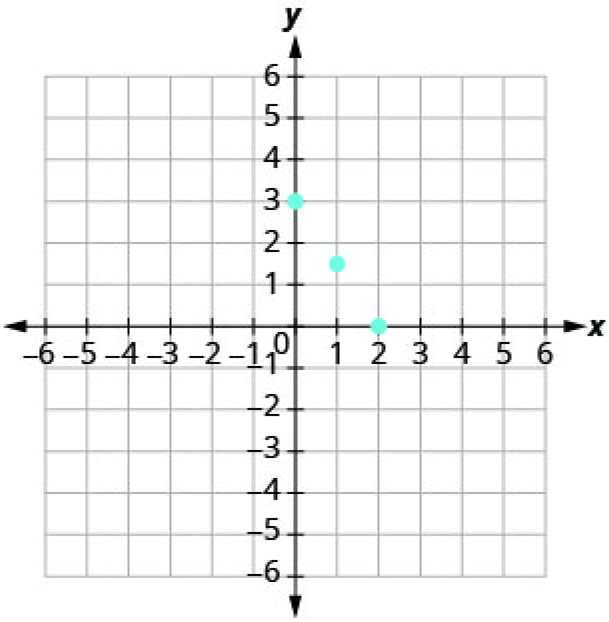
1. Evaluate $3x+2$ when $x=-1$.
   If you missed this problem, review [link].
2. Solve $3x+2y=12$ for y in general.
   If you missed this problem, review [link].

Recognize the Relationship Between the Solutions of an Equation and its Graph

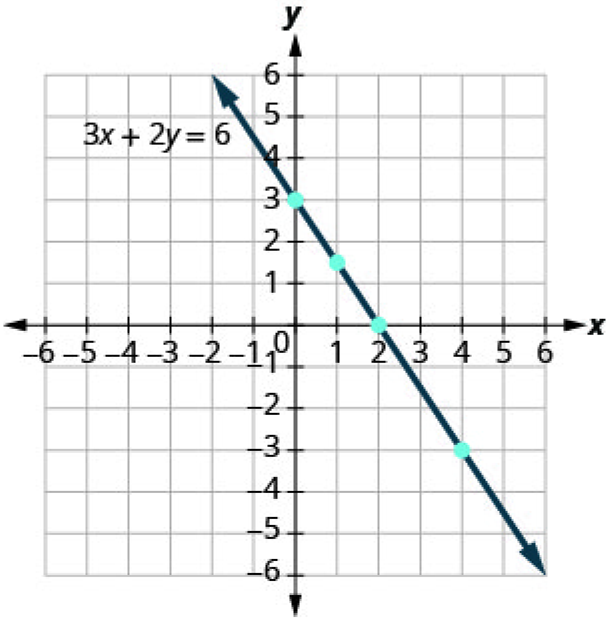
In the previous section, we found several solutions to the equation $3x+2y=6$. They are listed in [link]. So, the ordered pairs $(0,3)$, $(2,0)$, and $\left(1,\frac{3}{2}\right)$ are some solutions to the equation $3x+2y=6$. We can plot these solutions in the rectangular coordinate system as shown in [link].

$3x+2y=6$		
x	y	(x,y)

0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$\left(1, \frac{3}{2}\right)$

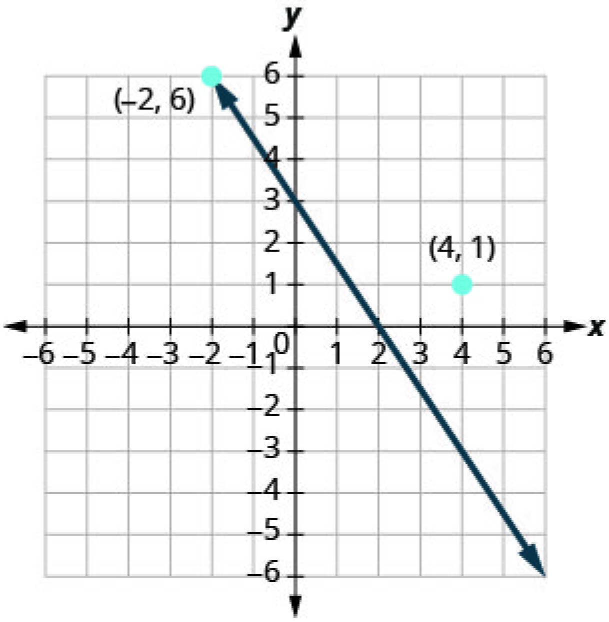


Notice how the points line up perfectly? We connect the points with a line to get the graph of the equation $3x + 2y = 6$. See [link]. Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.



Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points *not* on the line are not solutions.

Notice that the point whose coordinates are $(-2,6)$ is on the line shown in [link]. If you substitute $x=-2$ and $y=6$ into the equation, you find that it is a solution to the equation.



Test $(-2, 6)$

$$3x + 2y = 6$$

$$3(-2) + 2(6) = 6$$

$$-6 + 12 = 6$$

$$6 = 6 \checkmark$$

So the point $(-2, 6)$ is a solution to the equation $3x + 2y = 6$. (The phrase "the point whose coordinates are $(-2, 6)$" is often shortened to "the point $(-2, 6)$.")

What about $(4, 1)$?

$$3x + 2y = 6$$

$$3 \cdot 4 + 2 \cdot 1 = 6$$

$$12 + 2 \overset{?}{=} 6$$

$$14 \neq 6$$

So $(4, 1)$ is not a solution to the equation $3x + 2y = 6$. Therefore, the point $(4, 1)$ is not on the line. See [link]. This is an example of the saying, "A picture is worth a thousand words." The line shows you *all* the solutions to the equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the *graph* of the equation $3x + 2y = 6$.

Note:

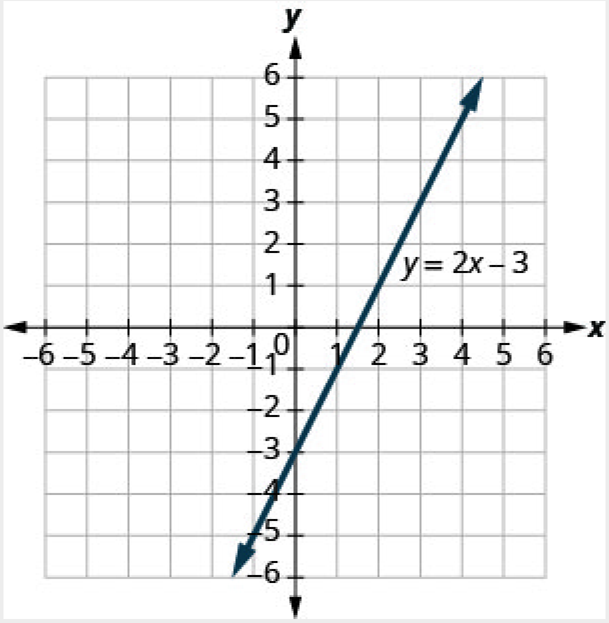
Graph of a Linear Equation

The **graph of a linear equation** $Ax + By = C$ is a line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

Example:
Exercise:

Problem: **The graph of $y = 2x - 3$ is shown.



For each ordered pair, decide:

ⓐ Is the ordered pair a solution to the equation?
ⓑ Is the point on the line?

A $(0, -3)$ B $(3, 3)$ C $(2, -3)$ D $(-1, -5)$

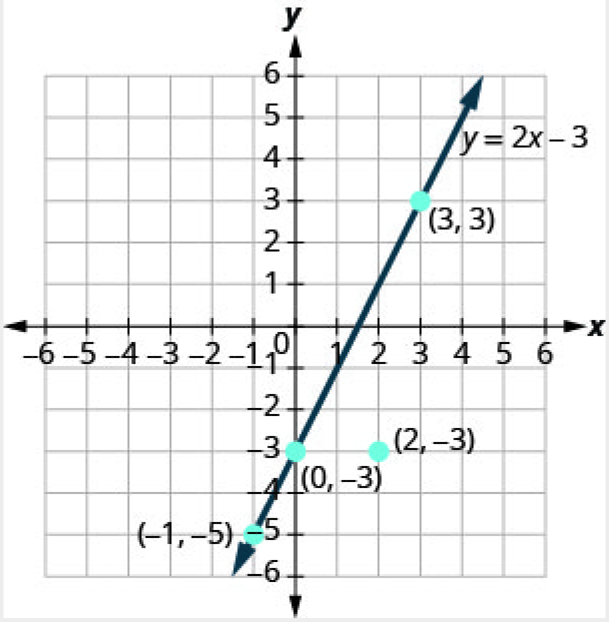
Solution:
Solution

Substitute the *x*- and *y*- values into the equation to check if the ordered pair is a solution to the equation.

ⓐ

A: $(0, -3)$	B: $(3, 3)$	C: $(2, -3)$	D: $(-1, -5)$
$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$
$-3 \overset{?}{=} 2(0) - 3$	$3 \overset{?}{=} 2(3) - 3$	$-3 \overset{?}{=} 2(2) - 3$	$-5 \overset{?}{=} 2(-1) - 3$
$-3 = -3$ ✓	$3 = 3$ ✓	$-3 \neq 1$	$-5 = -5$ ✓
$(0, -3)$ is a solution.	$(3, 3)$ is a solution.	$(2, -3)$ is not a solution.	$(-1, -5)$ is a solution.

ⓑ Plot the points A $(0, 3)$, B $(3, 3)$, C $(2, -3)$, and D $(-1, -5)$.



The points $(0, 3)$, $(3, 3)$, and $(-1, -5)$ are on the line $y = 2x - 3$, and the point $(2, -3)$ is not on the line.

The points that are solutions to $y = 2x - 3$ are on the line, but the point that is not a solution is not on the line.

Note:
Exercise:

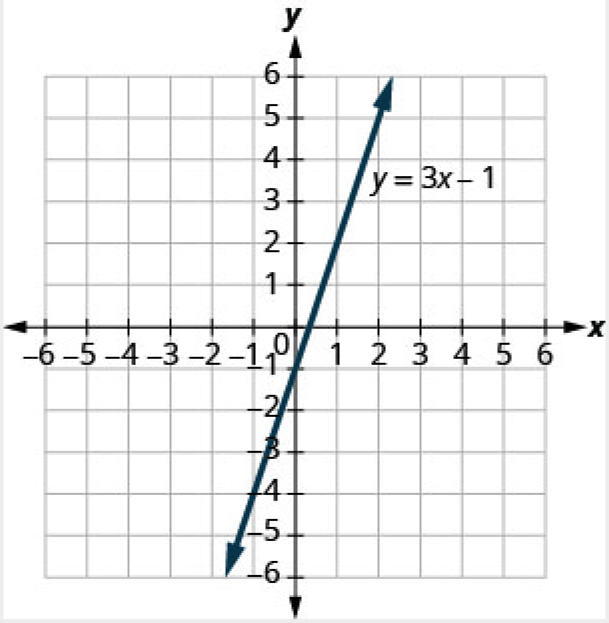
Problem:

TRY IT :: 4.19

Use the graph of $y = 3x - 1$ to decide whether each ordered pair is:

- a solution to the equation.
- on the line.

ⓐ $(0, -1)$ ⓑ $(2, 5)$



Solution:

ⓐ yes, yes ⓑ yes, yes

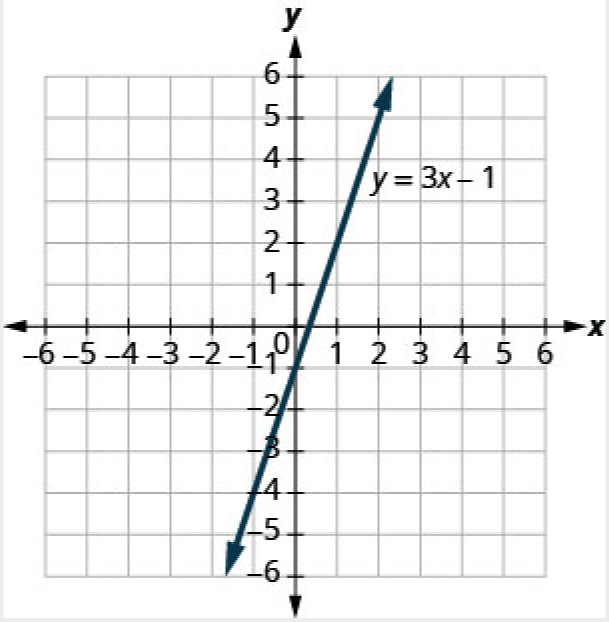
Note:
Exercise:

Problem:

TRY IT :: 4.20
Use graph of $y = 3x - 1$ to decide whether each ordered pair is:

- a solution to the equation
- on the line

ⓐ $(3, -1)$ ⓑ $(-1, -4)$



Solution:

ⓐ no, no ⓑ yes, yes

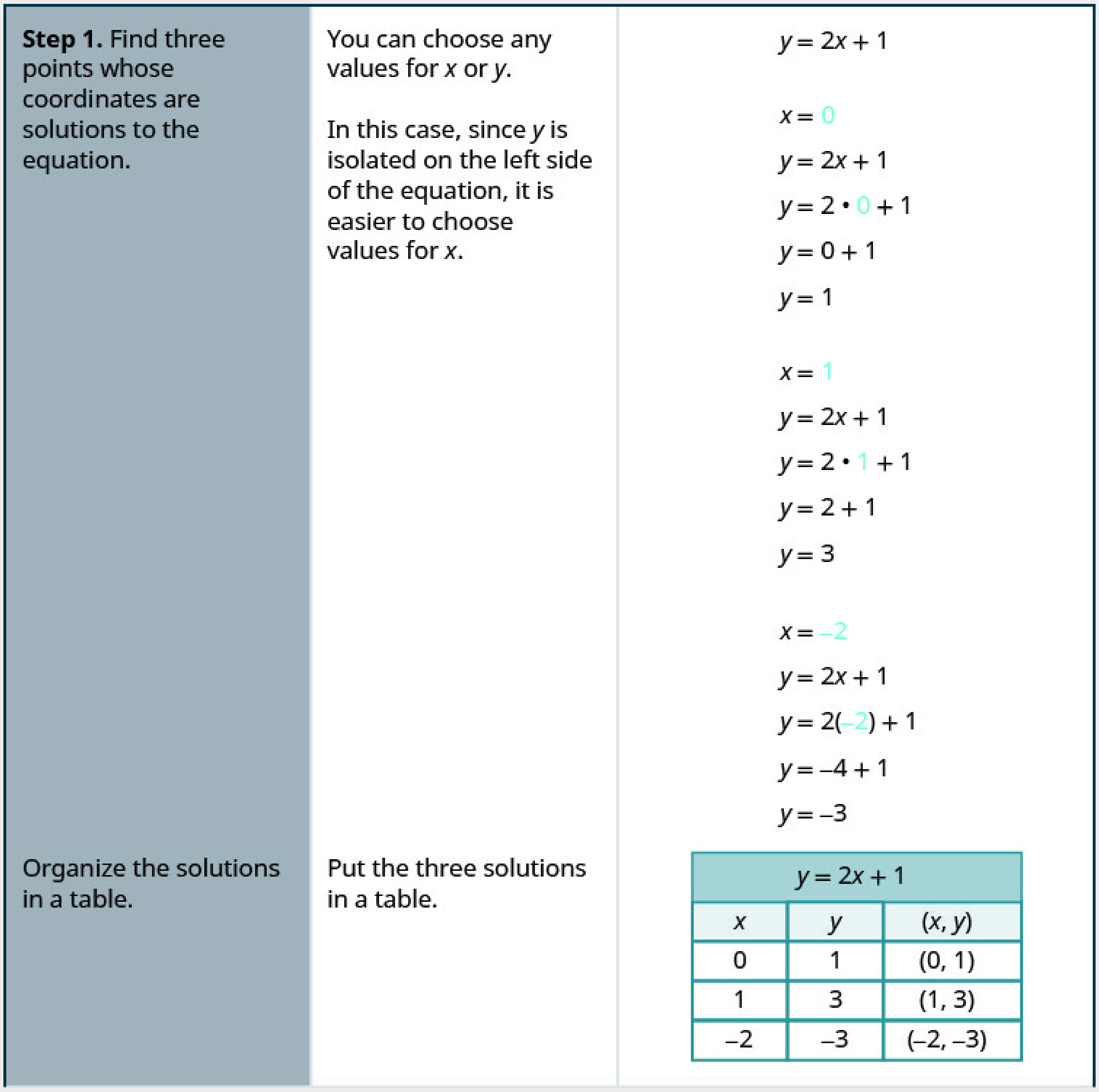
Graph a Linear Equation by Plotting Points

There are several methods that can be used to graph a linear equation. The method we used to graph $3x + 2y = 6$ is called plotting points, or the Point–Plotting Method.

Example:
How To Graph an Equation By Plotting Points
Exercise:

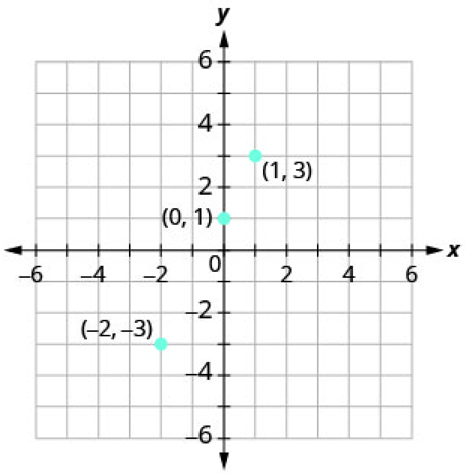
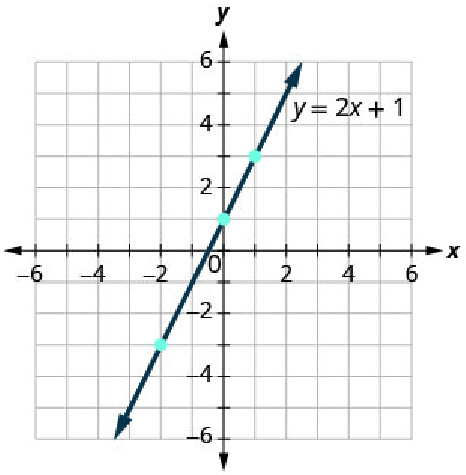
Problem: **Graph the equation $y = 2x + 1$ by plotting points.

Solution:
Solution



Step 1. Find three points whose coordinates are solutions to the equation.	You can choose any values for x or y. In this case, since y is isolated on the left side of the equation, it is easier to choose values for x.	$y = 2x + 1$ $x = 0$ $y = 2x + 1$ $y = 2 \cdot 0 + 1$ $y = 0 + 1$ $y = 1$ $x = 1$ $y = 2x + 1$ $y = 2 \cdot 1 + 1$ $y = 2 + 1$ $y = 3$ $x = -2$ $y = 2x + 1$ $y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$
Organize the solutions in a table.	Put the three solutions in a table.	$y = 2x + 1$

x	y	(x, y)
0	1	(0, 1)
1	3	(1, 3)
–2	–3	(–2, –3)

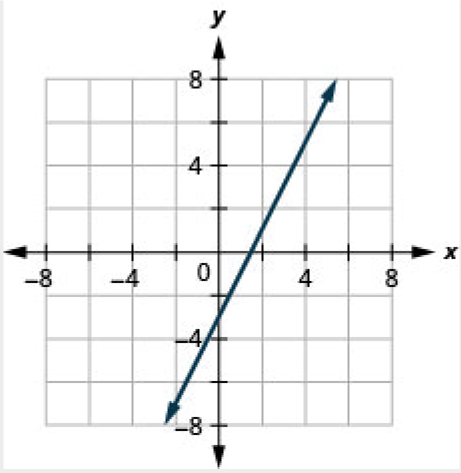
Step 2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!	Plot: (0, 1), (1, 3), (–2, –3). Do the points line up? Yes, the points line up.	
Step 3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.	This line is the graph of $y = 2x + 1$.	

Note:
Exercise:

TRY IT :: 4.21

Problem: Graph the equation by plotting points: $y = 2x - 3$.

Solution:

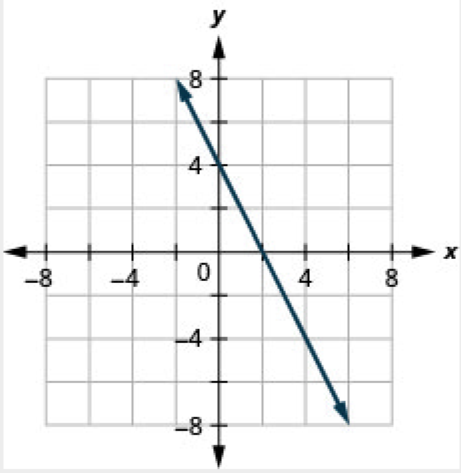


Note:
Exercise:

TRY IT :: 4.22

Problem: Graph the equation by plotting points: $y=-2x+4$.

Solution:



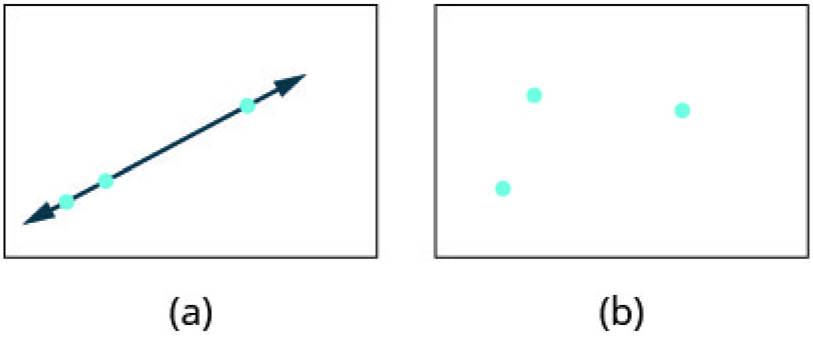
The steps to take when graphing a linear equation by plotting points are summarized below.

Note:
Graph a linear equation by plotting points.

Find three points whose coordinates are solutions to the equation. Organize them in a table.
Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. Look at the difference between part (a) and part (b) in [link].



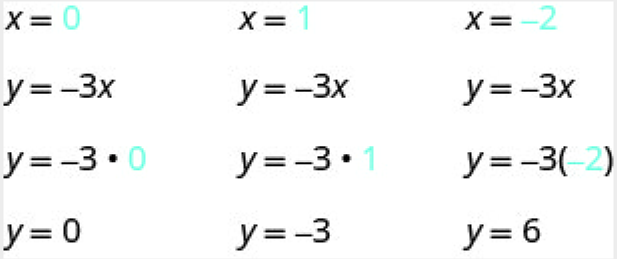
Let’s do another example. This time, we’ll show the last two steps all on one grid.

Example:
Exercise:

Problem: **Graph the equation $y = -3x$.

Solution:
Solution

Find three points that are solutions to the equation. Here, again, it’s easier to choose values for x. Do you see why?



$x = 0$
$y = -3x$
$y = -3 \cdot 0$
$y = 0$

$x = 1$
$y = -3x$
$y = -3 \cdot 1$
$y = -3$

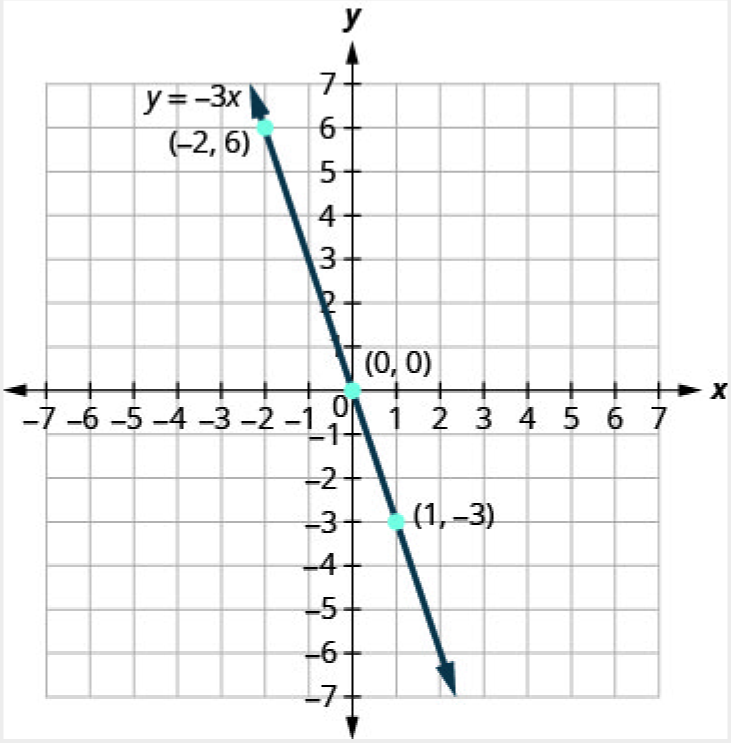
$x = -2$
$y = -3x$
$y = -3(-2)$
$y = 6$

We list the points in [link].

$y = -3x$		
x	y	(x, y)
0	0	$(0, 0)$
1	-3	$(1, -3)$

-2	6	$(-2, 6)$

Plot the points, check that they line up, and draw the line.

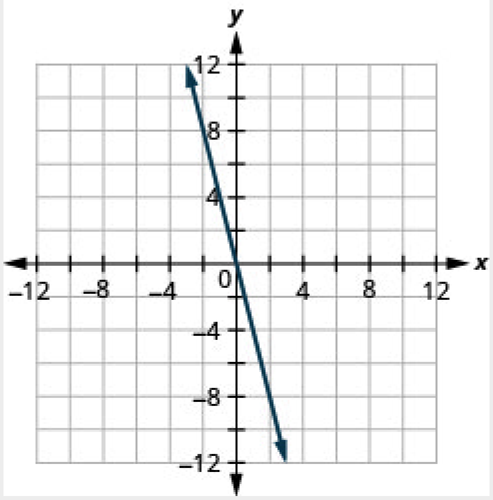


Note:
Exercise:

TRY IT :: 4.23

Problem: Graph the equation by plotting points: $y = -4x$.

Solution:

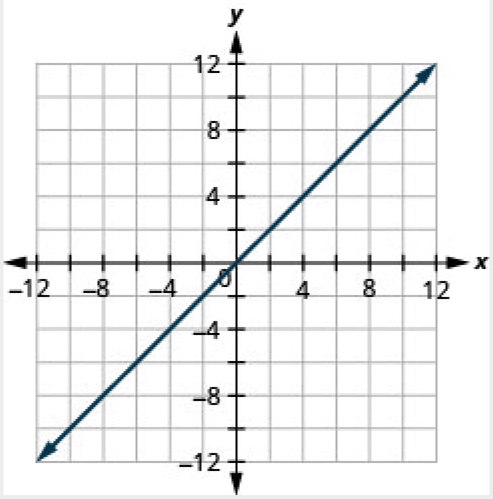


Note:
Exercise:

TRY IT :: 4.24

Problem: Graph the equation by plotting points: $y = x$.

Solution:



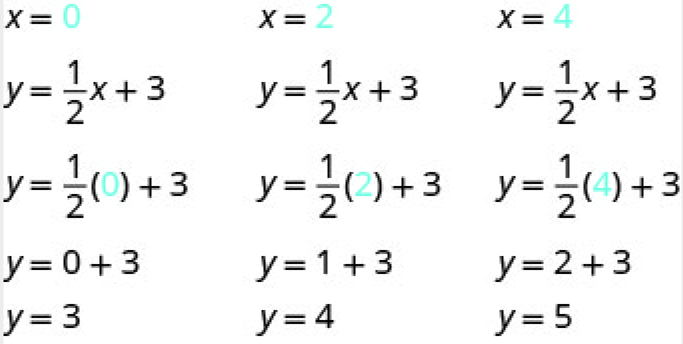
When an equation includes a fraction as the coefficient of x, we can still substitute any numbers for x. But the math is easier if we make ‘good’ choices for the values of x. This way we will avoid fraction answers, which are hard to graph precisely.

Example:
Exercise:

Problem: **Graph the equation $y = \frac{1}{2}x + 3$.

Solution:
Solution

Find three points that are solutions to the equation. Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x, we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of 2 a good choice for values of x?



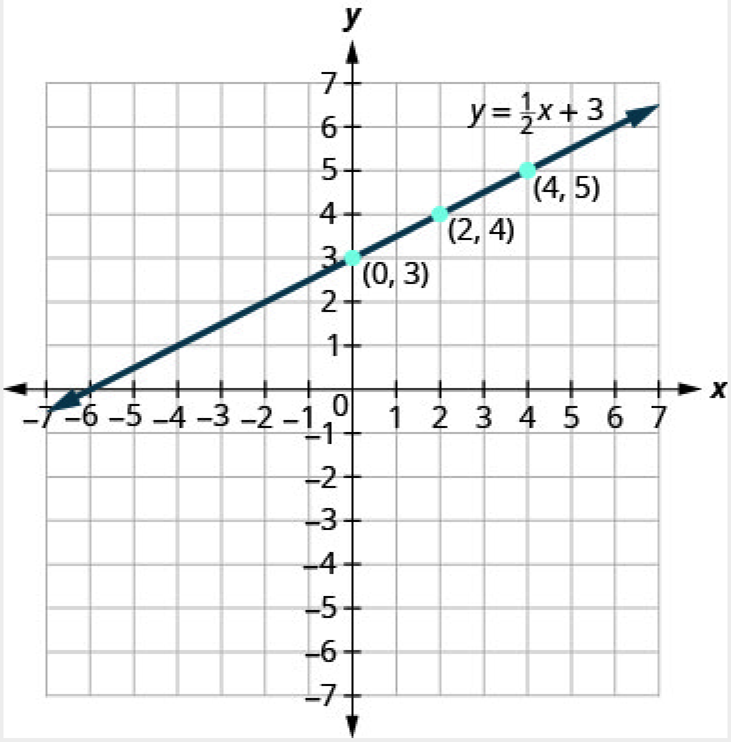
$x = 0$	$x = 2$	$x = 4$
$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$
$y = \frac{1}{2}(0) + 3$	$y = \frac{1}{2}(2) + 3$	$y = \frac{1}{2}(4) + 3$
$y = 0 + 3$	$y = 1 + 3$	$y = 2 + 3$
$y = 3$	$y = 4$	$y = 5$

The points are shown in [link].

$y = \frac{1}{2}x + 3$		

x	y	(x, y)
0	3	$(0, 3)$
2	4	$(2, 4)$
4	5	$(4, 5)$

Plot the points, check that they line up, and draw the line.

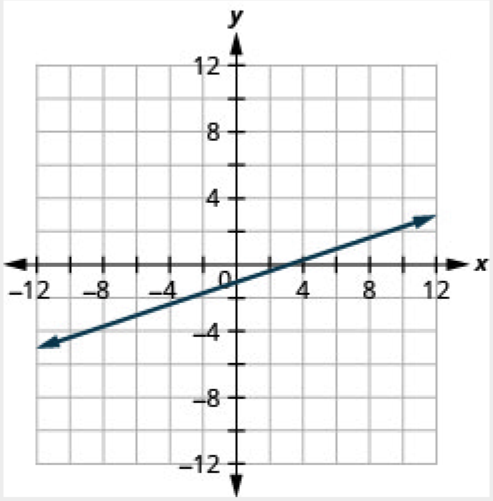


Note:
Exercise:

TRY IT :: 4.25

Problem: Graph the equation $y = \frac{1}{3}x - 1$.

Solution:

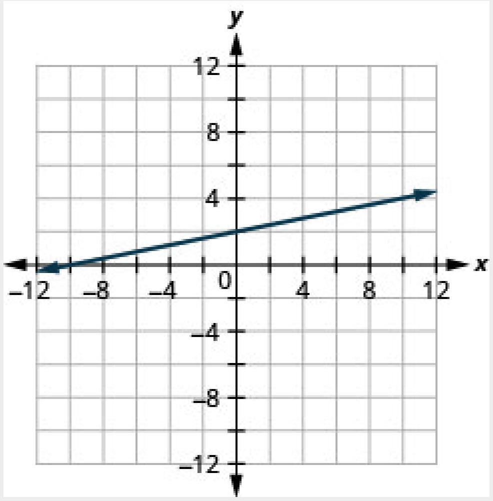


Note:
Exercise:

TRY IT :: 4.26

Problem: Graph the equation $y = \frac{1}{4}x + 2$.

Solution:



So far, all the equations we graphed had y given in terms of x. Now we'll graph an equation with x and y on the same side. Let's see what happens in the equation $2x + y = 3$. If $y = 0$ what is the value of x?

$$\begin{aligned} y &= 0 \\ 2x + y &= 3 \\ 2x + 0 &= 3 \\ 2x &= 3 \\ x &= \frac{3}{2} \\ &\left(\frac{3}{2}, 0\right) \end{aligned}$$

This point has a fraction for the *x*- coordinate and, while we could graph this point, it is hard to be precise graphing fractions. Remember in the example $y = \frac{1}{2}x + 3$, we carefully chose values for x so as not to graph fractions at all. If we solve the equation $2x + y = 3$ for y, it will be easier to find three solutions to the equation.

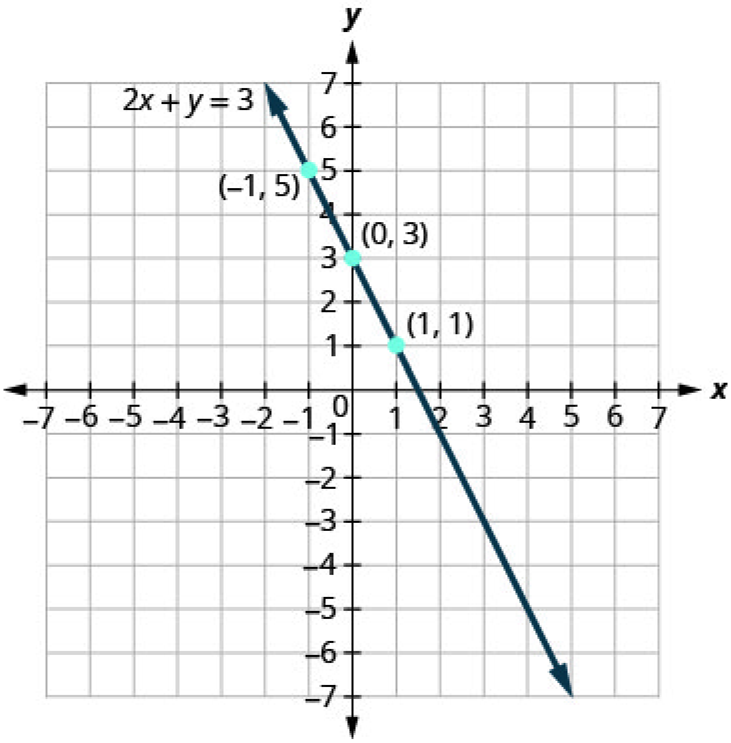
Equation:

$$\begin{aligned} 2x + y &= 3 \\ y &= -2x + 3 \end{aligned}$$

The solutions for $x = 0$, $x = 1$, and $x = -1$ are shown in the [link]. The graph is shown in [link].

$2x + y = 3$		

x	y	(x, y)
0	3	$(0, 3)$
1	1	$(1, 1)$
-1	5	$(-1, 5)$



Can you locate the point $\left(\frac{3}{2}, 0\right)$, which we found by letting $y = 0$, on the line?

Example:
Exercise:

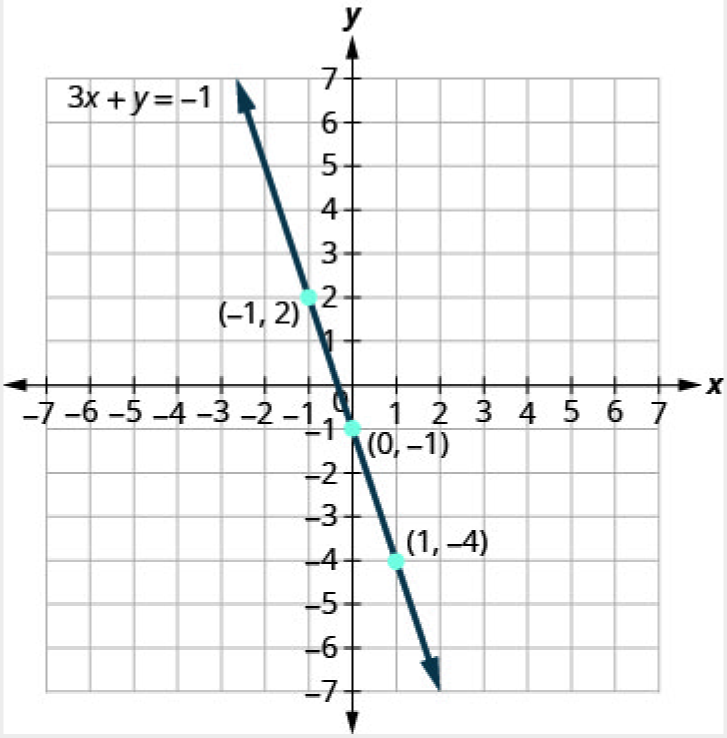
Problem: **Graph the equation $3x + y = -1$.

Solution:
Solution

Find three points that are solutions to the equation.	$3x + y = -1$
First, solve the equation for y.	$y = -3x - 1$

We'll let x be 0, 1, and -1 to find 3 points. The ordered pairs are shown in [link]. Plot the points, check that they line up, and draw the line. See [link].

$3x + y = -1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	-4	$(1, -4)$
-1	2	$(-1, 2)$

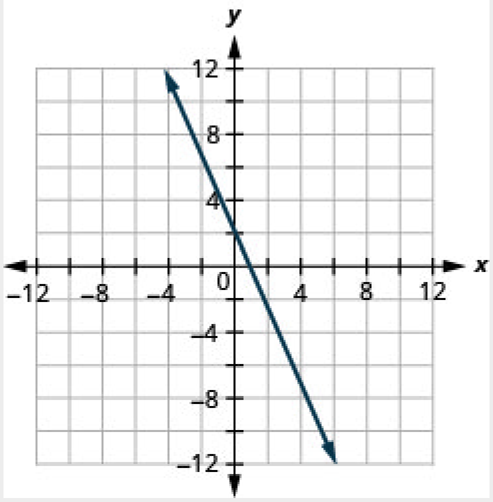


Note:
Exercise:

TRY IT :: 4.27

Problem: Graph the equation $2x + y = 2$.

Solution:

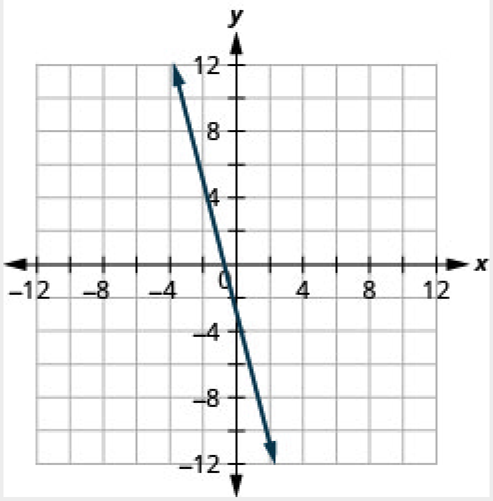


Note:
Exercise:

TRY IT :: 4.28

Problem: Graph the equation $4x + y = -3$.

Solution:



If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the *x*- and *y*-axis are the same, the graphs match!

The equation in [link] was written in standard form, with both x and y on the same side. We solved that equation for y in just one step. But for other equations in standard form it is not that easy to solve for y, so we will leave them in standard form. We can still find a first point to plot by letting $x = 0$ and solving for y. We can plot a second point by letting $y = 0$ and then solving for x. Then we will plot a third point by using some other value for x or y.

Example:
Exercise:

Problem: **Graph the equation $2x - 3y = 6$.

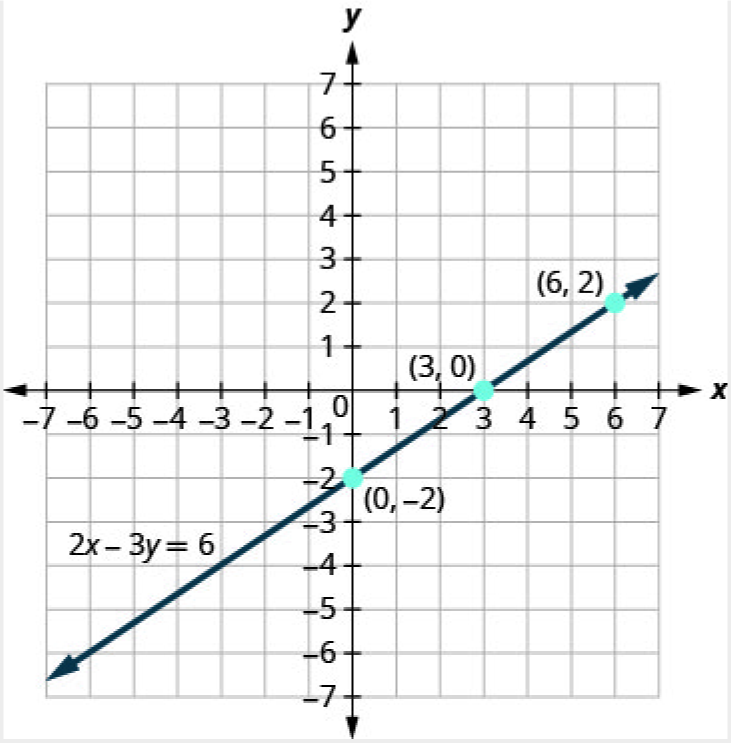
Solution:
Solution

Find three points that are solutions to the equation.	$2x - 3y \quad = \quad 6$
First, let $x = 0$.	$2(0) - 3y \quad = \quad 6$
Solve for y.	$\begin{aligned} -3y &= 6 \\ y &= -2 \end{aligned}$
Now let $y = 0$.	$2x - 3(0) \quad = \quad 6$

Solve for x.	$$\begin{aligned} 2x &= 6 \\ x &= 3 \end{aligned}$$
We need a third point. Remember, we can choose any value for x or y. We'll let $x = 6$.	$$2(6) - 3y = 6$$
Solve for y.	$$\begin{aligned} 12 - 3y &= 6 \\ -3y &= -6 \\ y &= 2 \end{aligned}$$

We list the ordered pairs in [link]. Plot the points, check that they line up, and draw the line. See [link].

$2x - 3y = 6$		
x	y	(x, y)
0	-2	$(0, -2)$
3	0	$(3, 0)$
6	2	$(6, 2)$

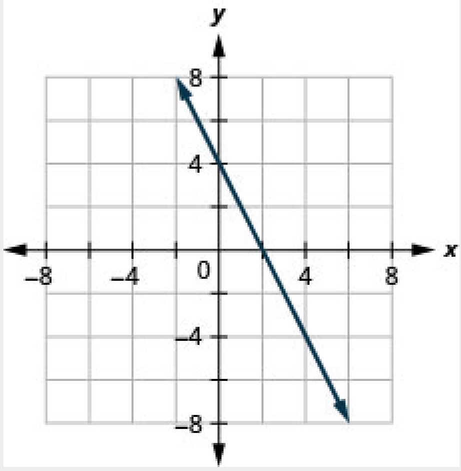


Note:
Exercise:

TRY IT :: 4.29

Problem: Graph the equation $4x + 2y = 8$.

Solution:

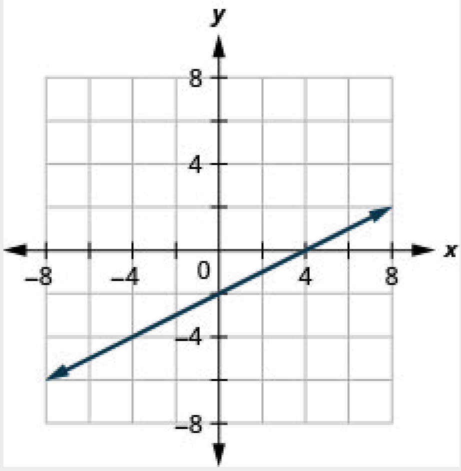


Note:
Exercise:

TRY IT :: 4.30

Problem: Graph the equation $2x - 4y = 8$.

Solution:



Graph Vertical and Horizontal Lines

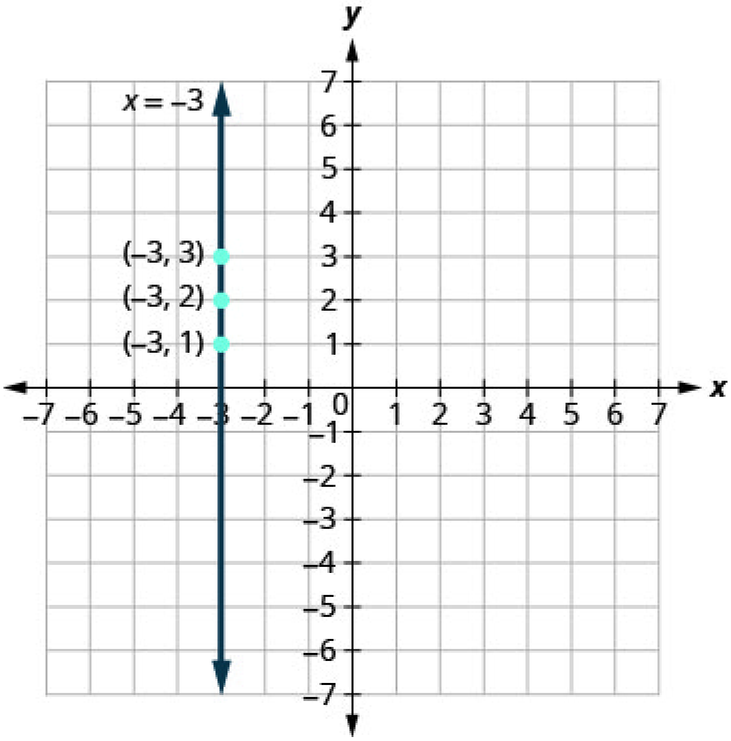
Can we graph an equation with only one variable? Just x and no y, or just y without an x? How will we make a table of values to get the points to plot?

Let’s consider the equation $x = -3$. This equation has only one variable, x. The equation says that x is *always* equal to -3, so its value does not depend on y. No matter what y is, the value of x is always -3.

So to make a table of values, write -3 in for all the x values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we’ll use 1, 2, and 3 for the y-coordinates. See [link].

$x = -3$		
x	y	(x, y)
-3	1	$(-3, 1)$
-3	2	$(-3, 2)$
-3	3	$(-3, 3)$

Plot the points from [link] and connect them with a straight line. Notice in [link] that we have graphed a *vertical line*.



Note:
Vertical Line
A **vertical line** is the graph of an equation of the form $x = a$.
The line passes through the *x*-axis at $(a, 0)$.

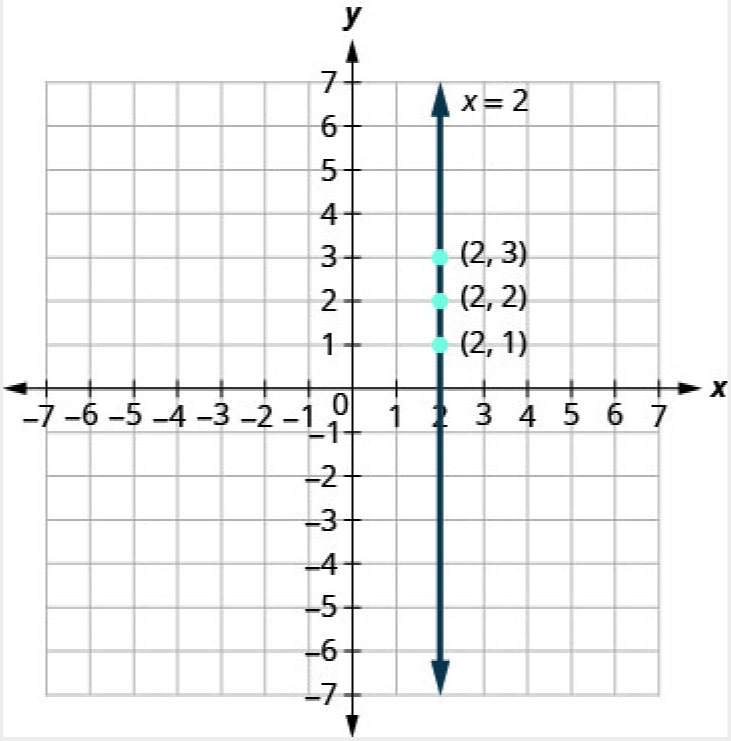
Example:
Exercise:

Problem: **Graph the equation $x = 2$.

Solution:
Solution

The equation has only one variable, x, and x is always equal to 2. We create [link] where x is always 2 and then put in any values for y. The graph is a vertical line passing through the x-axis at 2. See [link].

$x = 2$		
x	y	(x, y)
2	1	$(2, 1)$
2	2	$(2, 2)$
2	3	$(2, 3)$

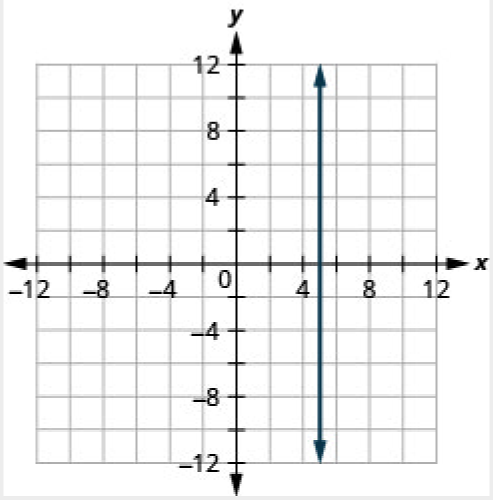


Note:
Exercise:

TRY IT :: 4.31

Problem: Graph the equation $x = 5$.

Solution:

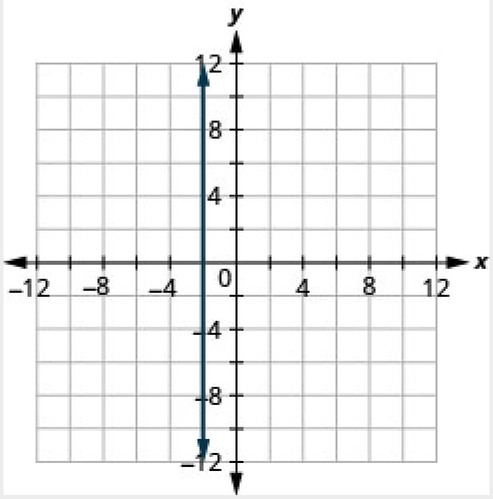


Note:
Exercise:

TRY IT :: 4.32

Problem: Graph the equation $x = -2$.

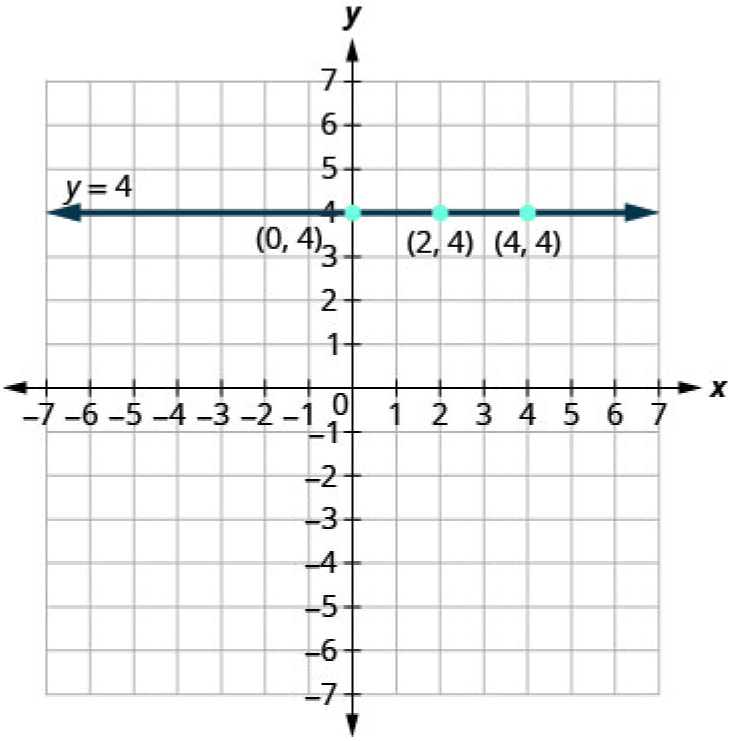
Solution:



What if the equation has y but no x? Let’s graph the equation $y = 4$. This time the *y*- value is a constant, so in this equation, y does not depend on x. Fill in 4 for all the y’s in [link] and then choose any values for x. We’ll use 0, 2, and 4 for the *x*-coordinates.

$y = 4$		
x	y	(x, y)
0	4	$(0, 4)$
2	4	$(2, 4)$
4	4	$(4, 4)$

The graph is a horizontal line passing through the *y*-axis at 4. See [link].



Note:
Horizontal Line
A **horizontal line** is the graph of an equation of the form $y = b$.
The line passes through the *y*-axis at $(0, b)$.

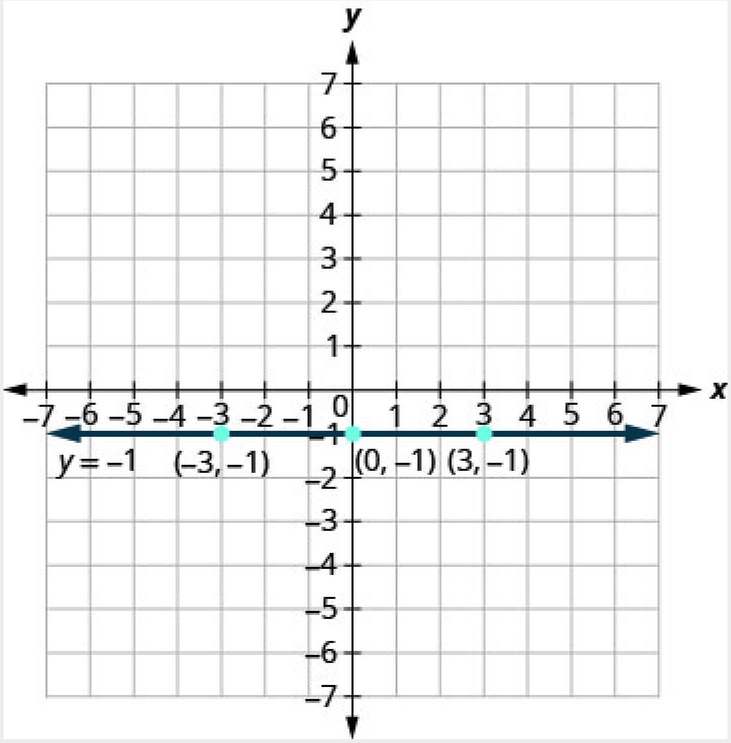
Example:
Exercise:

Problem: **Graph the equation $y = -1$.

Solution:
Solution

The equation $y = -1$ has only one variable, y. The value of y is constant. All the ordered pairs in [link] have the same *y*-coordinate. The graph is a horizontal line passing through the *y*-axis at -1, as shown in [link].

$y = -1$		
x	y	(x, y)
0	-1	$(0, -1)$
3	-1	$(3, -1)$
-3	-1	$(-3, -1)$

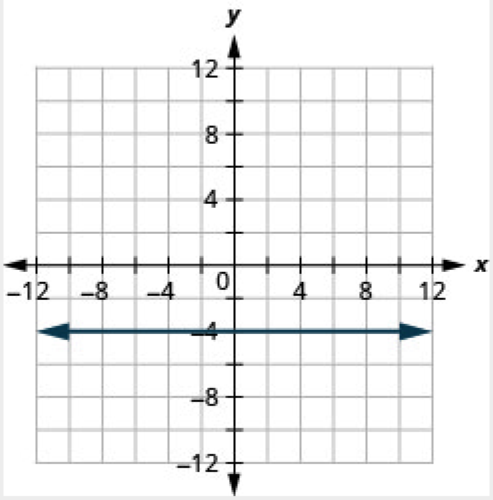


Note:
Exercise:

TRY IT :: 4.33

Problem: Graph the equation $y = -4$.

Solution:

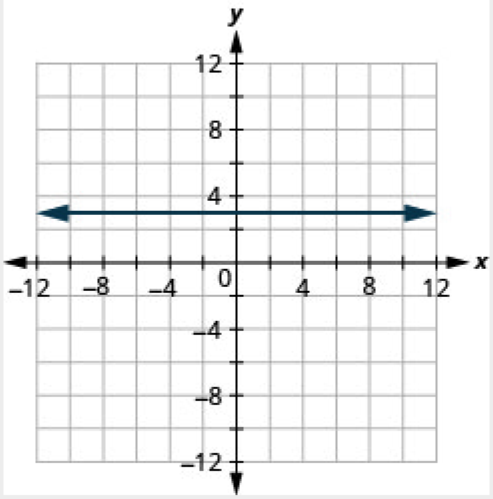


Note:
Exercise:

TRY IT :: 4.34

Problem: Graph the equation $y = 3$.

Solution:



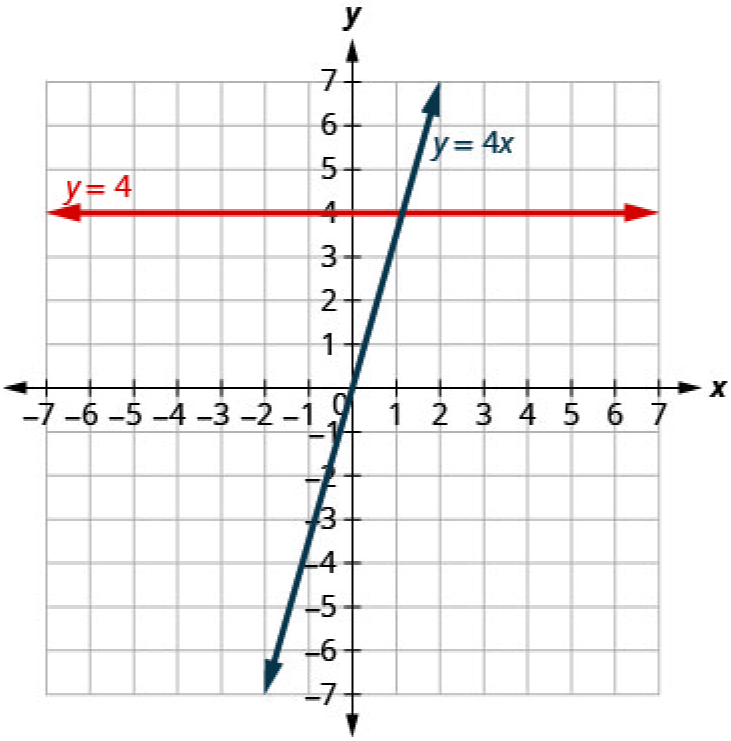
The equations for vertical and horizontal lines look very similar to equations like $y = 4x$. What is the difference between the equations $y = 4x$ and $y = 4$?

The equation $y = 4x$ has both x and y. The value of y depends on the value of x. The y-coordinate changes according to the value of x. The equation $y = 4$ has only one variable. The value of y is constant. The y-coordinate is always 4. It does not depend on the value of x. See [link].

$y = 4x$		
x	y	(x, y)
0	0	$(0, 0)$
1	4	$(1, 4)$

$y = 4$		
x	y	(x, y)
0	4	$(0, 4)$
1	4	$(1, 4)$

2	8	$(2, 8)$		2	4	$(2, 4)$



Notice, in [link], the equation $y = 4x$ gives a slanted line, while $y = 4$ gives a horizontal line.

Example:
Exercise:

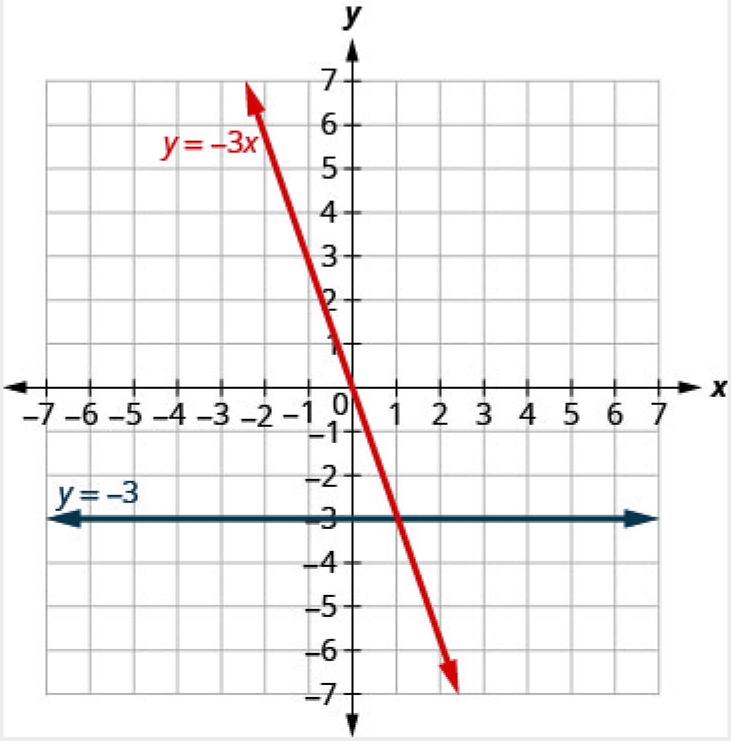
Problem:

**Graph $y = -3x$ and $y = -3$ in the same rectangular coordinate system.

Solution:
Solution

Notice that the first equation has the variable x, while the second does not. See [link]. The two graphs are shown in [link].

$y = -3x$				$y = -3$		
x	y	(x, y)		x	y	(x, y)
0	0	$(0, 0)$		0	-3	$(0, -3)$
1	-3	$(1, -3)$		1	-3	$(1, -3)$
2	-6	$(2, -6)$		2	-3	$(2, -3)$

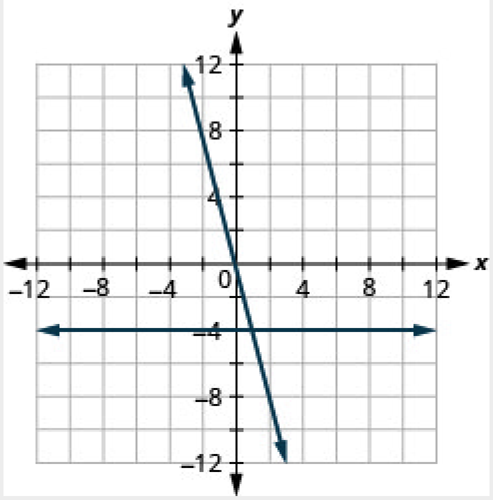


Note:
Exercise:

Problem:

TRY IT :: 4.35
Graph $y = -4x$ and $y = -4$ in the same rectangular coordinate system.

Solution:

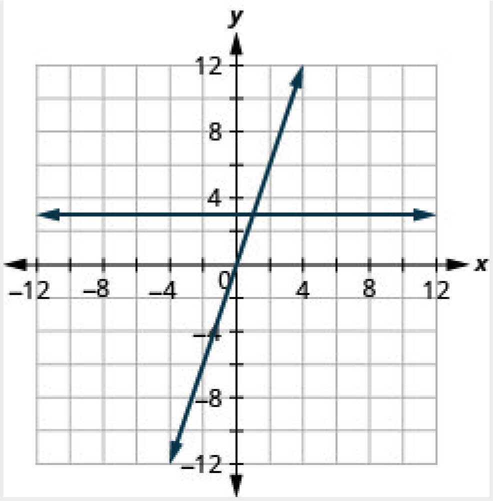


Note:
Exercise:

Problem:

TRY IT :: 4.36
Graph $y = 3$ and $y = 3x$ in the same rectangular coordinate system.

Solution:



Key Concepts

- **Graph a Linear Equation by Plotting Points**

  Find three points whose coordinates are solutions to the equation. Organize them in a table.
  Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!
  Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

Practice Makes Perfect

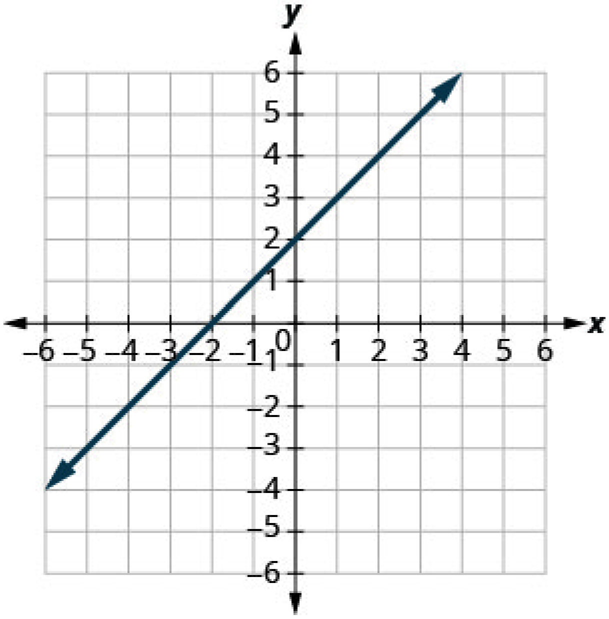
Recognize the Relationship Between the Solutions of an Equation and its Graph

In the following exercises, for each ordered pair, decide:

ⓐ Is the ordered pair a solution to the equation? ⓑ Is the point on the line?
Exercise:

Problem: $y = x + 2$

ⓐ $(0, 2)$
ⓑ $(1, 2)$
ⓒ $(-1, 1)$
ⓓ $(-3, -1)$



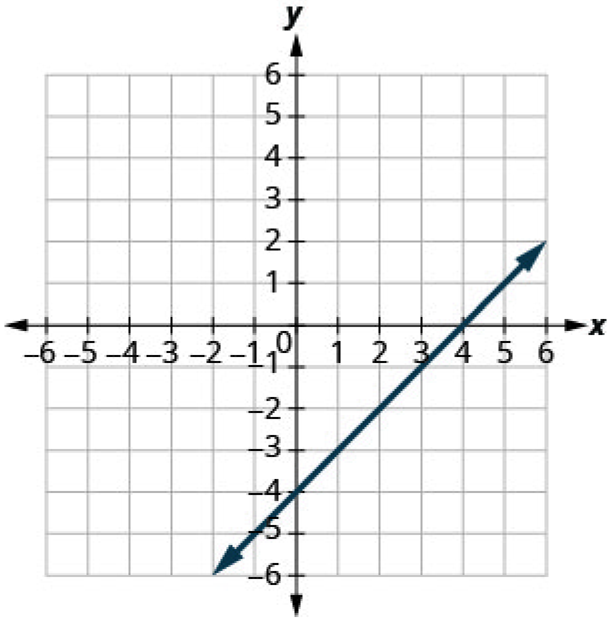
Solution:

ⓐ yes; no ⓑ no; no ⓒ yes; yes ⓓ yes; yes

Exercise:

Problem: $y = x - 4$

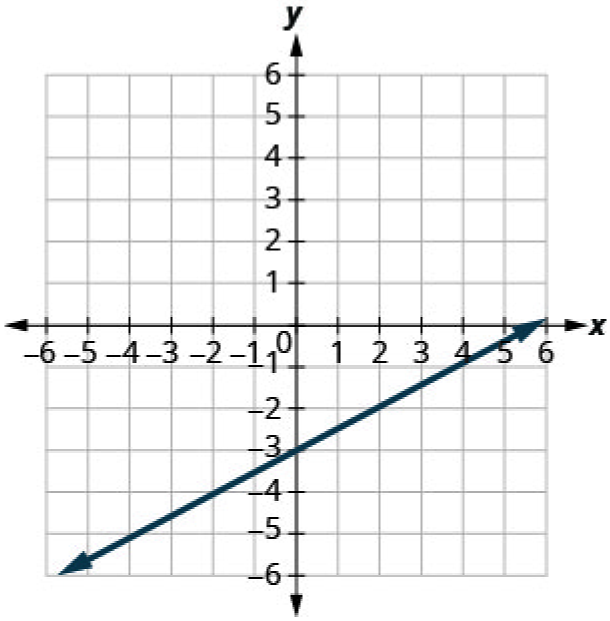
ⓐ $(0, -4)$
ⓑ $(3, -1)$
ⓒ $(2, 2)$
ⓓ $(1, -5)$



Exercise:

Problem: $y = \frac{1}{2}x - 3$

- ⓐ $(0, -3)$
- ⓑ $(2, -2)$
- ⓒ $(-2, -4)$
- ⓓ $(4, 1)$



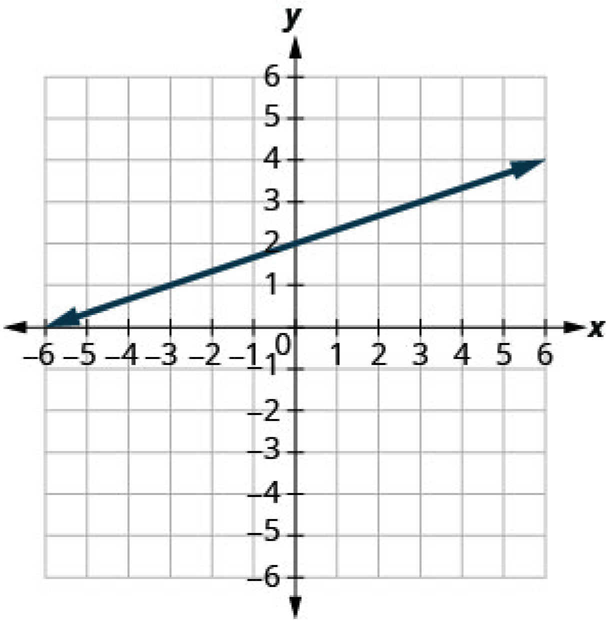
Solution:

ⓐ yes; yes ⓑ yes; yes ⓒ yes; yes ⓓ no; no

Exercise:

Problem: $y = \frac{1}{3}x + 2$

ⓐ $(0, 2)$
ⓑ $(3, 3)$
ⓒ $(-3, 2)$
ⓓ $(-6, 0)$



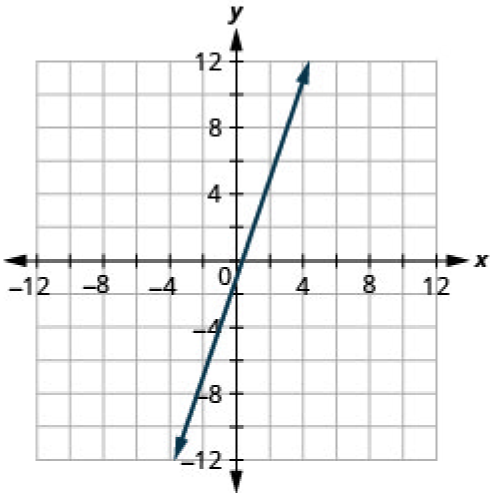
Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

Exercise:

Problem: $y = 3x - 1$

Solution:



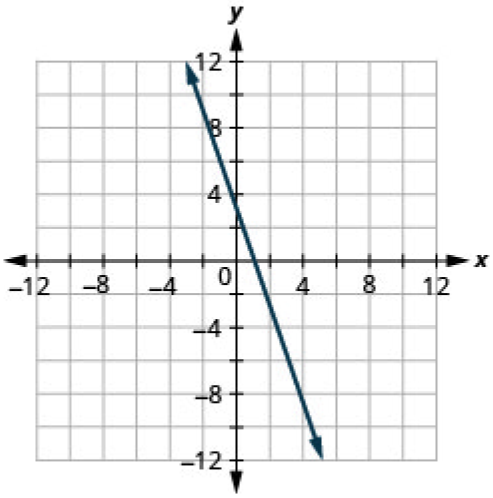
Exercise:

Problem: $y = 2x + 3$

Exercise:

Problem: $y = -2x + 2$

Solution:



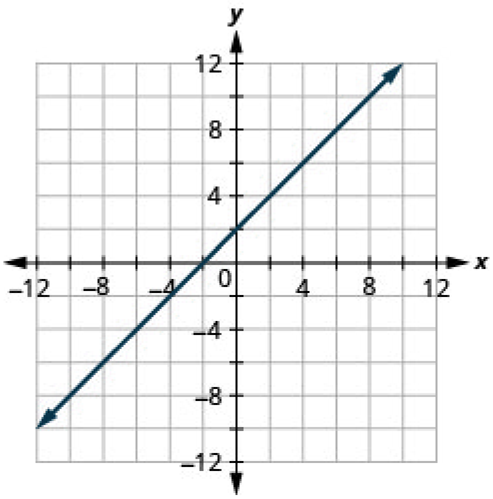
Exercise:

Problem: $y = -3x + 1$

Exercise:

Problem: $y = x + 2$

Solution:



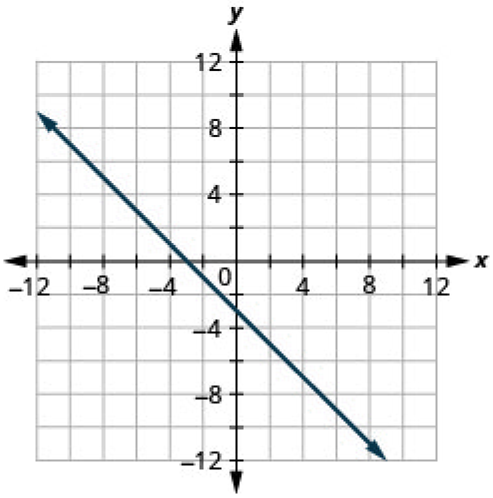
Exercise:

Problem: $y = x - 3$

Exercise:

Problem: $y = -x - 3$

Solution:



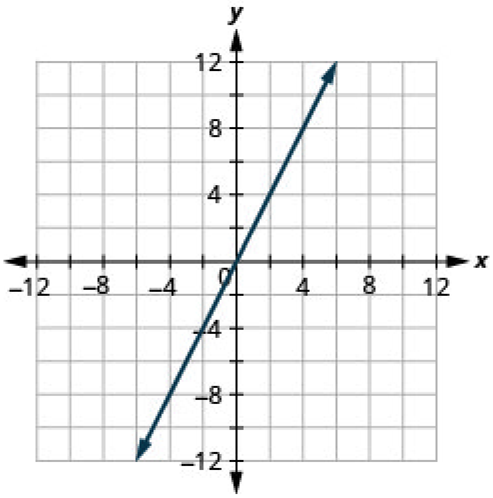
Exercise:

Problem: $y = -x - 2$

Exercise:

Problem: $y = 2x$

Solution:



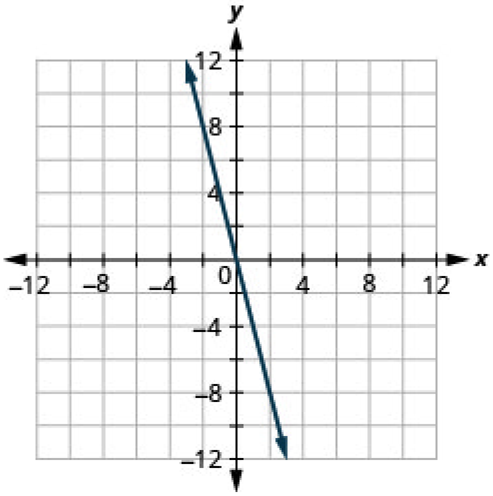
Exercise:

Problem: $y = 3x$

Exercise:

Problem: $y = -4x$

Solution:



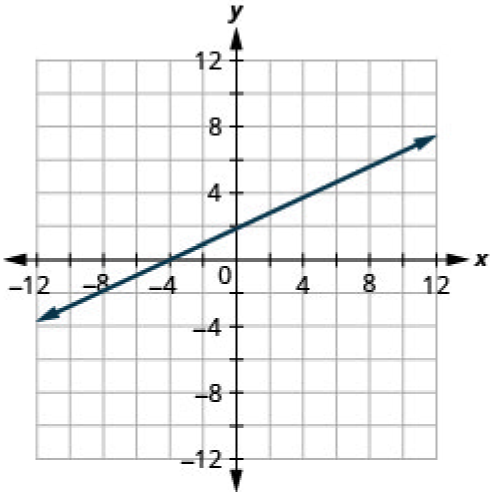
Exercise:

Problem: $y = -2x$

Exercise:

Problem: $y = \frac{1}{2}x + 2$

Solution:



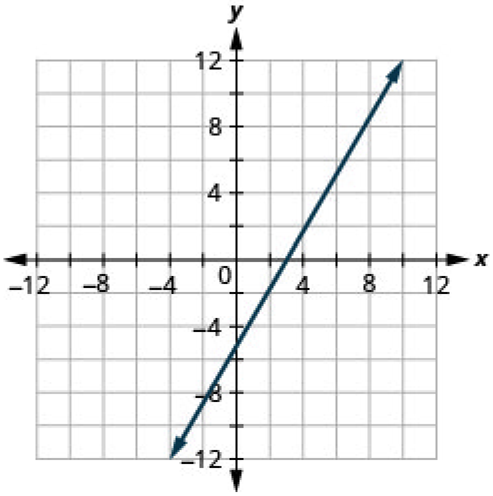
Exercise:

Problem: $y = \frac{1}{3}x - 1$

Exercise:

Problem: $y = \frac{4}{3}x - 5$

Solution:



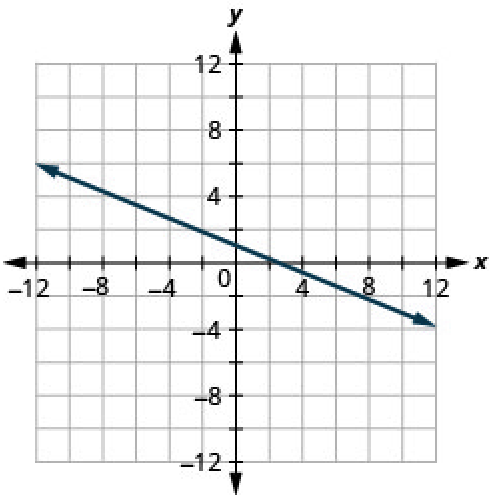
Exercise:

Problem: $y = \frac{3}{2}x - 3$

Exercise:

Problem: $y = -\frac{2}{5}x + 1$

Solution:



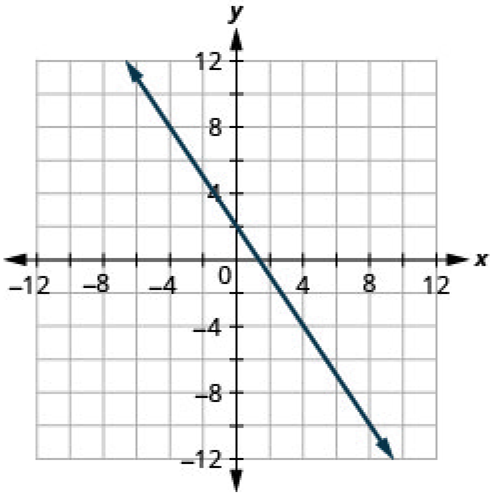
Exercise:

Problem: $y = -\frac{4}{5}x - 1$

Exercise:

Problem: $y = -\frac{3}{2}x + 2$

Solution:



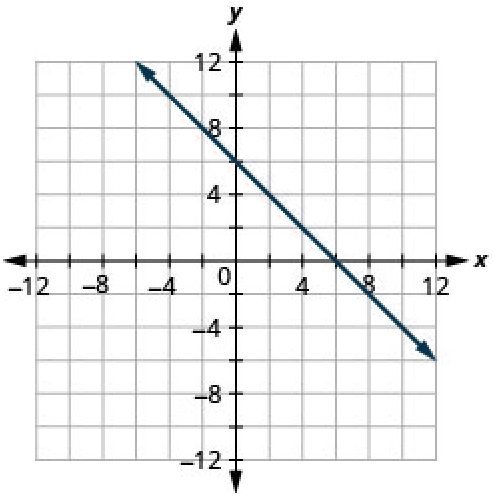
Exercise:

Problem: $y = -\frac{5}{3}x + 4$

Exercise:

Problem: $x + y = 6$

Solution:



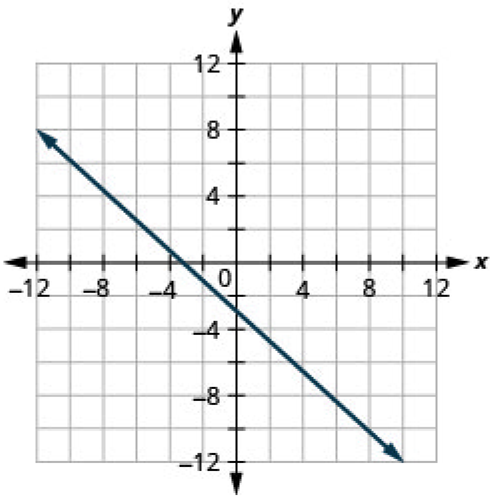
Exercise:

Problem: $x + y = 4$

Exercise:

Problem: $x + y = -3$

Solution:



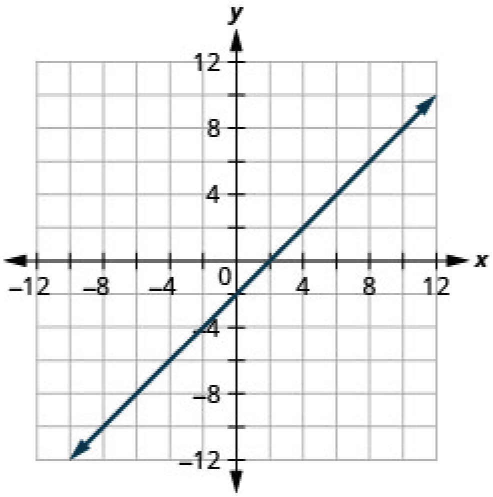
Exercise:

Problem: $x + y = -2$

Exercise:

Problem: $x - y = 2$

Solution:



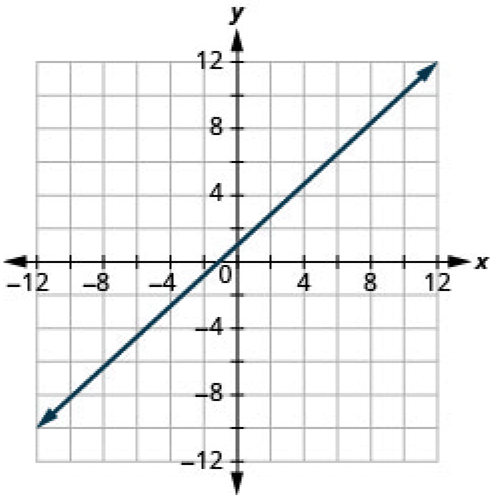
Exercise:

Problem: $x - y = 1$

Exercise:

Problem: $x - y = -1$

Solution:



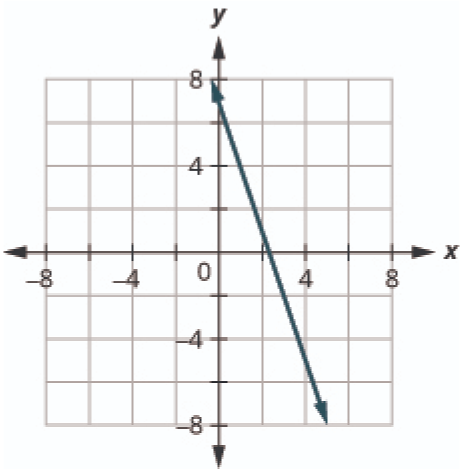
Exercise:

Problem: $x - y = -3$

Exercise:

Problem: $3x + y = 7$

Solution:



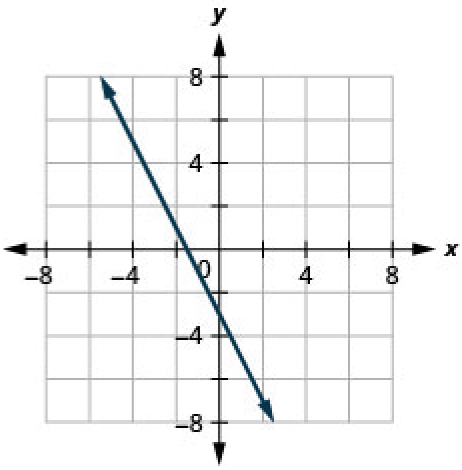
Exercise:

Problem: $5x+y=6$

Exercise:

Problem: $2x+y=-3$

Solution:



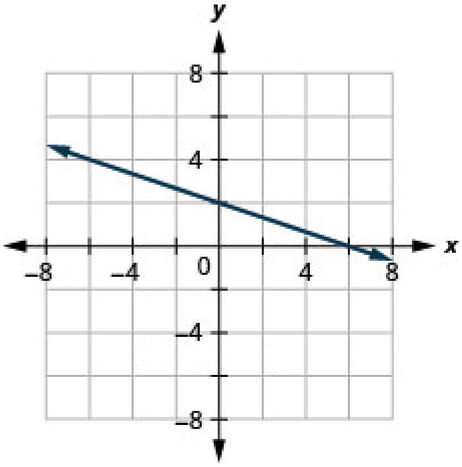
Exercise:

Problem: $4x+y=-5$

Exercise:

Problem: $\frac{1}{3}x+y=2$

Solution:



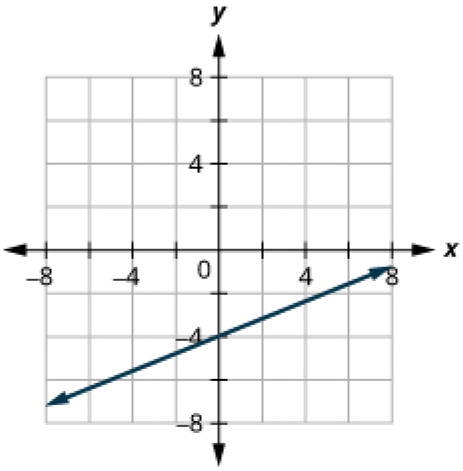
Exercise:

Problem: $\frac{1}{2}x+y=3$

Exercise:

Problem: $\frac{2}{5}x-y=4$

Solution:



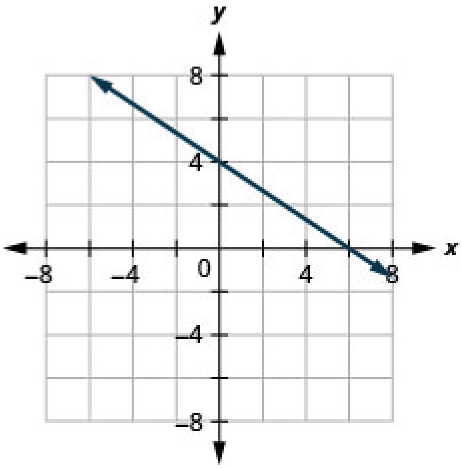
Exercise:

Problem: $\frac{3}{4}x - y = 6$

Exercise:

Problem: $2x + 3y = 12$

Solution:



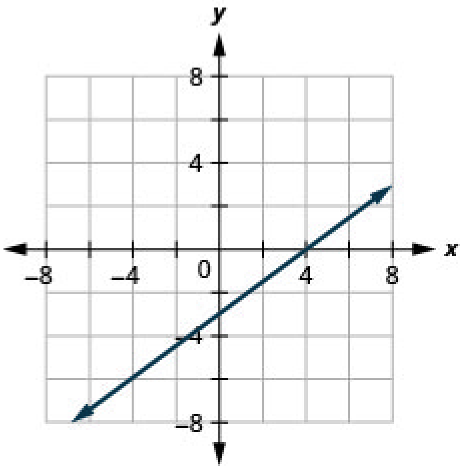
Exercise:

Problem: $4x + 2y = 12$

Exercise:

Problem: $3x - 4y = 12$

Solution:



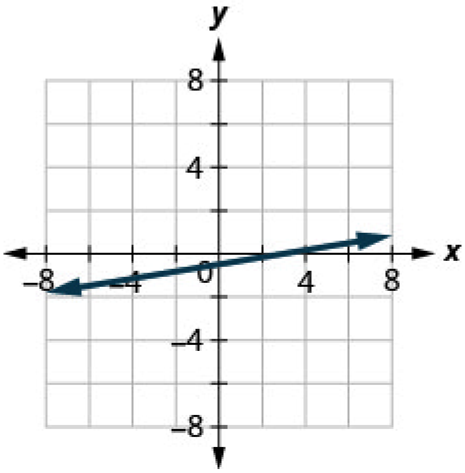
Exercise:

Problem: $2x - 5y = 10$

Exercise:

Problem: $x - 6y = 3$

Solution:



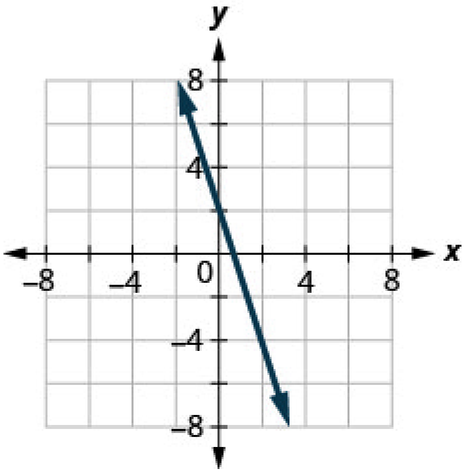
Exercise:

Problem: $x - 4y = 2$

Exercise:

Problem: $5x + 2y = 4$

Solution:



Exercise:

Problem: $3x + 5y = 5$

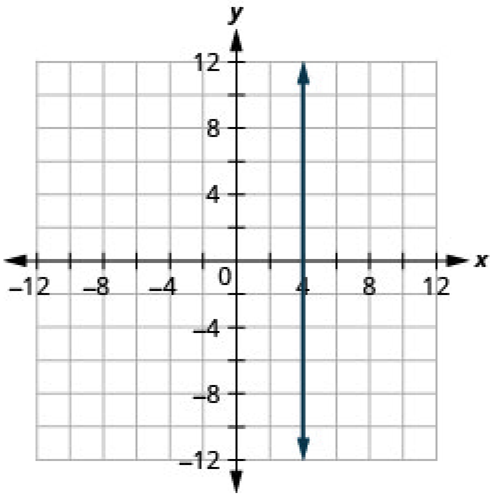
Graph Vertical and Horizontal Lines

In the following exercises, graph each equation.

Exercise:

Problem: $x = 4$

Solution:



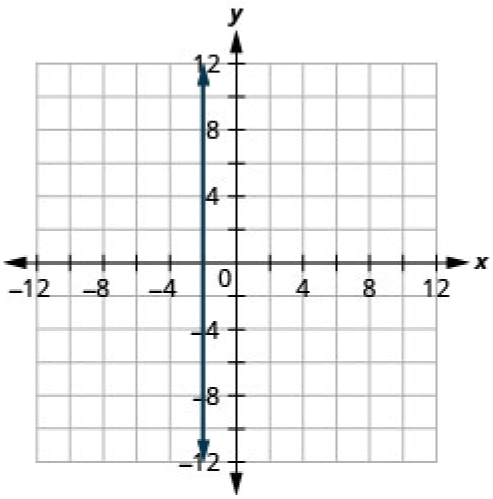
Exercise:

Problem: $x = 3$

Exercise:

Problem: $x = -2$

Solution:



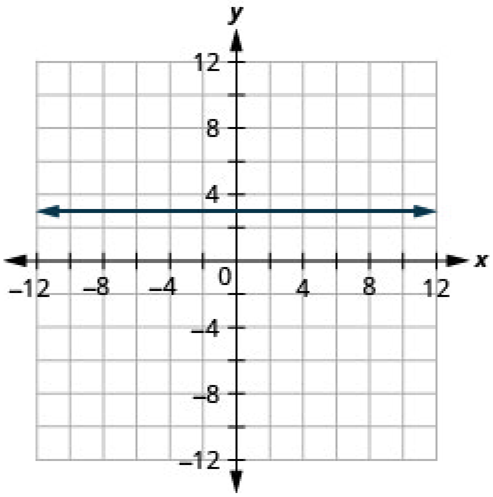
Exercise:

Problem: $x = -5$

Exercise:

Problem: $y = 3$

Solution:



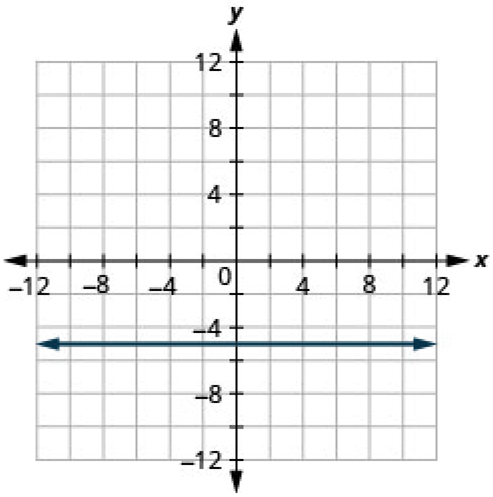
Exercise:

Problem: $y = 1$

Exercise:

Problem: $y = -5$

Solution:



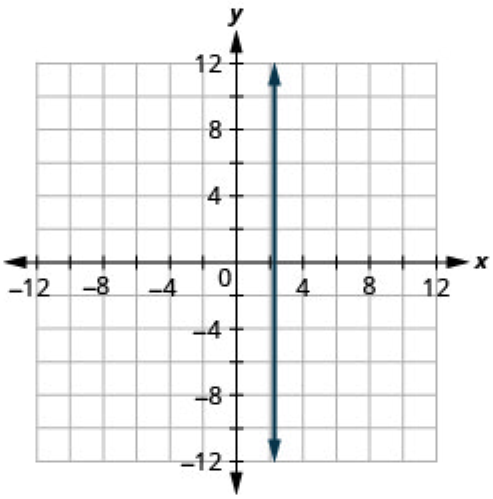
Exercise:

Problem: $y = -2$

Exercise:

Problem: $x = \frac{7}{3}$

Solution:



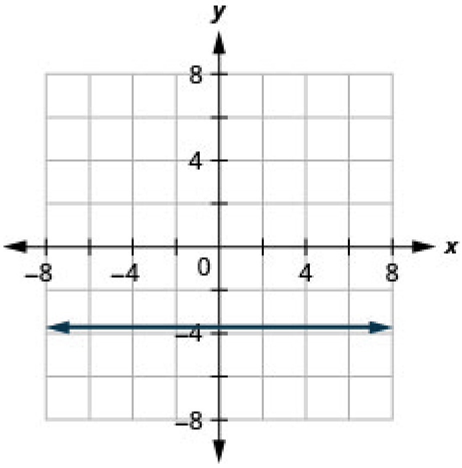
Exercise:

Problem: $x = \frac{5}{4}$

Exercise:

Problem: $y = -\frac{15}{4}$

Solution:



Exercise:

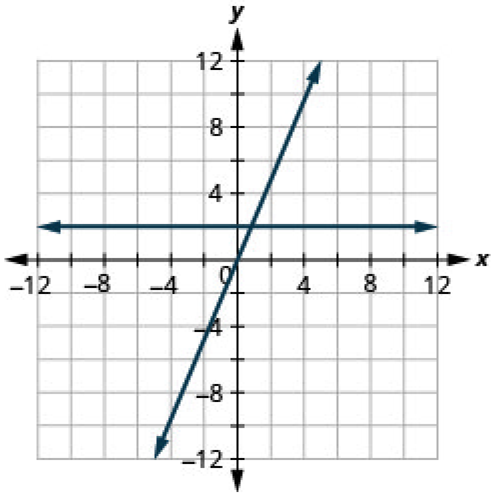
Problem: $y = -\frac{5}{3}$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.

Exercise:

Problem: $y = 2x$ and $y = 2$

Solution:



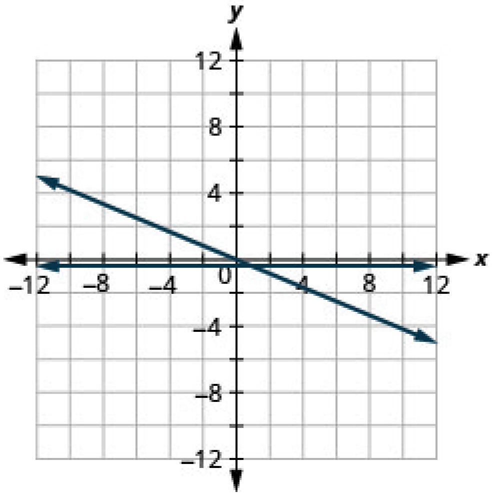
Exercise:

Problem: $y = 5x$ and $y = 5$

Exercise:

Problem: $y = -\frac{1}{2}x$ and $y = -\frac{1}{2}$

Solution:



Exercise:

Problem: $y = -\frac{1}{3}x$ and $y = -\frac{1}{3}$

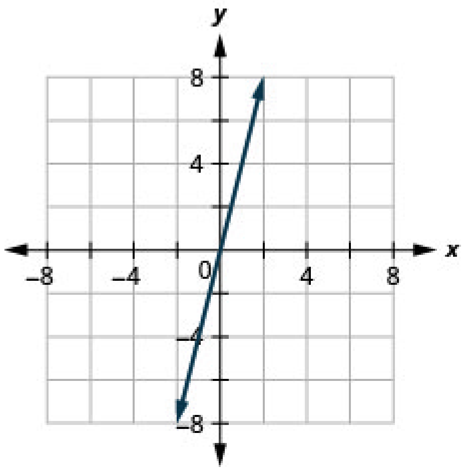
Mixed Practice

In the following exercises, graph each equation.

Exercise:

Problem: $y = 4x$

Solution:



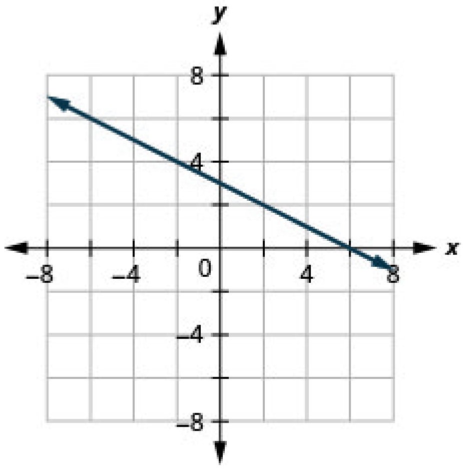
Exercise:

Problem: $y = 2x$

Exercise:

Problem: $y = -\frac{1}{2}x + 3$

Solution:



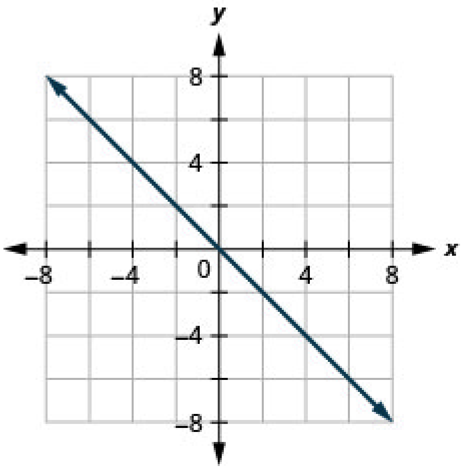
Exercise:

Problem: $y = \frac{1}{4}x - 2$

Exercise:

Problem: $y = -x$

Solution:



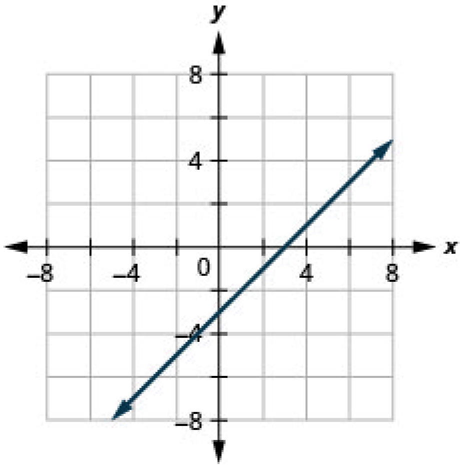
Exercise:

Problem: $y = x$

Exercise:

Problem: $x - y = 3$

Solution:



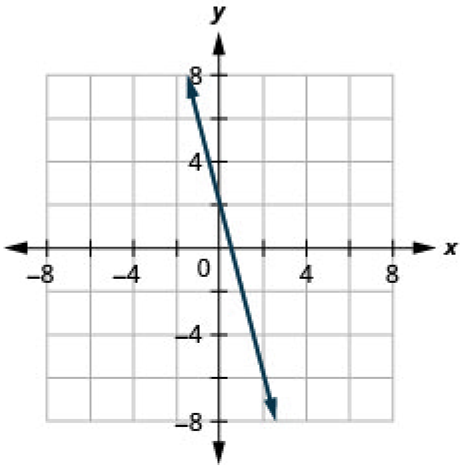
Exercise:

Problem: $x + y = -5$

Exercise:

Problem: $4x + y = 2$

Solution:



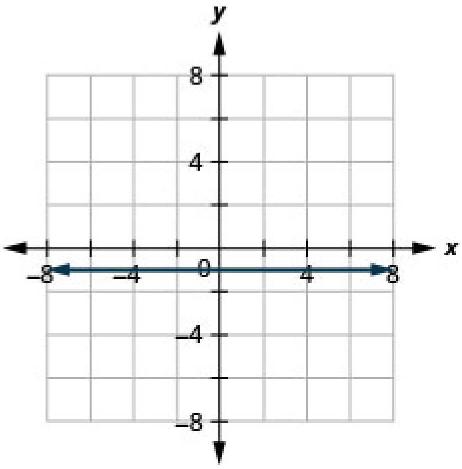
Exercise:

Problem: $2x + y = 6$

Exercise:

Problem: $y = -1$

Solution:



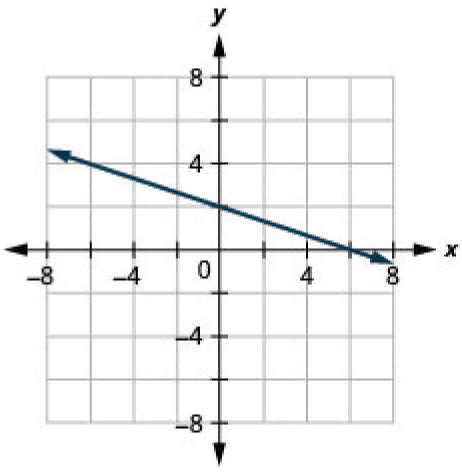
Exercise:

Problem: $y = 5$

Exercise:

Problem: $2x + 6y = 12$

Solution:



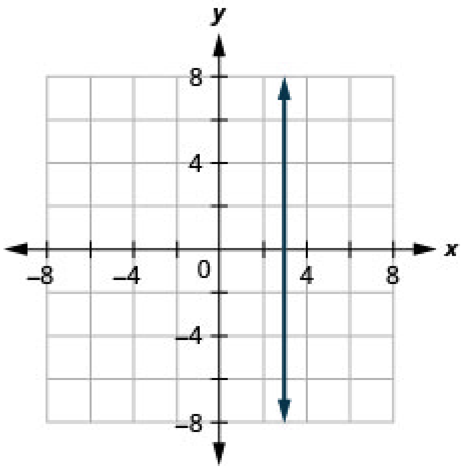
Exercise:

Problem: $5x + 2y = 10$

Exercise:

Problem: $x = 3$

Solution:



Exercise:

Problem: $x = -4$

Everyday Math

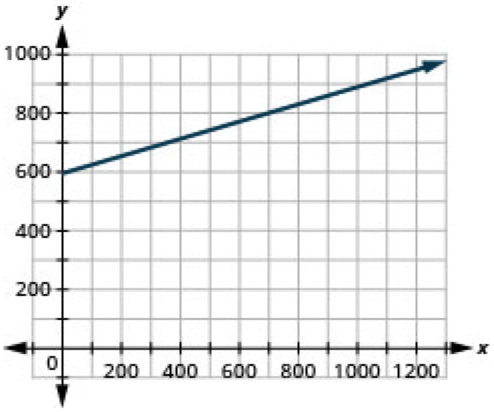
Exercise:

Problem:

Motor home cost. The Robinsons rented a motor home for one week to go on vacation. It cost them $594 plus $0.32 per mile to rent the motor home, so the linear equation $y = 594 + 0.32x$ gives the cost, y, for driving x miles. Calculate the rental cost for driving 400, 800, and 1200 miles, and then graph the line.

Solution:

$722, $850, $978



Exercise:

Problem:

Weekly earnings. At the art gallery where he works, Salvador gets paid $200 per week plus 15% of the sales he makes, so the equation $y = 200 + 0.15x$ gives the amount, y, he earns for selling x dollars of artwork. Calculate the amount Salvador earns for selling $900, $1600, and $2000, and then graph the line.

Writing Exercises

Exercise:

Problem:

Explain how you would choose three x- values to make a table to graph the line $y=\frac{1}{5}x-2$.

Solution:

Answers will vary.

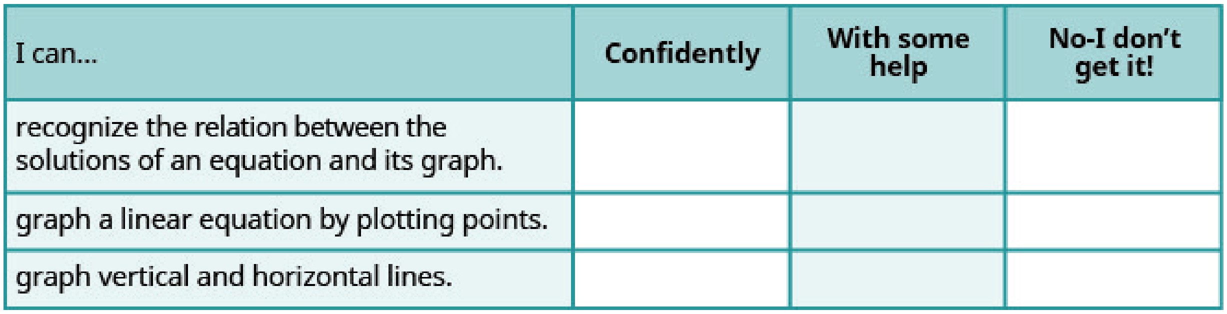
Exercise:

Problem:

What is the difference between the equations of a vertical and a horizontal line?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
recognize the relation between the solutions of an equation and its graph.			
graph a linear equation by plotting points.			
graph vertical and horizontal lines.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

Glossary

graph of a linear equation

The graph of a linear equation $Ax + By = C$ is a straight line. Every point on the line is a solution of the equation. Every solution of this equation is a point on this line.

horizontal line

A horizontal line is the graph of an equation of the form $y = b$. The line passes through the *y*-axis at $(0, b)$.

vertical line

A vertical line is the graph of an equation of the form $x = a$. The line passes through the *x*-axis at $(a, 0)$.

Graph with Intercepts
By the end of this section, you will be able to:

- Identify the x- and y- intercepts on a graph
- Find the x- and y- intercepts from an equation of a line
- Graph a line using the intercepts

Note:
Before you get started, take this readiness quiz.

1. Solve: $3 \cdot 0 + 4y = -2$.
   If you missed this problem, review [link].

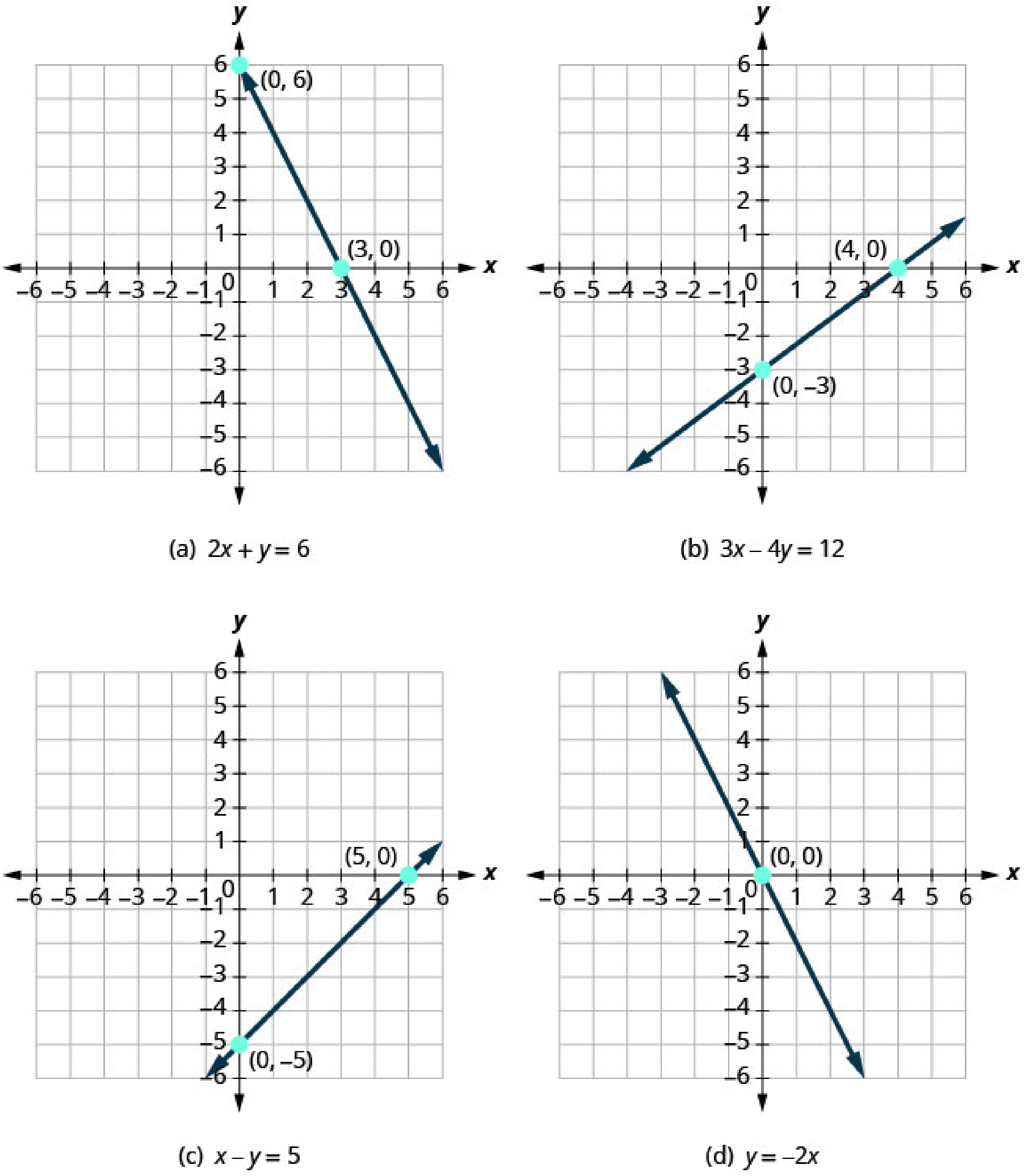
Identify the *x*- and *y*- Intercepts on a Graph

Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the *x*- axis and the *y*- axis. These points are called the *intercepts* of the line.

Note:
Intercepts of a Line
The points where a line crosses the *x*- axis and the *y*- axis are called the **intercepts of a line**.

Let’s look at the graphs of the lines in [link].



(a) $2x + y = 6$

(b) $3x - 4y = 12$

(c) $x - y = 5$

(d) $y = -2x$

Examples of graphs crossing the x-negative axis.

First, notice where each of these lines crosses the x negative axis. See [link].

Figure	**The line crosses the *x*- axis at:**	**Ordered pair of this point**
Figure (a)	3	$(3,0)$
Figure (b)	4	$(4,0)$
Figure (c)	5	$(5,0)$
Figure (d)	0	$(0,0)$

Do you see a pattern?

For each row, the *y*- coordinate of the point where the line crosses the *x*- axis is zero. The point where the line crosses the *x*- axis has the form $(a,0)$ and is called the ***x*- intercept of a line**. The *x*- intercept occurs when y is zero.

Now, let's look at the points where these lines cross the *y*- axis. See [link].

Figure	**The line crosses the *y*- axis at:**	**Ordered pair for this point**
Figure (a)	6	$(0,6)$
Figure (b)	-3	$(0,-3)$
Figure (c)	-5	$(0,5)$
Figure (d)	0	$(0,0)$

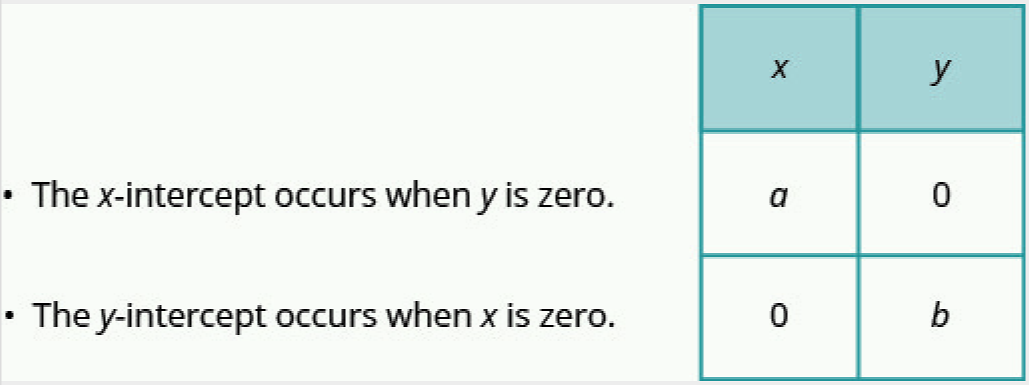
What is the pattern here?

In each row, the *x*- coordinate of the point where the line crosses the *y*- axis is zero. The point where the line crosses the *y*- axis has the form $(0,b)$ and is called the *y*- *intercept* of the line. The *y*- intercept occurs when x is zero.

Note:

x- intercept and ***y*- intercept of a line**

The *x*- intercept is the point $(a,0)$ where the line crosses the *x*- axis.
The *y*- intercept is the point $(0,b)$ where the line crosses the *y*- axis.

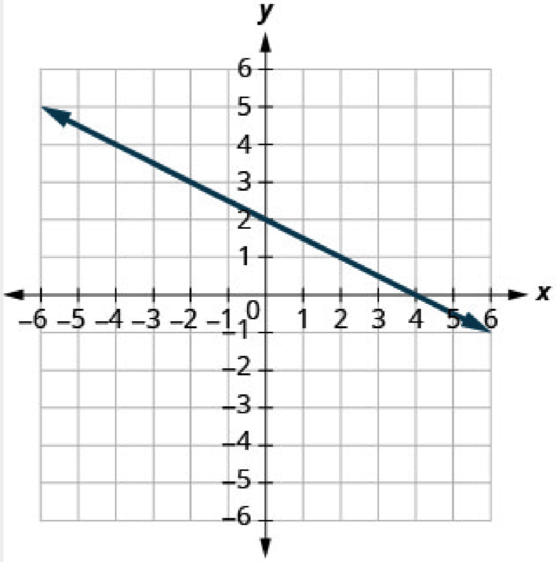


- The *x*-intercept occurs when *y* is zero.
- The *y*-intercept occurs when *x* is zero.

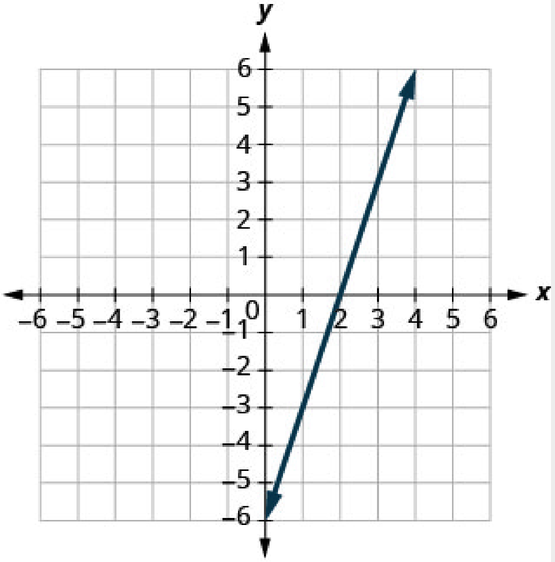
x	*y*
a	0
0	*b*

Example:
Exercise:

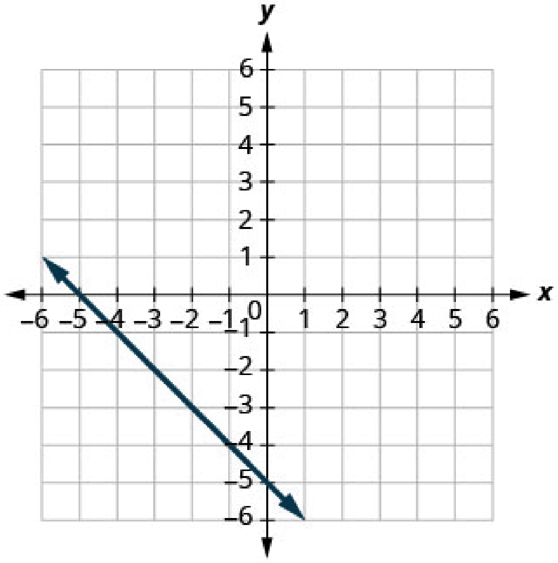
Problem: **Find the *x*- and *y*- intercepts on each graph.



(a)



(b)



(c)

Solution:
Solution

ⓐ The graph crosses the *x*- axis at the point $(4, 0)$. The *x*- intercept is $(4, 0)$.
The graph crosses the *y*- axis at the point $(0, 2)$. The *y*- intercept is $(0, 2)$.

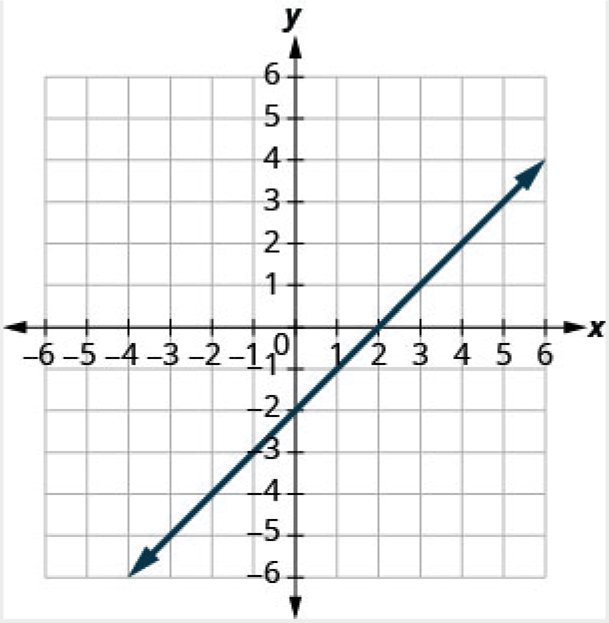
ⓑ The graph crosses the *x*- axis at the point $(2, 0)$. The *x*- intercept is $(2, 0)$
The graph crosses the *y*- axis at the point $(0, -6)$. The *y*- intercept is $(0, -6)$.

ⓒ The graph crosses the *x*- axis at the point $(-5, 0)$. The *x*- intercept is $(-5, 0)$.
The graph crosses the *y*- axis at the point $(0, -5)$. The *y*- intercept is $(0, -5)$.

Note:
Exercise:

TRY IT :: 4.37

Problem: Find the *x*- and *y*- intercepts on the graph.



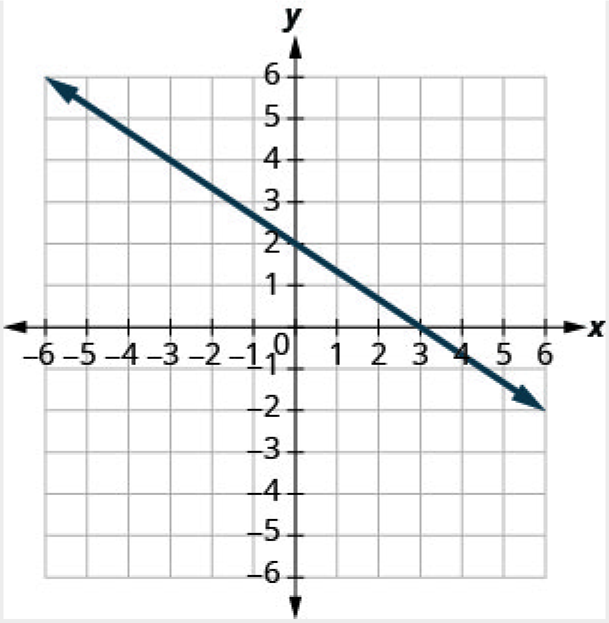
Solution:

x- intercept: $(2, 0)$; *y*- intercept: $(0, -2)$

Note:
Exercise:

TRY IT :: 4.38

Problem: Find the *x*- and *y*- intercepts on the graph.



Solution:

x- intercept: $(3, 0)$, *y*- intercept: $(0, 2)$

Find the *x*- and *y*- Intercepts from an Equation of a Line

Recognizing that the *x*- intercept occurs when *y* is zero and that the *y*- intercept occurs when *x* is zero, gives us a method to find the intercepts of a line from its equation. To find the *x*- intercept, let $y = 0$ and solve for *x*. To find the *y*- intercept, let $x = 0$ and solve for *y*.

Note:
Find the *x*- and *y*- Intercepts from the Equation of a Line
Use the equation of the line. To find:

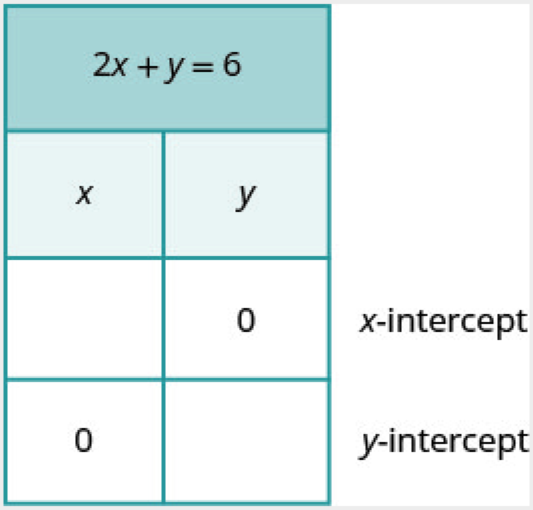
- the *x*- intercept of the line, let $y = 0$ and solve for x.
- the *y*- intercept of the line, let $x = 0$ and solve for y.

Example:
Exercise:

Problem: **Find the intercepts of $2x + y = 6$.

Solution:
Solution

We will let $y = 0$ to find the *x*- intercept, and let $x = 0$ to find the *y*-intercept. We will fill in the table, which reminds us of what we need to find.



$2x + y = 6$		
x	y	
	0	*x*-intercept
0		*y*-intercept

To find the *x*- intercept, let $y = 0$.

	$2x + y = 6$

Let $y = 0$.	$2x + 0 = 6$
Simplify.	$2x = 6$
	$x = 3$
The x-intercept is	(3, 0)
To find the y-intercept, let $x = 0$.	
	$2x + y = 6$
Let $x = 0$.	$2 \cdot 0 + y = 6$
Simplify.	$0 + y = 6$
	$y = 6$
The y-intercept is	(0, 6)

The intercepts are the points $(3, 0)$ and $(0, 6)$ as shown in [link].

$2x + y = 6$	
x	y
3	0
0	6

Note:
Exercise:

TRY IT :: 4.39

Problem: Find the intercepts of $3x + y = 12.$

Solution:

x- intercept: $(4, 0)$, *y*- intercept: $(0, 12)$

Note:
Exercise:

TRY IT :: 4.40

Problem: Find the intercepts of $x + 4y = 8.$

Solution:

x- intercept: $(8, 0)$, *y*- intercept: $(0, 2)$

Example:
Exercise:

Problem: **Find the intercepts of $4x-3y=12$.

Solution:
Solution

To find the *x*-intercept, let *y* = 0.	
	$4x-3y=12$
Let *y* = 0.	$4x-3\cdot 0=12$
Simplify.	$4x-0=12$
	$4x=12$
	$x=3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let *x* = 0.	
	$4x-3y=12$

Let *x* = 0.	$4 \cdot 0 - 3y = 12$
Simplify.	$0 - 3y = 12$
	$-3y = 12$
	$y = -4$
The *y*-intercept is	(0, −4)

The intercepts are the points (3, 0) and (0, −4) as shown in the following table.

$4x - 3y = 12$	
x	y
3	0
0	-4

Note:
Exercise:

TRY IT :: 4.41

Problem: Find the intercepts of $3x-4y=12$.

Solution:

x- intercept: $(4,0)$, *y*- intercept: $(0,-3)$

Note:
Exercise:

TRY IT :: 4.42

Problem: Find the intercepts of $2x-4y=8$.

Solution:

x- intercept: $(4,0)$, *y*- intercept: $(0,-2)$

Graph a Line Using the Intercepts

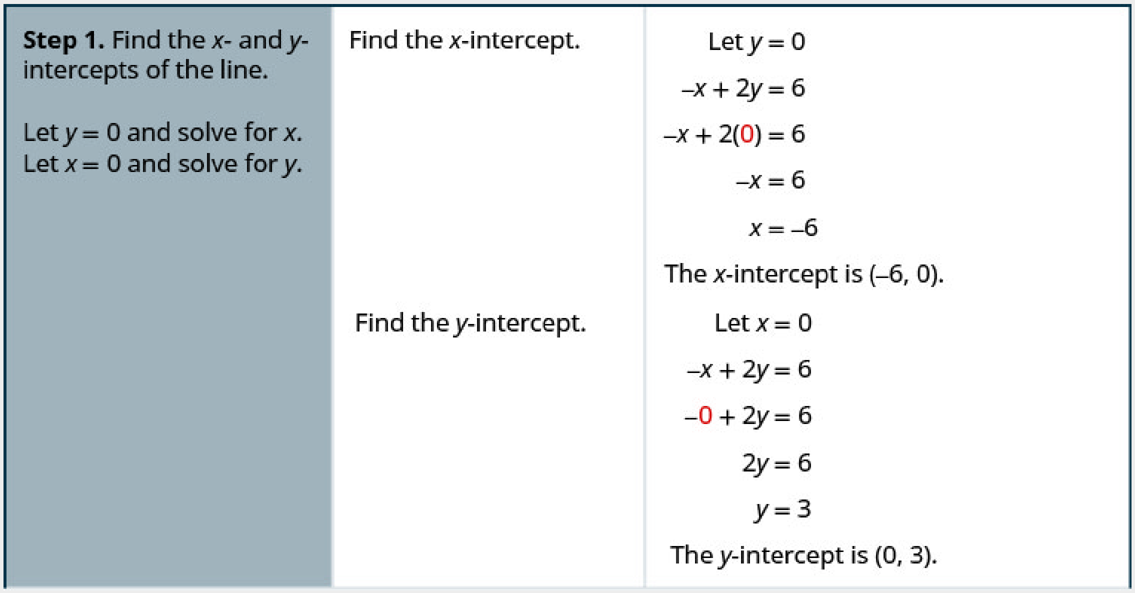
To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the *x*- and *y*-intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

Example:
How to Graph a Line Using Intercepts

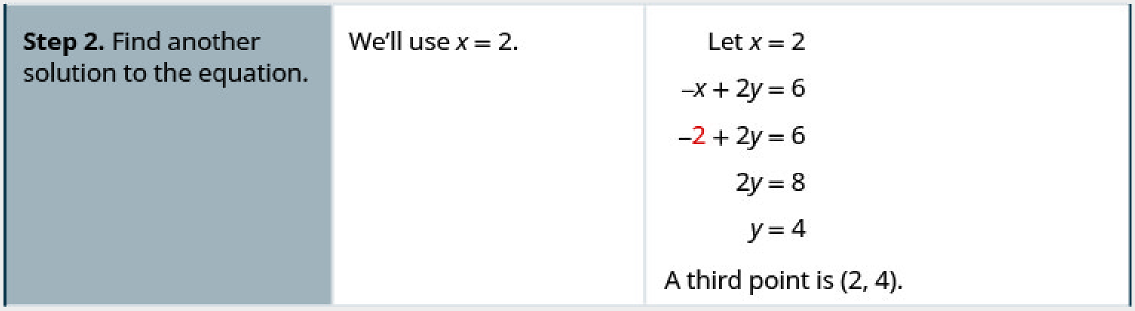
Exercise:

Problem: **Graph $-x + 2y = 6$ using the intercepts.

Solution:
Solution



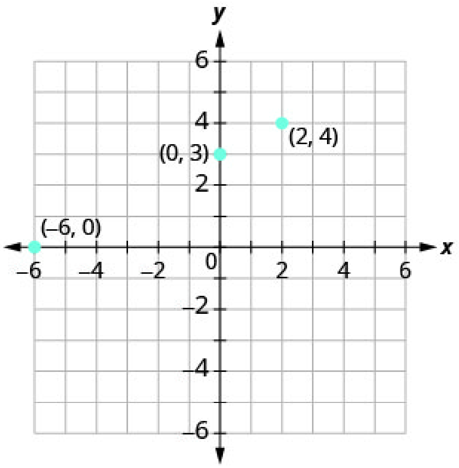
Step 1. Find the *x*- and *y*-intercepts of the line. Let $y = 0$ and solve for *x*. Let $x = 0$ and solve for *y*.	Find the *x*-intercept.	Let $y = 0$ $-x + 2y = 6$ $-x + 2(0) = 6$ $-x = 6$ $x = -6$ The *x*-intercept is (–6, 0).
	Find the *y*-intercept.	Let $x = 0$ $-x + 2y = 6$ $-0 + 2y = 6$ $2y = 6$ $y = 3$ The *y*-intercept is (0, 3).



Step 2. Find another solution to the equation.	We'll use $x = 2$.	Let $x = 2$ $-x + 2y = 6$ $-2 + 2y = 6$ $2y = 8$ $y = 4$ A third point is (2, 4).

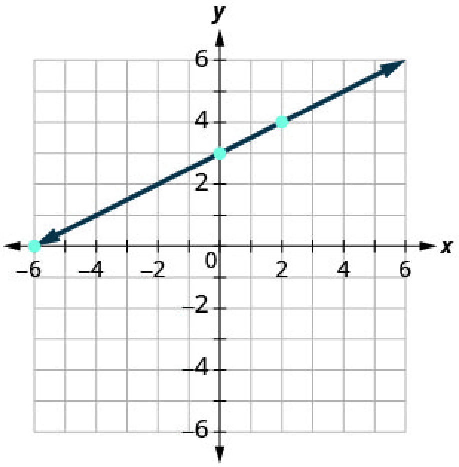
Step 3. Plot the three points. Check that the points line up.

x	y	(x, y)
–6	0	(–6, 0)
0	3	(0, 3)
2	4	(2, 4)



Step 4. Draw the line.

See the graph.



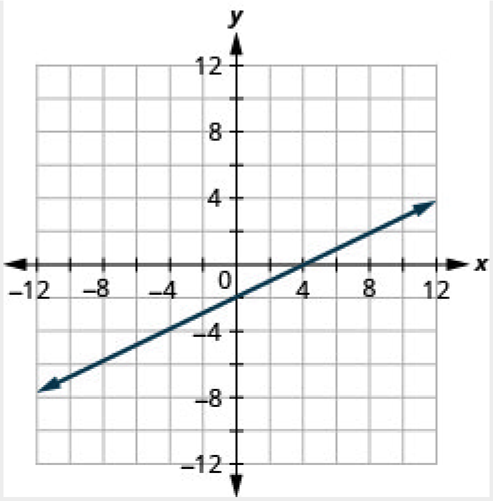
Note:

Exercise:

TRY IT :: 4.43

Problem: Graph $x - 2y = 4$ using the intercepts.

Solution:

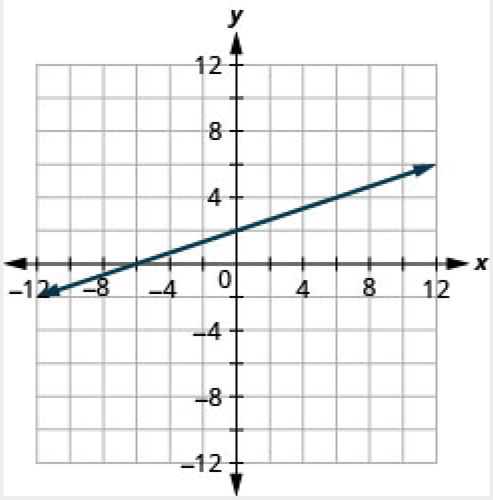


Note:
Exercise:

TRY IT :: 4.44

Problem: Graph $-x+3y=6$ using the intercepts.

Solution:



The steps to graph a linear equation using the intercepts are summarized below.

Note:
Graph a linear equation using the intercepts.

Find the*x*- and*y*- intercepts of the line.

- Let $y = 0$ and solve for x
- Let $x = 0$ and solve for y.

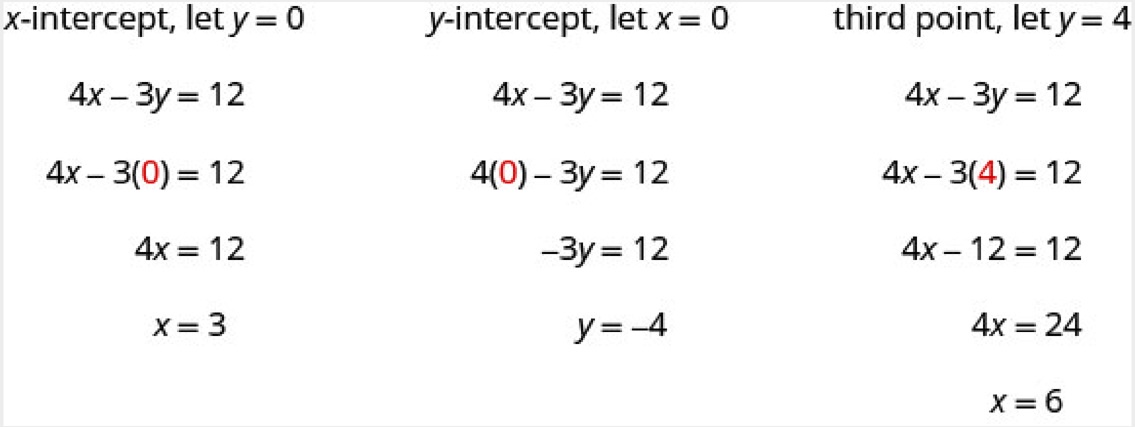
Find a third solution to the equation.
Plot the three points and check that they line up.
Draw the line.

Example:
Exercise:

Problem: **Graph $4x – 3y = 12$ using the intercepts.

Solution:
Solution

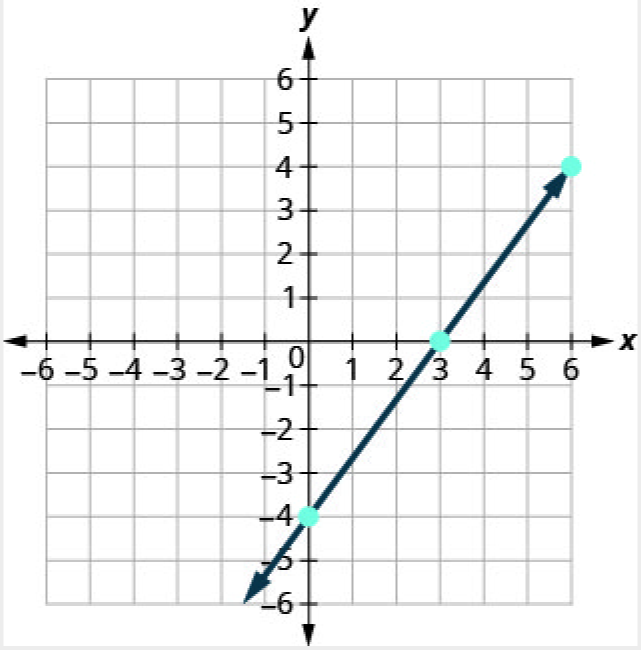
Find the intercepts and a third point.



x-intercept, let $y = 0$	y-intercept, let $x = 0$	third point, let $y = 4$
$4x - 3y = 12$	$4x - 3y = 12$	$4x - 3y = 12$
$4x - 3(0) = 12$	$4(0) - 3y = 12$	$4x - 3(4) = 12$
$4x = 12$	$-3y = 12$	$4x - 12 = 12$
$x = 3$	$y = -4$	$4x = 24$
		$x = 6$

We list the points in [link] and show the graph below.

$4x - 3y = 12$		
x	y	(x, y)
3	0	$(3, 0)$
0	-4	$(0, -4)$
6	4	$(6, 4)$



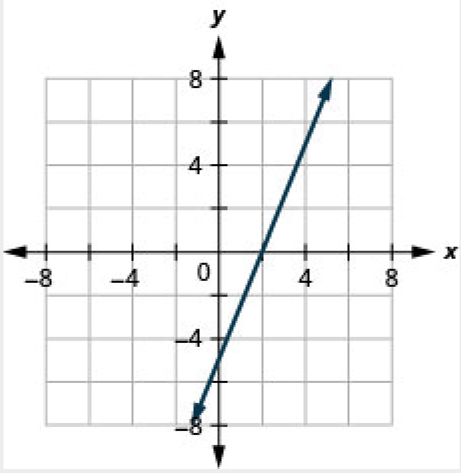
Note:

Exercise:

TRY IT :: 4.45

Problem: Graph $5x - 2y = 10$ using the intercepts.

Solution:



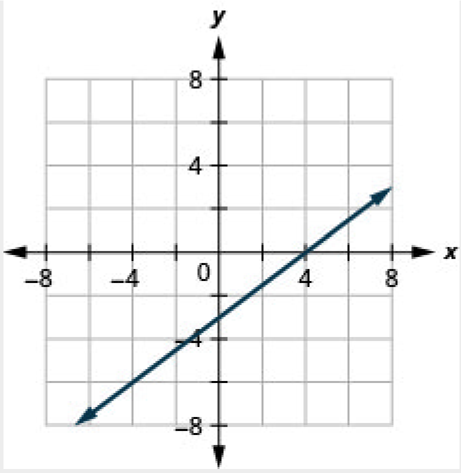
Note:

Exercise:

TRY IT :: 4.46

Problem: Graph $3x - 4y = 12$ using the intercepts.

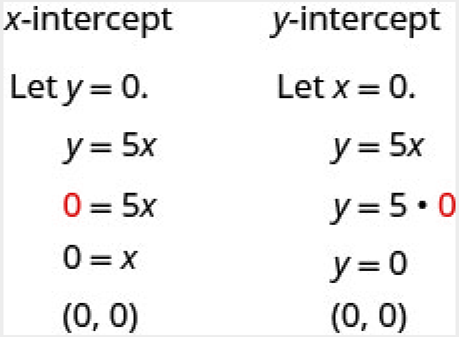
Solution:



Example:
Exercise:

Problem: **Graph $y = 5x$ using the intercepts.

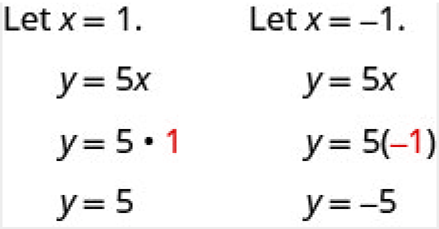
Solution:
Solution



x-intercept	y-intercept
Let $y = 0$.	Let $x = 0$.
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
$(0, 0)$	$(0, 0)$

This line has only one intercept. It is the point $(0, 0)$.

To ensure accuracy we need to plot three points. Since the *x*- and *y*-intercepts are the same point, we need *two* more points to graph the line.

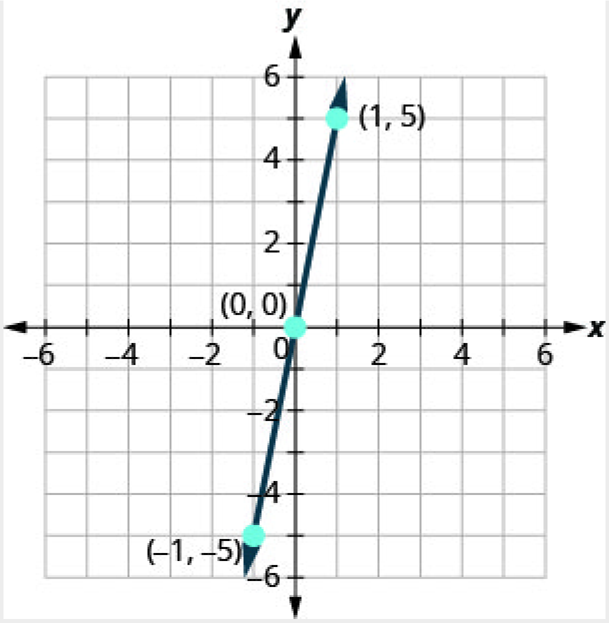


Let $x = 1$.	Let $x = -1$.
$y = 5x$	$y = 5x$
$y = 5 \cdot 1$	$y = 5(-1)$
$y = 5$	$y = -5$

See [link].

$y = 5x$		
x	y	(x, y)
0	0	$(0, 0)$
1	5	$(1, 5)$
-1	-5	$(-1, -5)$

Plot the three points, check that they line up, and draw the line.

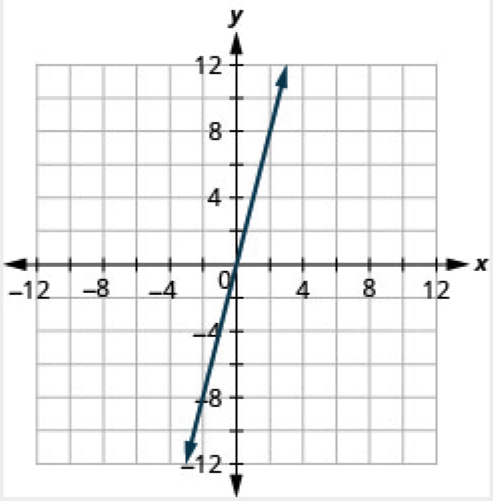


Note:
Exercise:

TRY IT :: 4.47

Problem: Graph $y = 4x$ using the intercepts.

Solution:

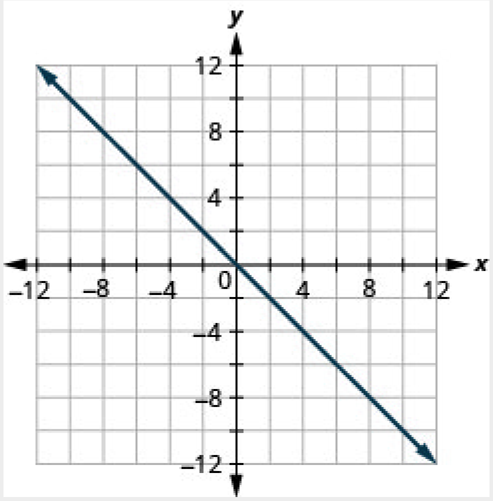


Note:
Exercise:

TRY IT :: 4.48

Problem: Graph $y = -x$ the intercepts.

Solution:



Key Concepts

- **Find the *x*- and *y*- Intercepts from the Equation of a Line**
    - Use the equation of the line to find the *x*- intercept of the line, let $y = 0$ and solve for *x*.
    - Use the equation of the line to find the *y*- intercept of the line, let $x = 0$ and solve for *y*.
- **Graph a Linear Equation using the Intercepts**

Find the *x*- and *y*- intercepts of the line.	Let $y = 0$ and solve for *x*.	Let $x = 0$ and solve for *y*.

Find a third solution to the equation.
Plot the three points and then check that they line up.
Draw the line.

- **Strategy for Choosing the Most Convenient Method to Graph a Line:**
    - Consider the form of the equation.
    - If it only has one variable, it is a vertical or horizontal line.
      $x = a$ is a vertical line passing through the *x*- axis at a
      $y = b$ is a horizontal line passing through the *y*- axis at b.
    - If *y* is isolated on one side of the equation, graph by plotting points.
    - Choose any three values for *x* and then solve for the corresponding *y*- values.
    - If the equation is of the form $ax + by = c$, find the intercepts.
      Find the *x*- and *y*- intercepts and then a third point.

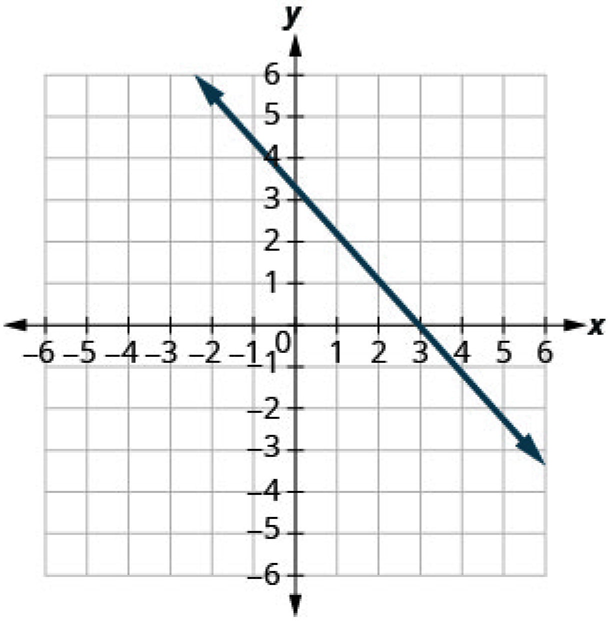
Practice Makes Perfect

Identify the *x*- and *y*- Intercepts on a Graph

In the following exercises, find the *x*- and *y*- intercepts on each graph.

Exercise:

Problem:

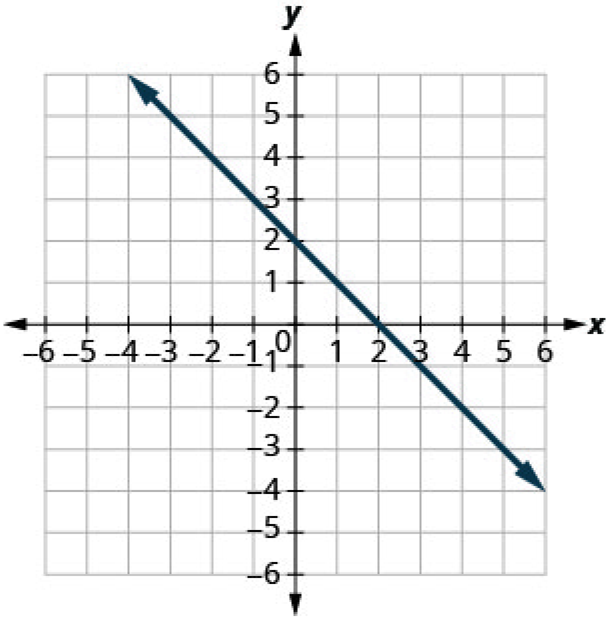


Solution:

$(3, 0), (0, 3)$

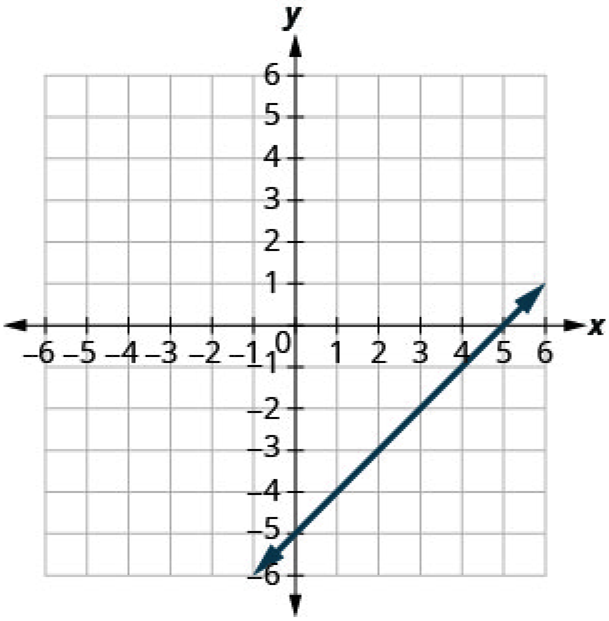
Exercise:

Problem:



Exercise:

Problem:

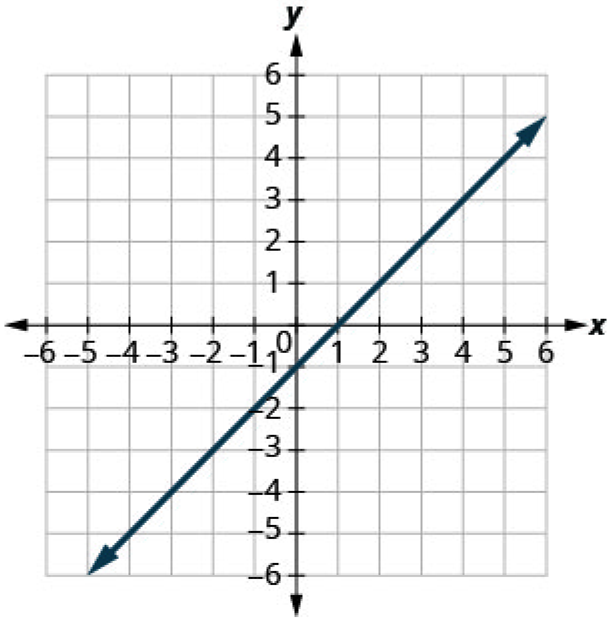


Solution:

$(5, 0), (0, -5)$

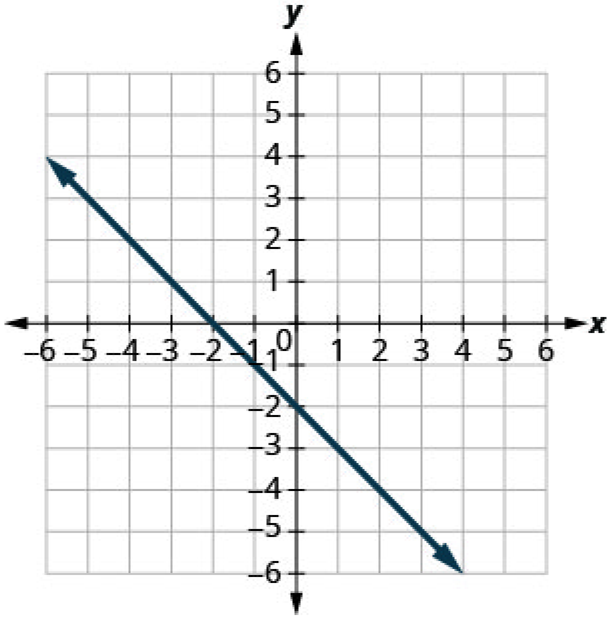
Exercise:

Problem:



Exercise:

Problem:

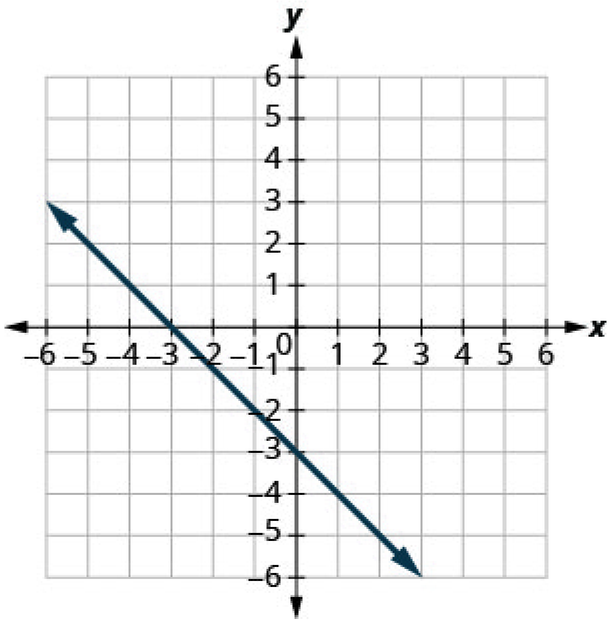


Solution:

$(-2, 0), (0, -2)$

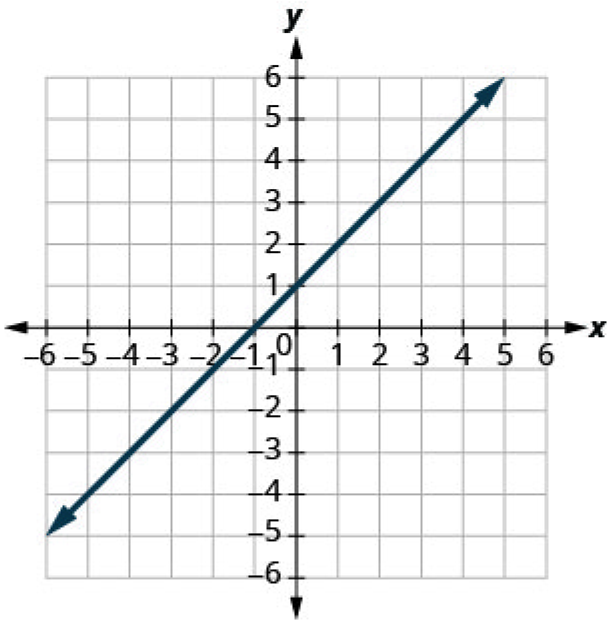
Exercise:

Problem:



Exercise:

Problem:

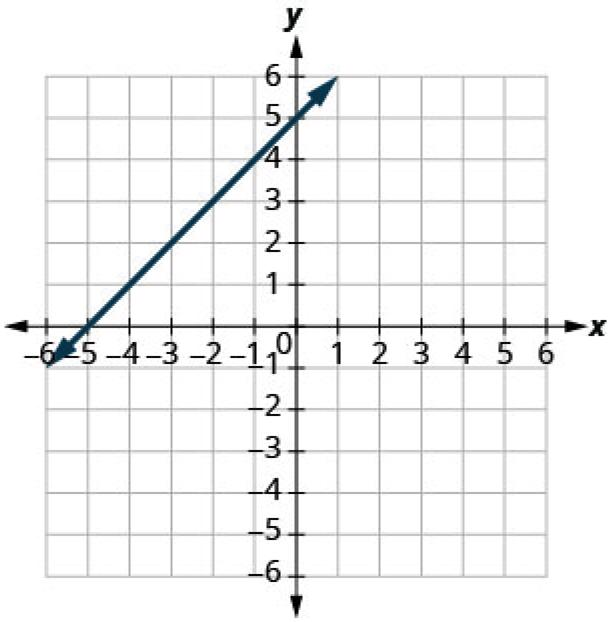


Solution:

$(-1, 0), (0, 1)$

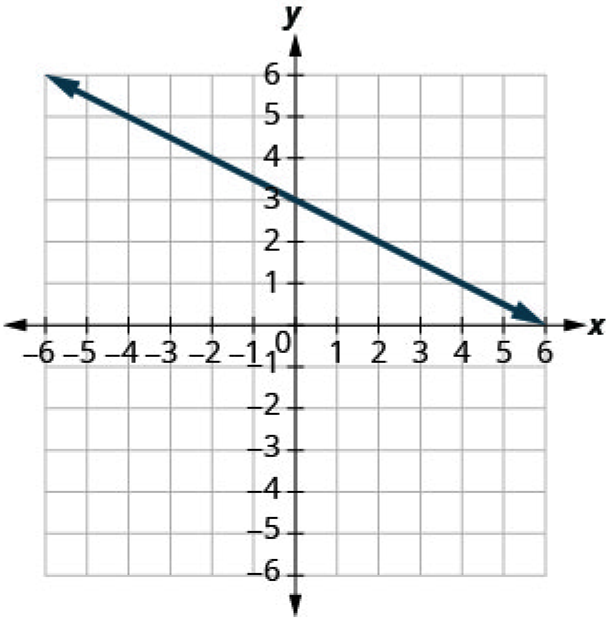
Exercise:

Problem:



Exercise:

Problem:

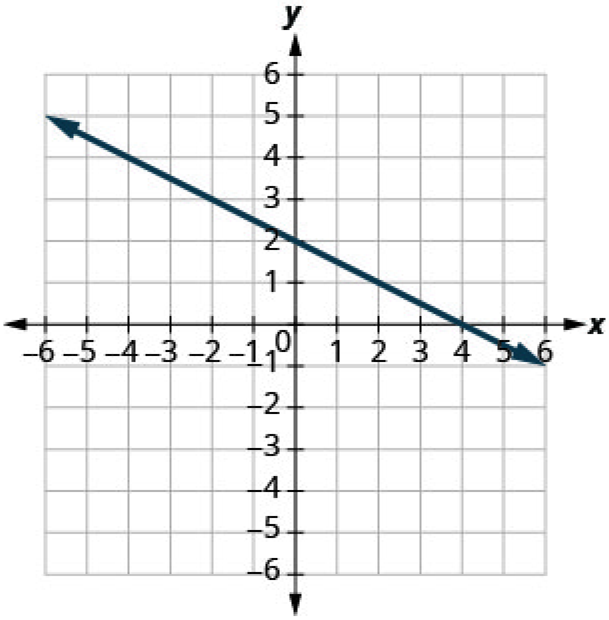


Solution:

$(6, 0), (0, 3)$

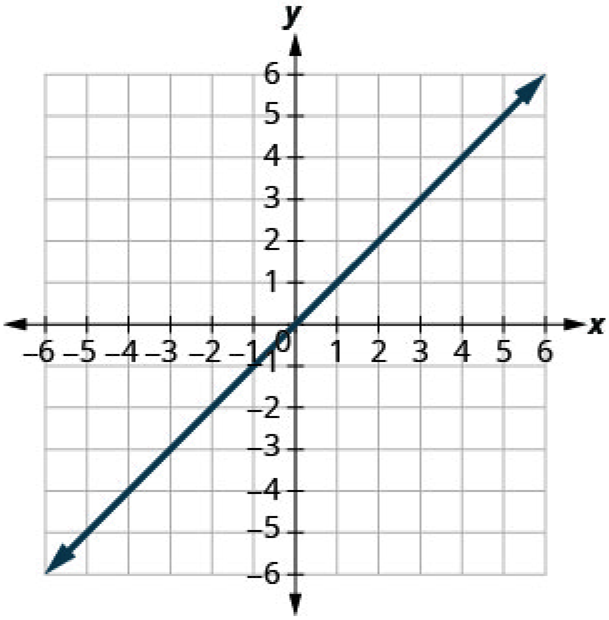
Exercise:

Problem:



Exercise:

Problem:

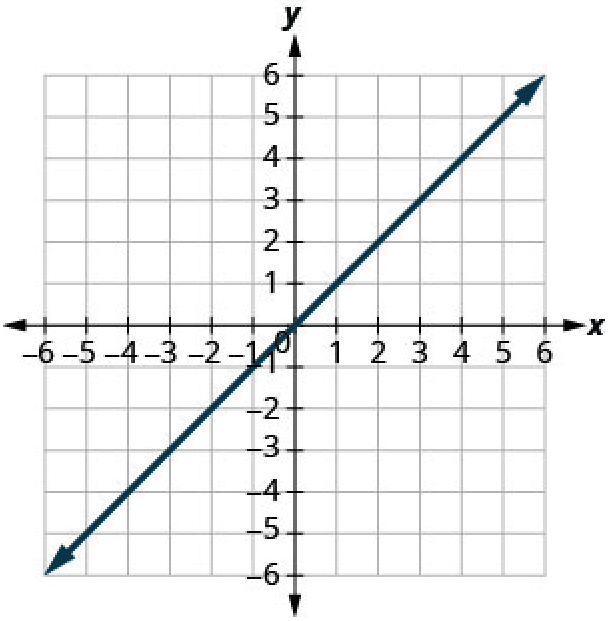


Solution:

$(0, 0)$

Exercise:

Problem:



Find the *x*- and *y*- Intercepts from an Equation of a Line

In the following exercises, find the intercepts for each equation.

Exercise:

Problem: $x + y = 4$

Solution:

$(4,0),(0,4)$

Exercise:

Problem: $x + y = 3$

Exercise:

Problem: $x + y = -2$

Solution:

$(-2,0),(0,-2)$

Exercise:

Problem: $x + y = -5$

Exercise:

Problem: $x - y = 5$

Solution:

$(5, 0), (0, -5)$

Exercise:

Problem: $x - y = 1$

Exercise:

Problem: $x - y = -3$

Solution:

$(-3, 0), \quad (0, 3)$

Exercise:

Problem: $x - y = -4$

Exercise:

Problem: $x + 2y = 8$

Solution:

$(8, 0), (0, 4)$

Exercise:

Problem: $x + 2y = 10$

Exercise:

Problem: $3x + y = 6$

Solution:

$(2, 0), (0, 6)$

Exercise:

Problem: $3x + y = 9$

Exercise:

Problem: $x - 3y = 12$

Solution:

$(12, 0), (0, -4)$

Exercise:

Problem: $x - 2y = 8$

Exercise:

Problem: $4x - y = 8$

Solution:

$(2, 0), (0, -8)$

Exercise:

Problem: $5x - y = 5$

Exercise:

Problem: $2x + 5y = 10$

Solution:

$(5, 0), (0, 2)$

Exercise:

Problem: $2x + 3y = 6$

Exercise:

Problem: $3x – 2y = 12$

Solution:

$(4, 0), (0, -6)$

Exercise:

Problem: $3x – 5y = 30$

Exercise:

Problem: $y = \frac{1}{3}x + 1$

Solution:

$(3, 0), (0, 1)$

Exercise:

Problem: $y = \frac{1}{4}x - 1$

Exercise:

Problem: $y = \frac{1}{5}x + 2$

Solution:

$(-10, 0), (0, 2)$

Exercise:

Problem: $y = \frac{1}{3}x + 4$

Exercise:

Problem: $y = 3x$

Solution:

$(0, 0)$

Exercise:

Problem: $y = -2x$

Exercise:

Problem: $y = -4x$

Solution:

$(0, 0)$

Exercise:

Problem: $y = 5x$

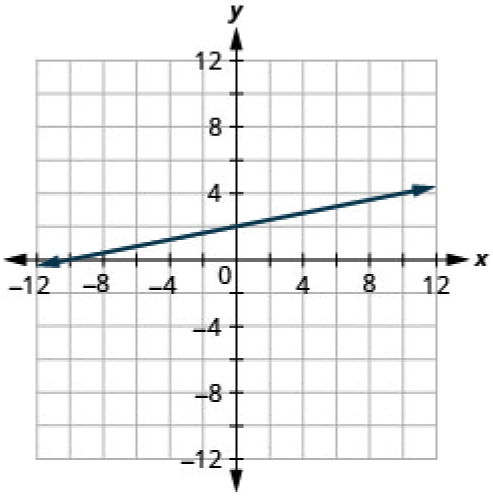
Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

Exercise:

Problem: $-x + 5y = 10$

Solution:



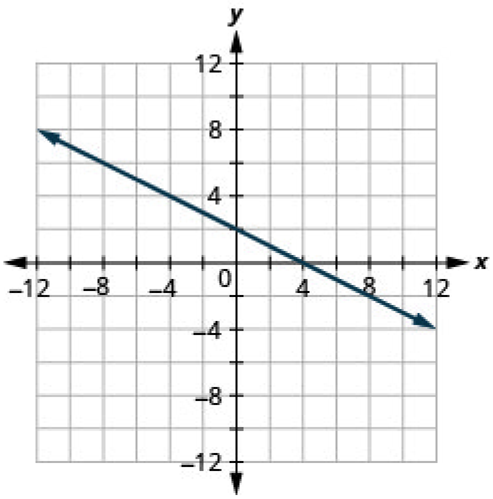
Exercise:

Problem: $-x + 4y = 8$

Exercise:

Problem: $x + 2y = 4$

Solution:



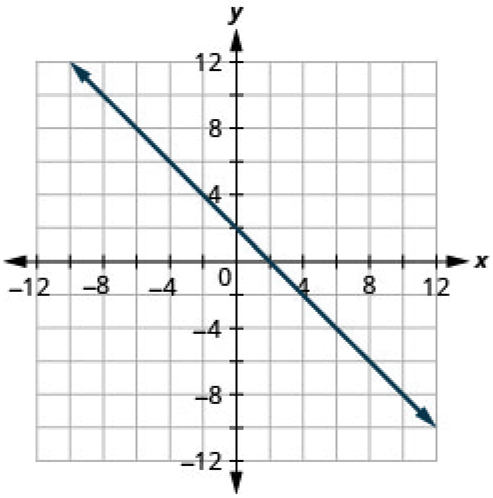
Exercise:

Problem: $x + 2y = 6$

Exercise:

Problem: $x + y = 2$

Solution:



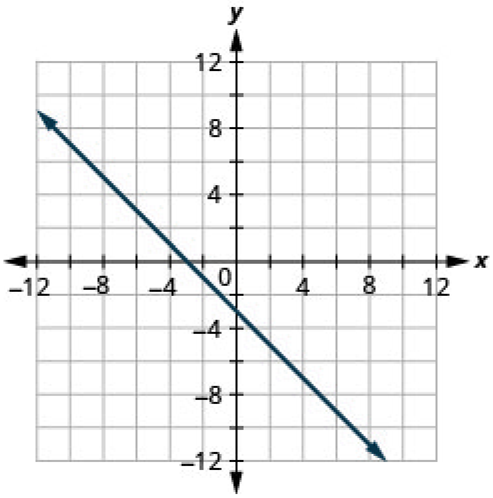
Exercise:

Problem: $x + y = 5$

Exercise:

Problem: $x + y = -3$

Solution:



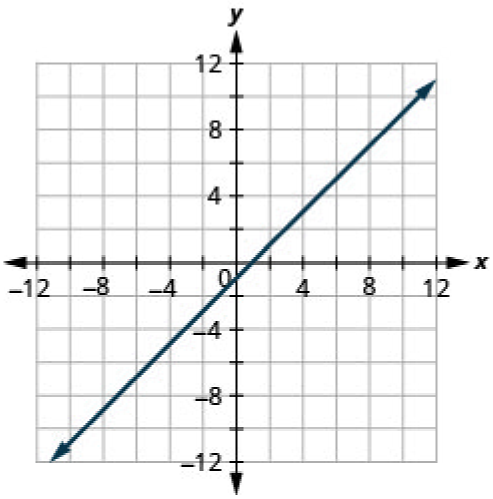
Exercise:

Problem: $x + y = -1$

Exercise:

Problem: $x - y = 1$

Solution:



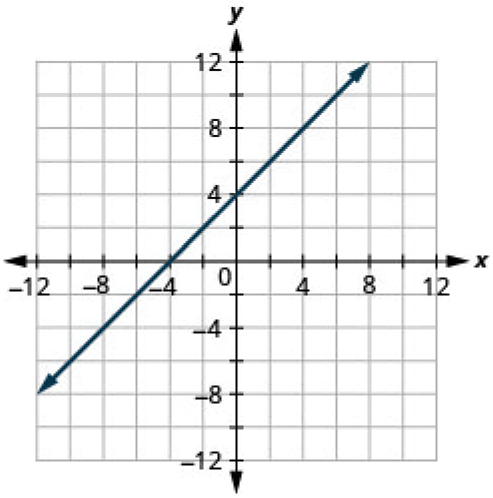
Exercise:

Problem: $x - y = 2$

Exercise:

Problem: $x - y = -4$

Solution:



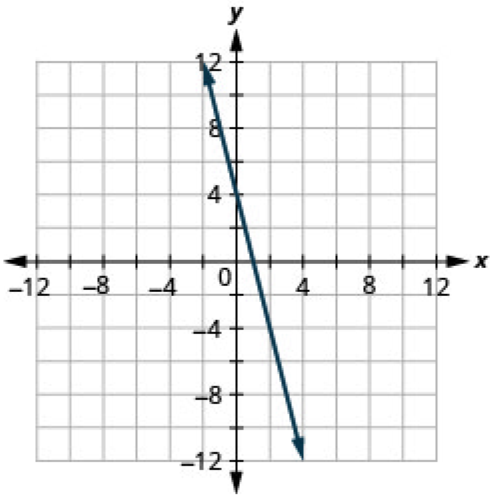
Exercise:

Problem: $x - y = -3$

Exercise:

Problem: $4x + y = 4$

Solution:



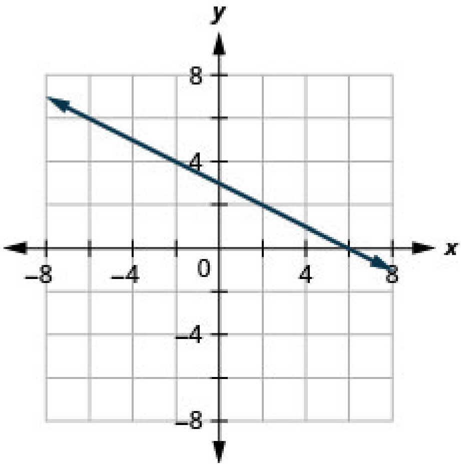
Exercise:

Problem: $3x + y = 3$

Exercise:

Problem: $2x + 4y = 12$

Solution:



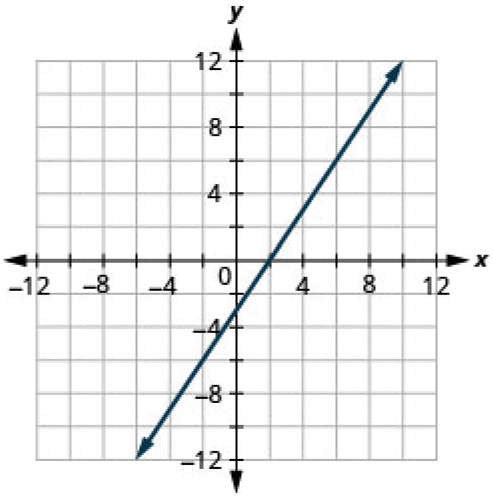
Exercise:

Problem: $3x + 2y = 12$

Exercise:

Problem: $3x – 2y = 6$

Solution:



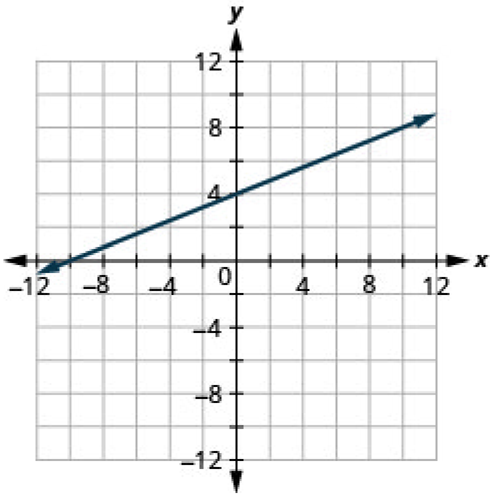
Exercise:

Problem: $5x – 2y = 10$

Exercise:

Problem: $2x – 5y = -20$

Solution:



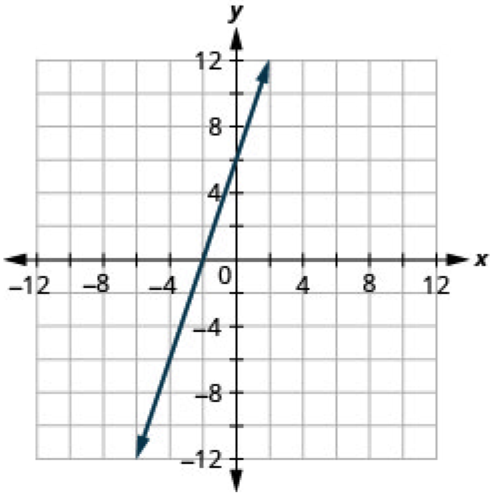
Exercise:

Problem: $3x – 4y = -12$

Exercise:

Problem: $3x – y = -6$

Solution:



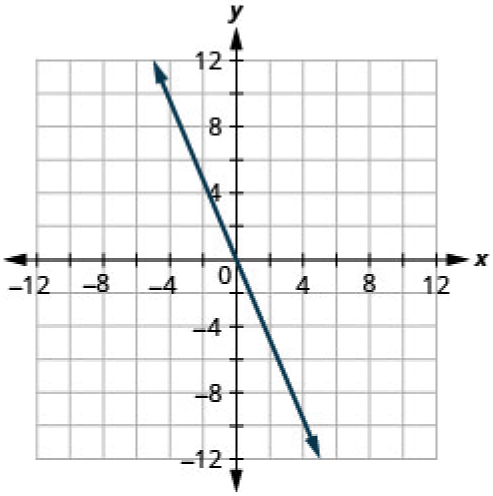
Exercise:

Problem: $2x – y = -8$

Exercise:

Problem: $y = -2x$

Solution:



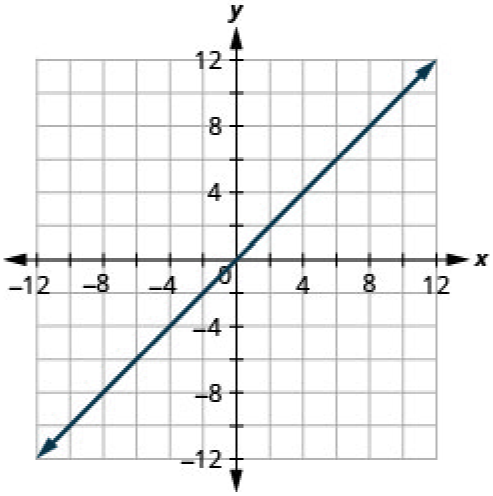
Exercise:

Problem: $y = -4x$

Exercise:

Problem: $y = x$

Solution:



Exercise:

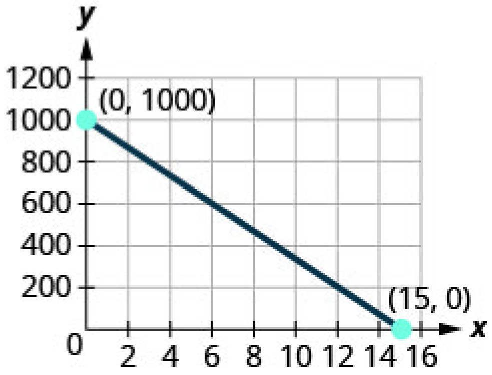
Problem: $y = 3x$

Everyday Math

Exercise:

Problem:

Road trip. Damien is driving from Chicago to Denver, a distance of 1000 miles. The *x*- axis on the graph below shows the time in hours since Damien left Chicago. The *y*- axis represents the distance he has left to drive.



ⓐ Find the *x*- and *y*- intercepts.
ⓑ Explain what the *x*- and *y*- intercepts mean for Damien.

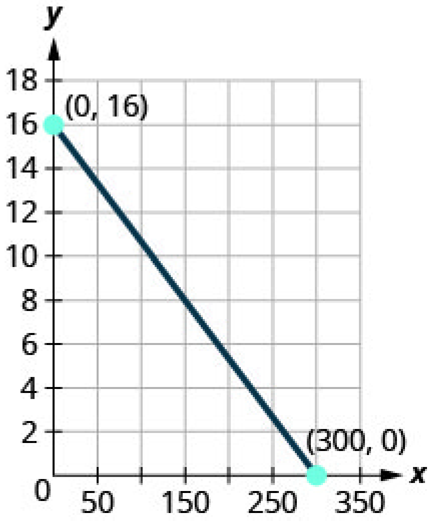
Solution:

ⓐ $(0, 1000), (15, 0)$
ⓑ At $(0, 1000)$, he has been gone 0 hours and has 1000 miles left. At $(15, 0)$, he has been gone 15 hours and has 0 miles left to go.

Exercise:

Problem:

Road trip. Ozzie filled up the gas tank of his truck and headed out on a road trip. The *x*- axis on the graph below shows the number of miles Ozzie drove since filling up. The *y*- axis represents the number of gallons of gas in the truck's gas tank.



ⓐ Find the *x*- and *y*- intercepts.
ⓑ Explain what the *x*- and *y*- intercepts mean for Ozzie.

Writing Exercises

Exercise:

Problem:

How do you find the *x*- intercept of the graph of $3x - 2y = 6$?

Solution:

Answers will vary.

Exercise:

Problem:

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $4x + y = -4$? Why?

Exercise:

Problem:

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = \frac{2}{3}x - 2$? Why?

Solution:

Answers will vary.

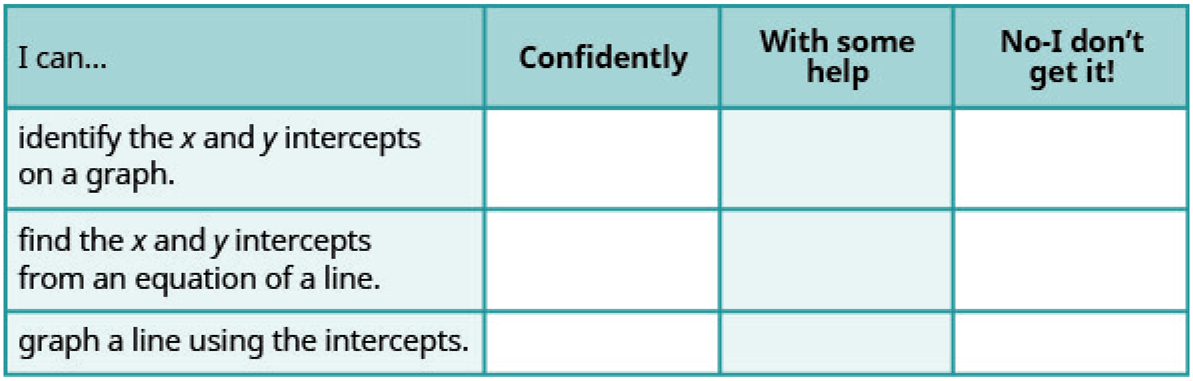
Exercise:

Problem:

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = 6$? Why?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
identify the *x* and *y* intercepts on a graph.			
find the *x* and *y* intercepts from an equation of a line.			
graph a line using the intercepts.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

intercepts of a line

The points where a line crosses the *x*- axis and the *y*- axis are called the intercepts of the line.

x- intercept

The point $(a, 0)$ where the line crosses the *x*- axis; the *x*- intercept occurs when y is zero.

y-intercept

The point $(0, b)$ where the line crosses the *y*- axis; the *y*- intercept occurs when x is zero.

Understand Slope of a Line

By the end of this section, you will be able to:

- Use geoboards to model slope
- Use $m = \frac{rise}{run}$ to find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications

Note:

Before you get started, take this readiness quiz.

1. Simplify: $\frac{1-4}{8-2}$.
   If you missed this problem, review [link].
2. Divide: $\frac{0}{4}, \frac{4}{0}$.
   If you missed this problem, review [link].
3. Simplify: $\frac{15}{-3}, \frac{-15}{3}, \frac{-15}{-3}$.
   If you missed this problem, review [link].

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down or if it is steep or flat?

In mathematics, the ‘tilt’ of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, and a ramp for a wheelchair are some examples where you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

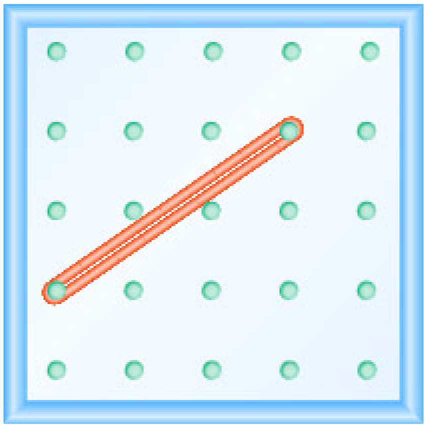
In this section, we will explore the concept of slope.

Use Geoboards to Model Slope

A **geoboard** is a board with a grid of pegs on it. Using rubber bands on a geoboard gives us a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line.

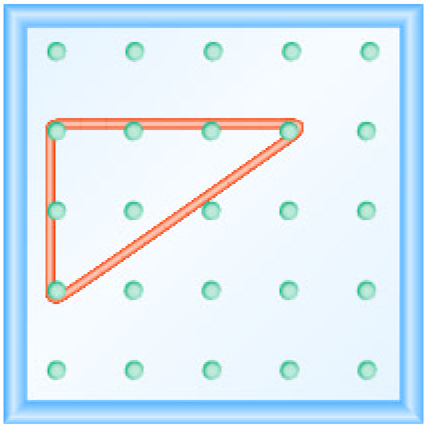
Note: Doing the Manipulative Mathematics activity “Exploring Slope” will help you develop a better understanding of the slope of a line. (Graph paper can be used instead of a geoboard, if needed.)

We’ll start by stretching a rubber band between two pegs as shown in [link].



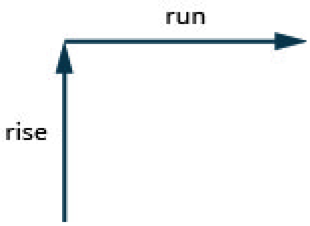
Doesn’t it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, as shown in [link]

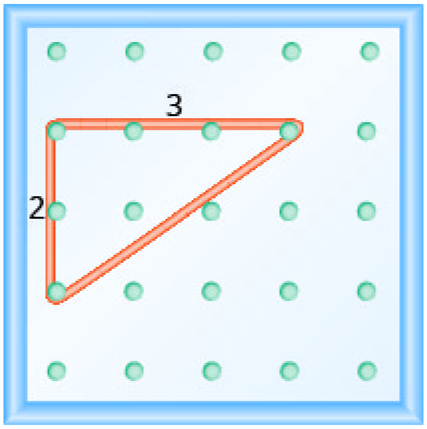


We carefully make a 90º angle around the third peg, so one of the newly formed lines is vertical and the other is horizontal.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the rise and the horizontal distance is called the run, as shown in [link].



If our geoboard and rubber band look just like the one shown in [link], the rise is 2. The rubber band goes up 2 units. (Each space is one unit.)



The rise on this geoboard is 2, as the rubber band goes up two units.

What is the run?

The rubber band goes across 3 units. The run is 3 (see [link]).

The slope of a line is the ratio of the rise to the run. In mathematics, it is always referred to with the letter m.

Note:
Slope of a Line
The **slope of a line** of a line is $m = \frac{\text{rise}}{\text{run}}$.
The **rise** measures the vertical change and the **run** measures the horizontal change between two points on the line.

What is the slope of the line on the geoboard in [link]?
Equation:

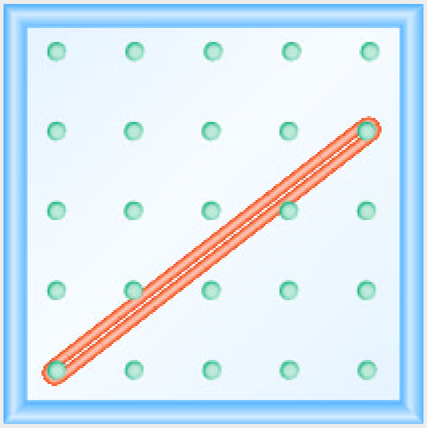
$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ m &= \frac{2}{3} \end{aligned}$$

The line has slope $\frac{2}{3}$. This means that the line rises 2 units for every 3 units of run.

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. If the rise goes up it is positive and if it goes down it is negative. The run will go from left to right and be positive.

Example:
Exercise:

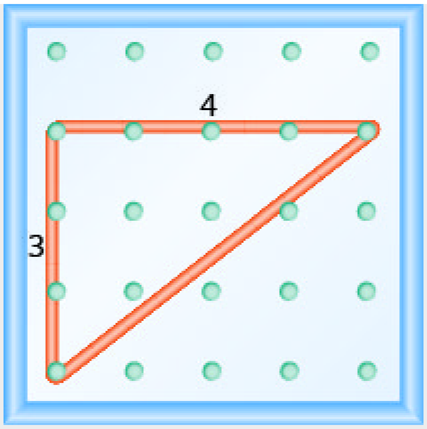
Problem: **What is the slope of the line on the geoboard shown?



Solution:
Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the spaces up and to the right to reach the second peg.



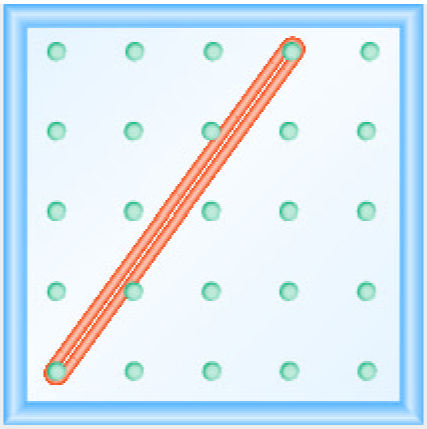
The rise is 3.	$m = \frac{3}{\text{run}}$
The run is 4.	$m = \frac{3}{4}$
	The slope is $\frac{3}{4}$.

This means that the line rises 3 units for every 4 units of run.

Note:
Exercise:

TRY IT :: 4.49

Problem: What is the slope of the line on the geoboard shown?



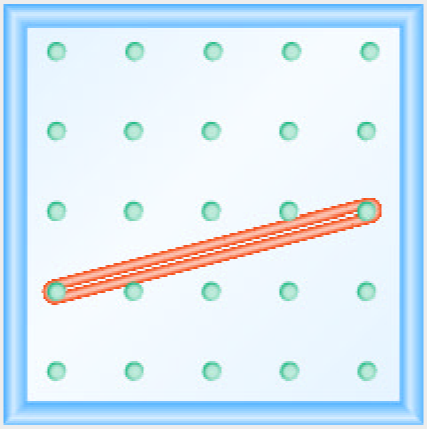
Solution:

$\frac{4}{3}$

Note:
Exercise:

TRY IT :: 4.50

Problem: What is the slope of the line on the geoboard shown?

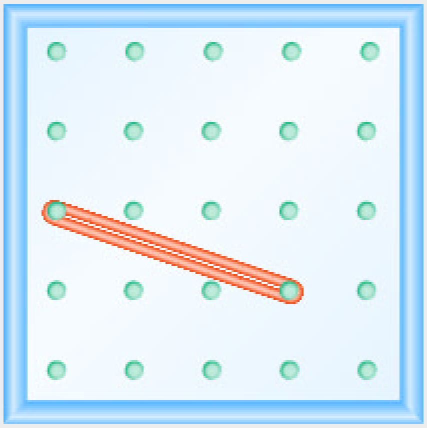


Solution:

$\frac{1}{4}$

Example:
Exercise:

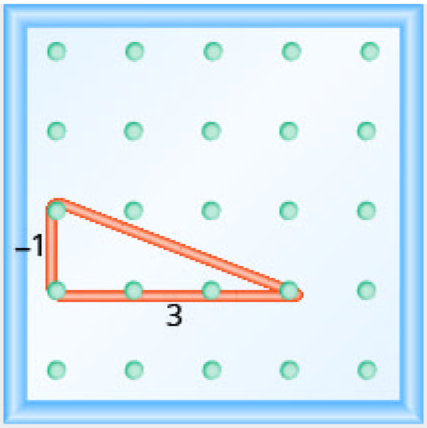
Problem: **What is the slope of the line on the geoboard shown?



Solution:
Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the units down and to the right to reach the second peg.



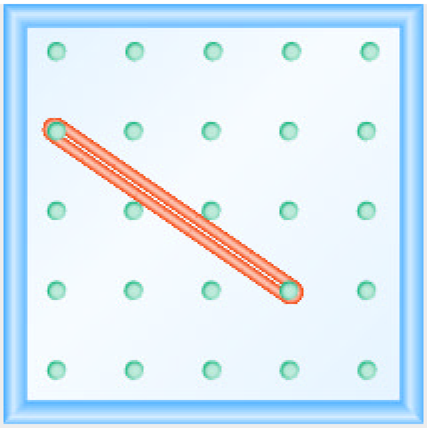
The rise is −1.	$= \frac{-1}{\text{run}}$
The run is 3.	$m = \frac{-1}{3}$ $m = -\frac{1}{3}$
	The slope is $-\frac{1}{3}$.

This means that the line drops 1 unit for every 3 units of run.

Note:
Exercise:

TRY IT :: 4.51

Problem: What is the slope of the line on the geoboard?



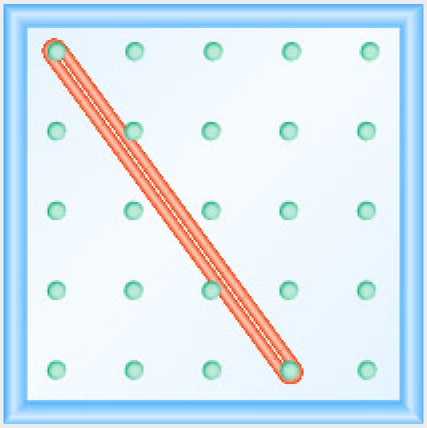
Solution:

$-\frac{2}{3}$

Note:
Exercise:

TRY IT :: 4.52

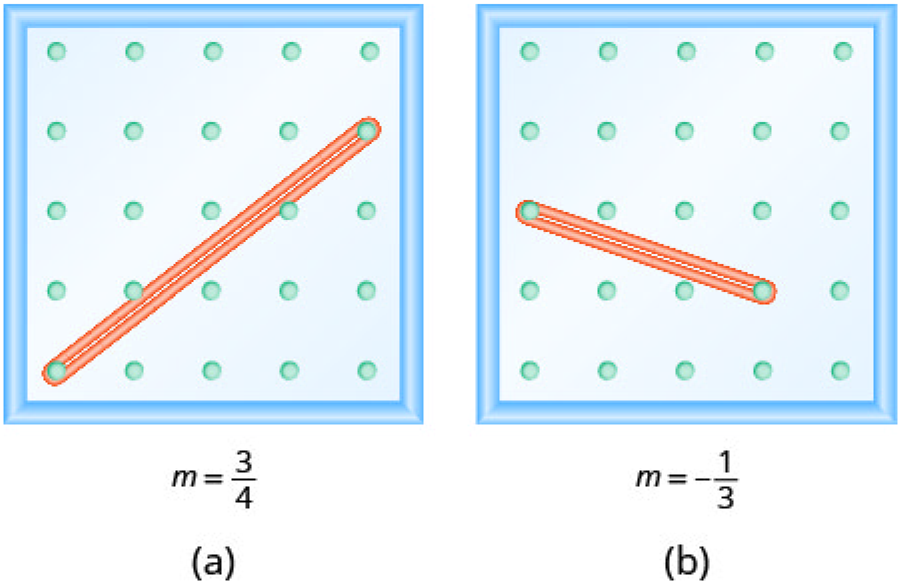
Problem: What is the slope of the line on the geoboard?



Solution:

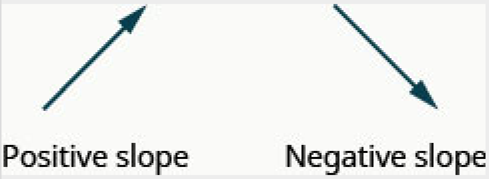
$-\frac{4}{3}$

Notice that in [link] the slope is positive and in [link] the slope is negative. Do you notice any difference in the two lines shown in [link](a) and [link](b)?



We ‘read’ a line from left to right just like we read words in English. As you read from left to right, the line in [link](a) is going up; it has **positive slope**. The line in [link](b) is going down; it has **negative slope**.

Note:
Positive and Negative Slopes



Example:
Exercise:

Problem: **Use a geoboard to model a line with slope $\frac{1}{2}$.

Solution:
Solution

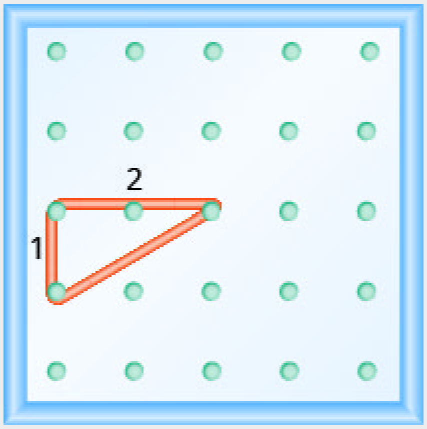
To model a line on a geoboard, we need the rise and the run.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $\frac{1}{2}$.	$\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

So, the rise is 1 and the run is 2.

Start at a peg in the lower left of the geoboard.

Stretch the rubber band up 1 unit, and then right 2 units.



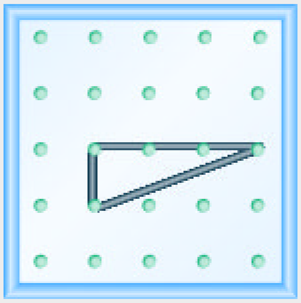
The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{1}{2}$.

Note:
Exercise:

Problem:

TRY IT :: 4.53
Model the slope $m = \frac{1}{3}$. Draw a picture to show your results.

Solution:

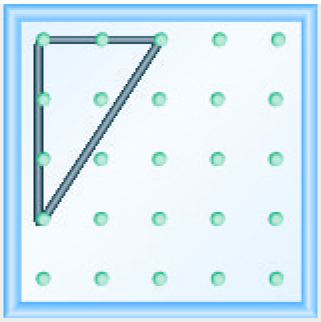


Note:
Exercise:

Problem:

TRY IT :: 4.54
Model the slope $m = \frac{3}{2}$. Draw a picture to show your results.

Solution:



Example:
Exercise:

Problem: **Use a geoboard to model a line with slope $\frac{-1}{4}$.

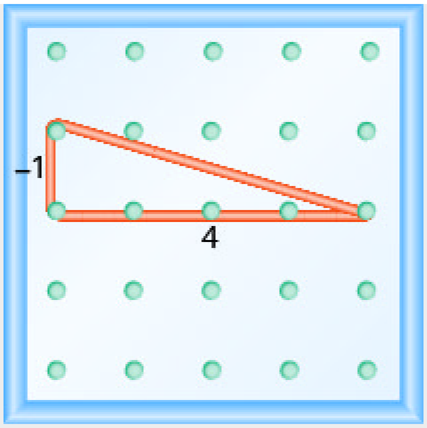
Solution:
Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $\frac{-1}{4}$.	$\frac{-1}{4} = \frac{\text{rise}}{\text{run}}$

So, the rise is -1 and the run is 4.

Since the rise is negative, we choose a starting peg on the upper left that will give us room to count down.

We stretch the rubber band down 1 unit, then go to the right 4 units, as shown.



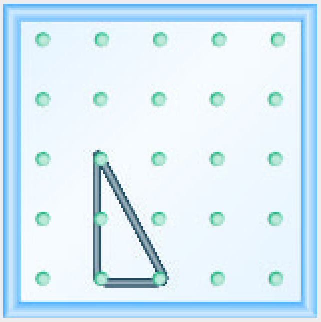
The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{-1}{4}$.

Note:
Exercise:

Problem:

TRY IT :: 4.55
Model the slope $m = -2$. Draw a picture to show your results.

Solution:



Note:

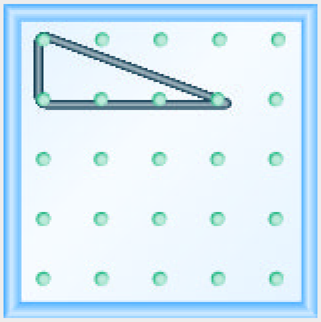
Exercise:

Problem:

TRY IT :: 4.56

Model the slope $m = \frac{-1}{3}$. Draw a picture to show your results.

Solution:



Use $m = \frac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

Now, we’ll look at some graphs on the xy-coordinate plane and see how to find their slopes. The method will be very similar to what we just modeled on our geoboards.

To find the slope, we must count out the rise and the run. But where do we start?

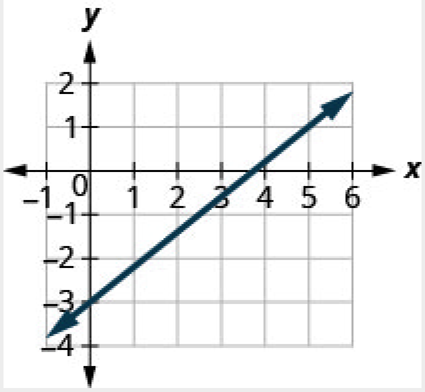
We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

Example:

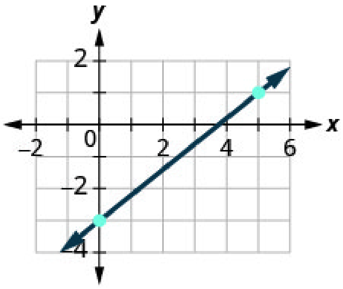
How to Use $m = \frac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

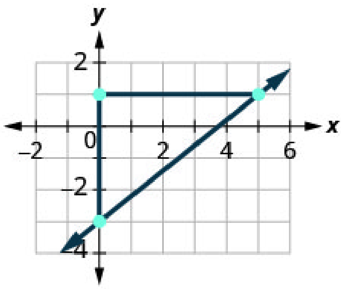
Exercise:

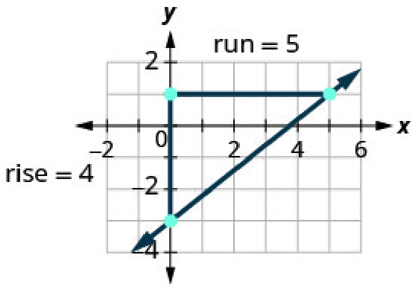
Problem: **Find the slope of the line shown.



Solution:
Solution

Step 1. Locate two points on the graph whose coordinates are integers.	Mark (0, –3) and (5, 1).	

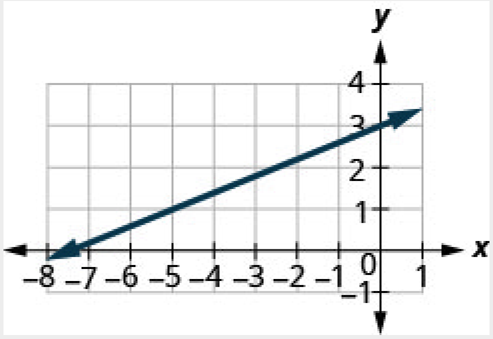
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.	Starting at (0, –3), sketch a right triangle to (5, 1).	

Step 3. Count the rise and the run on the legs of the triangle.	Count the rise. Count the run.	 The rise is 4. The run is 5.
Step 4. Take the ratio of rise to run to find the slope. $m = \frac{\text{rise}}{\text{run}}$	Use the slope formula. Substitute the values of the rise and run.	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{4}{5}$ The slope of the line is $\frac{4}{5}$. This means that y increases 4 units as x increases 5 units.

Note:
Exercise:

TRY IT :: 4.57

Problem: Find the slope of the line shown.



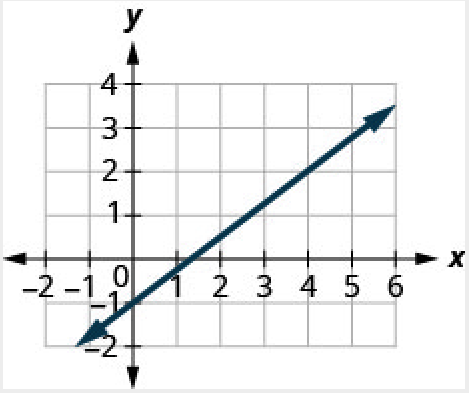
Solution:

$\frac{2}{5}$

Note:
Exercise:

TRY IT :: 4.58

Problem: Find the slope of the line shown.



Solution:

$\frac{3}{4}$

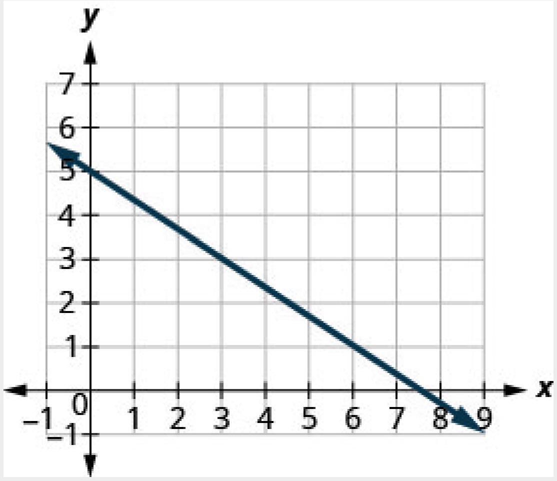
Note:
Find the slope of a line from its graph using $m = \frac{\text{rise}}{\text{run}}$.

Locate two points on the line whose coordinates are integers.
Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
Count the rise and the run on the legs of the triangle.

Take the ratio of rise to run to find the slope,$m = \frac{\text{rise}}{\text{run}}$.

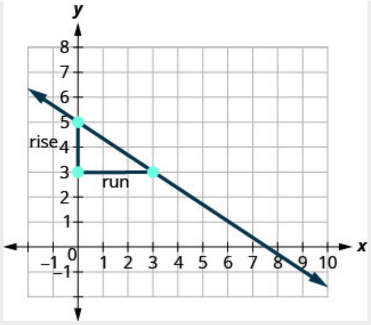
Example:
Exercise:

Problem: **Find the slope of the line shown.



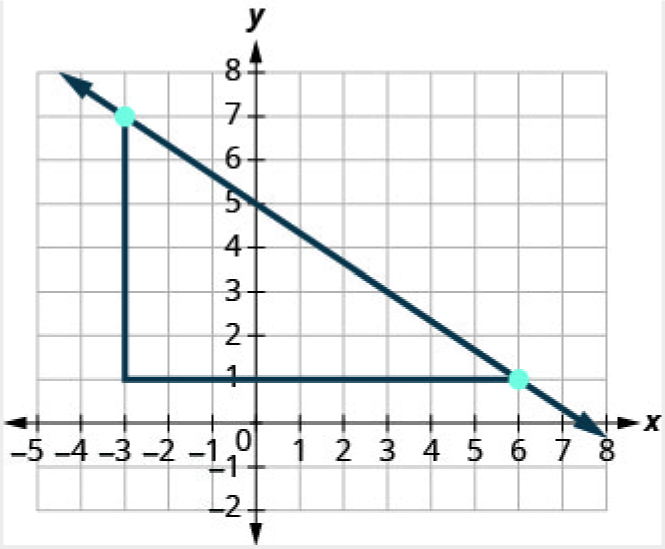
Solution:
Solution

Locate two points on the graph whose coordinates are integers.	$(0,5)$ and $(3,3)$
Which point is on the left?	$(0,5)$
Starting at $(0,5)$, sketch a right triangle to $(3,3)$.	

	
Count the rise—it is negative.	The rise is -2.
Count the run.	The run is 3.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-2}{3}$
Simplify.	$m = -\frac{2}{3}$
	The slope of the line is $-\frac{2}{3}$.

So y increases by 3 units as x decreases by 2 units.

What if we used the points $(-3, 7)$ and $(6, 1)$ to find the slope of the line?

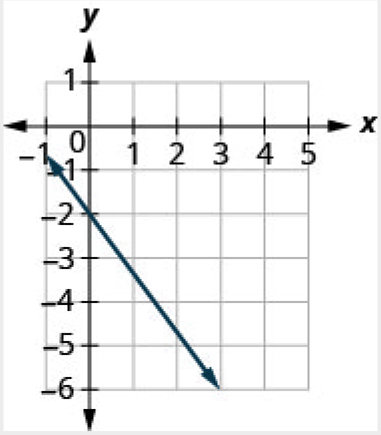


The rise would be -6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$. Remember, it does not matter which points you use—the slope of the line is always the same.

Note:
Exercise:

TRY IT :: 4.59

Problem: Find the slope of the line shown.



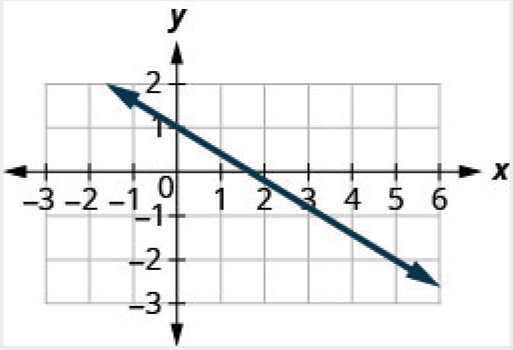
Solution:

$-\frac{4}{3}$

Note:
Exercise:

TRY IT :: 4.60

Problem: Find the slope of the line shown.



Solution:

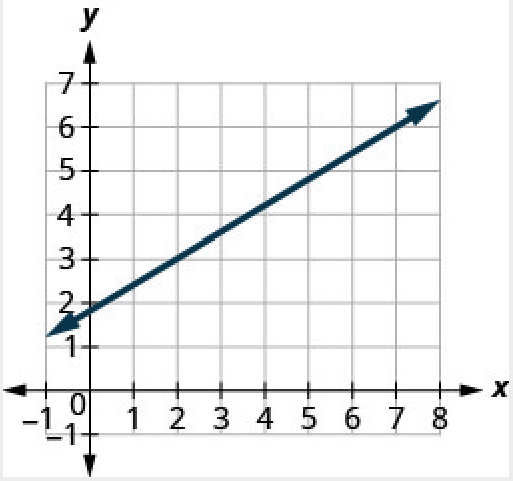
$-\frac{3}{5}$

In the last two examples, the lines had *y*-intercepts with integer values, so it was convenient to use the *y*-intercept as one of the points to find the slope. In the next example, the *y*-intercept is a fraction. Instead of using that point, we’ll look for two other points whose coordinates are integers. This will make the slope calculations easier.

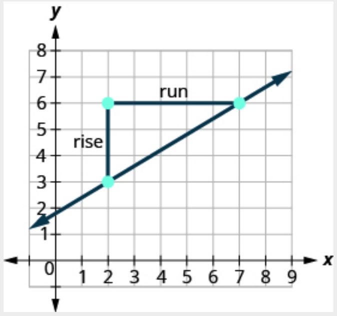
Example:

Exercise:

Problem: **Find the slope of the line shown.



Solution:
Solution

Locate two points on the graph whose coordinates are integers.	$(2,3)$ and $(7,6)$
Which point is on the left?	$(2,3)$
Starting at $(2,3)$, sketch a right triangle to $(7,6)$.	

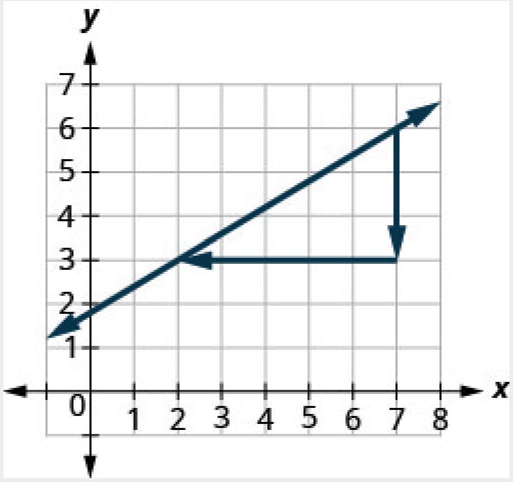
Count the rise.	The rise is 3.
Count the run.	The run is 5.
Use the slope formula.	$m=\frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m=\frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

This means that y increases 5 units as x increases 3 units.

When we used geoboards to introduce the concept of slope, we said that we would always start with the point on the left and count the rise and the run to get to the point on the right. That way the run was always positive and the rise determined whether the slope was positive or negative.

What would happen if we started with the point on the right?

Let’s use the points $(2,3)$ and $(7,6)$ again, but now we’ll start at $(7,6)$.



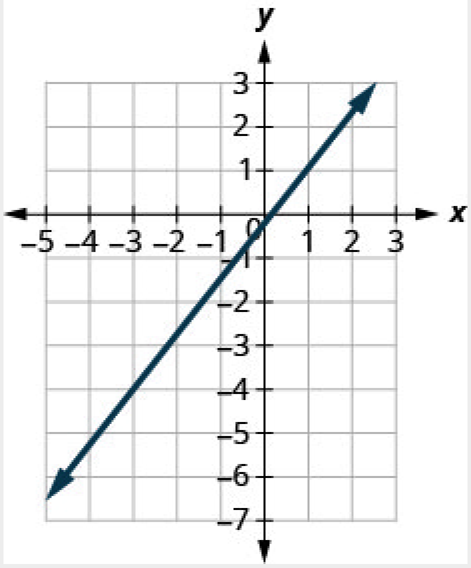
Count the rise.	The rise is -3.
Count the run. It goes from right to left, so it is negative.	The run is -5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-3}{-5} = \frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

It does not matter where you start—the slope of the line is always the same.

Note:
Exercise:

TRY IT :: 4.61

Problem: Find the slope of the line shown.



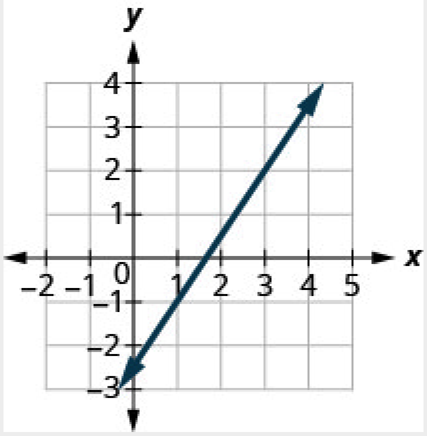
Solution:

$\frac{5}{4}$

Note:
Exercise:

TRY IT :: 4.62

Problem: Find the slope of the line shown.



Solution:

$\frac{3}{2}$

Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Equation:

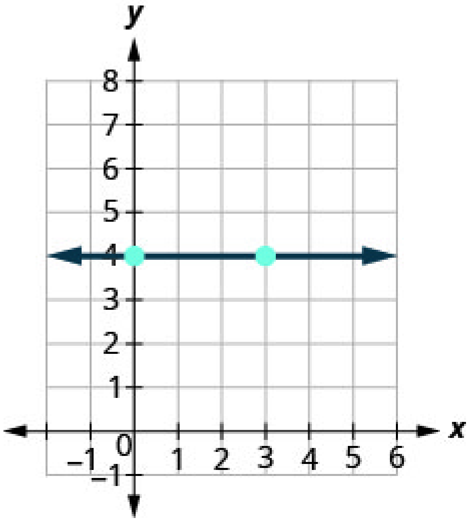
Horizontal line $y = b$

y-coordinates are the same.

Vertical line $x = a$

x-coordinates are the same.

So how do we find the slope of the horizontal line $y = 4$? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens when we do this.



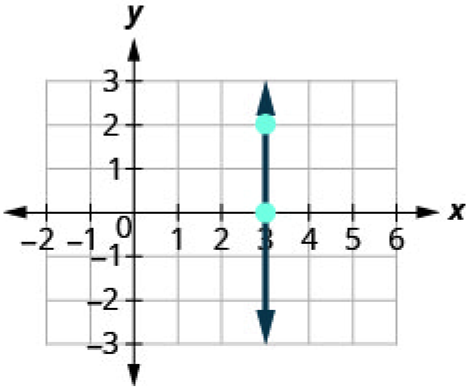
What is the rise?	The rise is 0.
Count the run.	The run is 3.
What is the slope?	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{0}{3}$ $m = 0$
	The slope of the horizontal line $y = 4$ is 0.

All horizontal lines have slope 0. When the *y*-coordinates are the same, the rise is 0.

Note:
Slope of a Horizontal Line
The slope of a horizontal line, $y = b$, is 0.

The floor of your room is horizontal. Its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Now, we'll consider a vertical line, the line.



What is the rise?	The rise is 2.
Count the run.	The run is 0.
What is the slope?	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line $x = 3$ is undefined.

The slope of any vertical line is undefined. When the *x*-coordinates of a line are all the same, the run is 0.

Note:
Slope of a Vertical Line
The slope of a vertical line, $x = a$, is undefined.

Example:
Exercise:

Problem: **Find the slope of each line:

ⓐ $x = 8$ ⓑ $y = -5$.

Solution:
Solution

ⓐ$x = 8$
This is a vertical line.
Its slope is undefined.

ⓑ $y = -5$
This is a horizontal line.
It has slope 0.

Note:
Exercise:

TRY IT :: 4.63

Problem: Find the slope of the line: $x = -4$.

Solution:

undefined

Note:
Exercise:

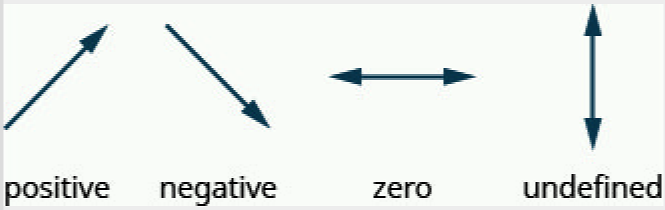
TRY IT :: 4.64

Problem: Find the slope of the line: $y = 7$.

Solution:

0

Note:
Quick Guide to the Slopes of Lines



Remember, we ‘read’ a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Note: Doing the Manipulative Mathematics activity “Slope of Lines Between Two Points” will help you develop a better understanding of how to find the slope of a line between two points.

Sometimes we’ll need to find the slope of a line between two points when we don’t have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but as we’ll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x, y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x, y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

Equation:

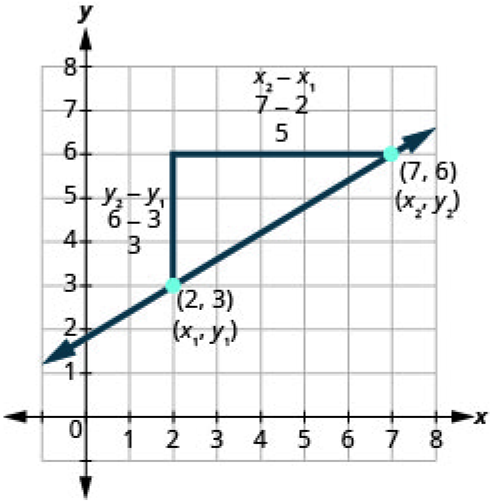
$$\begin{array}{ll} (x_1, y_1) & \text{read ‘} x \text{ sub } 1, y \text{ sub } 1\text{’} \\ (x_2, y_2) & \text{read ‘} x \text{ sub } 2, y \text{ sub } 2\text{’} \end{array}$$

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let’s see how the rise and run relate to the coordinates of the two points by taking another look at the slope of the line between the points $(2, 3)$ and $(7, 6)$.



Since we have two points, we will use subscript notation, $\binom{x_1,y_1}{2, 3}\binom{x_2,y_2}{7, 6}$.

On the graph, we counted the rise of 3 and the run of 5.

Notice that the rise of 3 can be found by subtracting the *y*-coordinates 6 and 3.
Equation:

$$3 = 6 - 3$$

And the run of 5 can be found by subtracting the *x*-coordinates 7 and 2.
Equation:

$$5 = 7 - 2$$

We know $m = \frac{\text{rise}}{\text{run}}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$.

But 6 is y_2, the *y*-coordinate of the second point and 3 is y_1, the *y*-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y_2 - y_1}{7 - 2}$

Also, 7 is x_2, the *x*-coordinate of the second point and 2 is x_1, the *x*-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y_2 - y_1}{x_2 - x_1}$

We've shown that $m = \frac{y_2 - y_1}{x_2 - x_1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

Note:
Slope Formula
The slope of the line between two points (x_1, y_1) and (x_2, y_2) is
Equation:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This is the **slope formula**.
The slope is:
Equation:

$$\begin{array}{c} y \text{ of the second point minus } y \text{ of the first point} \\ \text{over} \\ x \text{ of the second point minus } x \text{ of the first point.} \end{array}$$

Example:

Exercise:

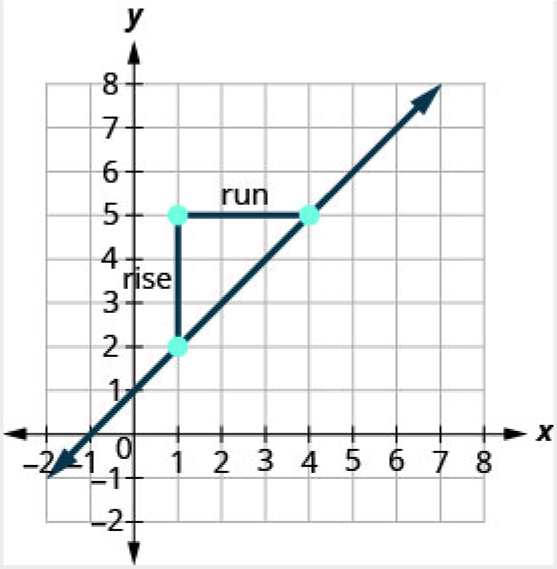
Problem:

**Use the slope formula to find the slope of the line between the points $(1,2)$ and $(4,5)$.

Solution:
Solution

We'll call $(1,2)$ point #1 and $(4,5)$ point #2.	$\left(\overset{x_1,y_1}{1,2}\right)\ \left(\overset{x_2,y_2}{4,5}\right)$
Use the slope formula.	$m=\frac{y_2-y_1}{x_2-x_1}.$
Substitute the values.	
y of the second point minus y of the first point	$m=\frac{5-2}{x_2-x_1}.$
x of the second point minus x of the first point	$m=\frac{5-2}{4-1}.$
Simplify the numerator and the denominator.	$m=\frac{3}{3}.$
Simplify.	$m=1.$

Let’s confirm this by counting out the slope on a graph using $m=\frac{\text{rise}}{\text{run}}.$



It doesn’t matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

Note:
Exercise:

Problem:

TRY IT :: 4.65
Use the slope formula to find the slope of the line through the points: $(8, 5)$ and $(6, 3)$.

Solution:

1

Note:
Exercise:

Problem:

TRY IT :: 4.66
Use the slope formula to find the slope of the line through the points: $(1, 5)$ and $(5, 9)$.

Solution:

1

Example:
Exercise:

Problem:

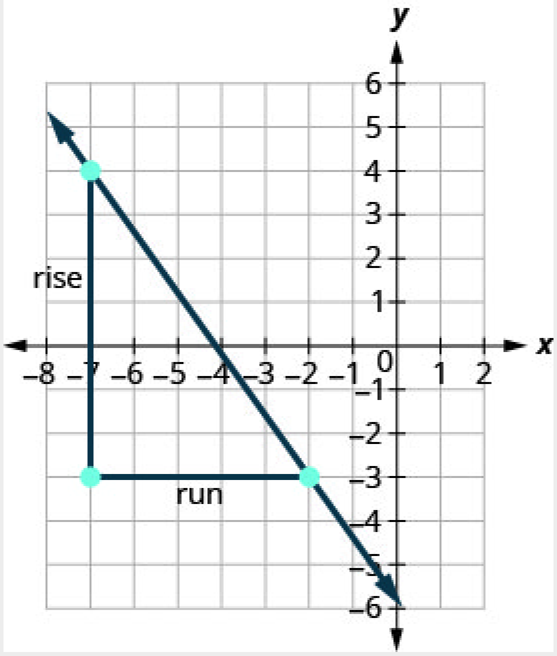
**Use the slope formula to find the slope of the line through the points $(-2, -3)$ and $(-7, 4)$.

Solution:
Solution

We'll call $(-2, -3)$ point #1 and $(-7, 4)$ point #2.	$\left(\overset{x_1,y_1}{-2,-3}\right)\ \left(\overset{x_2,y_2}{-7,4}\right)$
Use the slope formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}$.
Substitute the values.	
y of the second point minus y of the first point	$m = \frac{4-(-3)}{x_2 - x_1}$.

x of the second point minus x of the first point	$m = \frac{4-(-3)}{-7-(-2)}.$
Simplify.	$m = \frac{7}{-5}$ $m = -\frac{7}{5}$

Let’s verify this slope on the graph shown.



Equation:

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ m &= \frac{-7}{5} \\ m &= -\frac{7}{5} \end{aligned}$$

Note:
Exercise:

Problem:

TRY IT :: 4.67
Use the slope formula to find the slope of the line through the points: $(-3, 4)$ and $(2, -1)$.

Solution:

-1

Note:
Exercise:

Problem:

TRY IT :: 4.68
Use the slope formula to find the slope of the line through the pair of points: $(-2, 6)$ and $(-3, -4)$.

Solution:

10

Graph a Line Given a Point and the Slope

Up to now, in this chapter, we have graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

One other method we can use to graph lines is called the **point–slope method**. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the definition of slope to draw the graph of the line.

Example:

How To Graph a Line Given a Point and The Slope

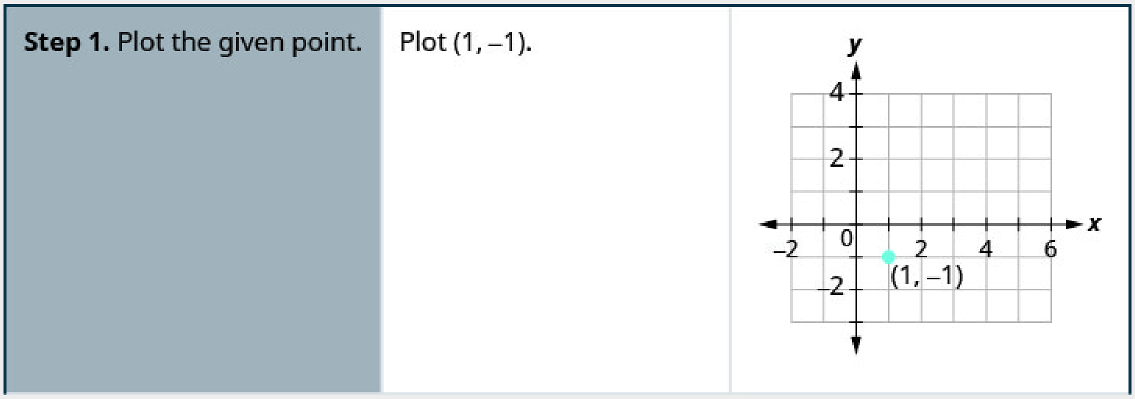
Exercise:

Problem:

**Graph the line passing through the point $(1, -1)$ whose slope is $m = \frac{3}{4}$.

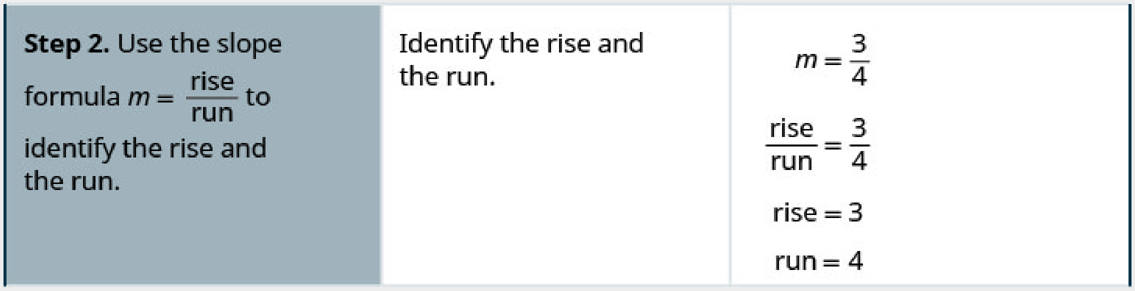
Solution:

Solution

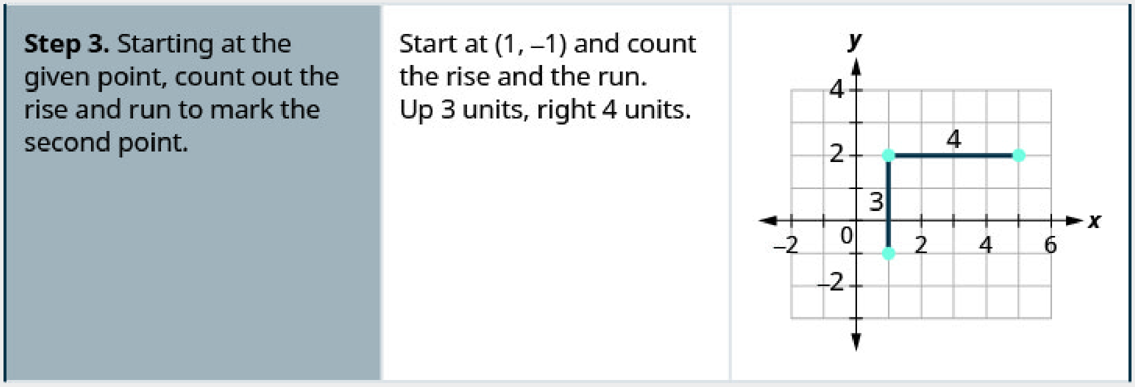


Step 1. Plot the given point.	Plot (1, –1).	

y
4
2
0
–2
2
4
6
x
–2
(1, –1)

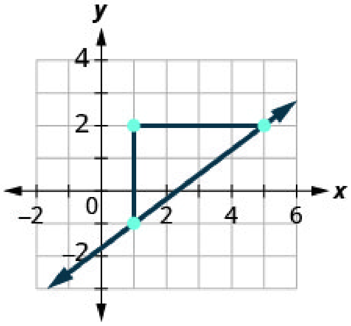


Step 2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = \frac{3}{4}$ $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ rise = 3 run = 4



Step 3. Starting at the given point, count out the rise and run to mark the second point.	Start at (1, –1) and count the rise and the run. Up 3 units, right 4 units.	

y
4
2
4
3
0
–2
2
4
6
x
–2

Step 4. Connect the points with a line.	Connect the two points with a line.	

Note:

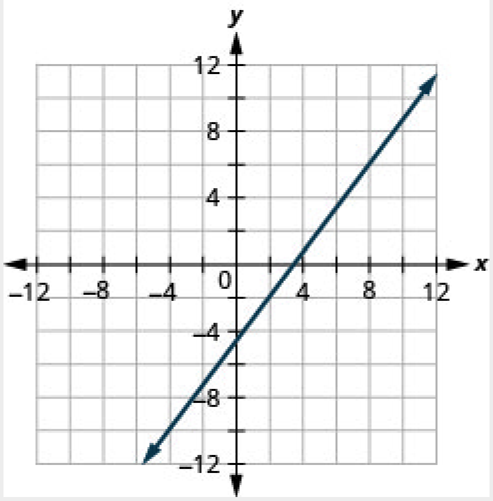
Exercise:

Problem:

TRY IT :: 4.69

Graph the line passing through the point $(2, -2)$ with the slope $m = \frac{4}{3}$.

Solution:

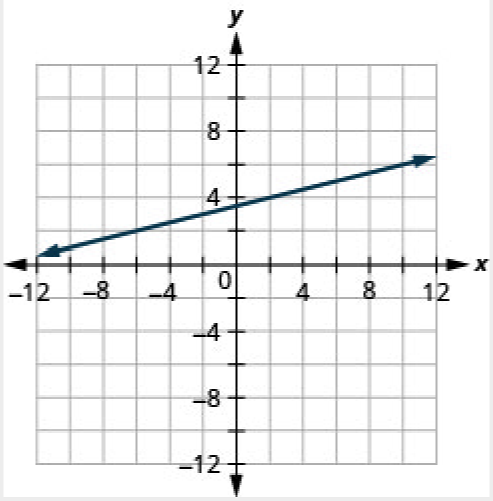


Note:
Exercise:

Problem:

TRY IT :: 4.70
Graph the line passing through the point $(-2, 3)$ with the slope $m = \frac{1}{4}$.

Solution:



Note:
Graph a line given a point and the slope.

Plot the given point.
Use the slope formula$m = \frac{\text{rise}}{\text{run}}$to identify the rise and the run.
Starting at the given point, count out the rise and run to mark the second point.
Connect the points with a line.

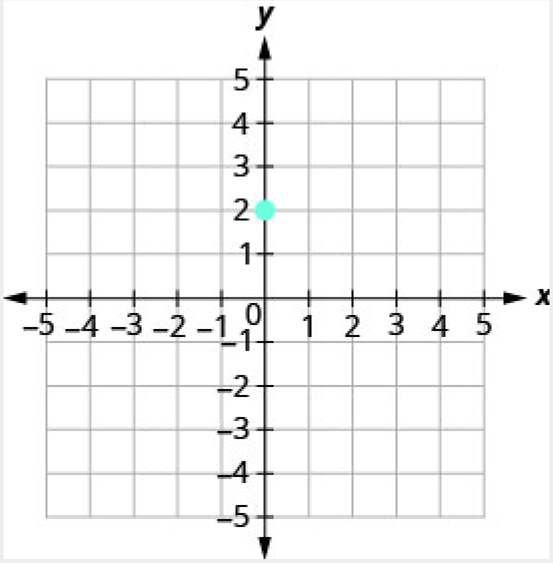
Example:
Exercise:

Problem:

**Graph the line with *y*-intercept 2 whose slope is $m = -\frac{2}{3}$.

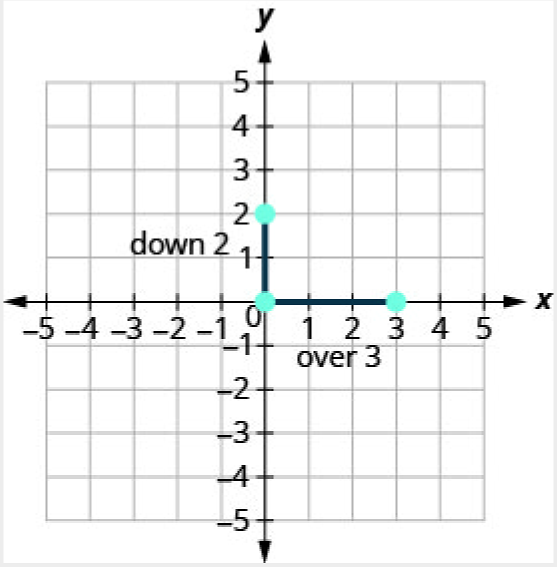
Solution:
Solution

Plot the given point, the *y*-intercept, $(0, 2)$.

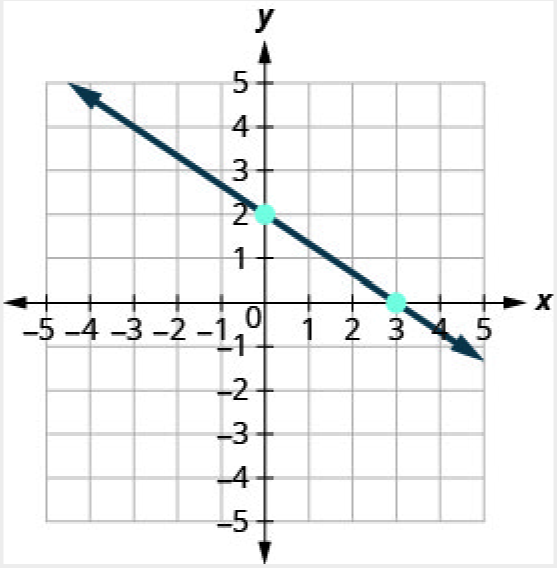


Identify the rise and the run.	$m = -\frac{2}{3}$
	$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$
	$\text{rise} = -2$
	$\text{run} = 3$

Count the rise and the run. Mark the second point.



Connect the two points with a line.



You can check your work by finding a third point. Since the slope is $m=-\frac{2}{3}$, it can be written as $m=\frac{2}{-3}$. Go back to $(0,2)$ and count out the rise, 2, and the run, -3.

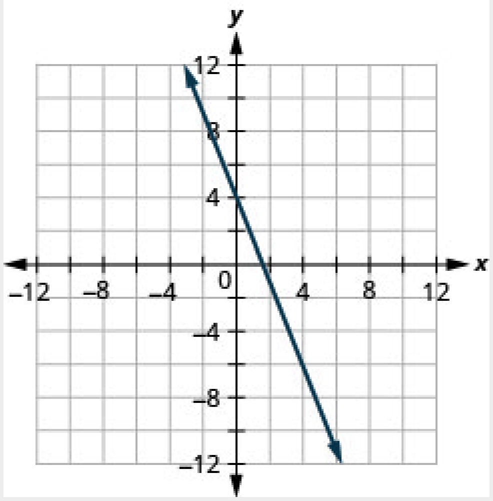
Note:

Exercise:

TRY IT :: 4.71

Problem: Graph the line with the *y*-intercept 4 and slope $m = -\frac{5}{2}$.

Solution:

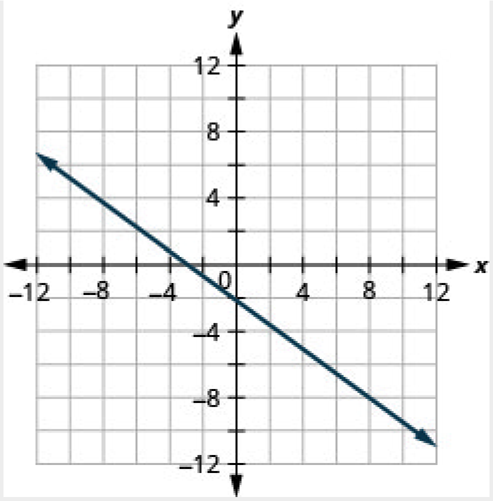


Note:
Exercise:

TRY IT :: 4.72

Problem: Graph the line with the *x*-intercept -3 and slope $m = -\frac{3}{4}$.

Solution:



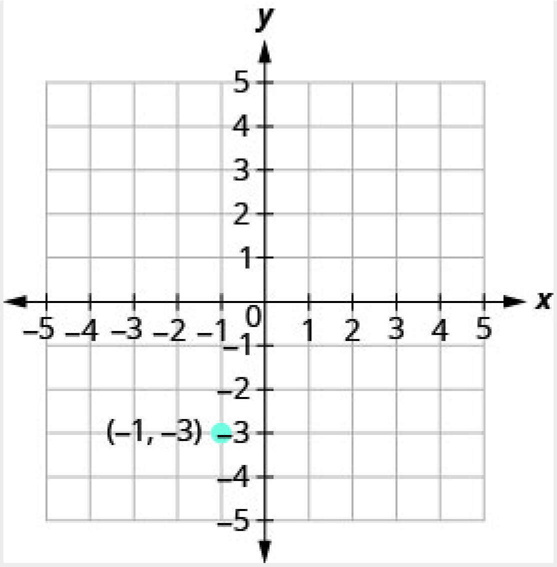
Example:
Exercise:

Problem:

**Graph the line passing through the point $(-1, -3)$ whose slope is $m = 4$.

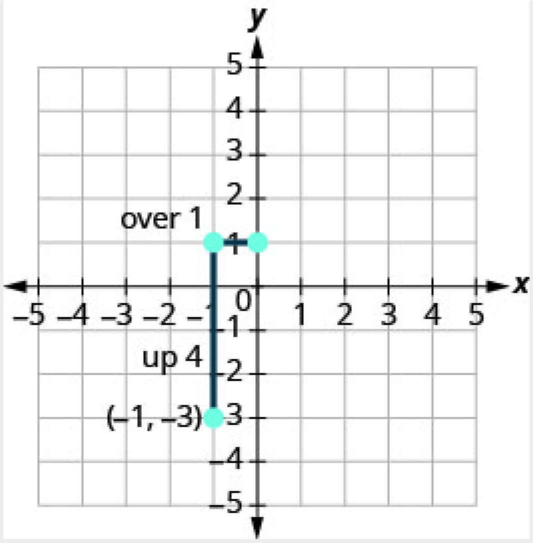
Solution:
Solution

Plot the given point.

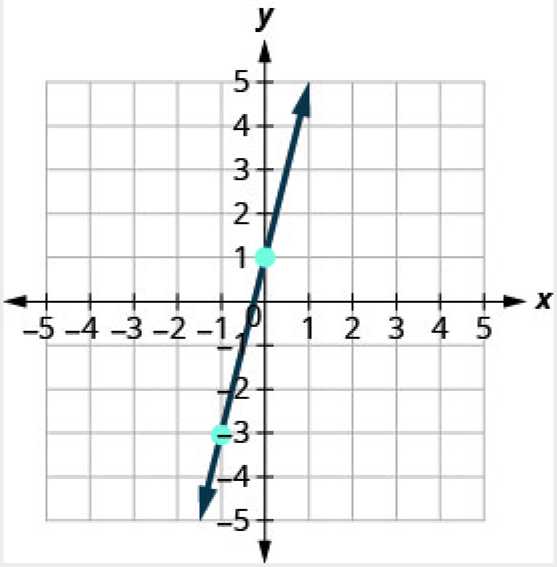


Identify the rise and the run.	$m = 4$
Write 4 as a fraction.	$\frac{\text{rise}}{\text{run}} = \frac{4}{1}$
	$\text{rise} = 4, \text{run} = 1$

Count the rise and run and mark the second point.



Connect the two points with a line.



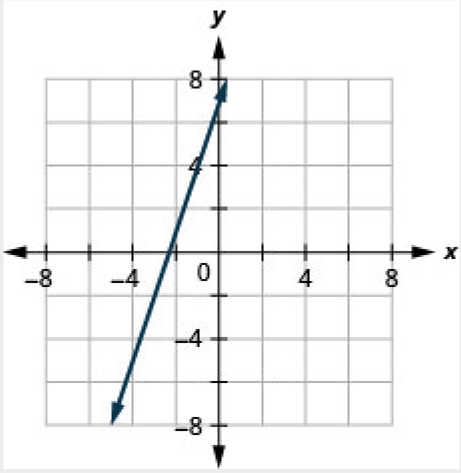
You can check your work by finding a third point. Since the slope is $m = 4$, it can be written as $m = \frac{-4}{-1}$. Go back to $(-1, -3)$ and count out the rise, -4, and the run, -1.

Note:
Exercise:

TRY IT :: 4.73

Problem: Graph the line with the point $(-2, 1)$ and slope $m = 3$.

Solution:

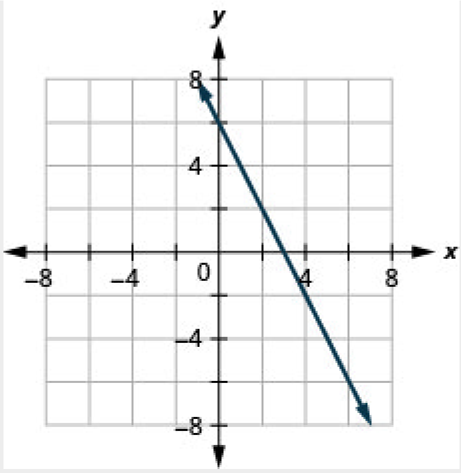


Note:
Exercise:

TRY IT :: 4.74

Problem: Graph the line with the point $(4, -2)$ and slope $m = -2$.

Solution:



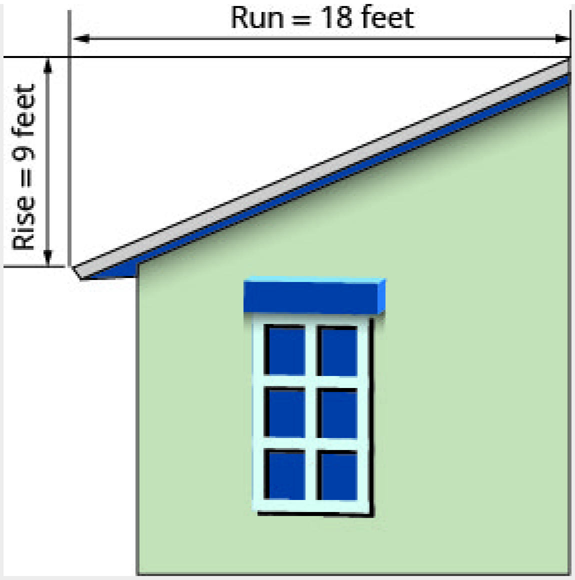
Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let’s look at a few now.

Example:
Exercise:

Problem:

**The ‘pitch’ of a building’s roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Solution:
Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values for rise and run.	$m = \frac{9}{18}$
Simplify.	$m = \frac{1}{2}$
The slope of the roof is $\frac{1}{2}$.	
	The roof rises 1 foot for every 2 feet of horizontal run.

Note:
Exercise:

TRY IT :: 4.75

Problem: Use [link], substituting the rise = 14 and run = 24.

Solution:

$\frac{7}{12}$

Note:
Exercise:

TRY IT :: 4.76

Problem: Use [link], substituting rise = 15 and run = 36.

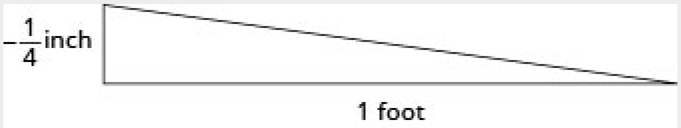
Solution:

$\frac{5}{12}$

Example:
Exercise:

Problem:

**Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Solution:
Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{-\frac{1}{4}\text{inch}}{1\text{ foot}}$ $m = \frac{-\frac{1}{4}\text{inch}}{12\text{ inches}}$
Simplify.	$m = -\frac{1}{48}$
	The slope of the pipe is $-\frac{1}{48}$.

The pipe drops 1 inch for every 48 inches of horizontal run.

Note:
Exercise:

TRY IT :: 4.77

Problem: Find the slope of a pipe that slopes down $\frac{1}{3}$ inch per foot.

Solution:

$-\frac{1}{36}$

Note:

Exercise:

TRY IT :: 4.78

Problem: Find the slope of a pipe that slopes down $\frac{3}{4}$ inch per yard.

Solution:

$-\frac{1}{48}$

Note:
Access these online resources for additional instruction and practice with understanding slope of a line.

- Practice Slope with a Virtual Geoboard
- Small, Medium, and Large Virtual Geoboards
- Explore Area and Perimeter with a Geoboard

Key Concepts

- **Find the Slope of a Line from its Graph using** $m = \frac{\text{rise}}{\text{run}}$

  Locate two points on the line whose coordinates are integers.
  Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
  Count the rise and the run on the legs of the triangle.
  Take the ratio of rise to run to find the slope.

- **Graph a Line Given a Point and the Slope**

  Plot the given point.

Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
Starting at the given point, count out the rise and run to mark the second point.
Connect the points with a line.

- **Slope of a Horizontal Line**
  - The slope of a horizontal line, $y = b$, is 0.
- **Slope of a vertical line**
  - The slope of a vertical line, $x = a$, is undefined

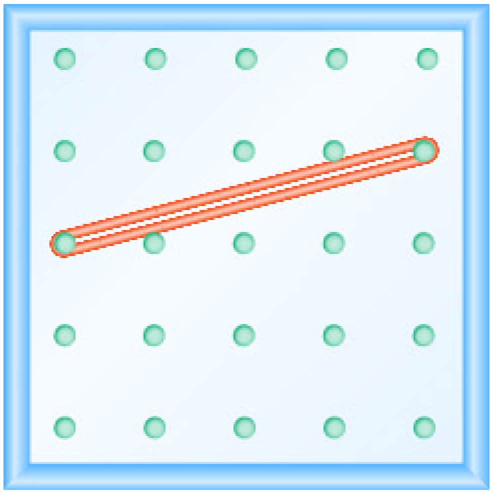
Practice Makes Perfect

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

Exercise:

Problem:

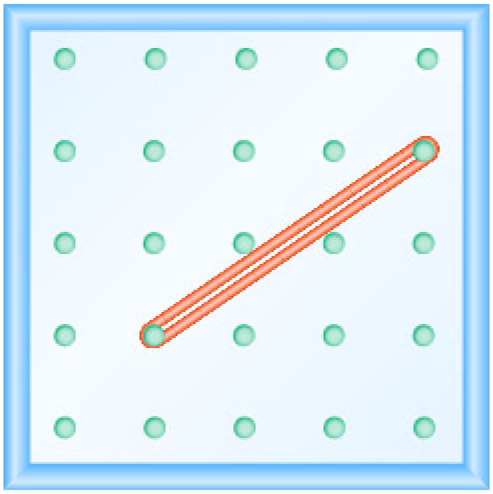


Solution:

$\frac{1}{4}$

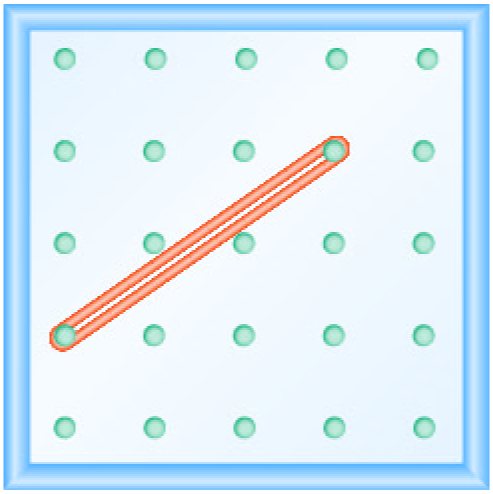
Exercise:

Problem:



Exercise:

Problem:

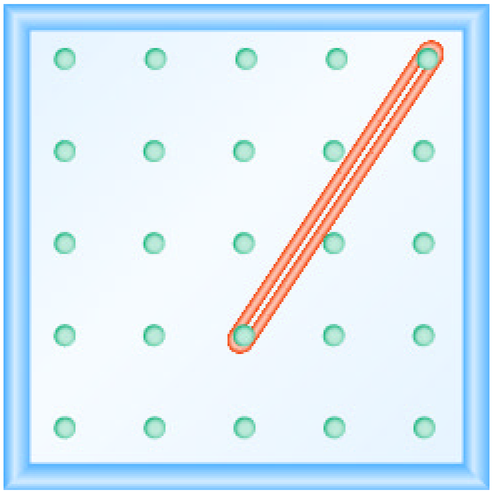


Solution:

$\frac{2}{3}$

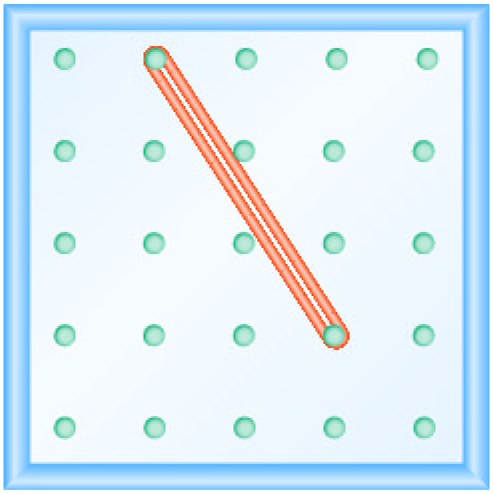
Exercise:

Problem:



Exercise:

Problem:

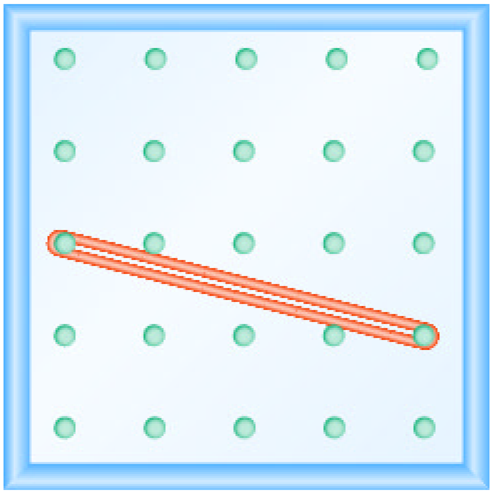


Solution:

$\frac{-3}{2} = -\frac{3}{2}$

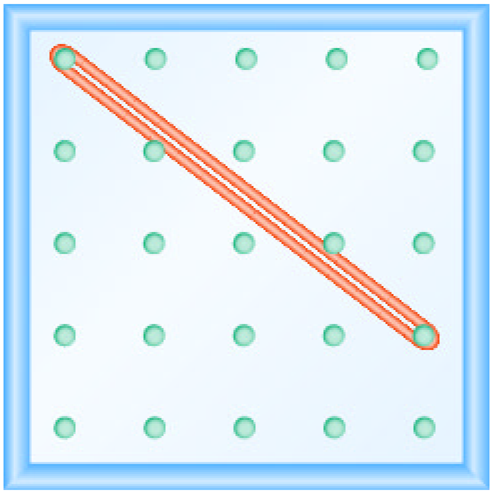
Exercise:

Problem:



Exercise:

Problem:

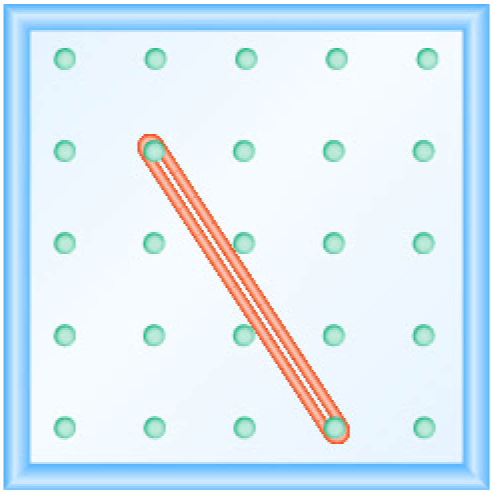


Solution:

$-\frac{2}{3}$

Exercise:

Problem:

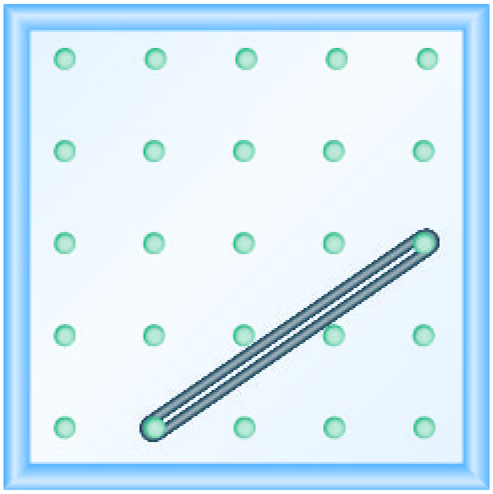


In the following exercises, model each slope. Draw a picture to show your results.

Exercise:

Problem: $\frac{2}{3}$

Solution:



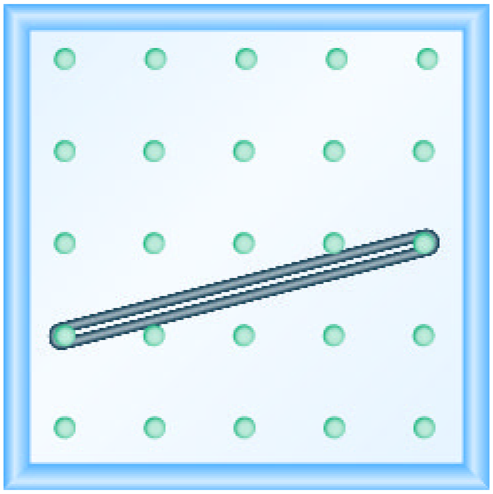
Exercise:

Problem: $\frac{3}{4}$

Exercise:

Problem: $\frac{1}{4}$

Solution:



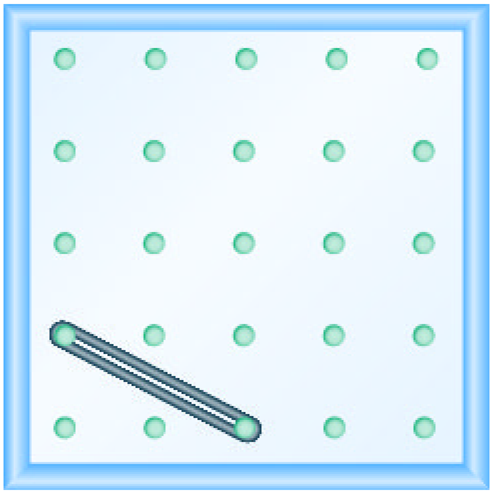
Exercise:

Problem: $\frac{4}{3}$

Exercise:

Problem: $-\frac{1}{2}$

Solution:



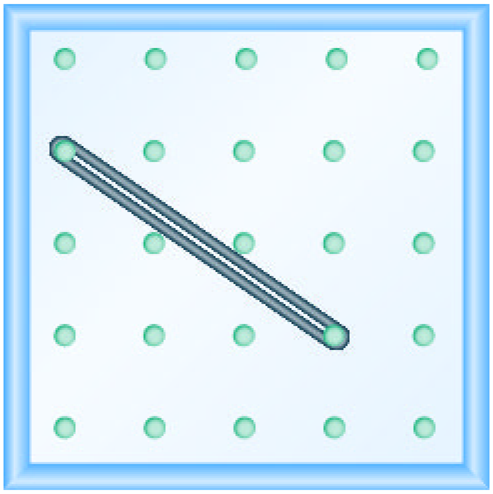
Exercise:

Problem: $-\frac{3}{4}$

Exercise:

Problem: $-\frac{2}{3}$

Solution:



Exercise:

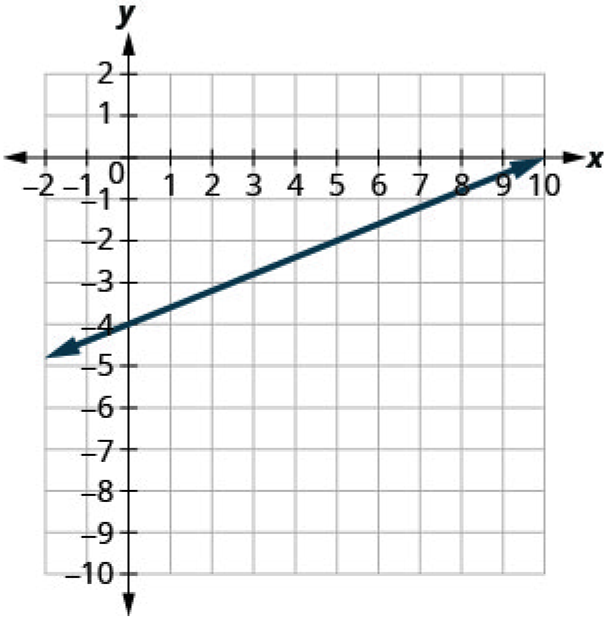
Problem: $-\frac{3}{2}$

Use $m = \frac{\text{rise}}{\text{run}}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

Exercise:

Problem:

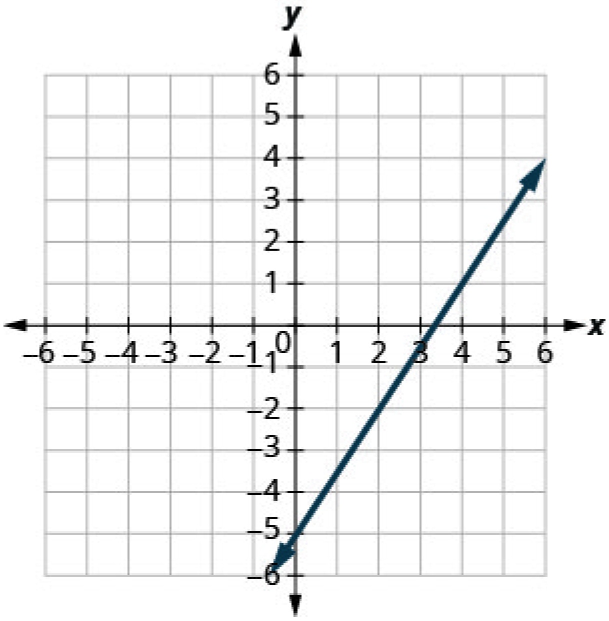


Solution:

$\frac{2}{5}$

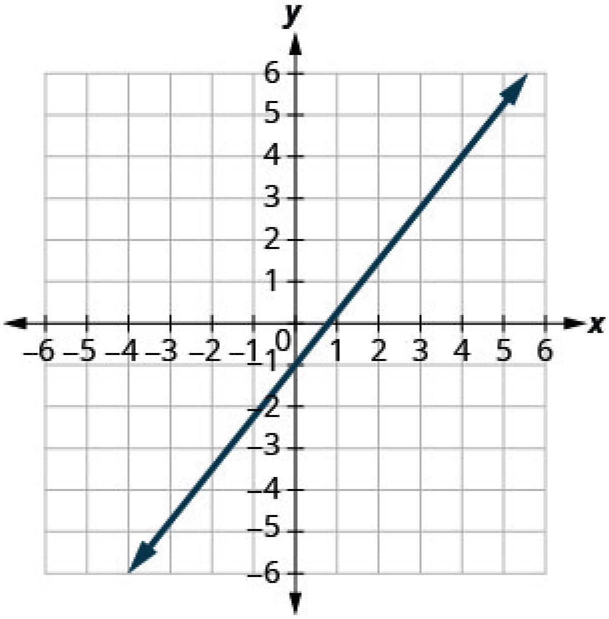
Exercise:

Problem:



Exercise:

Problem:

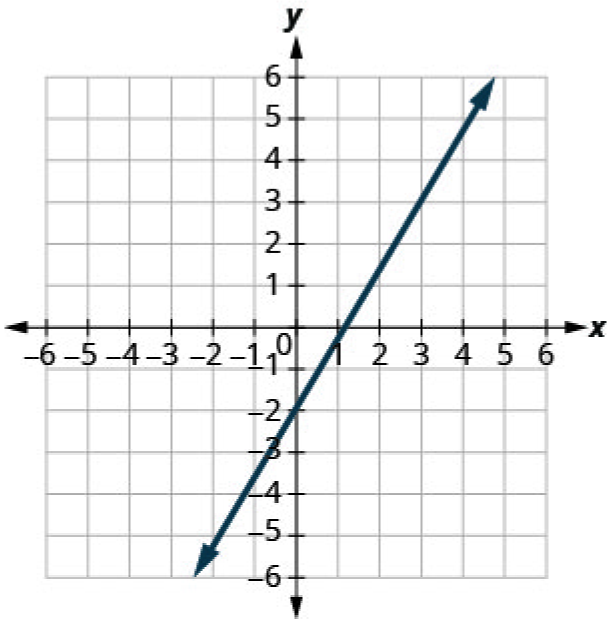


Solution:

$\frac{5}{4}$

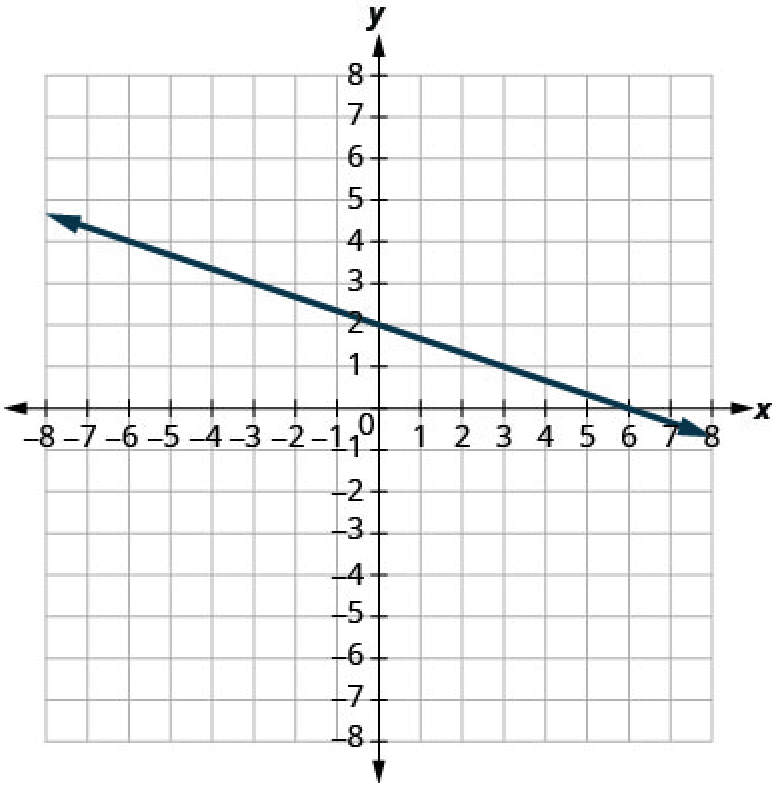
Exercise:

Problem:



Exercise:

Problem:

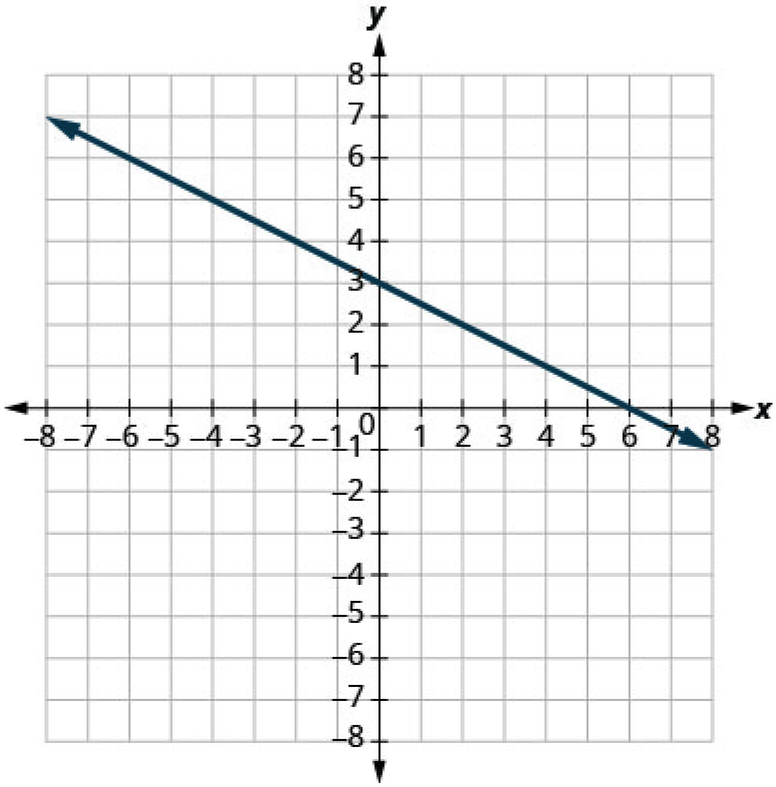


Solution:

$-\frac{1}{3}$

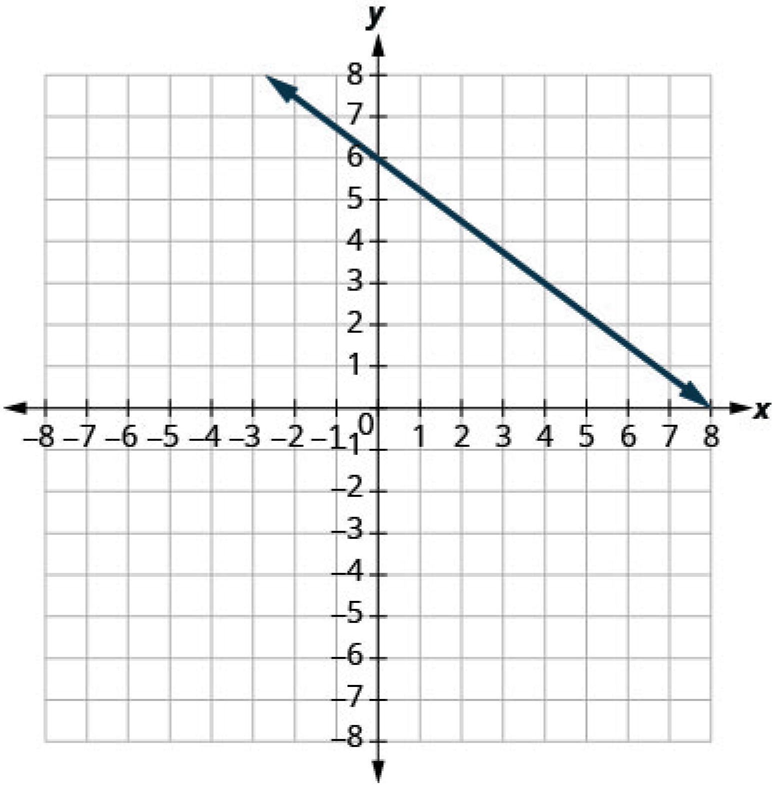
Exercise:

Problem:



Exercise:

Problem:



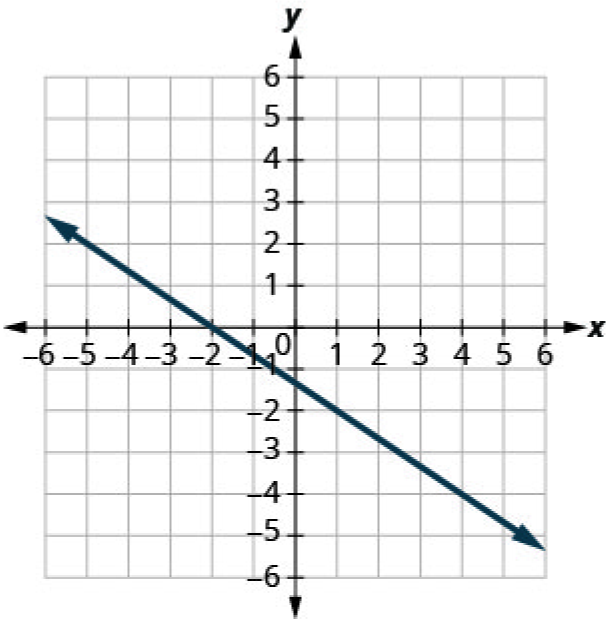
Solution:

$-\frac{3}{4}$

Exercise:

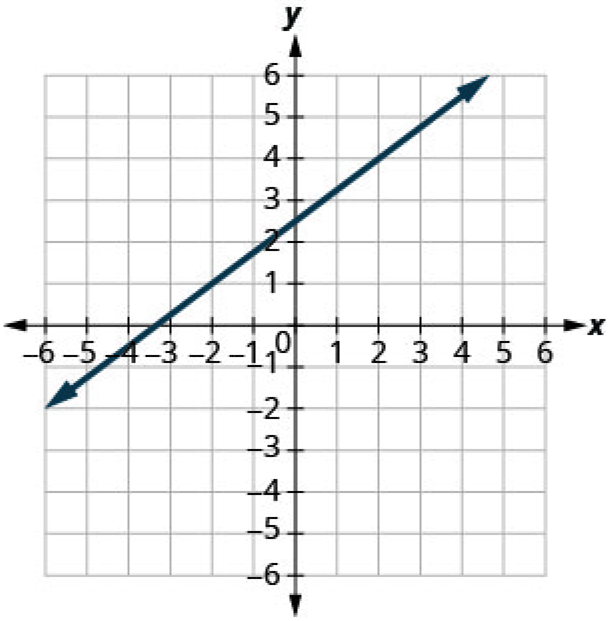
Problem:

$\frac{3}{4}$



Exercise:

Problem:

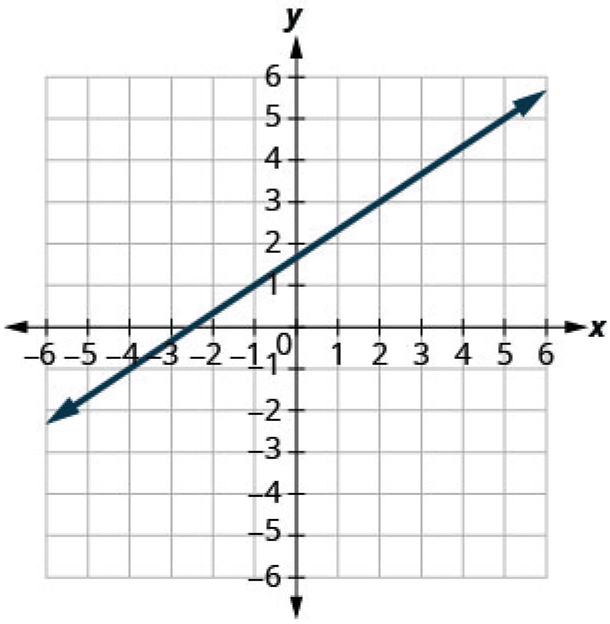


Solution:

$\frac{3}{4}$

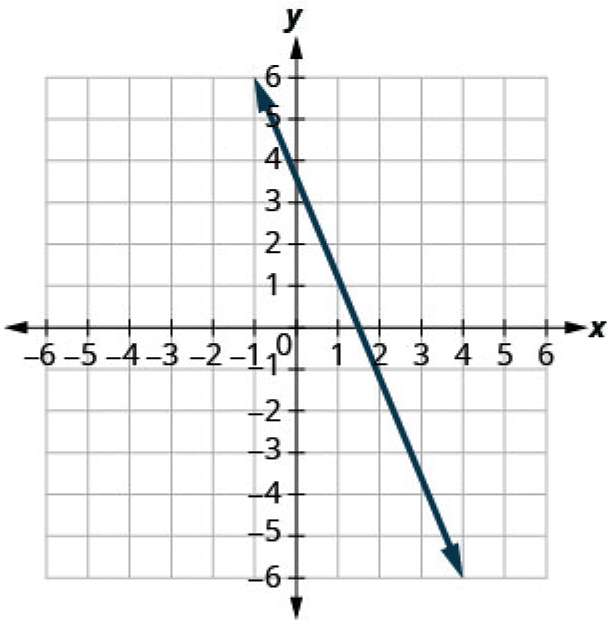
Exercise:

Problem:



Exercise:

Problem:

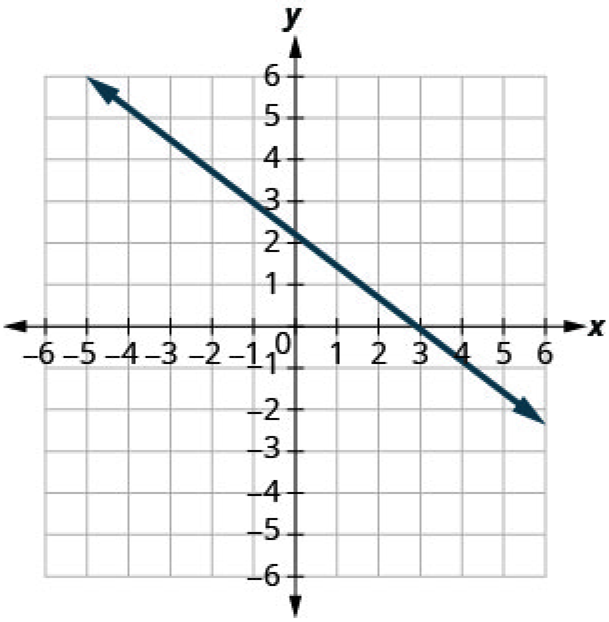


Solution:

$-\frac{5}{2}$

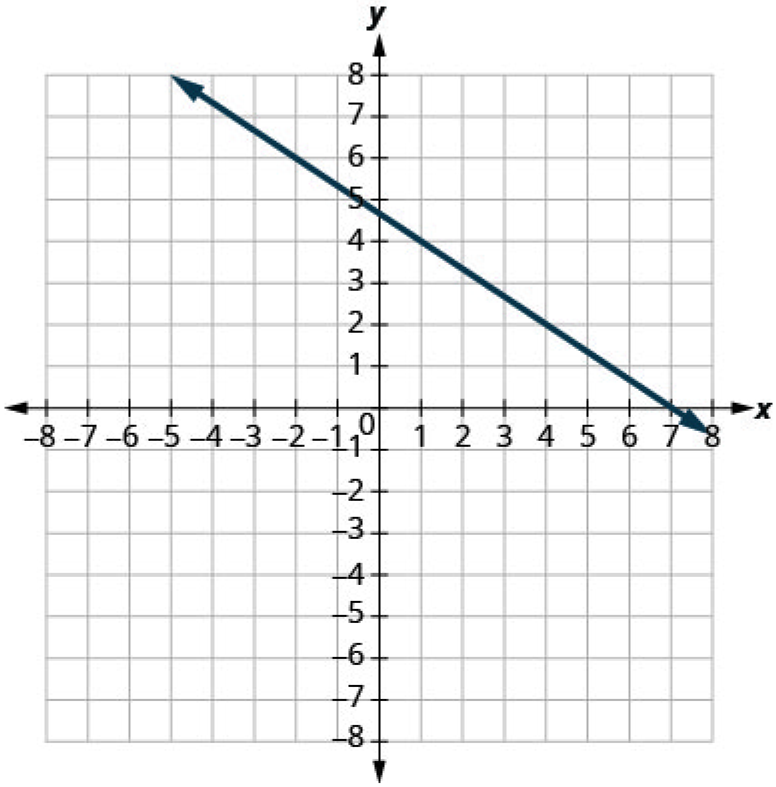
Exercise:

Problem:



Exercise:

Problem:

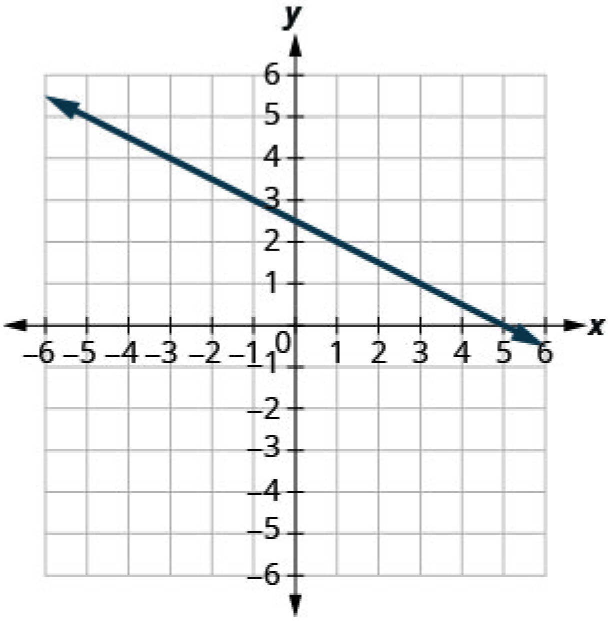


Solution:

$-\frac{2}{3}$

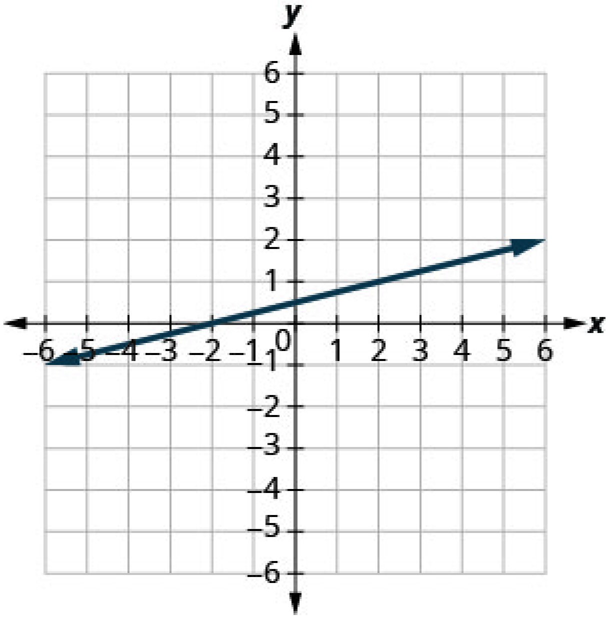
Exercise:

Problem:



Exercise:

Problem:

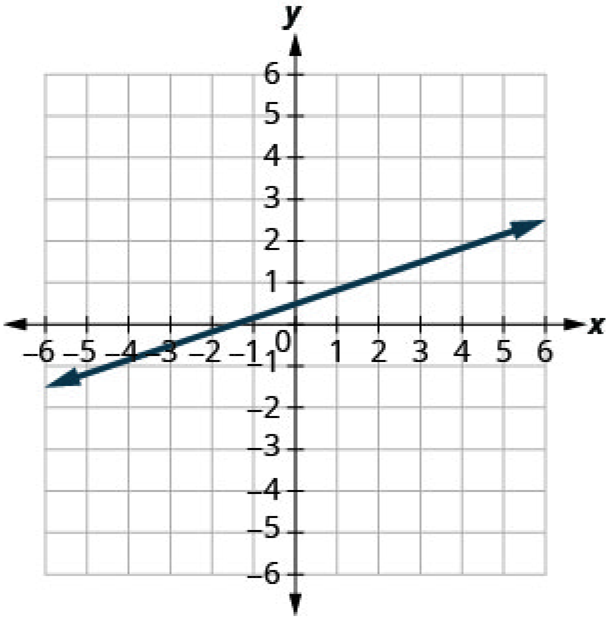


Solution:

$\frac{1}{4}$

Exercise:

Problem:



Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise:

Problem: $y = 3$

Solution:

0

Exercise:

Problem: $y = 1$

Exercise:

Problem: $x = 4$

Solution:

undefined

Exercise:

Problem: $x = 2$

Exercise:

Problem: $y = -2$

Solution:

0

Exercise:

Problem: $y = -3$

Exercise:

Problem: $x = -5$

Solution:

undefined

Exercise:

Problem: $x = -4$

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise:

Problem: $(1,4),(3,9)$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $(2,3),(5,7)$

Exercise:

Problem: $(0,3),(4,6)$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $(0,1),(5,4)$

Exercise:

Problem: $(2,5),(4,0)$

Solution:

$-\frac{5}{2}$

Exercise:

Problem: $(3,6),(8,0)$

Exercise:

Problem: $(-3,3),(4,-5)$

Solution:

$-\frac{8}{7}$

Exercise:

Problem: $(-2,4),(3,-1)$

Exercise:

Problem: $(-1,-2),(2,5)$

Solution:

$\frac{7}{3}$

Exercise:

Problem: $(-2,-1),(6,5)$

Exercise:

Problem: $(4,-5),(1,-2)$

Solution:

-1

Exercise:

Problem: $(3,-6),(2,-2)$

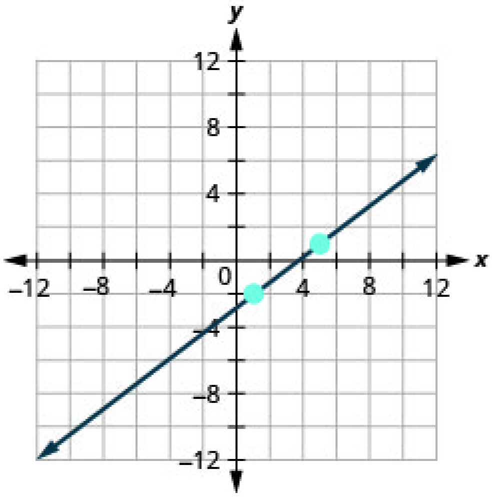
Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

Exercise:

Problem: $(1,-2); m=\frac{3}{4}$

Solution:



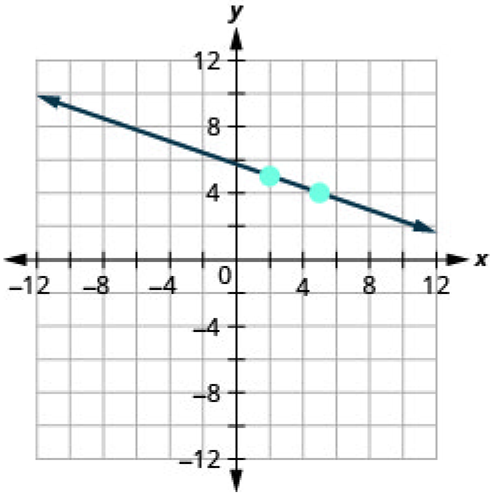
Exercise:

Problem: $(1,-1); m=\frac{2}{3}$

Exercise:

Problem: $(2,5); m=-\frac{1}{3}$

Solution:



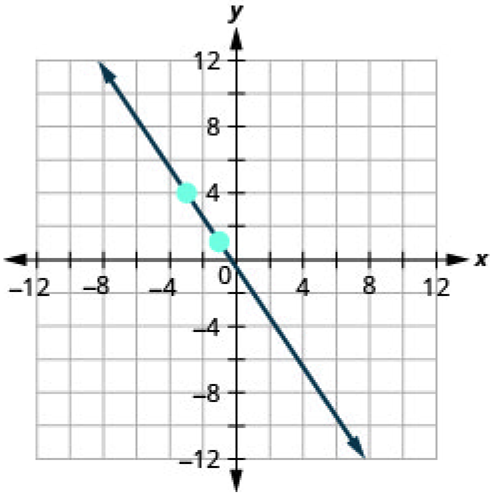
Exercise:

Problem: $(1, 4); m = -\frac{1}{2}$

Exercise:

Problem: $(-3, 4); m = -\frac{3}{2}$

Solution:



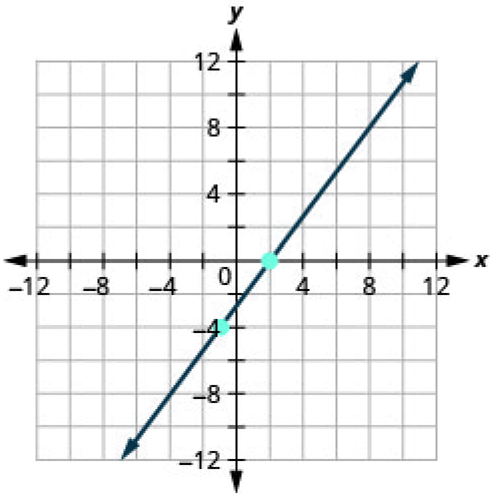
Exercise:

Problem: $(-2, 5); m = -\frac{5}{4}$

Exercise:

Problem: $(-1, -4); m = \frac{4}{3}$

Solution:



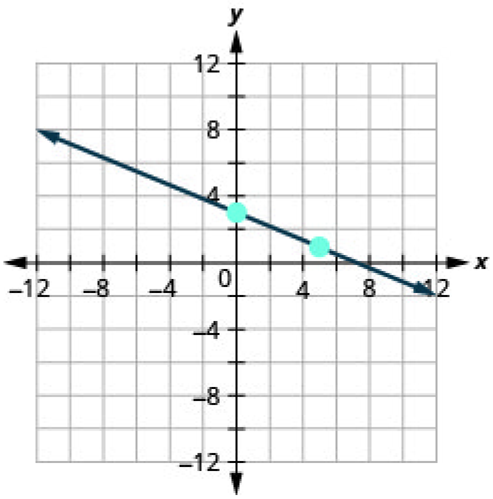
Exercise:

Problem: $(-3, -5); m = \frac{3}{2}$

Exercise:

Problem: *y*-intercept 3; $m = -\frac{2}{5}$

Solution:



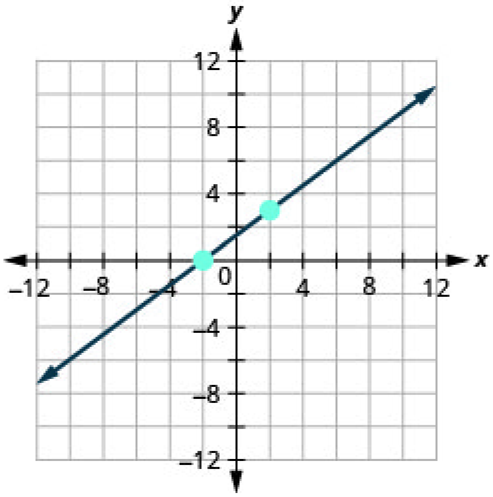
Exercise:

Problem: y-intercept 5; $m = -\frac{4}{3}$

Exercise:

Problem: x-intercept -2; $m = \frac{3}{4}$

Solution:



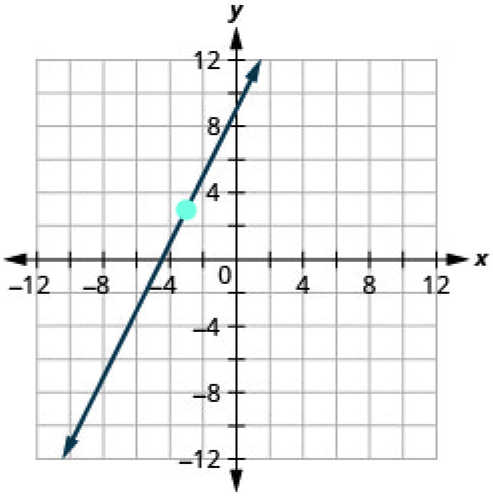
Exercise:

Problem: x-intercept -1; $m = \frac{1}{5}$

Exercise:

Problem: $(-3, 3)$; $m = 2$

Solution:



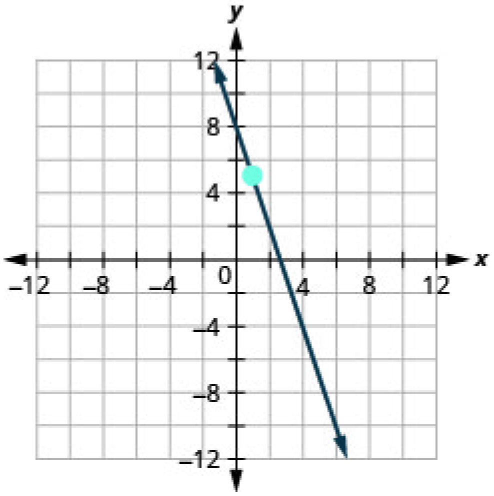
Exercise:

Problem: $(-4, 2)$; $m = 4$

Exercise:

Problem: $(1, 5)$; $m = -3$

Solution:



Exercise:

Problem: $(2,3)$; $m=-1$

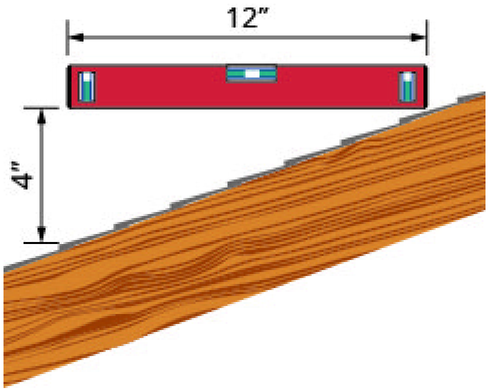
Everyday Math

Exercise:

Problem:

Slope of a roof. An easy way to determine the slope of a roof is to set one end of a 12 inch level on the roof surface and hold it level. Then take a tape measure or ruler and measure from the other end of the level down to the roof surface. This will give you the slope of the roof. Builders, sometimes, refer to this as pitch and state it as an “x 12 pitch” meaning $\frac{x}{12}$, where x is the measurement from the roof to the level—the rise. It is also sometimes stated as an “x-in-12 pitch”.

ⓐ What is the slope of the roof in this picture?
ⓑ What is the pitch in construction terms?



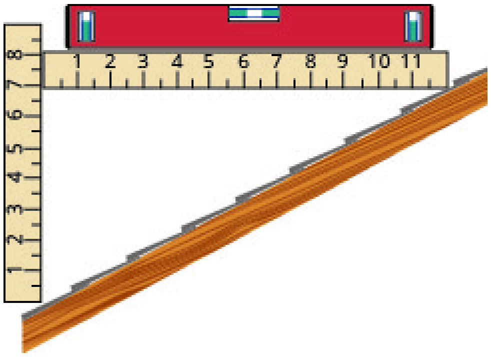
Solution:

ⓐ $\frac{1}{3}$ ⓑ 4 12 pitch or 4-in-12 pitch

Exercise:

Problem:

The slope of the roof shown here is measured with a 12” level and a ruler. What is the slope of this roof?



Exercise:

Problem:

Road grade. A local road has a grade of 6%. The grade of a road is its slope expressed as a percent. Find the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

Solution:

$\frac{3}{50}$; rise = 3, run = 50

Exercise:

Problem:

Highway grade. A local road rises 2 feet for every 50 feet of highway.

ⓐ What is the slope of the highway?
ⓑ The grade of a highway is its slope expressed as a percent. What is the grade of this highway?

Exercise:

Problem:

Wheelchair ramp. The rules for wheelchair ramps require a maximum 1-inch rise for a 12-inch run.

ⓐ How long must the ramp be to accommodate a 24-inch rise to the door?
ⓑ Create a model of this ramp.

Solution:

ⓐ 288 inches (24 feet) ⓑ Models will vary.

Exercise:

Problem:

Wheelchair ramp. A 1-inch rise for a 16-inch run makes it easier for the wheelchair rider to ascend a ramp.

ⓐ How long must a ramp be to easily accommodate a 24-inch rise to the door?
ⓑ Create a model of this ramp.

Writing Exercises

Exercise:

Problem: What does the sign of the slope tell you about a line?

Solution:

When the slope is a positive number the line goes up from left to right. When the slope is a negative number the line goes down from left to right.

Exercise:

Problem:

How does the graph of a line with slope $m = \frac{1}{2}$ differ from the graph of a line with slope $m = 2$?

Exercise:

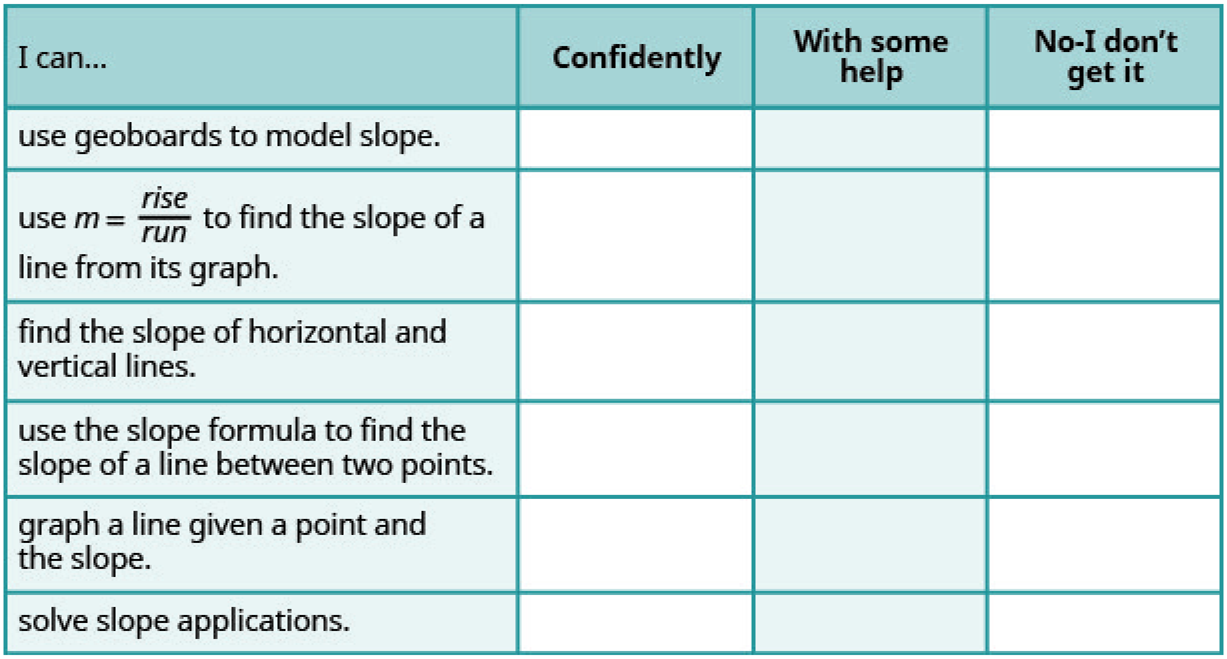
Problem: Why is the slope of a vertical line “undefined”?

Solution:

A vertical line has 0 run and since division by 0 is undefined the slope is undefined.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it
use geoboards to model slope.			
use $m=\frac{rise}{run}$ to find the slope of a line from its graph.			
find the slope of horizontal and vertical lines.			
use the slope formula to find the slope of a line between two points.			
graph a line given a point and the slope.			
solve slope applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

geoboard

A geoboard is a board with a grid of pegs on it.

negative slope

A negative slope of a line goes down as you read from left to right.

positive slope

A positive slope of a line goes up as you read from left to right.

rise

The rise of a line is its vertical change.

run

The run of a line is its horizontal change.

slope formula

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of a line

The slope of a line is $m = \frac{\text{rise}}{\text{run}}$. The rise measures the vertical change and the run measures the horizontal change.

Use the Slope–Intercept Form of an Equation of a Line
By the end of this section, you will be able to:

- Recognize the relation between the graph and the slope–intercept form of an equation of a line
- Identify the slope and y-intercept form of an equation of a line
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel lines
- Use slopes to identify perpendicular lines

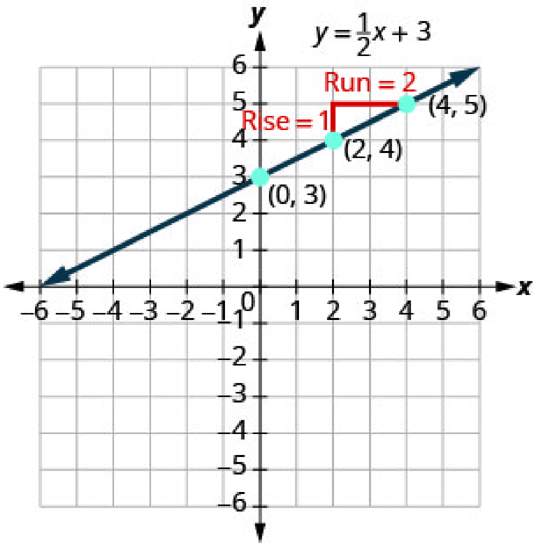
Note:
Before you get started, take this readiness quiz.

1. Add: $\frac{x}{4} + \frac{1}{4}$.
   If you missed this problem, review [link].
2. Find the reciprocal of $\frac{3}{7}$.
   If you missed this problem, review [link].
3. Solve $2x - 3y = 12$ for y.
   If you missed this problem, review [link].

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point–slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method we can use to graph lines.

In Graph Linear Equations in Two Variables, we graphed the line of the equation $y = \frac{1}{2}x + 3$ by plotting points. See [link]. Let's find the slope of this line.



The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

Equation:

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ m &= \frac{1}{2} \end{aligned}$$

What is the *y*-intercept of the line? The *y*-intercept is where the line crosses the *y*-axis, so *y*-intercept is $(0, 3)$. The equation of this line is:

$$y = \frac{1}{2}x + 3$$

Notice, the line has:

slope $m = \frac{1}{2}$ and *y*-intercept (0, 3)

When a linear equation is solved for y, the coefficient of the x term is the slope and the constant term is the *y*-coordinate of the *y*-intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

$m = \frac{1}{2}$; *y*-intercept is (0, 3)

$$y = \frac{1}{2}x + 3$$

$$y = mx + b$$

Note:
Slope-Intercept Form of an Equation of a Line
The **slope–intercept form** of an equation of a line with slope m and y-intercept, $(0, b)$ is,
Equation:

$$y = mx + b$$

Sometimes the slope–intercept form is called the “y-form.”

Example:
Exercise:

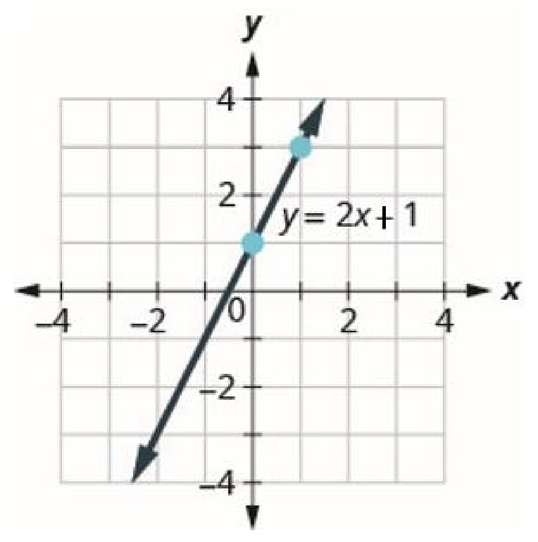
Problem:

**Use the graph to find the slope and y-intercept of the line, $y = 2x + 1$.

Compare these values to the equation$y = mx + b$.

Solution:
Solution

To find the slope of the line, we need to choose two points on the line. We’ll use the points $(0, 1)$ and $(1, 3)$.

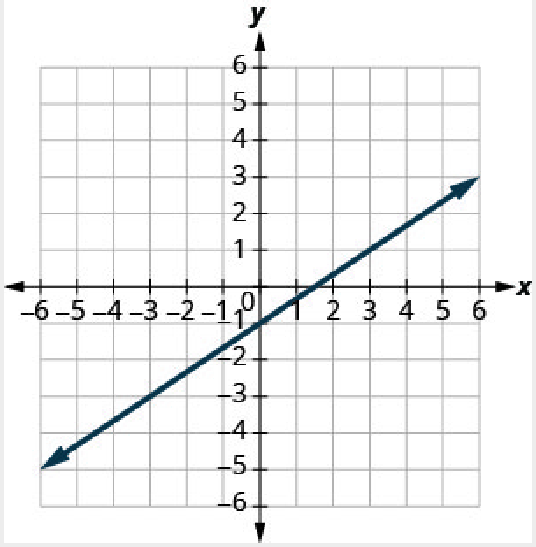
	
Find the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{1}$
	$m = 2$
Find the *y*-intercept of the line.	The *y*-intercept is the point (0, 1).
We found slope $m = 2$ and *y*-intercept (0, 1).	$y = 2x + 1$ $y = mx + b$

The slope is the same as the coefficient of x and the *y*-coordinate of the *y*-intercept is the same as the constant term.

Note:
Exercise:

Problem:

TRY IT :: 4.79
Use the graph to find the slope and y-intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.



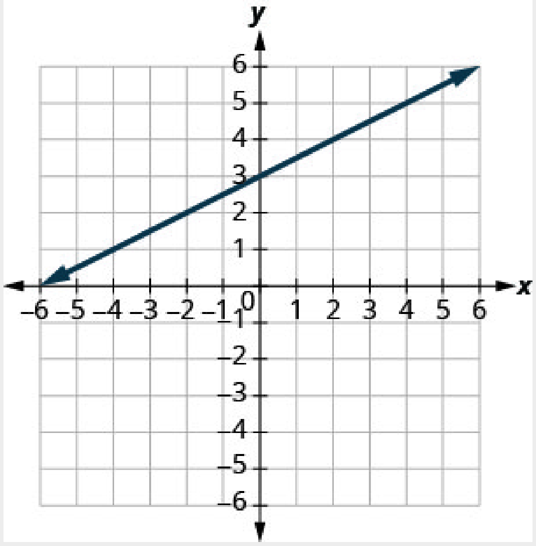
Solution:

slope $m = \frac{2}{3}$ and y-intercept $(0, -1)$

Note:
Exercise:

Problem:

TRY IT :: 4.80
Use the graph to find the slope and y-intercept of the line $y = \frac{1}{2}x + 3$. Compare these values to the equation $y = mx + b$.



Solution:

slope $m = \frac{1}{2}$ and *y*-intercept $(0, 3)$

Identify the Slope and *y*-Intercept From an Equation of a Line

In Understand Slope of a Line, we graphed a line using the slope and a point. When we are given an equation in slope–intercept form, we can use the *y*-intercept as the point, and then count out the slope from there. Let’s practice finding the values of the slope and *y*-intercept from the equation of a line.

Example:
Exercise:

Problem:

**Identify the slope and *y*-intercept of the line with equation $y = -3x + 5$.

Solution:
Solution

We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the *y*-intercept.	*y*-intercept is $(0, 5)$

Note:
Exercise:

TRY IT :: 4.81

Problem: Identify the slope and *y*-intercept of the line $y = \frac{2}{5}x - 1$.

Solution:

$\frac{2}{5}; (0, -1)$

Note:
Exercise:

TRY IT :: 4.82

Problem: Identify the slope and *y*-intercept of the line $y = -\frac{4}{3}x + 1$.

Solution:

$-\frac{4}{3}; (0, 1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y.

Example:
Exercise:

Problem:

**Identify the slope and *y*-intercept of the line with equation $x + 2y = 6$.

Solution:
Solution

This equation is not in slope–intercept form. In order to compare it to the slope–intercept form we must first solve the equation for y.

Solve for *y*.	$x + 2y = 6$
Subtract *x* from each side.	$2y = -x + 6$
Divide both sides by 2.	$\frac{2y}{2} = \frac{-x+6}{2}$
Simplify.	$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$
$\left(\text{Remember: } \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\right)$	
Simplify.	$y = -\frac{1}{2}x + 3$

Write the slope–intercept form of the equation of the line.	$y = mx + b$
Write the equation of the line.	$y = -\frac{1}{2}x + 3$
Identify the slope.	$m = -\frac{1}{2}$
Identify the *y*-intercept.	*y*-intercept is (0, 3)

Note:
Exercise:

TRY IT :: 4.83

Problem: Identify the slope and *y*-intercept of the line $x + 4y = 8$.

Solution:

$-\frac{1}{4}; (0, 2)$

Note:
Exercise:

TRY IT :: 4.84

Problem: Identify the slope and *y*-intercept of the line $3x + 2y = 12$.

Solution:

$-\frac{3}{2}; (0, 6)$

Graph a Line Using its Slope and Intercept

Now that we know how to find the slope and *y*-intercept of a line from its equation, we can graph the line by plotting the *y*-intercept and then using the slope to find another point.

Example:
How to Graph a Line Using its Slope and Intercept
Exercise:

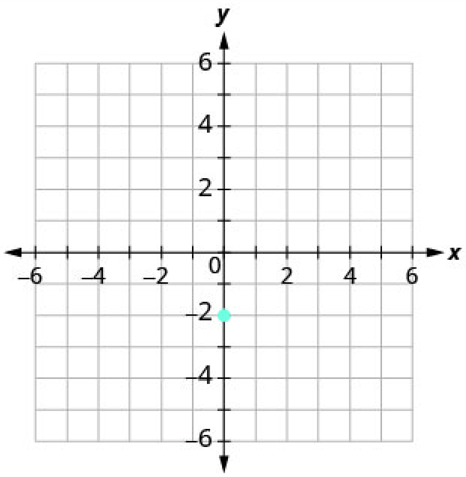
Problem:

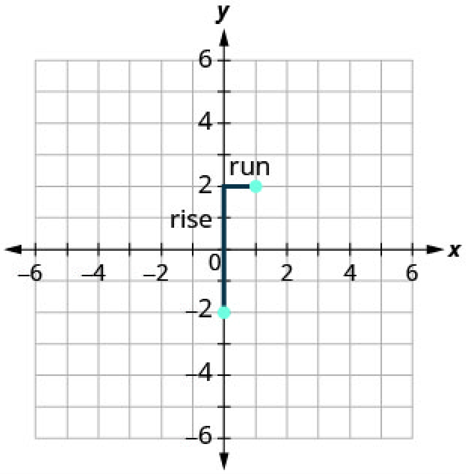
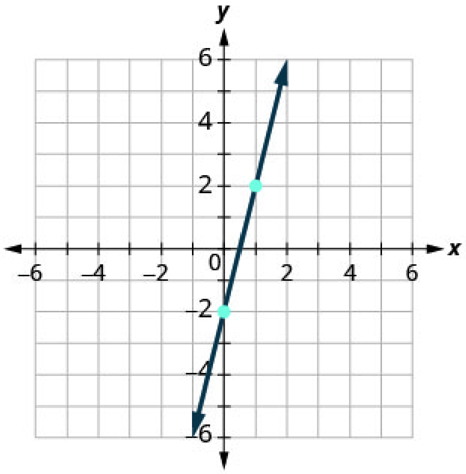
**Graph the line of the equation $y = 4x - 2$ using its slope and *y*-intercept.

Solution:
Solution

Step 1. Find the slope–intercept form of the equation.	This equation is in slope–intercept form.	$y = 4x - 2$

Step 2. Identify the slope and *y*-intercept.	Use $y = mx + b$ Find the slope. Find the *y*-intercept.	$y = mx + b$ $y = 4x + (-2)$ $m = 4$ $b = -2$, $(0, -2)$

Step 3. Plot the *y*-intercept.	Plot (0, –2).	

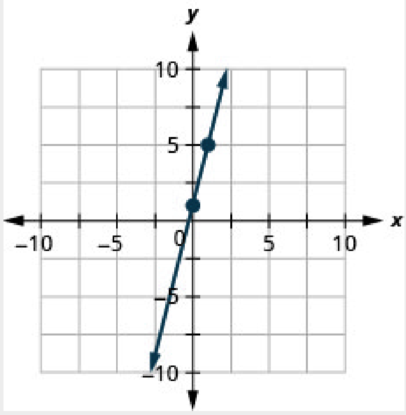
Step 4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = 4$ $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$ $\text{rise} = 4$ $\text{run} = 1$
Step 5. Starting at the *y*-intercept, count out the rise and run to mark the second point.	Start at (0, –2) and count the rise and the run. Up 4, right 1.	
Step 6. Connect the points with a line.	Connect the two points with a line.	

Note:
Exercise:

Problem:

TRY IT :: 4.85
Graph the line of the equation $y = 4x + 1$ using its slope and *y*-intercept.

Solution:

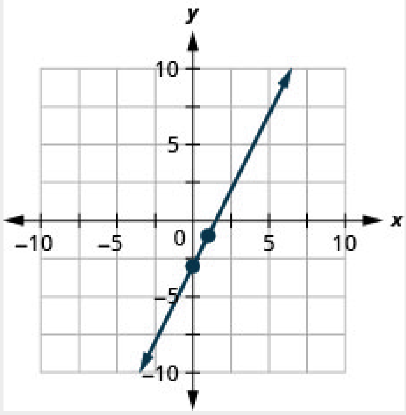


Note:
Exercise:

Problem:

TRY IT :: 4.86
Graph the line of the equation $y = 2x - 3$ using its slope and *y*-intercept.

Solution:



Note:
Graph a line using its slope and *y*-intercept.

Find the slope-intercept form of the equation of the line.
Identify the slope and*y*-intercept.
Plot the*y*-intercept.
Use the slope formula$m = \frac{\text{rise}}{\text{run}}$to identify the rise and the run.
Starting at the*y*-intercept, count out the rise and run to mark the second point.
Connect the points with a line.

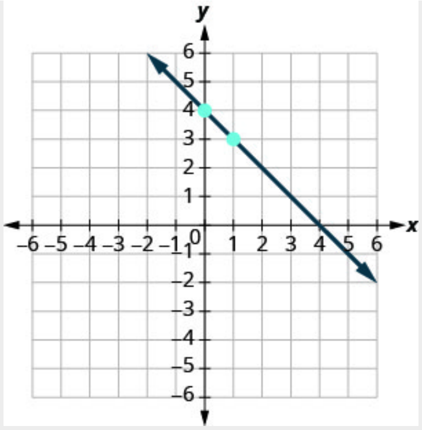
Example:
Exercise:

Problem:

**Graph the line of the equation $y = -x + 4$ using its slope and *y*-intercept.

Solution:
Solution

	$y = mx + b$
The equation is in slope–intercept form.	$y = -x + 4$
Identify the slope and *y*-intercept.	$m = -1$
	y-intercept is (0, 4)
Plot the *y*-intercept.	See graph below.
Identify the rise and the run.	$m = \frac{-1}{1}$
Count out the rise and run to mark the second point.	rise −1, run 1

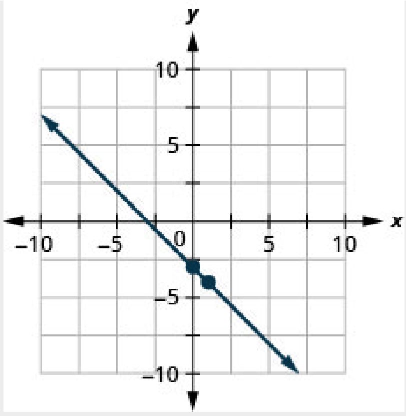
Draw the line.	
To check your work, you can find another point on the line and make sure it is a solution of the equation. In the graph we see the line goes through (4, 0).	
Check. $\begin{array}{rcl} y & = & -x+4 \\ 0 & \overset{?}{=} & -4+4 \\ 0 & = & 0\checkmark \end{array}$	

Note:
Exercise:

Problem:

TRY IT :: 4.87
Graph the line of the equation $y = -x - 3$ using its slope and *y*-intercept.

Solution:

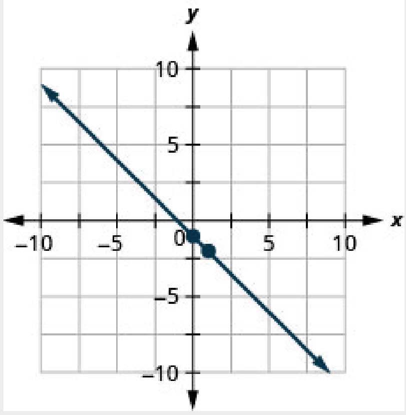


Note:
Exercise:

Problem:

TRY IT :: 4.88
Graph the line of the equation $y = -x - 1$ using its slope and y-intercept.

Solution:

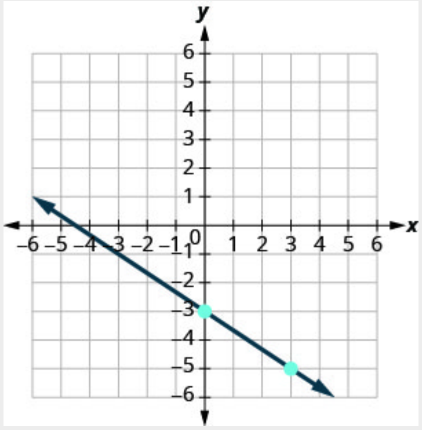


Example:
Exercise:

Problem:

**Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and *y*-intercept.

Solution:
Solution

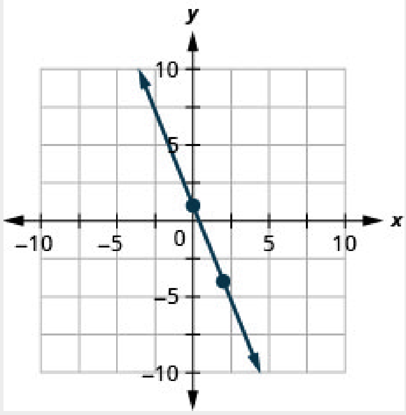
	$y = mx + b$
The equation is in slope–intercept form.	$y = -\frac{2}{3}x - 3$
Identify the slope and *y*-intercept.	$m = -\frac{2}{3}$; *y*-intercept is (0, −3)
Plot the *y*-intercept.	See graph below.
Identify the rise and the run.	
Count out the rise and run to mark the second point.	
Draw the line.	

Note:
Exercise:

Problem:

TRY IT :: 4.89
Graph the line of the equation $y = -\frac{5}{2}x + 1$ using its slope and *y*-intercept.

Solution:

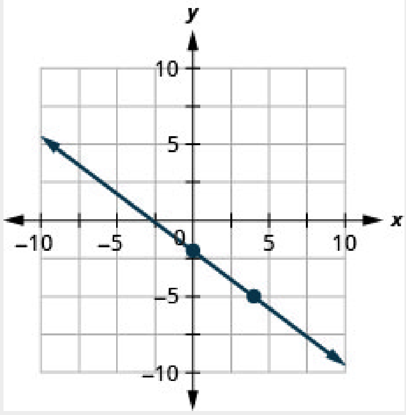


Note:
Exercise:

Problem:

TRY IT :: 4.90
Graph the line of the equation $y = -\frac{3}{4}x - 2$ using its slope and *y*-intercept.

Solution:



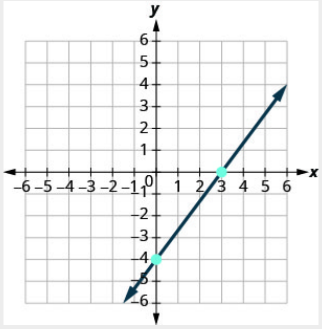
Example:
Exercise:

Problem:

**Graph the line of the equation $4x - 3y = 12$ using its slope and *y*-intercept.

Solution:
Solution

	$4x - 3y = 12$
Find the slope–intercept form of the equation.	$-3y = -4x + 12$
	$-\frac{3y}{3} = \frac{-4x+12}{-3}$
The equation is now in slope–intercept form.	$y = \frac{4}{3}x - 4$
Identify the slope and *y*-intercept.	$m = \frac{4}{3}$
	y-intercept is (0, −4)
Plot the *y*-intercept.	See graph below.

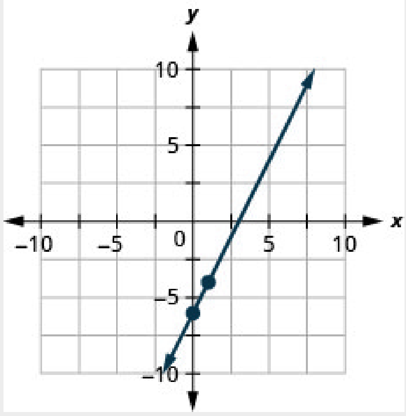
Identify the rise and the run; count out the rise and run to mark the second point.	
Draw the line.	

Note:
Exercise:

Problem:

TRY IT :: 4.91
Graph the line of the equation $2x - y = 6$ using its slope and y-intercept.

Solution:



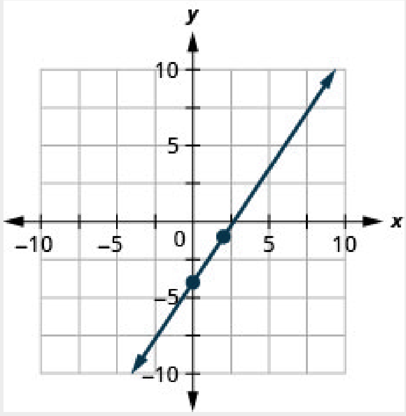
Note:

Exercise:

Problem:

TRY IT :: 4.92
Graph the line of the equation $3x - 2y = 8$ using its slope and *y*-intercept.

Solution:



We have used a grid with x and y both going from about -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications with real-world data, we'll need to extend the axes to bigger positive or smaller negative numbers.

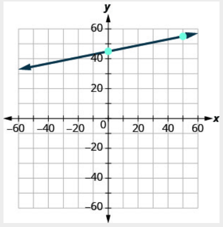
Example:
Exercise:

Problem:

**Graph the line of the equation $y = 0.2x + 45$ using its slope and *y*-intercept.

Solution:
Solution

We'll use a grid with the axes going from about -80 to 80.

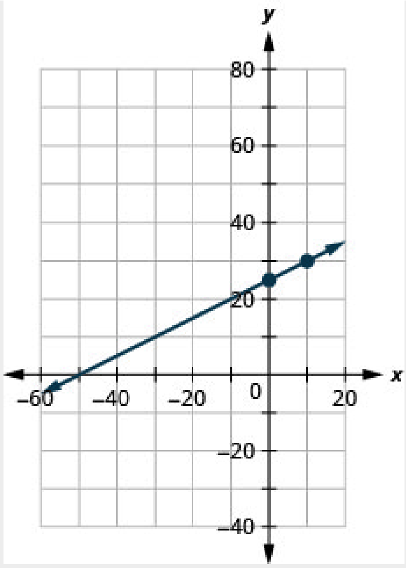
	$y = mx + b$
The equation is in slope–intercept form.	$y = 0.2x + 45$
Identify the slope and *y*-intercept.	$m = 0.2$
	The *y*-intercept is (0, 45)
Plot the *y*-intercept.	See graph below.
Count out the rise and run to mark the second point. The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.	
Draw the line.	

Note:
Exercise:

Problem:

TRY IT :: 4.93
Graph the line of the equation $y = 0.5x + 25$ using its slope and *y*-intercept.

Solution:



Note:

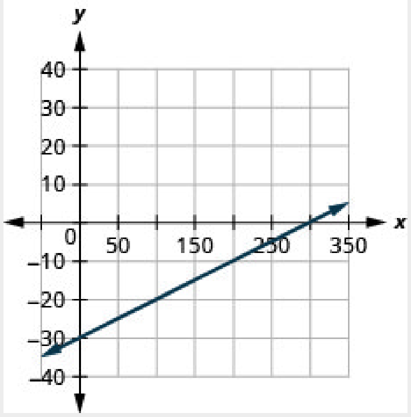
Exercise:

Problem:

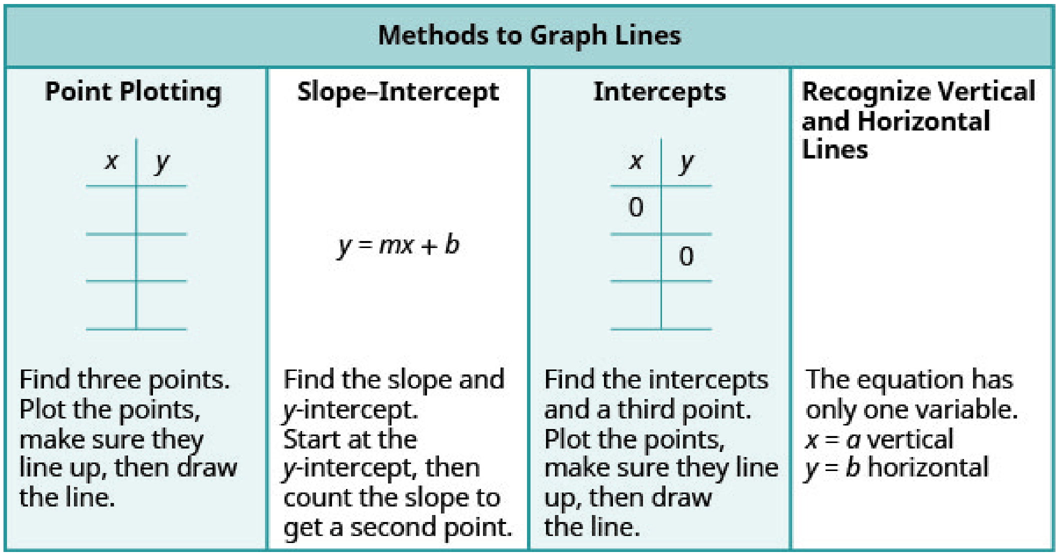
TRY IT :: 4.94

Graph the line of the equation $y = 0.1x - 30$ using its slope and y-intercept.

Solution:



Now that we have graphed lines by using the slope and *y*-intercept, let’s summarize all the methods we have used to graph lines. See [link].



<table>
<tr><th colspan="4">Methods to Graph Lines</th></tr>
<tr><th>Point Plotting</th><th>Slope–Intercept</th><th>Intercepts</th><th>Recognize Vertical and Horizontal Lines</th></tr>
<tr><td>$\begin{array}{c|c} x & y \\ \hline & \\ \hline & \\ \hline & \\ \hline \end{array}$</td><td>$y = mx + b$</td><td>$\begin{array}{c|c} x & y \\ \hline 0 & \\ \hline & 0 \\ \hline & \\ \hline \end{array}$</td><td></td></tr>
<tr><td>Find three points. Plot the points, make sure they line up, then draw the line.</td><td>Find the slope and y-intercept.
Start at the y-intercept, then count the slope to get a second point.</td><td>Find the intercepts and a third point.
Plot the points, make sure they line up, then draw the line.</td><td>The equation has only one variable.
$x = a$ vertical
$y = b$ horizontal</td></tr>
</table>

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods we can use to graph lines, how do we know which method to use for a given equation?

While we could plot points, use the slope–intercept form, or find the intercepts for *any* equation, if we recognize the most convenient way to graph a certain type of equation, our work will be easier. Generally, plotting points is not the most efficient way to graph a line. We saw better methods in sections 4.3, 4.4, and earlier in this section. Let’s look for some patterns to help determine the most convenient method to graph a line.

Here are six equations we graphed in this chapter, and the method we used to graph each of them.

Equation:

	Equation	**Method**
#1	$x = 2$	Vertical line
#2	$y = 4$	Horizontal line
#3	$-x + 2y = 6$	Intercepts
#4	$4x - 3y = 12$	Intercepts
#5	$y = 4x - 2$	Slope–intercept
#6	$y = -x + 4$	Slope–intercept

Equations #1 and #2 each have just one variable. Remember, in equations of this form the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #3 and #4, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the *x*-intercept and $x = 0$ to find the *y*-intercept, and then found a third point by choosing another value for x or y.

Equations #5 and #6 are written in slope–intercept form. After identifying the slope and *y*-intercept from the equation we used them to graph the line.

This leads to the following strategy.

Note:
Strategy for Choosing the Most Convenient Method to Graph a Line
Consider the form of the equation.

- If it only has one variable, it is a vertical or horizontal line.
    - $x = a$ is a vertical line passing through the *x*-axis at a.
    - $y = b$ is a horizontal line passing through the *y*-axis at b.
- If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
    - Identify the slope and *y*-intercept and then graph.
- If the equation is of the form $Ax + By = C$, find the intercepts.
    - Find the *x*- and *y*-intercepts, a third point, and then graph.

Example:
Exercise:

Problem: **Determine the most convenient method to graph each line.

ⓐ $y=-6$ ⓑ $5x-3y=15$ ⓒ $x=7$ ⓓ $y=\frac{2}{5}x-1$.

Solution:
Solution

ⓐ $y=-6$
This equation has only one variable,y. Its graph is a horizontal line crossing the *y*-axis at -6.
ⓑ $5x-3y=15$
This equation is of the form $Ax+By=C$. The easiest way to graph it will be to find the intercepts and one more point.
ⓒ $x=7$
There is only one variable, x. The graph is a vertical line crossing the *x*-axis at 7.
ⓓ $y=\frac{2}{5}x-1$
Since this equation is in $y=mx+b$ form, it will be easiest to graph this line by using the slope and *y*-intercept.

Note:
Exercise:

Problem:

TRY IT :: 4.95
Determine the most convenient method to graph each line: ⓐ $3x+2y=12$
ⓑ $y=4$ ⓒ $y=\frac{1}{5}x-4$ ⓓ $x=-7$.

Solution:

ⓐ intercepts ⓑ horizontal line ⓒ slope–intercept ⓓ vertical line

Note:
Exercise:

Problem:

TRY IT :: 4.96
Determine the most convenient method to graph each line: ⓐ $x=6$ ⓑ $y=-\frac{3}{4}x+1$ ⓒ $y=-8$ ⓓ $4x-3y=-1$.

Solution:

ⓐ vertical line ⓑ slope–intercept ⓒ horizontal line ⓓ intercepts

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually when a linear equation models a real-world situation, different letters are used for the variables, instead of *x* and *y*. The variable names remind us of what quantities are being measured.

Example:
Exercise:

Problem:

**The equation $F=\frac{9}{5}C+32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

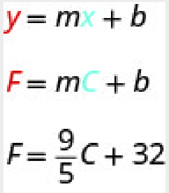
ⓐ Find the Fahrenheit temperature for a Celsius temperature of 0.
ⓑ Find the Fahrenheit temperature for a Celsius temperature of 20.
ⓒ Interpret the slope and F-intercept of the equation.
ⓓ Graph the equation.

Solution:
Solution

ⓐ Find the Fahrenheit temperature for a Celsius temperature of 0. Find F when $C = 0$. Simplify.	$F = \frac{9}{5}C + 32$ $F = \frac{9}{5}(0) + 32$ $F = 32$
ⓑ Find the Fahrenheit temperature for a Celsius temperature of 20. Find F when $C = 20$. Simplify. Simplify.	$F = \frac{9}{5}C + 32$ $F = \frac{9}{5}(20) + 32$ $F = 36 + 32$ $F = 68$

ⓒ Interpret the slope and F-intercept of the equation.

Even though this equation uses $F$and C, it is still in slope–intercept form.



$y = mx + b$

$F = mC + b$

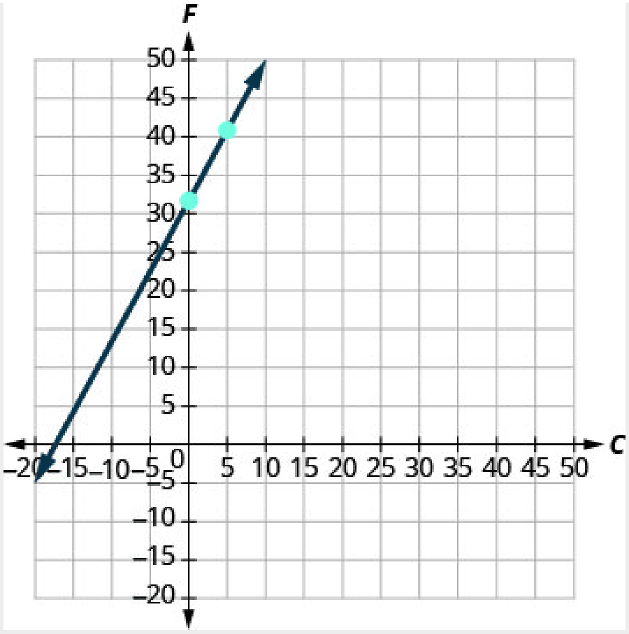
$F = \frac{9}{5}C + 32$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (F) increases 9 degrees when the temperature Celsius (C) increases 5 degrees.

The F-intercept means that when the temperature is 0° on the Celsius scale, it is 32° on the Fahrenheit scale.

ⓓ Graph the equation.

We'll need to use a larger scale than our usual. Start at the F-intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point. See [link].



Note:
Exercise:

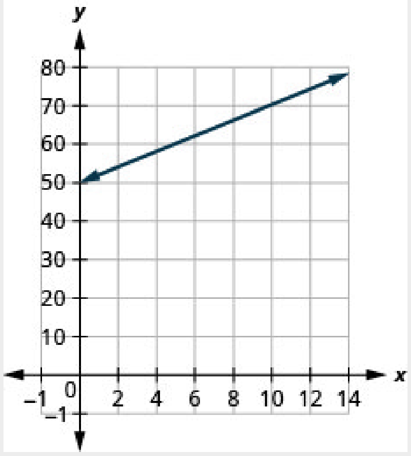
Problem:

TRY IT :: 4.97
The equation $h = 2s + 50$ is used to estimate a woman's height in inches, *h*, based on her shoe size, *s*.

ⓐ Estimate the height of a child who wears women's shoe size 0.
ⓑ Estimate the height of a woman with shoe size 8.
ⓒ Interpret the slope and *h*-intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ 50 inches
ⓑ 66 inches
ⓒ The slope, 2, means that the height, *h*, increases by 2 inches when the shoe size, *s*, increases by 1. The *h*-intercept means that when the shoe size is 0, the height is 50 inches.
ⓓ



Note:
Exercise:

Problem:

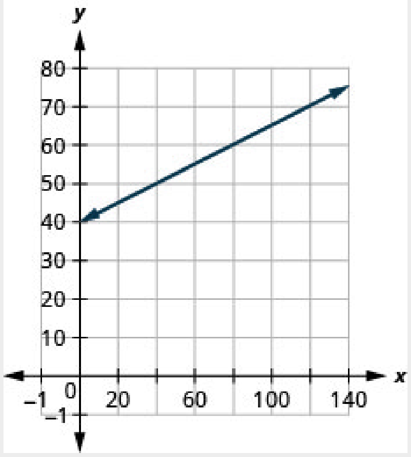
TRY IT :: 4.98

The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, *T*, based on the number of cricket chirps, *n*, in one minute.

ⓐ Estimate the temperature when there are no chirps.
ⓑ Estimate the temperature when the number of chirps in one minute is 100.
ⓒ Interpret the slope and *T*-intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ 40 degrees
ⓑ 65 degrees
ⓒ The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (*F*) increases 1 degree when the number of chirps, *n*, increases by 4. The *T*-intercept means that when the number of chirps is 0, the temperature is 40°.
ⓓ



The cost of running some types business has two components—a *fixed cost* and a *variable cost*. The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly. The variable cost depends on the number of units produced. It is for the material and labor needed to produce each item.

Example:
Exercise:

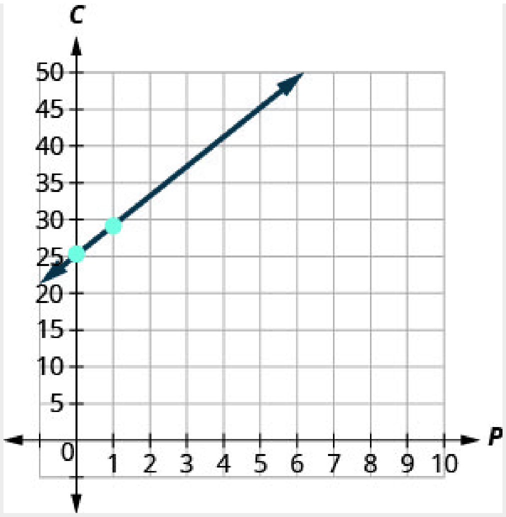
Problem:

**Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, C, in dollars and the number of pizzas, p, that she sells.

ⓐ Find Stella’s cost for a week when she sells no pizzas.
ⓑ Find the cost for a week when she sells 15 pizzas.
ⓒ Interpret the slope and C-intercept of the equation.
ⓓ Graph the equation.

Solution:
Solution

ⓐ Find Stella's cost for a week when she sells no pizzas.	$C = 4p + 25$
Find C when $p = 0$.	$C = 4(0) + 25$
Simplify.	$C = 25$
	Stella's fixed cost is $25 when she sells no pizzas.
ⓑ Find the cost for a week when she sells 15 pizzas.	$C = 4p + 25$
Find C when $p = 15$.	$C = 4(15) + 25$
Simplify.	$C = 60 + 25$
	$C = 85$
	Stella's costs are $85 when she sells 15 pizzas.
ⓒ Interpret the slope and *C*-intercept of the equation.	$y = mx + b$ $C = 4p + 25$
	The slope, 4, means that the cost increases by $4 for each pizza Stella sells. The *C*-intercept means that even when Stella sells no pizzas, her costs for the week are $25.

ⓓ Graph the equation. We'll need to use a larger scale than our usual. Start at the *C*-intercept (0, 25) then count out the rise of 4 and the run of 1 to get a second point.	

Note:
Exercise:

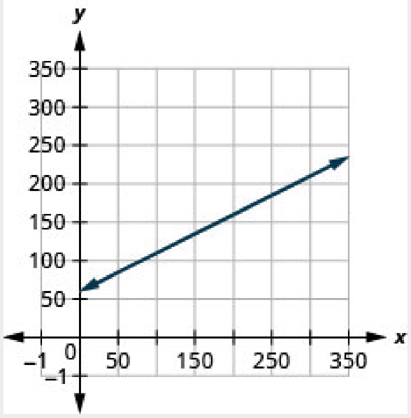
Problem:

TRY IT :: 4.99
Sam drives a delivery van. The equation $C = 0.5m + 60$ models the relation between his weekly cost, *C*, in dollars and the number of miles, *m*, that he drives.

ⓐ Find Sam’s cost for a week when he drives 0 miles.
ⓑ Find the cost for a week when he drives 250 miles.
ⓒ Interpret the slope and *C*-intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ \$60
ⓑ \$185
ⓒ The slope, 0.5, means that the weekly cost, *C*, increases by \$0.50 when the number of miles driven, *n,* increases by 1. The *C*-intercept means that when the number of miles driven is 0, the weekly cost is \$60
ⓓ



Note:
Exercise:

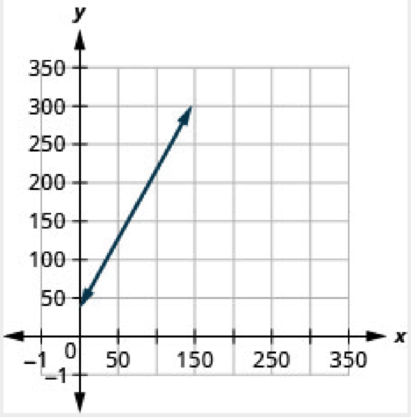
Problem:

TRY IT :: 4.100
Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, *C*, in dollars and the number of wedding invitations, *n*, that she writes.

ⓐFind Loreen’s cost for a week when she writes no invitations.
ⓑ Find the cost for a week when she writes 75 invitations.
ⓒ Interpret the slope and *C*-intercept of the equation.
ⓓ Graph the equation.

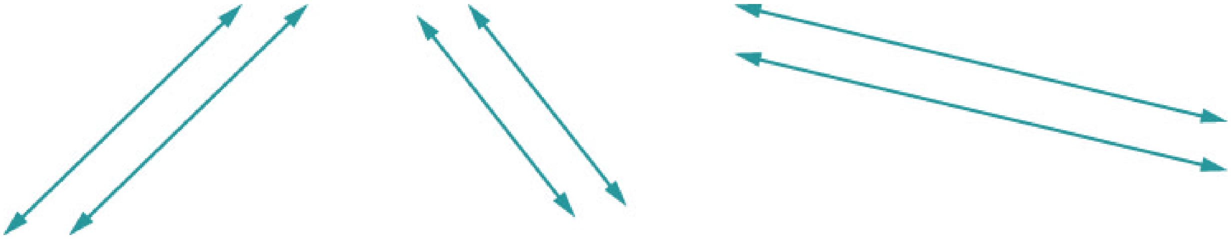
Solution:

ⓐ $35
ⓑ $170
ⓒ The slope, 1.8, means that the weekly cost, C, increases by $1.80 when the number of invitations, *n*, increases by 1.
The *C*-intercept means that when the number of invitations is 0, the weekly cost is $35.;
ⓓ

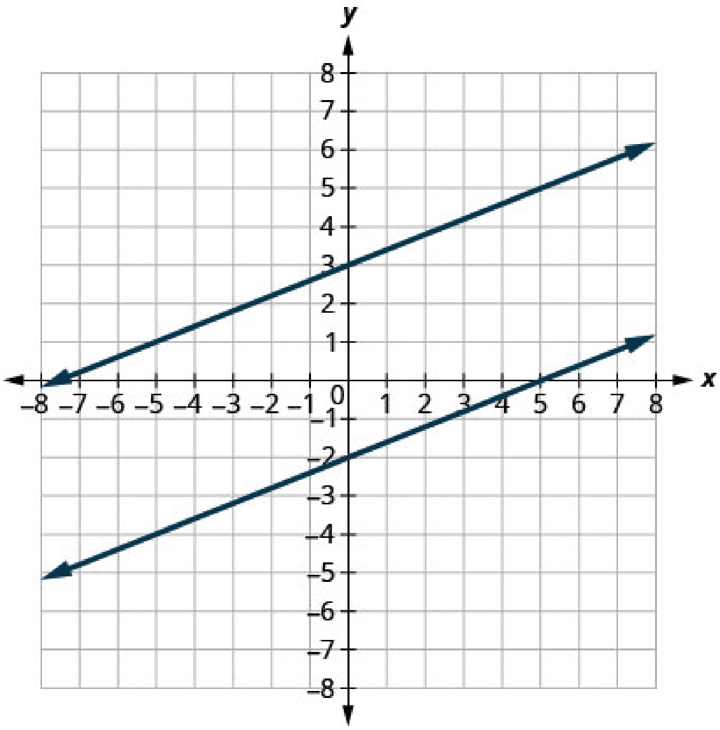


Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called parallel lines. Parallel lines never intersect.

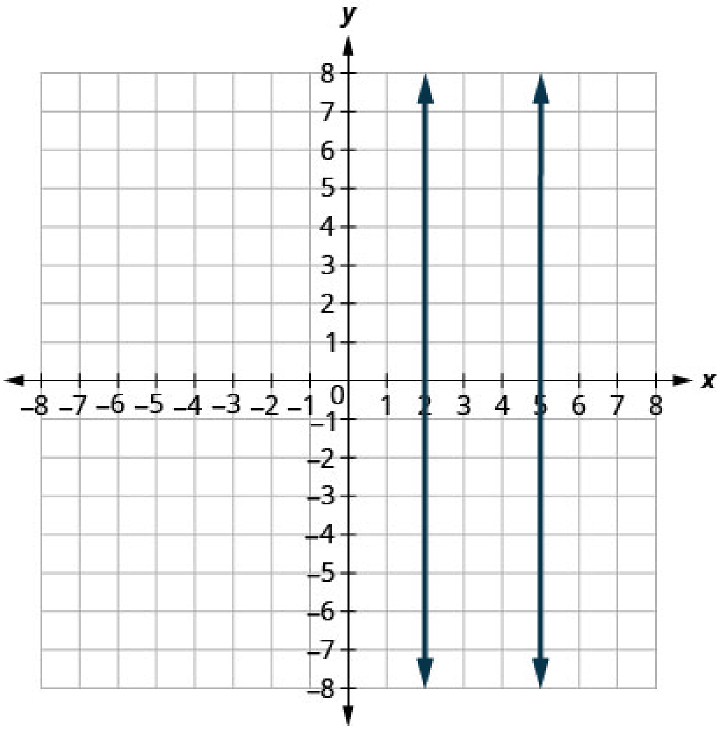


We say this more formally in terms of the rectangular coordinate system. Two lines that have the same slope and different *y*-intercepts are called **parallel lines**. See [link].



Verify that both lines have the same slope, $m = \frac{2}{5}$, and different *y*-intercepts.

What about vertical lines? The slope of a vertical line is undefined, so vertical lines don’t fit in the definition above. We say that vertical lines that have different *x*-intercepts are parallel. See [link].



Vertical lines with diferent *x*-intercepts are parallel.

Note:
Parallel Lines
Parallel lines are lines in the same plane that do not intersect.

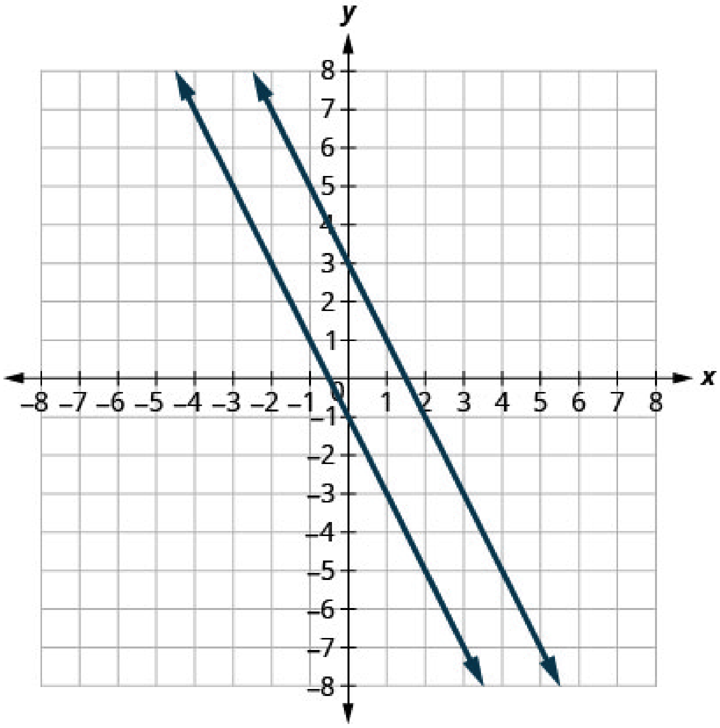
- Parallel lines have the same slope and different *y*-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then$m_1 = m_2$.
- Parallel vertical lines have different *x*-intercepts.

Let’s graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope–intercept form: $y = -2x + 3$. We solve the second equation for y:

Equation:

$$\begin{array}{rcl} 2x + y &=& -1 \\ y &=& -2x - 1 \end{array}$$

Graph the lines.



Notice the lines look parallel. What is the slope of each line? What is the *y*-intercept of each line?

Equation:

$$\begin{array}{rcl} y &=& mx + b \\ y &=& -2x + 3 \\ m &=& -2 \\ b &=& 3, (0, 3) \end{array} \qquad \begin{array}{rcl} y &=& mx + b \\ y &=& -2x - 1 \\ m &=& -2 \\ b &=& -1, (0, -1) \end{array}$$

The slopes of the lines are the same and the *y*-intercept of each line is different. So we know these lines are parallel.

Since parallel lines have the same slope and different *y*-intercepts, we can now just look at the slope–intercept form of the equations of lines and decide if the lines are parallel.

Example:
Exercise:

Problem:

**Use slopes and y-intercepts to determine if the lines $3x - 2y = 6$ and $y = \frac{3}{2}x + 1$ are parallel.

Solution:
Solution

Solve the first equation for y.	$\begin{aligned} 3x - 2y &= 6 \\ -2y &= -3x + 6 \\ \frac{-2y}{-2} &= \frac{-3x+6}{-2} \end{aligned}$	and	$y = \frac{3}{2}x + 1$
The equation is now in slope-intercept form.	$y = \frac{3}{2}x - 3$		
The equation of the second line is already in slope-intercept form.			$y = \frac{3}{2}x + 1$
Identify the slope and y-intercept			$y = \frac{3}{2}x + 1$ $y = mx + b$ $m = \frac{3}{2}$

of both lines.	$y = \frac{3}{2}x - 3$ $y = mx + b$ $m = \frac{3}{2}$		
	y-intercept is $(0, -3)$		y-intercept is $(0, 1)$

The lines have the same slope and different *y*-intercepts and so they are parallel. You may want to graph the lines to confirm whether they are parallel.

Note:
Exercise:

Problem:

TRY IT :: 4.101
Use slopes and *y*-intercepts to determine if the lines $2x + 5y = 5$ and $y = -\frac{2}{5}x - 4$ are parallel.

Solution:

parallel

Note:
Exercise:

Problem:

TRY IT :: 4.102
Use slopes and *y*-intercepts to determine if the lines $4x - 3y = 6$ and $y = \frac{4}{3}x - 1$ are parallel.

Solution:

parallel

Example:
Exercise:

Problem:

**Use slopes and y-intercepts to determine if the lines $y=-4$ and $y=3$ are parallel.

Solution:
Solution

	$y=-4$ $y=0x-4$	and	$y=3$ $y=0x+3$
Write each equation in slope-intercept form.	$y=0x-4$		$y=0x+3$
Since there is no x term we write $0x$.	$y=mx+b$		$y=mx+b$
Identify the slope and y-intercept of both lines.	$m=0$		$m=0$
	y-intercept is $(0, 4)$		y-intercept is $(0, 3)$

The lines have the same slope and different y-intercepts and so they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the y-axis at $y=-4$ and at $y=3$, we know the *y*-intercepts are $(0, -4)$ and $(0, 3)$. The lines have the same slope and different *y*-intercepts and so they are parallel.

Note:
Exercise:

Problem:

TRY IT :: 4.103
Use slopes and *y*-intercepts to determine if the lines $y = 8$ and $y = -6$ are parallel.

Solution:

parallel

Note:
Exercise:

Problem:

TRY IT :: 4.104
Use slopes and *y*-intercepts to determine if the lines $y = 1$ and $y = -5$ are parallel.

Solution:

parallel

Example:
Exercise:

Problem:

**Use slopes and *y*-intercepts to determine if the lines $x = -2$ and $x = -5$ are parallel.

Solution:

Solution
Equation:

$$x = -2 \text{ and } x = -5$$

Since there is no$y$, the equations cannot be put in slope–intercept form. But we recognize them as equations of vertical lines. Their *x*-intercepts are -2 and -5. Since their *x*-intercepts are different, the vertical lines are parallel.

Note:
Exercise:

Problem:

TRY IT :: 4.105
Use slopes and *y*-intercepts to determine if the lines $x = 1$ and $x = -5$ are parallel.

Solution:

parallel

Note:
Exercise:

Problem:

TRY IT :: 4.106
Use slopes and *y*-intercepts to determine if the lines $x = 8$ and $x = -6$ are parallel.

Solution:

parallel

Example:
Exercise:

Problem:

**Use slopes and y-intercepts to determine if the lines $y=2x-3$ and $-6x+3y=-9$ are parallel. You may want to graph these lines, too, to see what they look like.

Solution:
Solution

	$y=2x-3$	and	$-6x+3y=-9$
The first equation is already in slope-intercept form.	$y=2x-3$		
Solve the second equation for y.			$\begin{aligned} -6x+3y &= -9 \\ 3y &= 6x-9 \\ \frac{3y}{3} &= \frac{6x-9}{3} \\ y &= 2x-3 \end{aligned}$
The second equation is now in slope-intercept form.	$y=2x-3$		
Identify the slope and y-	$\begin{aligned} y &= 2x-3 \\ y &= mx+b \\ m &= 2 \end{aligned}$		$\begin{aligned} y &= 2x-3 \\ y &= mx+b \\ m &= 2 \end{aligned}$

intercept of both lines.			
	y-intercept is $(0, -3)$		y-intercept is $(0, -3)$

The lines have the same slope, but they also have the same *y*-intercepts. Their equations represent the same line. They are not parallel; they are the same line.

Note:
Exercise:

Problem:

TRY IT :: 4.107
Use slopes and *y*-intercepts to determine if the lines $y = -\frac{1}{2}x - 1$ and $x + 2y = -2$ are parallel.

Solution:

not parallel; same line

Note:
Exercise:

Problem:

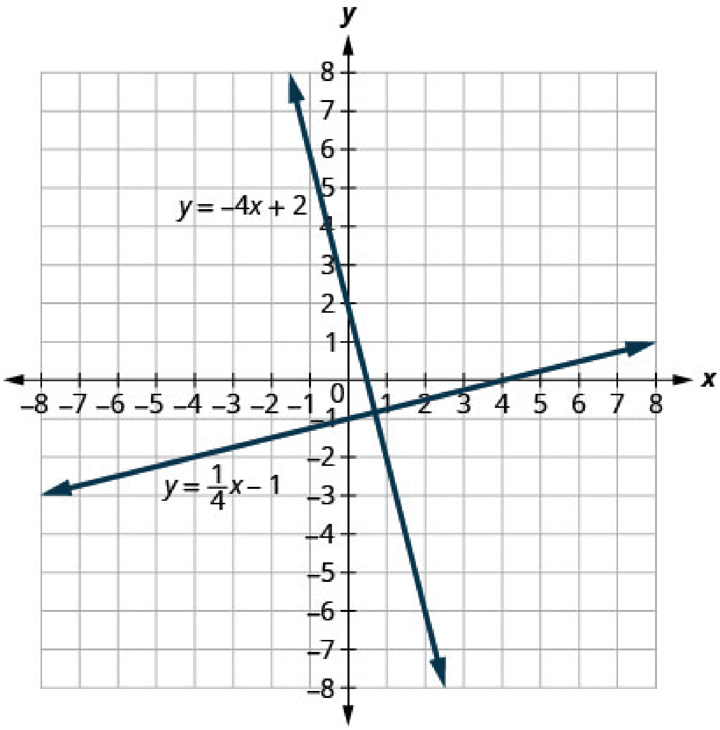
TRY IT :: 4.108
Use slopes and *y*-intercepts to determine if the lines $y = \frac{3}{4}x - 3$ and $3x - 4y = 12$ are parallel.

Solution:

not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let’s look at the lines whose equations are $y=\frac{1}{4}x-1$ and $y=-4x+2$, shown in [link].



These lines lie in the same plane and intersect in right angles. We call these lines **perpendicular**.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y=\frac{1}{4}x-1$ rises, so its slope is positive. The line$y=-4x+2$ drops from left to right, so it has a negative slope. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1=\frac{1}{4}$, and the slope of the second line, $m_2=-4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1.

Equation:

$$m_1\cdot m_2$$

$$\frac{1}{4}(-4)$$

$$-1$$

This is always true for perpendicular lines and leads us to this definition.

Note:
Perpendicular Lines
Perpendicular lines are lines in the same plane that form a right angle.
If m_1 and m_2 are the slopes of two perpendicular lines, then:
Equation:

$$m_1 \cdot m_2 = -1 \text{ and } m_1 = \frac{-1}{m_2}$$

Vertical lines and horizontal lines are always perpendicular to each other.

We were able to look at the slope–intercept form of linear equations and determine whether or not the lines were parallel. We can do the same thing for perpendicular lines.

We find the slope–intercept form of the equation, and then see if the slopes are negative reciprocals. If the product of the slopes is -1, the lines are perpendicular. Perpendicular lines may have the same *y*-intercepts.

Example:
Exercise:

Problem:

**Use slopes to determine if the lines, $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Solution:
Solution

The first equation is	$y = -5x - 4$	

already in slope-intercept form.		
Solve the second equation for y.	$x-5y = 5$ $-5y = -x+5$ $\frac{-5y}{-5} = \frac{-x+5}{-5}$ $y = \frac{1}{5}x-1$	
Identify the slope of each line.	$y = -5x-4$ $y = mx+b$ $m_1 = -5$	$y = \frac{1}{5}x-1$ $y = mx+b$ $m_2 = \frac{1}{5}$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

Equation:

$$\begin{array}{l} m_1 \cdot m_2 \\ -5\left(\frac{1}{5}\right) \\ -1\checkmark \end{array}$$

Note:

Exercise:

Problem:

TRY IT :: 4.109

Use slopes to determine if the lines $y=-3x+2$ and $x-3y=4$ are perpendicular.

Solution:

perpendicular

Note:

Exercise:

Problem:

TRY IT :: 4.110
Use slopes to determine if the lines $y = 2x - 5$ and $x + 2y = -6$ are perpendicular.

Solution:

perpendicular

Example:
Exercise:

Problem:

**Use slopes to determine if the lines, $7x + 2y = 3$ and $2x + 7y = 5$ are perpendicular.

Solution:
Solution

Solve the equations for y.	$$\begin{aligned} 7x + 2y &= 3 \\ 2y &= -7x + 3 \\ \frac{2y}{2} &= \frac{-7x+3}{2} \\ y &= -\frac{7}{2}x + \frac{3}{2} \end{aligned}$$	$$\begin{aligned} 2x + 7y &= 5 \\ 7y &= -2x + 5 \\ \frac{7y}{7} &= \frac{-2x+5}{7} \\ y &= -\frac{2}{7}x + \frac{5}{7} \end{aligned}$$
Identify the slope of each line.	$$\begin{aligned} y &= mx + b \\ m_1 &= -\frac{7}{2} \end{aligned}$$	$$\begin{aligned} y &= mx + b \\ m_2 &= -\frac{2}{7} \end{aligned}$$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

Note:
Exercise:

Problem:

TRY IT :: 4.111
Use slopes to determine if the lines $5x + 4y = 1$ and $4x + 5y = 3$ are perpendicular.

Solution:

not perpendicular

Note:
Exercise:

Problem:

TRY IT :: 4.112
Use slopes to determine if the lines $2x - 9y = 3$ and $9x - 2y = 1$ are perpendicular.

Solution:

not perpendicular

Note:
Access this online resource for additional instruction and practice with graphs.

- Explore the Relation Between a Graph and the Slope–Intercept Form of an Equation of a Line

Key Concepts

- The slope–intercept form of an equation of a line with slope m and *y*-intercept, $(0, b)$ is, $y = mx + b$.
- **Graph a Line Using its Slope and *y*-Intercept**

  Find the slope-intercept form of the equation of the line.
  Identify the slope and *y*-intercept.
  Plot the *y*-intercept.
  Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  Starting at the *y*-intercept, count out the rise and run to mark the second point.
  Connect the points with a line.

- **Strategy for Choosing the Most Convenient Method to Graph a Line:** Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    $x = a$ is a vertical line passing through the *x*-axis at a.
    $y = b$ is a horizontal line passing through the *y*-axis at b.
  - If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and *y*-intercept.
    Identify the slope and *y*-intercept and then graph.
  - If the equation is of the form $Ax + By = C$, find the intercepts.
    Find the *x*- and *y*-intercepts, a third point, and then graph.

- Parallel lines are lines in the same plane that do not intersect.
  - Parallel lines have the same slope and different *y*-intercepts.
  - If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
  - Parallel vertical lines have different *x*-intercepts.

- Perpendicular lines are lines in the same plane that form a right angle.
  - If m_1 and m_2 are the slopes of two perpendicular lines, then $m_1 \cdot m_2 = -1$ and $m_1 = \frac{-1}{m_2}$.
  - Vertical lines and horizontal lines are always perpendicular to each other.

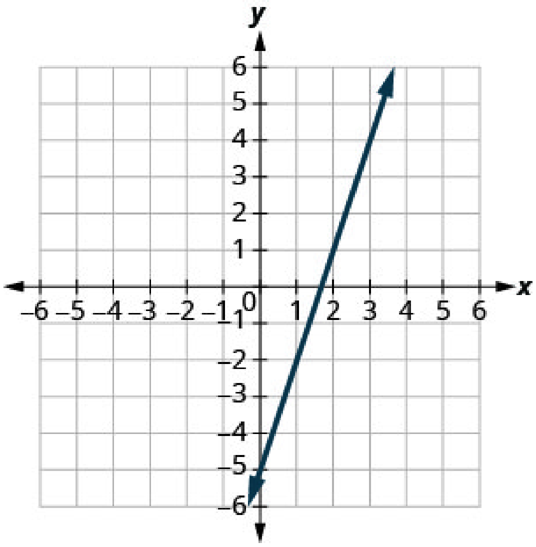
Practice Makes Perfect

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise:

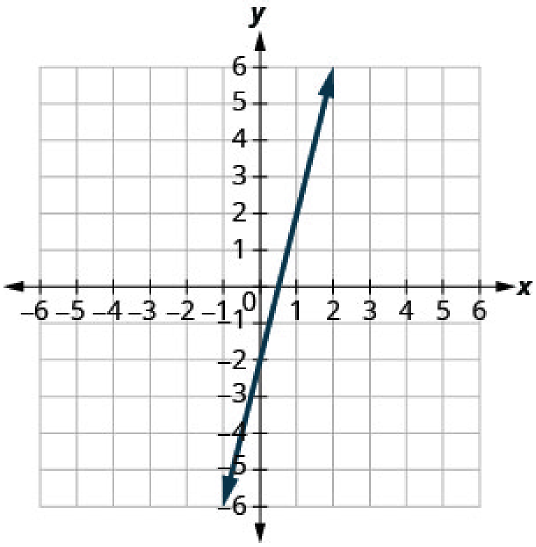
Problem:



$y = 3x - 5$

Exercise:

Problem:



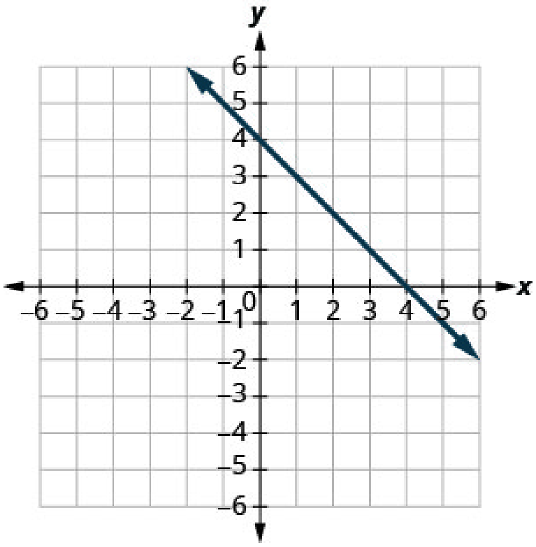
$y = 4x - 2$

Solution:

slope $m = 4$ and y-intercept $(0, -2)$

Exercise:

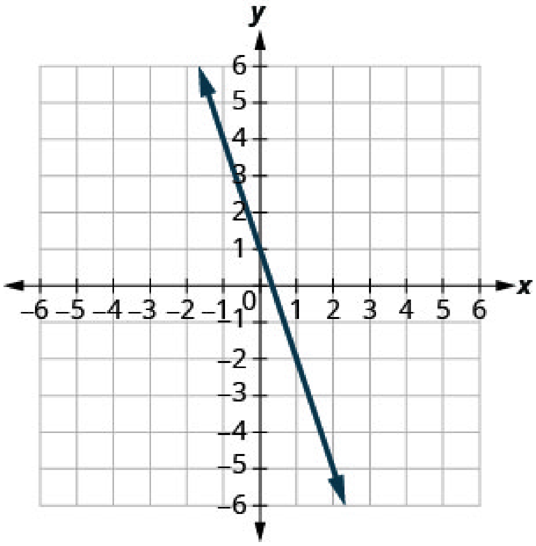
Problem:



$y = -x + 4$

Exercise:

Problem:



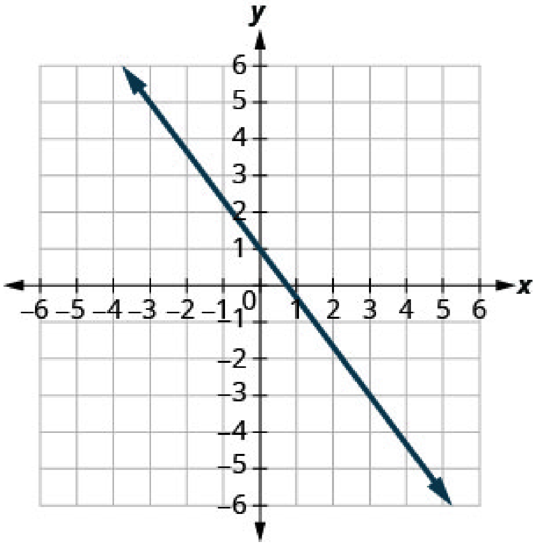
$y=-3x+1$

Solution:

slope $m=-3$ and y-intercept $(0,1)$

Exercise:

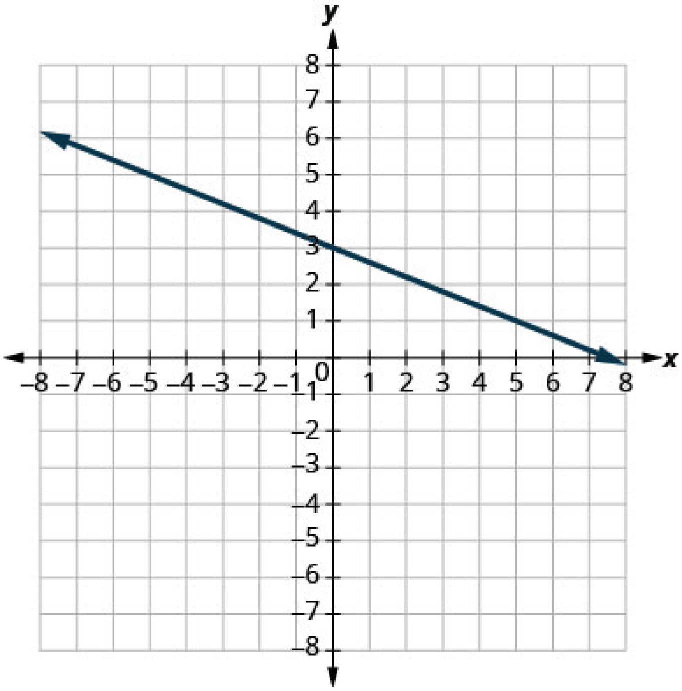
Problem:



$y=-\frac{4}{3}x+1$

Exercise:

Problem:



$y=-\frac{2}{5}x+3$

Solution:

slope $m=-\frac{2}{5}$ and y-intercept $(0,3)$

Identify the Slope and y-Intercept From an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

Exercise:

Problem: $y=-7x+3$

Exercise:

Problem: $y=-9x+7$

Solution:

$-9; (0,7)$

Exercise:

Problem: $y = 6x - 8$

Exercise:

Problem: $y = 4x - 10$

Solution:

$4; (0, -10)$

Exercise:

Problem: $3x + y = 5$

Exercise:

Problem: $4x + y = 8$

Solution:

$-4; (0, 8)$

Exercise:

Problem: $6x + 4y = 12$

Exercise:

Problem: $8x + 3y = 12$

Solution:

$-\frac{8}{3}; (0, 4)$

Exercise:

Problem: $5x - 2y = 6$

Exercise:

Problem: $7x - 3y = 9$

Solution:

$\frac{7}{3}$; $(0, -3)$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

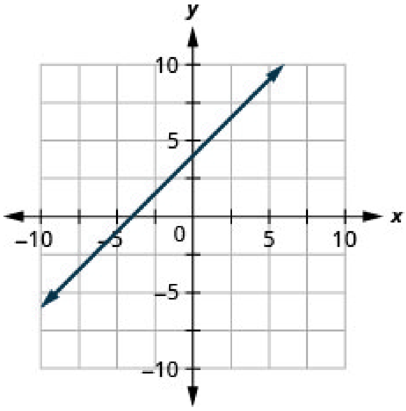
Exercise:

Problem: $y = x + 3$

Exercise:

Problem: $y = x + 4$

Solution:



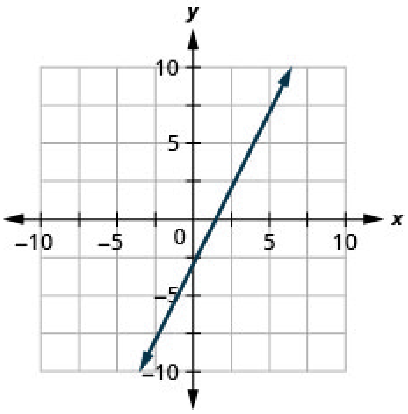
Exercise:

Problem: $y = 3x - 1$

Exercise:

Problem: $y = 2x - 3$

Solution:



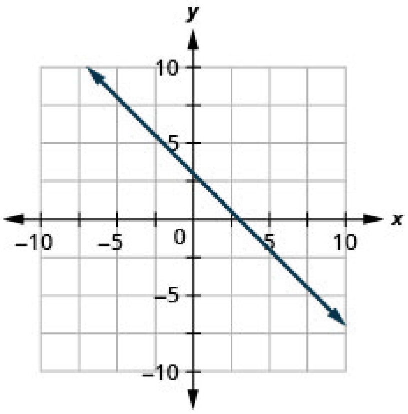
Exercise:

Problem: $y = -x + 2$

Exercise:

Problem: $y = -x + 3$

Solution:



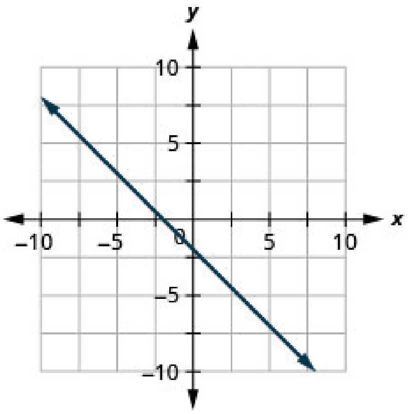
Exercise:

Problem: $y = -x - 4$

Exercise:

Problem: $y = -x - 2$

Solution:



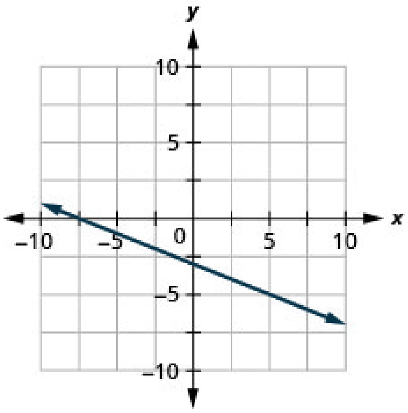
Exercise:

Problem: $y=-\frac{3}{4}x-1$

Exercise:

Problem: $y=-\frac{2}{5}x-3$

Solution:



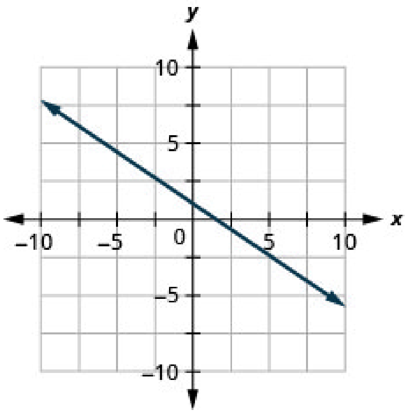
Exercise:

Problem: $y=-\frac{3}{5}x+2$

Exercise:

Problem: $y=-\frac{2}{3}x+1$

Solution:



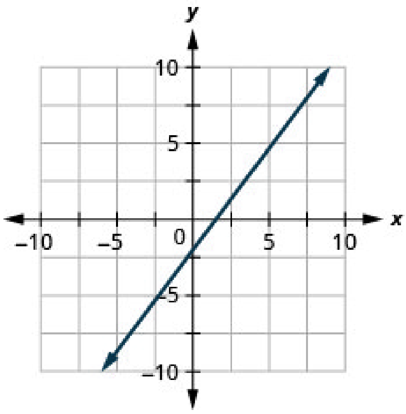
Exercise:

Problem: $3x-4y=8$

Exercise:

Problem: $4x-3y=6$

Solution:



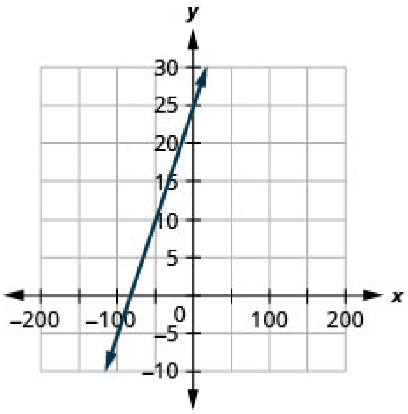
Exercise:

Problem: $y=0.1x+15$

Exercise:

Problem: $y=0.3x+25$

Solution:



Choose the Most Convenient Method to Graph a Line

In the following exercises, determine the most convenient method to graph each line.

Exercise:

Problem: $x = 2$

Exercise:

Problem: $y = 4$

Solution:

horizontal line

Exercise:

Problem: $y = 5$

Exercise:

Problem: $x = -3$

Solution:

vertical line

Exercise:

Problem: $y = -3x + 4$

Exercise:

Problem: $y=-5x+2$

Solution:

slope–intercept

Exercise:

Problem: $x-y=5$

Exercise:

Problem: $x-y=1$

Solution:

intercepts

Exercise:

Problem: $y=\frac{2}{3}x-1$

Exercise:

Problem: $y=\frac{4}{5}x-3$

Solution:

slope–intercept

Exercise:

Problem: $y=-3$

Exercise:

Problem: $y=-1$

Solution:

horizontal line

Exercise:

Problem: $3x-2y=-12$

Exercise:

Problem: $2x-5y=-10$

Solution:

intercepts

Exercise:

Problem: $y=-\frac{1}{4}x+3$

Exercise:

Problem: $y=-\frac{1}{3}x+5$

Solution:

slope–intercept

Graph and Interpret Applications of Slope–Intercept

Exercise:

Problem:

The equation $P=31+1.75w$ models the relation between the amount of Tuyet's monthly water bill payment, *P*, in dollars, and the number of units of water, *w*, used.

ⓐ Find Tuyet's payment for a month when 0 units of water are used.
ⓑ Find Tuyet's payment for a month when 12 units of water are used.
ⓒ Interpret the slope and *P*-intercept of the equation.
ⓓ Graph the equation.

Exercise:

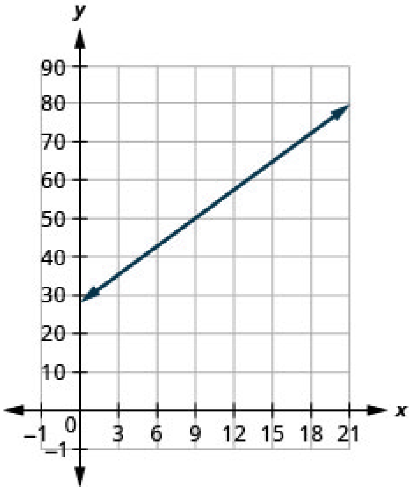
Problem:

The equation $P=28+2.54w$ models the relation between the amount of Randy's monthly water bill payment, *P*, in dollars, and the number of units of water, *w*, used.

ⓐ Find the payment for a month when Randy used 0 units of water.
ⓑ Find the payment for a month when Randy used 15 units of water.
ⓒ Interpret the slope and *P*-intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ $28
ⓑ $66.10
ⓒ The slope, 2.54, means that Randy's payment, *P*, increases by $2.54 when the number of units of water he used, *w*, increases by 1. The *P*–intercept means that if the number units of water Randy used was 0, the payment would be $28.
ⓓ



Exercise:

Problem:

Bruce drives his car for his job. The equation $R = 0.575m + 42$ models the relation between the amount in dollars, *R*, that he is reimbursed and the number of miles, *m*, he drives in one day.

ⓐ Find the amount Bruce is reimbursed on a day when he drives 0 miles.
ⓑ Find the amount Bruce is reimbursed on a day when he drives 220 miles.
ⓒ Interpret the slope and *R*-intercept of the equation.
ⓓ Graph the equation.

Exercise:

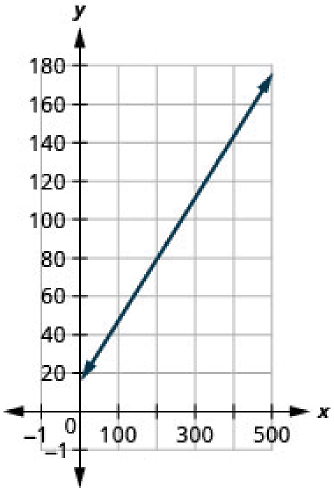
Problem:

Janelle is planning to rent a car while on vacation. The equation $C = 0.32m + 15$ models the relation between the cost in dollars, *C*, per day and the number of miles, *m*, she drives in one day.

ⓐ Find the cost if Janelle drives the car 0 miles one day.
ⓑ Find the cost on a day when Janelle drives the car 400 miles.
ⓒ Interpret the slope and *C*–intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ \$15
ⓑ \$143
ⓒ The slope, 0.32, means that the cost, *C*, increases by \$0.32 when the number of miles driven, *m*, increases by 1. The *C*-intercept means that if Janelle drives 0 miles one day, the cost would be \$15.
ⓓ



Exercise:

Problem:

Cherie works in retail and her weekly salary includes commission for the amount she sells. The equation $S = 400 + 0.15c$ models the relation between her weekly salary, *S*, in dollars and the amount of her sales, *c*, in dollars.

ⓐ Find Cherie’s salary for a week when her sales were 0.

ⓑ Find Cherie’s salary for a week when her sales were 3600.
ⓒ Interpret the slope and *S*–intercept of the equation.
ⓓ Graph the equation.

Exercise:

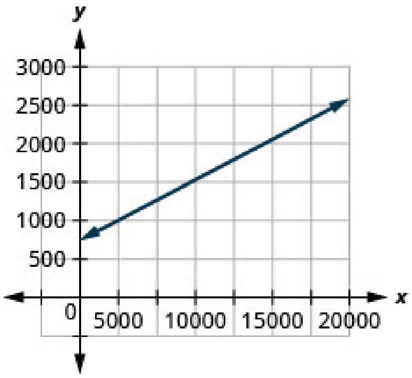
Problem:

Patel’s weekly salary includes a base pay plus commission on his sales. The equation $S = 750 + 0.09c$ models the relation between his weekly salary, *S*, in dollars and the amount of his sales, *c*, in dollars.

ⓐ Find Patel’s salary for a week when his sales were 0.
ⓑ Find Patel’s salary for a week when his sales were 18,540.
ⓒ Interpret the slope and *S*-intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ $750
ⓑ $2418.60
ⓒ The slope, 0.09, means that Patel’s salary, *S*, increases by $0.09 for every $1 increase in his sales. The *S*-intercept means that when his sales are $0, his salary is $750.
ⓓ



Exercise:

Problem:

Costa is planning a lunch banquet. The equation $C = 450 + 28g$ models the relation between the cost in dollars, C, of the banquet and the number of guests, g.

ⓐ Find the cost if the number of guests is 40.
ⓑ Find the cost if the number of guests is 80.
ⓒ Interpret the slope and C-intercept of the equation.
ⓓ Graph the equation.

Exercise:

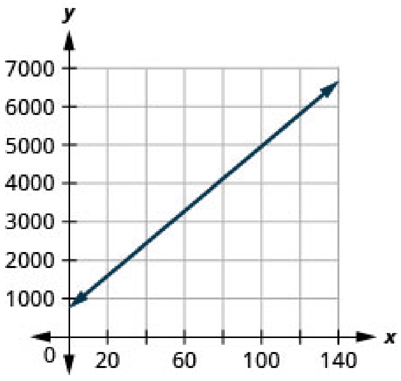
Problem:

Margie is planning a dinner banquet. The equation $C = 750 + 42g$ models the relation between the cost in dollars, C of the banquet and the number of guests, g.

ⓐ Find the cost if the number of guests is 50.
ⓑ Find the cost if the number of guests is 100.
ⓒ Interpret the slope and C–intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ $2850
ⓑ $4950
ⓒ The slope, 42, means that the cost, C, increases by $42 for when the number of guests increases by 1. The C-intercept means that when the number of guests is 0, the cost would be $750.
ⓓ



Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise:

Problem: $y = \frac{3}{4}x - 3;\;\; 3x - 4y = -2$

Exercise:

Problem: $y = \frac{2}{3}x - 1;\;\; 2x - 3y = -2$

Solution:

parallel

Exercise:

Problem: $2x - 5y = -3;\;\; y = \frac{2}{5}x + 1$

Exercise:

Problem: $3x - 4y = -2;\;\; y = \frac{3}{4}x - 3$

Solution:

parallel

Exercise:

Problem: $2x - 4y = 6;\;\; x - 2y = 3$

Exercise:

Problem: $6x - 3y = 9;\;\; 2x - y = 3$

Solution:

not parallel

Exercise:

Problem: $4x+2y=6;\;\;6x+3y=3$

Exercise:

Problem: $8x+6y=6;\;\;12x+9y=12$

Solution:

parallel

Exercise:

Problem: $x=5;\;\;x=-6$

Exercise:

Problem: $x=7;\;\;x=-8$

Solution:

parallel

Exercise:

Problem: $x=-4;\;\;x=-1$

Exercise:

Problem: $x=-3;\;\;x=-2$

Solution:

parallel

Exercise:

Problem: $y=2;\;\;y=6$

Exercise:

Problem: $y=5;\;\;y=1$

Solution:

parallel

Exercise:

Problem: $y=-4;\;\; y=3$

Exercise:

Problem: $y=-1;\;\; y=2$

Solution:

parallel

Exercise:

Problem: $x-y=2;\;\; 2x-2y=4$

Exercise:

Problem: $4x+4y=8;\;\; x+y=2$

Solution:

not parallel

Exercise:

Problem: $x-3y=6;\;\; 2x-6y=12$

Exercise:

Problem: $5x-2y=11;\;\; 5x-y=7$

Solution:

not parallel

Exercise:

Problem: $3x-6y=12;\;\; 6x-3y=3$

Exercise:

Problem: $4x-8y=16;\;\; x-2y=4$

Solution:

not parallel

Exercise:

Problem: $9x - 3y = 6; \;\; 3x - y = 2$

Exercise:

Problem: $x - 5y = 10; \;\; 5x - y = -10$

Solution:

not parallel

Exercise:

Problem: $7x - 4y = 8; \;\; 4x + 7y = 14$

Exercise:

Problem: $9x - 5y = 4; \;\; 5x + 9y = -1$

Solution:

not parallel

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise:

Problem: $3x - 2y = 8; 2x + 3y = 6$

Exercise:

Problem: $x - 4y = 8; 4x + y = 2$

Solution:

perpendicular

Exercise:

Problem: $2x + 5y = 3; 5x - 2y = 6$

Exercise:

Problem: $2x + 3y = 5; 3x - 2y = 7$

Solution:

perpendicular

Exercise:

Problem: $3x - 2y = 1; 2x - 3y = 2$

Exercise:

Problem: $3x - 4y = 8; 4x - 3y = 6$

Solution:

not perpendicular

Exercise:

Problem: $5x + 2y = 6; 2x + 5y = 8$

Exercise:

Problem: $2x + 4y = 3; 6x + 3y = 2$

Solution:

not perpendicular

Exercise:

Problem: $4x - 2y = 5; 3x + 6y = 8$

Exercise:

Problem: $2x - 6y = 4; 12x + 4y = 9$

Solution:

perpendicular

Exercise:

Problem: $6x-4y=5;8x+12y=3$

Exercise:

Problem: $8x-2y=7;3x+12y=9$

Solution:

perpendicular

Everyday Math

Exercise:

Problem:

The equation $C=\frac{5}{9}F-17.8$ can be used to convert temperatures *F*, on the Fahrenheit scale to temperatures, *C*, on the Celsius scale.

ⓐ Explain what the slope of the equation means.
ⓑ Explain what the *C*–intercept of the equation means.

Exercise:

Problem:

The equation $n=4T-160$ is used to estimate the number of cricket chirps, *n*, in one minute based on the temperature in degrees Fahrenheit, *T*.

ⓐ Explain what the slope of the equation means.
ⓑ Explain what the *n*–intercept of the equation means. Is this a realistic situation?

Solution:

ⓐ For every increase of one degree Fahrenheit, the number of chirps increases by four.
ⓑ There would be -160 chirps when the Fahrenheit temperature is 0°. (Notice that this does not make sense; this model cannot be used for all possible temperatures.)

Writing Exercises

Exercise:

Problem:

Explain in your own words how to decide which method to use to graph a line.

Exercise:

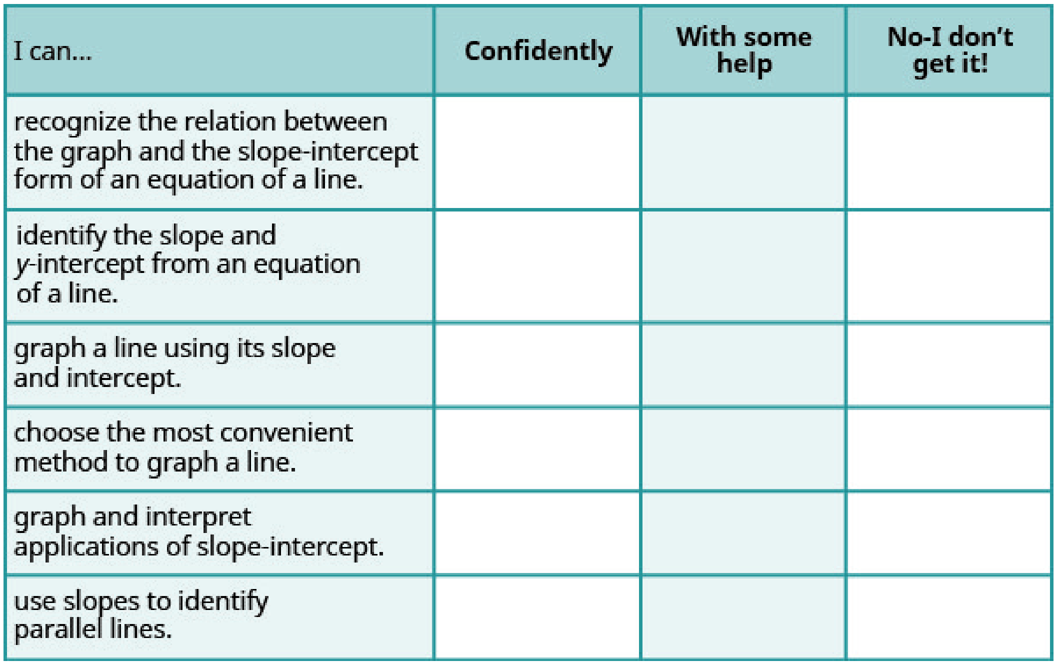
Problem: Why are all horizontal lines parallel?

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
recognize the relation between the graph and the slope-intercept form of an equation of a line.			
identify the slope and *y*-intercept from an equation of a line.			
graph a line using its slope and intercept.			
choose the most convenient method to graph a line.			
graph and interpret applications of slope-intercept.			
use slopes to identify parallel lines.			

ⓑ After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

Glossary

parallel lines
Lines in the same plane that do not intersect.

perpendicular lines
Lines in the same plane that form a right angle.

slope-intercept form of an equation of a line
The slope–intercept form of an equation of a line with slope m and y-intercept, $(0, b)$ is, $y = mx + b$.

Find the Equation of a Line

By the end of this section, you will be able to:

- Find an equation of the line given the slope and y-intercept
- Find an equation of the line given the slope and a point
- Find an equation of the line given two points
- Find an equation of a line parallel to a given line
- Find an equation of a line perpendicular to a given line

Note:
Before you get started, take this readiness quiz.

1. Solve: $\frac{2}{3} = \frac{x}{5}$.
   If you missed this problem, review [link].
2. Simplify: $-\frac{2}{5}(x - 15)$.
   If you missed this problem, review [link].

How do online retailers know that ‘you may also like’ a particular item based on something you just ordered? How can economists know how a rise in the minimum wage will affect the unemployment rate? How do medical researchers create drugs to target cancer cells? How can traffic engineers predict the effect on your commuting time of an increase or decrease in gas prices? It’s all mathematics.

You are at an exciting point in your mathematical journey as the mathematics you are studying has interesting applications in the real world.

The physical sciences, social sciences, and the business world are full of situations that can be modeled with linear equations relating two variables. Data is collected and graphed. If the data points appear to form a straight line, an equation of that line can be used to predict the value of one variable based on the value of the other variable.

To create a mathematical model of a linear relation between two variables, we must be able to find the equation of the line. In this section we will look at several ways to write the equation of a line. The specific method we use will be determined by what information we are given.

Find an Equation of the Line Given the Slope and *y*-Intercept

We can easily determine the slope and intercept of a line if the equation was written in slope–intercept form, $y = mx + b$. Now, we will do the reverse —we will start with the slope and *y*-intercept and use them to find the equation of the line.

Example:
Exercise:

Problem:

**Find an equation of a line with slope -7 and *y*-intercept $(0, -1)$.

Solution:
Solution

Since we are given the slope and *y*-intercept of the line, we can substitute the needed values into the slope–intercept form, $y = mx + b$.

Name the slope.	$m = -7$

Name the *y*-intercept.	*y*-intercept $(0, -1)$
Substitute the values into $y = mx + b$.	$y = mx + b$
	$y = -7x + (-1)$
Simplify.	$y = -7x - 1$

Note:
Exercise:

Problem:

TRY IT :: 4.113
Find an equation of a line with slope $\frac{2}{5}$ and *y*-intercept $(0, 4)$.

Solution:

$y = \frac{2}{5}x + 4$

Note:
Exercise:

Problem:

TRY IT :: 4.114
Find an equation of a line with slope -1 and *y*-intercept $(0, -3)$.

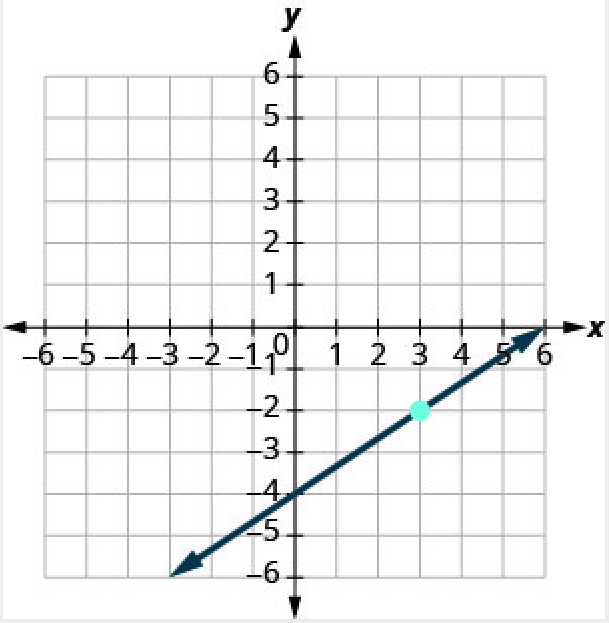
Solution:

$y = -x - 3$

Sometimes, the slope and intercept need to be determined from the graph.

Example:
Exercise:

Problem: **Find the equation of the line shown.



Solution:
Solution

We need to find the slope and *y*-intercept of the line from the graph so we can substitute the needed values into the slope–intercept form, $y = mx + b$.

To find the slope, we choose two points on the graph.

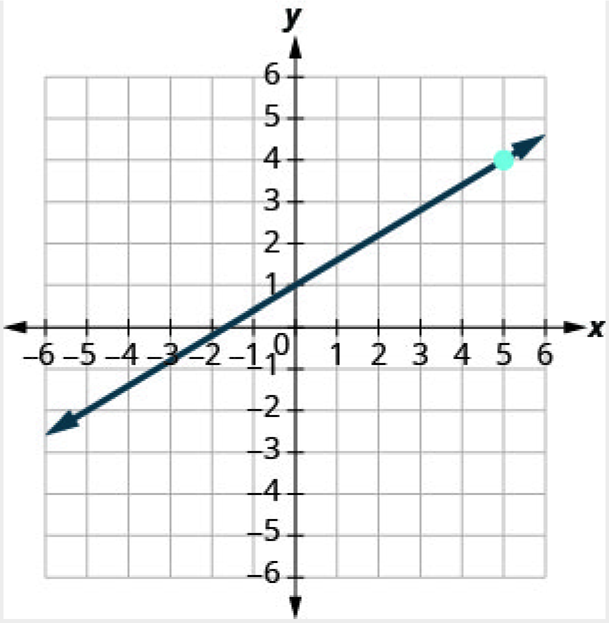
The y-intercept is $(0, -4)$ and the graph passes through $(3, -2)$.

Find the slope by counting the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{3}$
Find the y-intercept.	y-intercept $(0, -4)$
Substitute the values into $y = mx + b$.	$y = mx + b$
	$y = \frac{2}{3}x - 4$

Note:
Exercise:

TRY IT :: 4.115

Problem: Find the equation of the line shown in the graph.



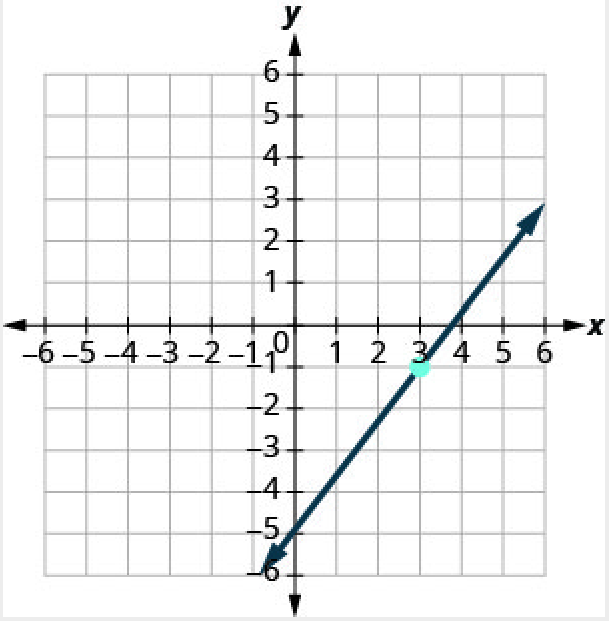
Solution:

$y = \frac{3}{5}x + 1$

Note:
Exercise:

TRY IT :: 4.116

Problem: Find the equation of the line shown in the graph.



Solution:

$y = \frac{4}{3}x - 5$

Find an Equation of the Line Given the Slope and a Point

Finding an equation of a line using the slope–intercept form of the equation works well when you are given the slope and *y*-intercept or when you read them off a graph. But what happens when you have another point instead of the *y*-intercept?

We are going to use the slope formula to derive another form of an equation of the line. Suppose we have a line that has slope m and that contains some specific point (x_1, y_1) and some other point, which we will just call (x, y). We can write the slope of this line and then change it to a different form.

	$m = \frac{y - y_1}{x - x_1}$
Multiply both sides of the equation by $x - x_1$.	$m\left(x - x_1\right) = \left(\frac{y - y_1}{x - x_1}\right)\left(x - x_1\right)$
Simplify.	$m\left(x - x_1\right) = y - y_1$
Rewrite the equation with the y terms on the left.	$y - y_1 = m\left(x - x_1\right)$

This format is called the point–slope form of an equation of a line.

Note:
Point–slope Form of an Equation of a Line
The **point–slope form** of an equation of a line with slope m and containing the point $\left(x_1, y_1\right)$ is

$$y - y_1 = m(x - x_1)$$

We can use the point–slope form of an equation to find an equation of a line when we are given the slope and one point. Then we will rewrite the equation in slope–intercept form. Most applications of linear equations use the the slope–intercept form.

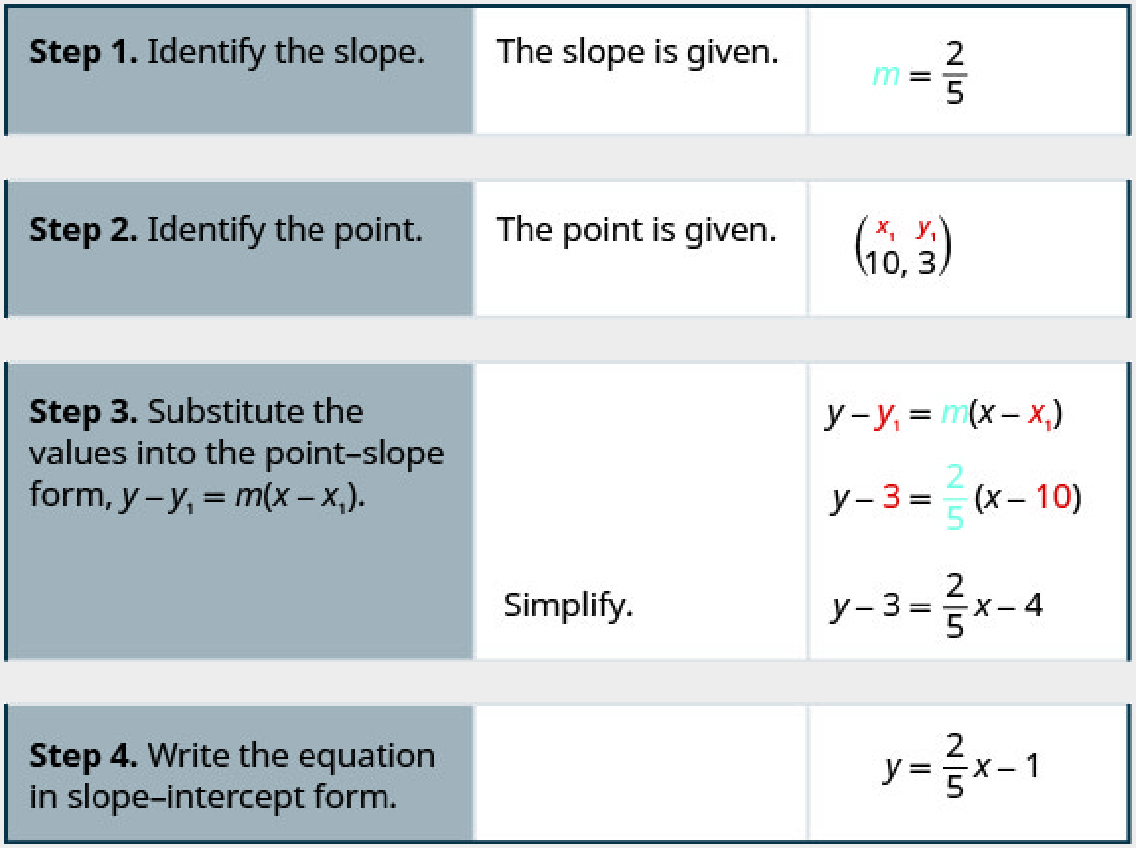
Example:
Find an Equation of a Line Given the Slope and a Point

Exercise:

Problem:

**Find an equation of a line with slope $m = \frac{2}{5}$ that contains the point $(10, 3)$. Write the equation in slope–intercept form.

Solution:
Solution



Step 1. Identify the slope.	The slope is given.	$m = \frac{2}{5}$
Step 2. Identify the point.	The point is given.	$(\overset{x_1}{10}, \overset{y_1}{3})$
Step 3. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.		$y - y_1 = m(x - x_1)$ $y - 3 = \frac{2}{5}(x - 10)$
	Simplify.	$y - 3 = \frac{2}{5}x - 4$
Step 4. Write the equation in slope–intercept form.		$y = \frac{2}{5}x - 1$

Note:
Exercise:

Problem:

TRY IT :: 4.117
Find an equation of a line with slope $m = \frac{5}{6}$ and containing the point $(6, 3)$.

Solution:

$y = \frac{5}{6}x - 2$

Note:
Exercise:

Problem:

TRY IT :: 4.118
Find an equation of a line with slope $m = \frac{2}{3}$ and containing thepoint $(9, 2)$.

Solution:

$y = \frac{2}{3}x - 4$

Note:
Find an equation of a line given the slope and a point.

Identify the slope.
Identify the point.
Substitute the values into the point-slope form,$y - y_1 = m\left(x - x_1\right)$.
Write the equation in slope–intercept form.

Example:
Exercise:

Problem:

**Find an equation of a line with slope $m = -\frac{1}{3}$ that contains the point $(6, -4)$. Write the equation in slope–intercept form.

Solution:
Solution

Since we are given a point and the slope of the line, we can substitute the needed values into the point–slope form, $y - y_1 = m\left(x - x_1\right)$.

Identify the slope.	$m = -\frac{1}{3}$
Identify the point.	$\left(\overset{x_1}{6}, \overset{y_1}{-4}\right)$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - (-4) = -\frac{1}{3}(x - 6)$
Simplify.	$y + 4 = -\frac{1}{3}x + 2$

Write in slope–intercept form.	$y=-\frac{1}{3}x-2$

Note:
Exercise:

Problem:

TRY IT :: 4.119
Find an equation of a line with slope $m=-\frac{2}{5}$ and containing the point $(10,-5)$.

Solution:

$y=-\frac{2}{5}x-1$

Note:
Exercise:

Problem:

TRY IT :: 4.120
Find an equation of a line with slope $m=-\frac{3}{4}$, and containing the point $(4,-7)$.

Solution:

$y=-\frac{3}{4}x-4$

Example:
Exercise:

Problem:

**Find an equation of a horizontal line that contains the point $(-1, 2)$. Write the equation in slope–intercept form.

Solution:
Solution

Every horizontal line has slope 0. We can substitute the slope and points into the point–slope form, $y - y_1 = m\left(x - x_1\right)$.

Identify the slope.	$m = 0$
Identify the point.	$\left(\overset{x_1}{-1}, \overset{y_1}{2}\right)$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - 2 = 0(x - (-1))$
Simplify.	$y - 2 = 0(x + 1)$

	$y - 2 = 0$
	$y = 2$
Write in slope–intercept form.	It is in *y*-form, but could be written $y = 0x + 2$.

Did we end up with the form of a horizontal line, $y = a$?

Note:
Exercise:

Problem:

TRY IT :: 4.121
Find an equation of a horizontal line containing the point $(-3, 8)$.

Solution:

$y = 8$

Note:
Exercise:

Problem:

TRY IT :: 4.122
Find an equation of a horizontal line containing the point $(-1, 4)$.

Solution:

$y = 4$

Find an Equation of the Line Given Two Points

When real-world data is collected, a linear model can be created from two data points. In the next example we'll see how to find an equation of a line when just two points are given.

We have two options so far for finding an equation of a line: slope–intercept or point–slope. Since we will know two points, it will make more sense to use the point–slope form.

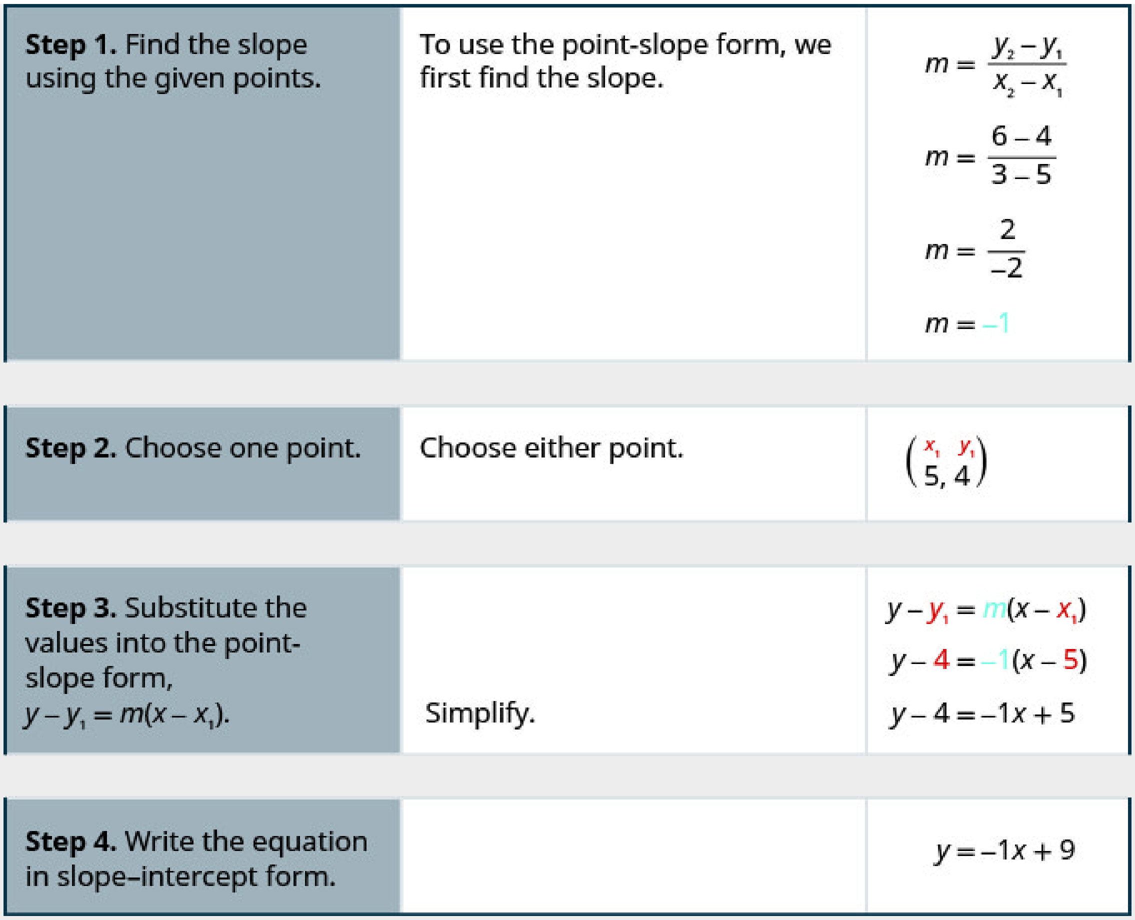
But then we need the slope. Can we find the slope with just two points? Yes. Then, once we have the slope, we can use it and one of the given points to find the equation.

Example:
Find an Equation of a Line Given Two Points
Exercise:

Problem:

**Find an equation of a line that contains the points $(5, 4)$ and $(3, 6)$. Write the equation in slope–intercept form.

Solution:
Solution



Step 1. Find the slope using the given points.	To use the point-slope form, we first find the slope.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{6-4}{3-5}$ $m = \frac{2}{-2}$ $m = -1$
Step 2. Choose one point.	Choose either point.	$\left(\overset{x_1}{5}, \overset{y_1}{4}\right)$
Step 3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 4 = -1(x - 5)$ $y - 4 = -1x + 5$
Step 4. Write the equation in slope–intercept form.		$y = -1x + 9$

Use the point $(3, 6)$ and see that you get the same equation.

Note:
Exercise:

Problem:

TRY IT :: 4.123
Find an equation of a line containing the points $(3, 1)$ and $(5, 6)$.

Solution:

$y = \frac{5}{2}x - \frac{13}{2}$

Note:
Exercise:

Problem:

TRY IT :: 4.124
Find an equation of a line containing the points $(1,4)$ and $(6,2)$.

Solution:

$y=-\frac{2}{5}x+\frac{22}{5}$

Note:
Find an equation of a line given two points.

Find the slope using the given points.
Choose one point.
Substitute the values into the point-slope form,$y-y_1=m\left(x-x_1\right)$.
Write the equation in slope–intercept form.

Example:
Exercise:

Problem:

**Find an equation of a line that contains the points $(-3,-1)$ and $(2,-2)$. Write the equation in slope–intercept form.

Solution:
Solution

Since we have two points, we will find an equation of the line using the point–slope form. The first step will be to find the slope.

Find the slope of the line through (−3, −1) and (2, −2).	$m = \frac{y_2 - y_1}{x_2 - x_1}$
	$m = \frac{-2 - (-1)}{2 - (-3)}$
	$m = \frac{-1}{5}$
	$m = -\frac{1}{5}$
Choose either point.	$\left(\overset{x_1}{2}, \overset{y_1}{-2}\right)$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - (-2) = -\frac{1}{5}(x - 2)$
	$y + 2 = -\frac{1}{5}x + \frac{2}{5}$

Write in slope–intercept form.	$y=-\frac{1}{5}x-\frac{8}{5}$

Note:
Exercise:

Problem:

TRY IT :: 4.125
Find an equation of a line containing the points $(-2,-4)$ and $(1,-3)$.

Solution:

$y=\frac{1}{3}x-\frac{10}{3}$

Note:
Exercise:

Problem:

TRY IT :: 4.126
Find an equation of a line containing the points $(-4,-3)$ and $(1,-5)$.

Solution:

$y=-\frac{2}{5}x-\frac{23}{5}$

Example:
Exercise:

Problem:

**Find an equation of a line that contains the points $(-2,4)$ and $(-2,-3)$. Write the equation in slope–intercept form.

Solution:
Solution

Again, the first step will be to find the slope.

Find the slope of the line through $(-2,4)$ and $(-2,-3)$.	$m=\frac{y_2-y_1}{x_2-x_1}$
	$m=\frac{-3-4}{-2-(-2)}$
	$m=\frac{-7}{0}$
	The slope is undefined.

This tells us it is a vertical line. Both of our points have an *x*-coordinate of -2. So our equation of the line is $x=-2$. Since there is no y, we cannot write it in slope–intercept form.

You may want to sketch a graph using the two given points. Does the graph agree with our conclusion that this is a vertical line?

Note:
Exercise:

Problem:

TRY IT :: 4.127
Find an equation of a line containing the points $(5, 1)$ and $(5, -4)$.

Solution:

$x = 5$

Note:
Exercise:

Problem:

TRY IT :: 4.128
Find an equaion of a line containing the points $(-4, 4)$ and $(-4, 3)$.

Solution:

$x = -4$

We have seen that we can use either the slope–intercept form or the point–slope form to find an equation of a line. Which form we use will depend on the information we are given. This is summarized in [link].

To Write an Equation of a Line		

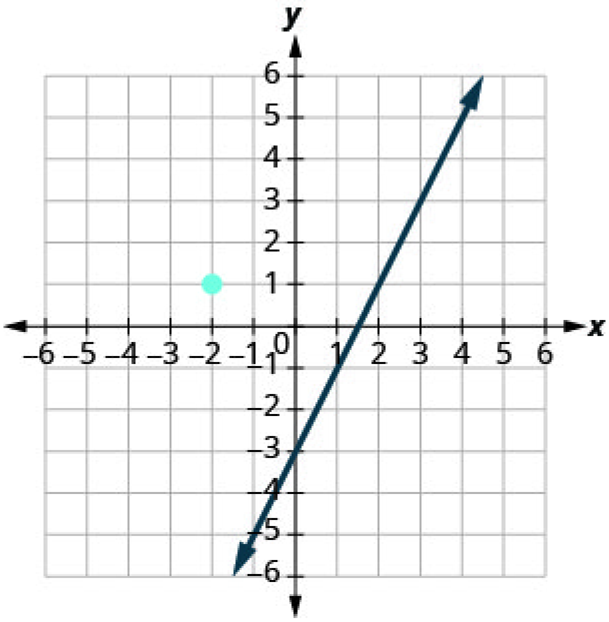
If given:	Use:	Form:
Slope and *y*-intercept	slope–intercept	$y = mx + b$
Slope and a point	point–slope	$y - y_1 = m\left(x - x_1\right)$
Two points	point–slope	$y - y_1 = m\left(x - x_1\right)$

Find an Equation of a Line Parallel to a Given Line

Suppose we need to find an equation of a line that passes through a specific point and is parallel to a given line. We can use the fact that parallel lines have the same slope. So we will have a point and the slope—just what we need to use the point–slope equation.

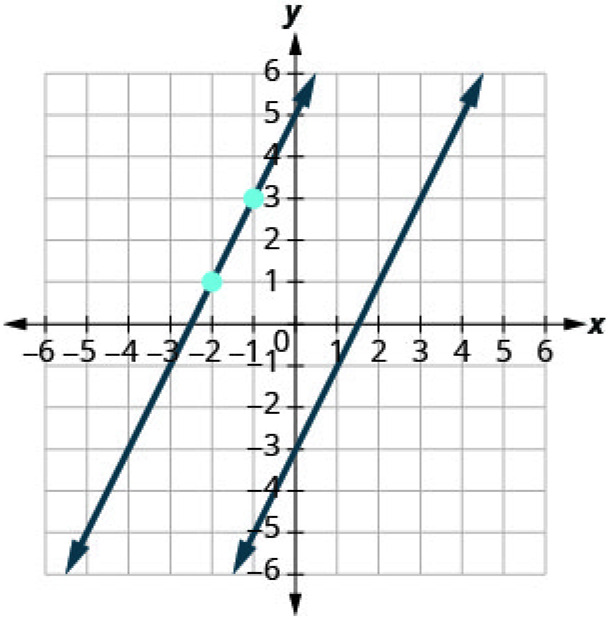
First let's look at this graphically.

The graph shows the graph of $y = 2x - 3$. We want to graph a line parallel to this line and passing through the point $\left(-2, 1\right)$.



We know that parallel lines have the same slope. So the second line will have the same slope as$y = 2x - 3$. That slope is$m_{\parallel} = 2$. We'll use the notation $m_{\|}$ to represent the slope of a line parallel to a line with slope m. (Notice that the subscript $\|$ looks like two parallel lines.)

The second line will pass through $(-2, 1)$ and have $m = 2$. To graph the line, we start at$(-2, 1)$ and count out the rise and run. With $m = 2$ (or $m = \frac{2}{1}$), we count out the rise 2 and the run 1. We draw the line.



Do the lines appear parallel? Does the second line pass through $(-2, 1)$?

Now, let's see how to do this algebraically.

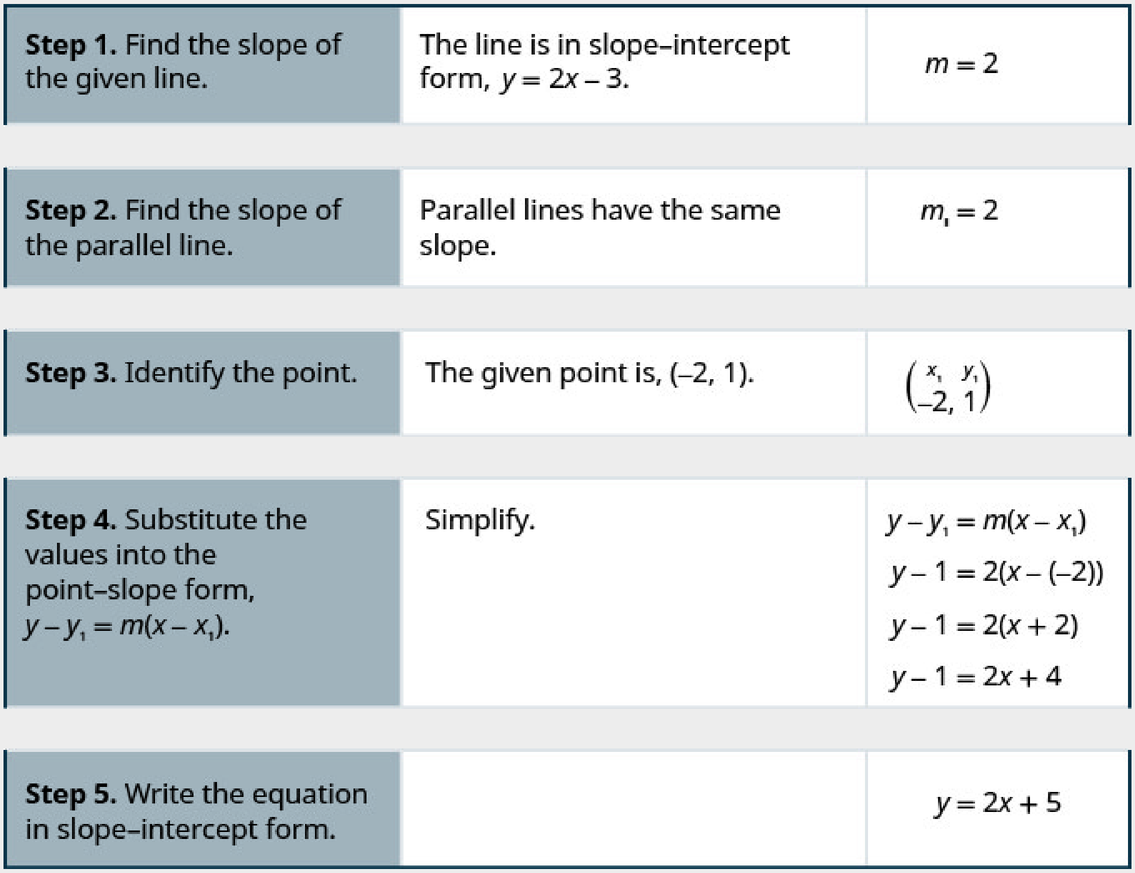
We can use either the slope–intercept form or the point–slope form to find an equation of a line. Here we know one point and can find the slope. So we will use the point–slope form.

Example:
How to Find an Equation of a Line Parallel to a Given Line
Exercise:

Problem:

**Find an equation of a line parallel to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution:
Solution



Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the parallel line.	Parallel lines have the same slope.	$m_{\parallel} = 2$
Step 3. Identify the point.	The given point is, $(-2, 1)$.	$\left(\overset{x_1}{-2}, \overset{y_1}{1}\right)$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = 2(x - (-2))$ $y - 1 = 2(x + 2)$ $y - 1 = 2x + 4$
Step 5. Write the equation in slope–intercept form.		$y = 2x + 5$

Does this equation make sense? What is the *y*-intercept of the line? What is the slope?

Note:
Exercise:

Problem:

TRY IT :: 4.129
Find an equation of a line parallel to the line $y = 3x + 1$ that contains the point $(4, 2)$. Write the equation in slope–intercept form.

Solution:

$y = 3x - 10$

Note:
Exercise:

Problem:

TRY IT :: 4.130
Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

Solution:

$y = \frac{1}{2}x + 1$

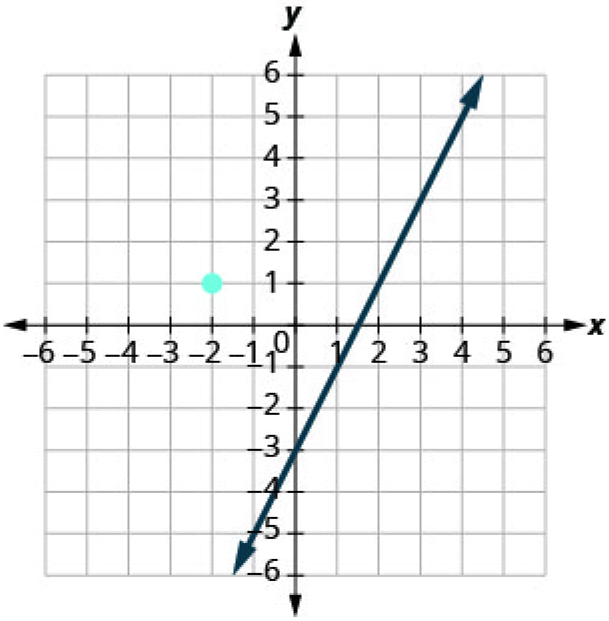
Note:
Find an equation of a line parallel to a given line.

Find the slope of the given line.
Find the slope of the parallel line.
Identify the point.
Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
Write the equation in slope–intercept form.

Find an Equation of a Line Perpendicular to a Given Line

Now, let's consider perpendicular lines. Suppose we need to find a line passing through a specific point and which is perpendicular to a given line. We can use the fact that perpendicular lines have slopes that are negative reciprocals. We will again use the point–slope equation, like we did with parallel lines.

The graph shows the graph of $y = 2x - 3$. Now, we want to graph a line perpendicular to this line and passing through $(-2, 1)$.

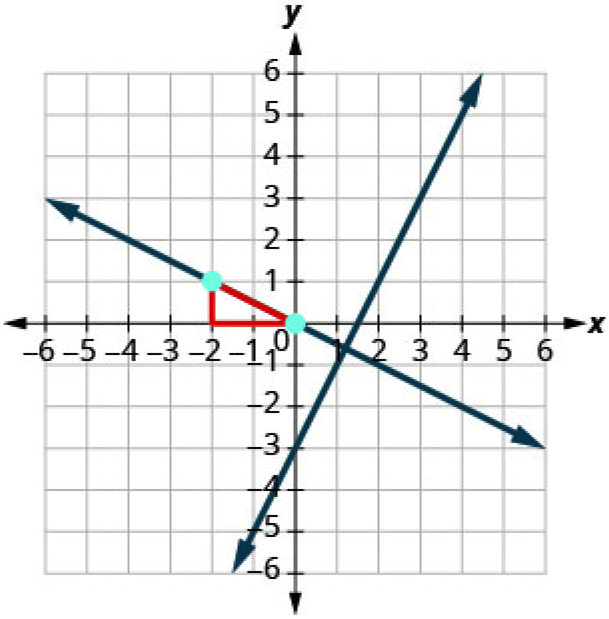


We know that perpendicular lines have slopes that are negative reciprocals. We'll use the notation $m_\perp$ to represent the slope of a line perpendicular to a line with slope m. (Notice that the subscript $\perp$ looks like the right angles made by two perpendicular lines.)
Equation:

$$\begin{array}{cc} y = 2x - 3 & \text{perpendicular line} \\ m = 2 & m_\perp = -\frac{1}{2} \end{array}$$

We now know the perpendicular line will pass through $(-2, 1)$ with $m_\perp = -\frac{1}{2}$.

To graph the line, we will start at $(-2, 1)$ and count out the rise -1 and the run 2. Then we draw the line.



Do the lines appear perpendicular? Does the second line pass through $(-2, 1)$?

Now, let’s see how to do this algebraically. We can use either the slope–intercept form or the point–slope form to find an equation of a line. In this example we know one point, and can find the slope, so we will use the point–slope form.

Example:
How to Find an Equation of a Line Perpendicular to a Given Line
Exercise:

Problem:

**Find an equation of a line perpendicular to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution:
Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the perpendicular line.	The slopes of perpendicular lines are negative reciprocals.	$m_{\perp} = -\frac{1}{2}$
Step 3. Identify the point.	The given point is, $(-2, 1)$	$\left(\overset{x_1}{-2}, \overset{y_1}{1}\right)$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = -\frac{1}{2}(x - (-2))$ $y - 1 = -\frac{1}{2}(x + 2)$ $y - 1 = -\frac{1}{2}x - 1$
Step 5. Write the equation in slope–intercept form.		$y = -\frac{1}{2}x$

Note:
Exercise:

Problem:

TRY IT :: 4.131

Find an equation of a line perpendicular to the line $y = 3x + 1$ that contains the point $(4, 2)$. Write the equation in slope–intercept form.

Solution:

$y = -\frac{1}{3}x + \frac{10}{3}$

Note:

Exercise:

Problem:

TRY IT :: 4.132

Find an equation of a line perpendicular to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

Solution:

$y = -2x + 16$

Note:

Find an equation of a line perpendicular to a given line.

Find the slope of the given line.
Find the slope of the perpendicular line.
Identify the point.
Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
Write the equation in slope–intercept form.

Example:
Exercise:

Problem:

**Find an equation of a line perpendicular to $x=5$ that contains the point $(3,-2)$. Write the equation in slope–intercept form.

Solution:
Solution

Again, since we know one point, the point–slope option seems more promising than the slope–intercept option. We need the slope to use this form, and we know the new line will be perpendicular to $x=5$. This line is vertical, so its perpendicular will be horizontal. This tells us the $m_{\perp}=0$.

Identify the point.	$(3,-2)$
Identify the slope of the perpendicular line.	$m_{\perp}=0$
Substitute the values into $y-y_1=m\left(x-x_1\right)$.	$y-y_1=m\left(x-x_1\right)$ $y-(-2)=0\left(x-3\right)$ $y+2=0$
Simplify.	$y=-2$

Sketch the graph of both lines. Do they appear to be perpendicular?

Note:
Exercise:

Problem:

TRY IT :: 4.133
Find an equation of a line that is perpendicular to the line $x = 4$ that contains the point $(4, -5)$. Write the equation in slope–intercept form.

Solution:

$y = -5$

Note:
Exercise:

Problem:

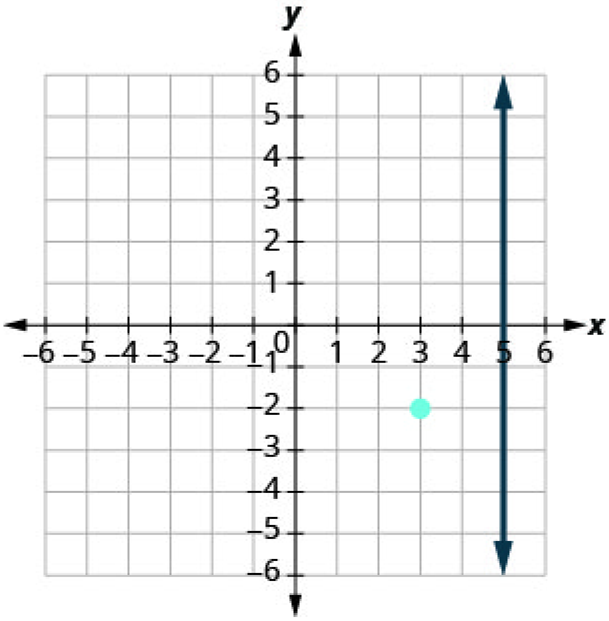
TRY IT :: 4.134
Find an equation of a line that is perpendicular to the line $x = 2$ that contains the point $(2, -1)$. Write the equation in slope–intercept form.

Solution:

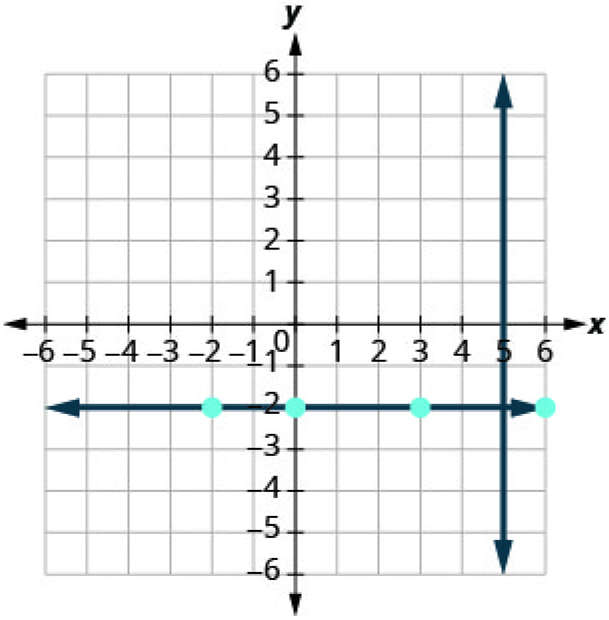
$y = -1$

In [link], we used the point–slope form to find the equation. We could have looked at this in a different way.

We want to find a line that is perpendicular to $x = 5$ that contains the point $(3, -2)$. The graph shows us the line$x = 5$ and the point $(3, -2)$.



We know every line perpendicular to a vetical line is horizontal, so we will sketch the horizontal line through $(3, -2)$.



Do the lines appear perpendicular?

If we look at a few points on this horizontal line, we notice they all have *y*-coordinates of -2. So, the equation of the line perpendicular to the vertical line $x = 5$ is $y = -2$.

Example:
Exercise:

Problem:

**Find an equation of a line that is perpendicular to $y = -4$ that contains the point $(-4, 2)$. Write the equation in slope–intercept form.

Solution:
Solution

The line $y = -4$ is a horizontal line. Any line perpendicular to it must be vertical, in the form $x = a$. Since the perpendicular line is vertical and passes through $(-4, 2)$, every point on it has an *x*-coordinate of -4. The equation of the perpendicular line is $x = -4$. You may want to sketch the lines. Do they appear perpendicular?

Note:
Exercise:

Problem:

TRY IT :: 4.135
Find an equation of a line that is perpendicular to the line $y = 1$ that contains the point $(-5, 1)$. Write the equation in slope–intercept form.

Solution:

$x = -5$

Note:
Exercise:

Problem:

TRY IT :: 4.136
Find an equation of a line that is perpendicular to the line $y = -5$ that contains the point $(-4, -5)$.

Solution:

$x = -4$

Note:
Access this online resource for additional instruction and practice with finding the equation of a line.

- Use the Point-Slope Form of an Equation of a Line

Key Concepts

- **To Find an Equation of a Line Given the Slope and a Point**

  Identify the slope.
  Identify the point.
  Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
  Write the equation in slope-intercept form.

- **To Find an Equation of a Line Given Two Points**

  Find the slope using the given points.
  Choose one point.
  Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.

Write the equation in slope-intercept form.

- **To Write and Equation of a Line**
  - If given slope and *y*-intercept, use slope–intercept form $y = mx + b$.
  - If given slope and a point, use point–slope form $y - y_1 = m\left(x - x_1\right)$.
  - If given two points, use point–slope form $y - y_1 = m\left(x - x_1\right)$.

- **To Find an Equation of a Line Parallel to a Given Line**

  Find the slope of the given line.
  Find the slope of the parallel line.
  Identify the point.
  Substitute the values into the point-slope form,$y - y_1 = m\left(x - x_1\right)$.
  Write the equation in slope-intercept form.

- **To Find an Equation of a Line Perpendicular to a Given Line**

  Find the slope of the given line.
  Find the slope of the perpendicular line.
  Identify the point.
  Substitute the values into the point-slope form,$y - y_1 = m\left(x - x_1\right)$.
  Write the equation in slope-intercept form.

Practice Makes Perfect

Find an Equation of the Line Given the Slope and *y*-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

Exercise:

Problem: slope 3 and *y*-intercept $(0,5)$

Exercise:

Problem: slope 4 and *y*-intercept $(0,1)$

Solution:

$y=4x+1$

Exercise:

Problem: slope 6 and *y*-intercept $(0,-4)$

Exercise:

Problem: slope 8 and *y*-intercept $(0,-6)$

Solution:

$y=8x-6$

Exercise:

Problem: slope -1 and *y*-intercept $(0,3)$

Exercise:

Problem: slope -1 and *y*-intercept $(0,7)$

Solution:

$y=-x+7$

Exercise:

Problem: slope -2 and *y*-intercept $(0,-3)$

Exercise:

Problem: slope -3 and *y*-intercept $(0,-1)$

Solution:

$y=-3x-1$

Exercise:

Problem: slope $\frac{3}{5}$ and *y*-intercept $(0,-1)$

Exercise:

Problem: slope $\frac{1}{5}$ and *y*-intercept $(0,-5)$

Solution:

$y=\frac{1}{5}x-5$

Exercise:

Problem: slope $-\frac{3}{4}$ and *y*-intercept $(0,-2)$

Exercise:

Problem: slope $-\frac{2}{3}$ and *y*-intercept $(0,-3)$

Solution:

$y=-\frac{2}{3}x-3$

Exercise:

Problem: slope 0 and *y*-intercept $(0, -1)$

Exercise:

Problem: slope 0 and *y*-intercept $(0, 2)$

Solution:

$y = 2$

Exercise:

Problem: slope -3 and *y*-intercept $(0, 0)$

Exercise:

Problem: slope -4 and *y*-intercept $(0, 0)$

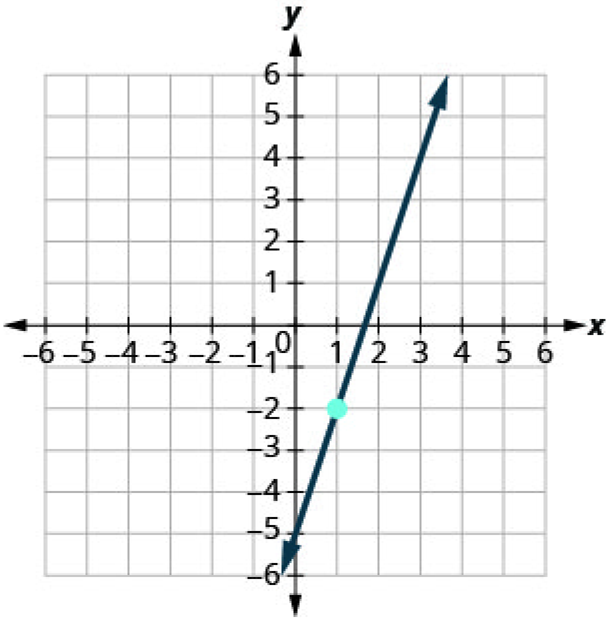
Solution:

$y = -4x$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

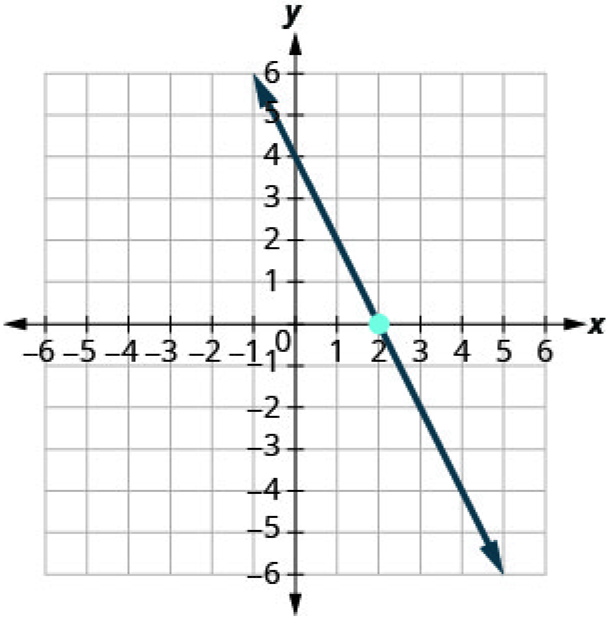
Exercise:

Problem:



Exercise:

Problem:

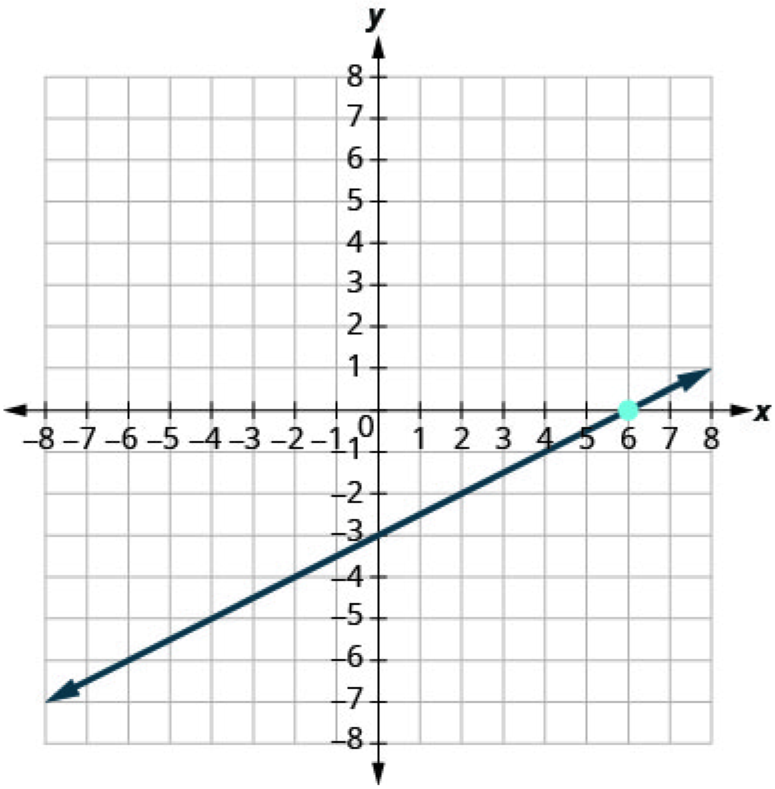


Solution:

$y = -2x + 4$

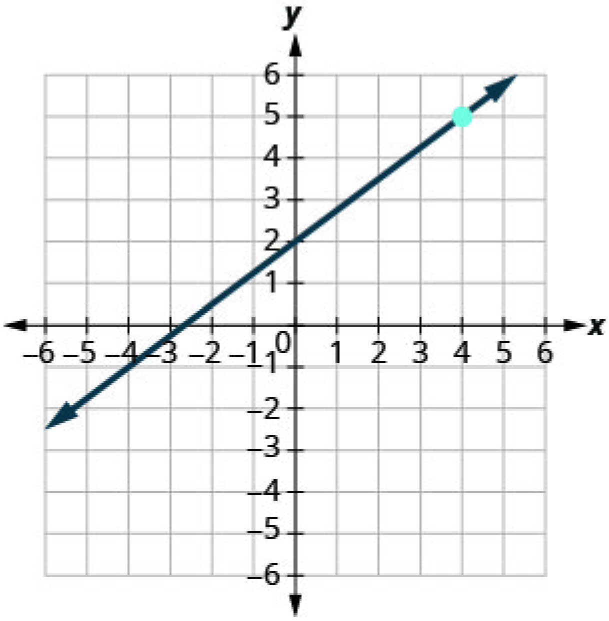
Exercise:

Problem:



Exercise:

Problem:

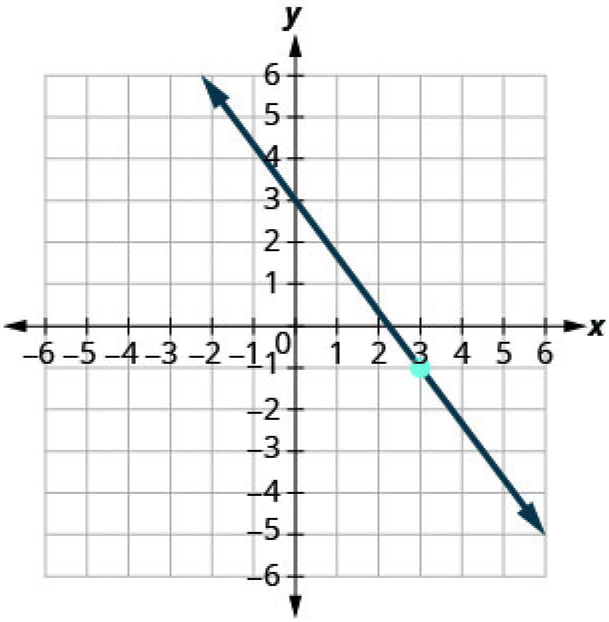


Solution:

$y=\frac{3}{4}x+2$

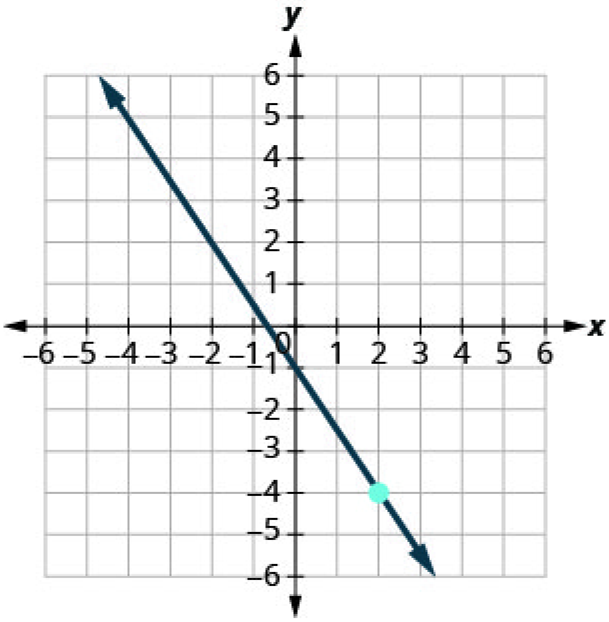
Exercise:

Problem:



Exercise:

Problem:

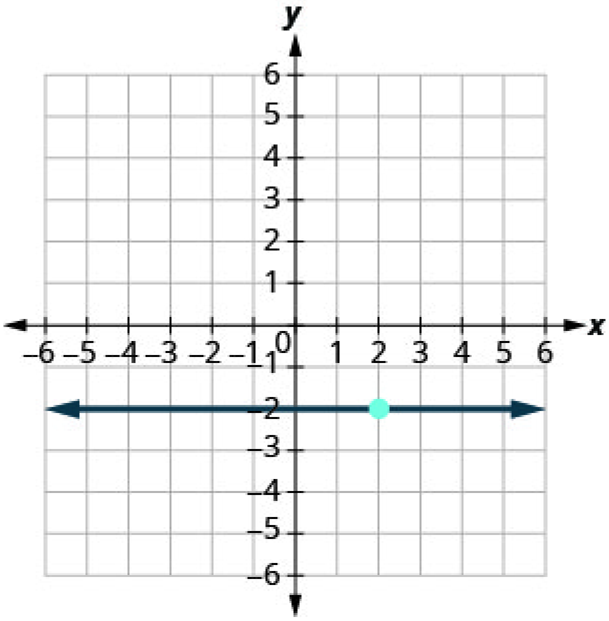


Solution:

$y=-\frac{3}{2}x-1$

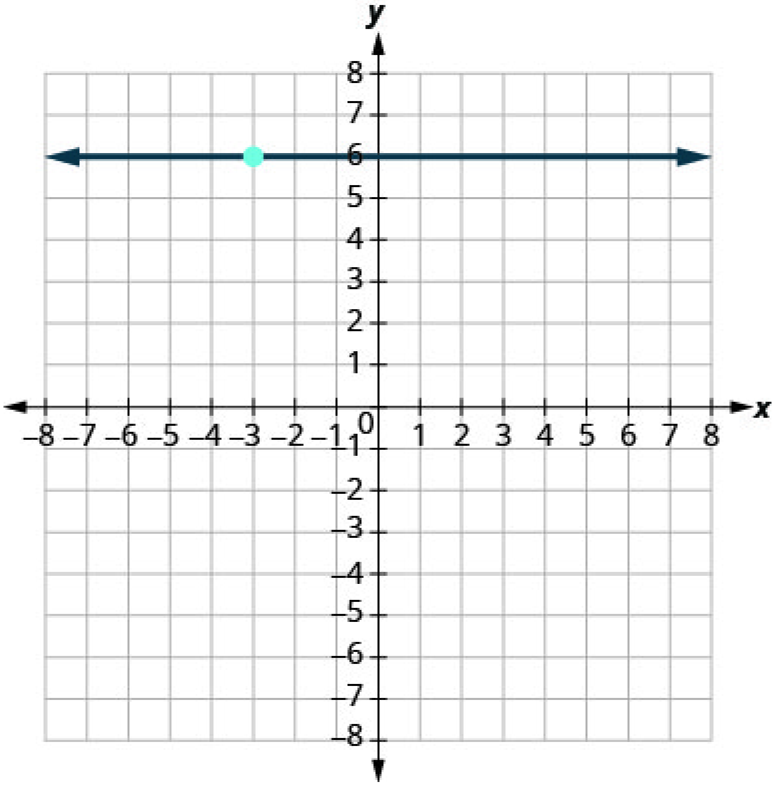
Exercise:

Problem:



Exercise:

Problem:



Solution:

$y = 6$

Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

Exercise:

Problem: $m = \frac{5}{8}$, point $(8, 3)$

Exercise:

Problem: $m = \frac{3}{8}$, point $(8, 2)$

Solution:

$y = \frac{3}{8}x - 1$

Exercise:

Problem: $m = \frac{1}{6}$, point $(6, 1)$

Exercise:

Problem: $m = \frac{5}{6}$, point $(6, 7)$

Solution:

$y = \frac{5}{6}x + 2$

Exercise:

Problem: $m = -\frac{3}{4}$, point $(8, -5)$

Exercise:

Problem: $m = -\frac{3}{5}$, point $(10, -5)$

Solution:

$y = -\frac{3}{5}x + 1$

Exercise:

Problem: $m = -\frac{1}{4}$, point $(-12, -6)$

Exercise:

Problem: $m = -\frac{1}{3}$, point $(-9, -8)$

Solution:

$y = -\frac{1}{3}x - 11$

Exercise:

Problem: Horizontal line containing $(-2, 5)$

Exercise:

Problem: Horizontal line containing $(-1, 4)$

Solution:

$y = 4$

Exercise:

Problem: Horizontal line containing $(-2, -3)$

Exercise:

Problem: Horizontal line containing $(-1,-7)$

Solution:

$y=-7$

Exercise:

Problem: $m=-\frac{3}{2}$, point $(-4,-3)$

Exercise:

Problem: $m=-\frac{5}{2}$, point $(-8,-2)$

Solution:

$y=-\frac{5}{2}x-22$

Exercise:

Problem: $m=-7$, point $(-1,-3)$

Exercise:

Problem: $m=-4$, point $(-2,-3)$

Solution:

$y=-4x-11$

Exercise:

Problem: Horizontal line containing $(2,-3)$

Exercise:

Problem: Horizontal line containing $(4,-8)$

Solution:

$y=-8$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

Exercise:

Problem: $(2,6)$ and $(5,3)$

Exercise:

Problem: $(3,1)$ and $(2,5)$

Solution:

$y=-4x+13$

Exercise:

Problem: $(4,3)$ and $(8,1)$

Exercise:

Problem: $(2,7)$ and $(3,8)$

Solution:

$y=x+5$

Exercise:

Problem: $(-3,-4)$ and $(5-2)$

Exercise:

Problem: $(-5,-3)$ and $(4,-6)$

Solution:

$y=-\frac{1}{3}x-\frac{14}{3}$

Exercise:

Problem: $(-1,3)$ and $(-6,-7)$

Exercise:

Problem: $(-2,8)$ and $(-4,-6)$

Solution:

$y=7x+22$

Exercise:

Problem: $(6,-4)$ and $(-2,5)$

Exercise:

Problem: $(3,-2)$ and $(-4,4)$

Solution:

$y=-\frac{6}{7}x+\frac{4}{7}$

Exercise:

Problem: $(0,4)$ and $(2,-3)$

Exercise:

Problem: $(0,-2)$ and $(-5,-3)$

Solution:

$y=\frac{1}{5}x-2$

Exercise:

Problem: $(7,2)$ and $(7,-2)$

Exercise:

Problem: $(4,2)$ and $(4,-3)$

Solution:

$x=4$

Exercise:

Problem: $(-7,-1)$ and $(-7,-4)$

Exercise:

Problem: $(-2,1)$ and $(-2,-4)$

Solution:

$x=-2$

Exercise:

Problem: $(6,1)$ and $(0,1)$

Exercise:

Problem: $(6,2)$ and $(-3,2)$

Solution:

$y = 2$

Exercise:

Problem: $(3, -4)$ and $(5, -4)$

Exercise:

Problem: $(-6, -3)$ and $(-1, -3)$

Solution:

$y = -3$

Exercise:

Problem: $(4, 3)$ and $(8, 0)$

Exercise:

Problem: $(0, 0)$ and $(1, 4)$

Solution:

$y = 4x$

Exercise:

Problem: $(-2, -3)$ and $(-5, -6)$

Exercise:

Problem: $(-3, 0)$ and $(-7, -2)$

Solution:

$y = \frac{1}{2}x + \frac{3}{2}$

Exercise:

Problem: $(8,-1)$ and $(8,-5)$

Exercise:

Problem: $(3,5)$ and $(-7,5)$

Solution:

$y=5$

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise:

Problem: line $y=4x+2$, point $(1,2)$

Exercise:

Problem: line $y=3x+4$, point $(2,5)$

Solution:

$y=3x-1$

Exercise:

Problem: line $y=-2x-3$, point $(-1,3)$

Exercise:

Problem: line $y=-3x-1$, point $(2,-3)$

Solution:

$y = -3x + 3$

Exercise:

Problem: line $3x - y = 4$, point $(3, 1)$

Exercise:

Problem: line $2x - y = 6$, point $(3, 0)$

Solution:

$y = 2x - 6$

Exercise:

Problem: line $4x + 3y = 6$, point $(0, -3)$

Exercise:

Problem: line $2x + 3y = 6$, point $(0, 5)$

Solution:

$y = -\frac{2}{3}x + 5$

Exercise:

Problem: line $x = -3$, point $(-2, -1)$

Exercise:

Problem: line $x = -4$, point $(-3, -5)$

Solution:

$x = -3$

Exercise:

Problem: line $x - 2 = 0$, point $(1, -2)$

Exercise:

Problem: line $x - 6 = 0$, point $(4, -3)$

Solution:

$x = 4$

Exercise:

Problem: line $y = 5$, point $(2, -2)$

Exercise:

Problem: line $y = 1$, point $(3, -4)$

Solution:

$y = -4$

Exercise:

Problem: line $y + 2 = 0$, point $(3, -3)$

Exercise:

Problem: line $y + 7 = 0$, point $(1, -1)$

Solution:

$y = -1$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise:

Problem: line $y=-2x+3$, point $(2,2)$

Exercise:

Problem: line $y=-x+5$, point $(3,3)$

Solution:

$y=x$

Exercise:

Problem: line $y=\frac{3}{4}x-2$, point $(-3,4)$

Exercise:

Problem: line $y=\frac{2}{3}x-4$, point $(2,-4)$

Solution:

$y=-\frac{3}{2}x-1$

Exercise:

Problem: line $2x-3y=8$, point $(4,-1)$

Exercise:

Problem: line $4x-3y=5$, point $(-3,2)$

Solution:

$y=-\frac{3}{4}x-\frac{1}{4}$

Exercise:

Problem: line $2x+5y=6$, point $(0,0)$

Exercise:

Problem: line $4x+5y=-3$, point $(0,0)$

Solution:

$y=\frac{5}{4}x$

Exercise:

Problem: line $y-3=0$, point $(-2,-4)$

Exercise:

Problem: line $y-6=0$, point $(-5,-3)$

Solution:

$x=-5$

Exercise:

Problem: line *y*-axis, point $(3,4)$

Exercise:

Problem: line *y*-axis, point $(2,1)$

Solution:

$y=1$

Mixed Practice

In the following exercises, find the equation of each line. Write the equation in slope–intercept form.

Exercise:

Problem: Containing the points $(4, 3)$ and $(8, 1)$

Exercise:

Problem: Containing the points $(2, 7)$ and $(3, 8)$

Solution:

$y = x + 5$

Exercise:

Problem: $m = \frac{1}{6}$, containing point $(6, 1)$

Exercise:

Problem: $m = \frac{5}{6}$, containing point $(6, 7)$

Solution:

$y = \frac{5}{6}x + 2$

Exercise:

Problem: Parallel to the line $4x + 3y = 6$, containing point $(0, -3)$

Exercise:

Problem: Parallel to the line $2x + 3y = 6$, containing point $(0, 5)$

Solution:

$y = -\frac{2}{3}x + 5$

Exercise:

Problem: $m = -\frac{3}{4}$, containing point $(8, -5)$

Exercise:

Problem: $m = -\frac{3}{5}$, containing point $(10, -5)$

Solution:

$y = -\frac{3}{5}x + 1$

Exercise:

Problem: Perpendicular to the line $y - 1 = 0$, point $(-2, 6)$

Exercise:

Problem: Perpendicular to the line *y*-axis, point $(-6, 2)$

Solution:

$y = 2$

Exercise:

Problem: Containing the points $(4, 3)$ and $(8, 1)$

Exercise:

Problem: Containing the points $(-2, 0)$ and $(-3, -2)$

Solution:

$y = x + 2$

Exercise:

Problem: Parallel to the line $x=-3$, containing point $(-2,-1)$

Exercise:

Problem: Parallel to the line $x=-4$, containing point $(-3,-5)$

Solution:

$x=-3$

Exercise:

Problem: Containing the points $(-3,-4)$ and $(2,-5)$

Exercise:

Problem: Containing the points $(-5,-3)$ and $(4,-6)$

Solution:

$y=-\frac{1}{3}x-\frac{14}{3}$

Exercise:

Problem:

Perpendicular to the line $x-2y=5$, containing point $(-2,2)$

Exercise:

Problem:

Perpendicular to the line $4x+3y=1$, containing point $(0,0)$

Solution:

$y=\frac{3}{4}x$

Everyday Math

Exercise:

Problem:

Cholesterol. The age, x, and LDL cholesterol level, y, of two men are given by the points $(18, 68)$ and $(27, 122)$. Find a linear equation that models the relationship between age and LDL cholesterol level.

Exercise:

Problem:

Fuel consumption. The city mpg, x, and highway mpg, y, of two cars are given by the points $(29, 40)$ and$(19, 28)$. Find a linear equation that models the relationship between city mpg and highway mpg.

Solution:

$y = 1.2x + 5.2$

Writing Exercises

Exercise:

Problem: Why are all horizontal lines parallel?

Exercise:

Problem:

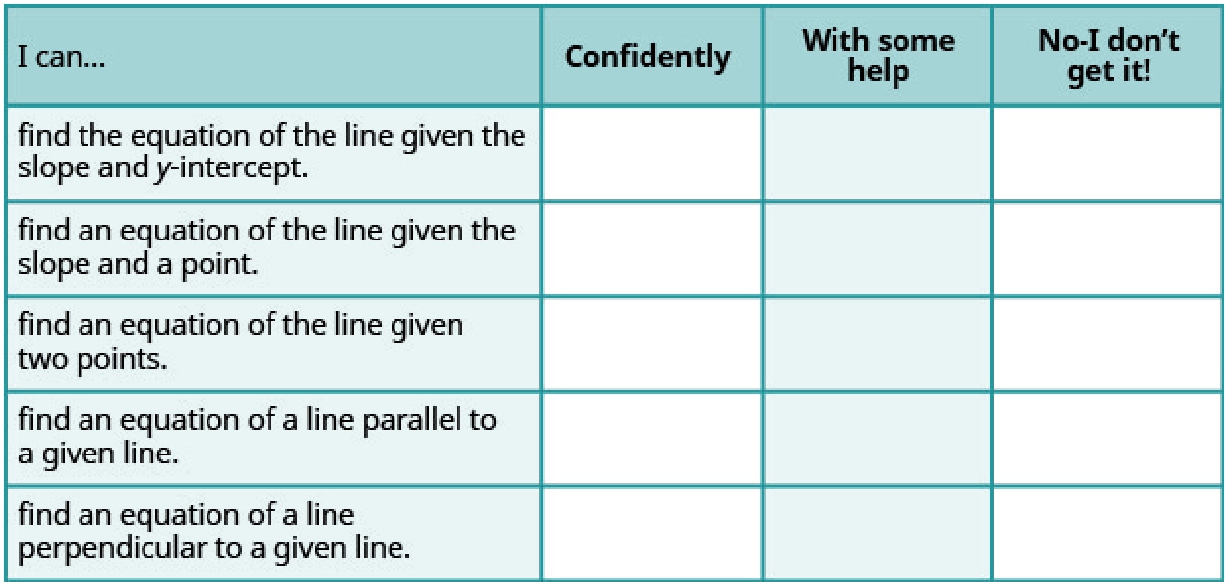
Explain in your own words why the slopes of two perpendicular lines must have opposite signs.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
find the equation of the line given the slope and *y*-intercept.			
find an equation of the line given the slope and a point.			
find an equation of the line given two points.			
find an equation of a line parallel to a given line.			
find an equation of a line perpendicular to a given line.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

point–slope form

The point–slope form of an equation of a line with slope m and containing the point (x_1, y_1) is $y - y_1 = m\,(x - x_1)$.

Review & Practice Test
By the end of this section, you will be prepared to take your Chapter 4 Assessment.

Chapter 4 Review Exercises

Rectangular Coordinate System

Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system.

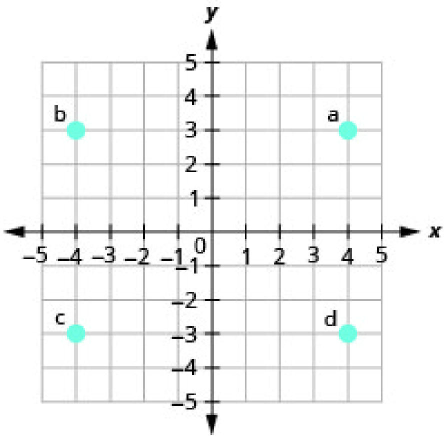
Exercise:

ⓐ $(-1,-5)$
ⓑ $(-3,4)$
ⓒ $(2,-3)$
Problem: ⓓ $\left(1,\frac{5}{2}\right)$

Exercise:

ⓐ $(4,3)$
ⓑ $(-4,3)$
ⓒ $(-4,-3)$
Problem: ⓓ $(4,-3)$

Solution:



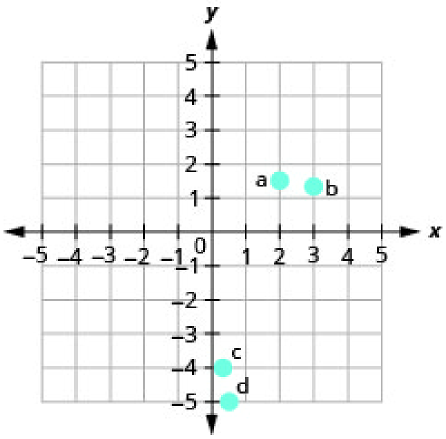
Exercise:

ⓐ $(-2, 0)$
ⓑ $(0, -4)$
ⓒ $(0, 5)$
Problem: ⓓ $(3, 0)$

Exercise:

ⓐ $\left(2, \frac{3}{2}\right)$
ⓑ $\left(3, \frac{4}{3}\right)$
ⓒ $\left(\frac{1}{3}, -4\right)$
Problem: ⓓ $\left(\frac{1}{2}, -5\right)$

Solution:

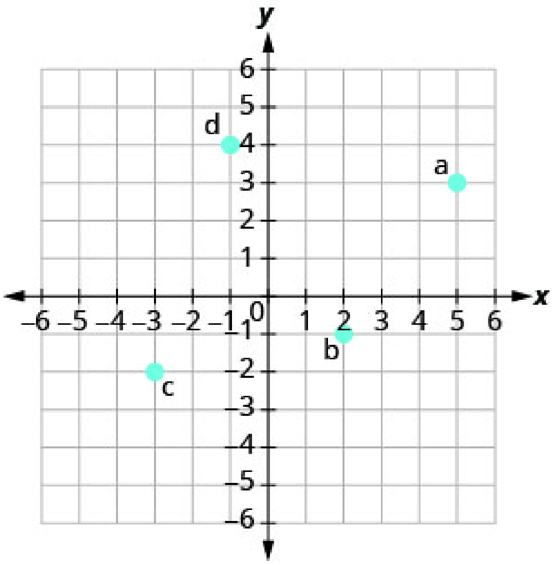


Identify Points on a Graph

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

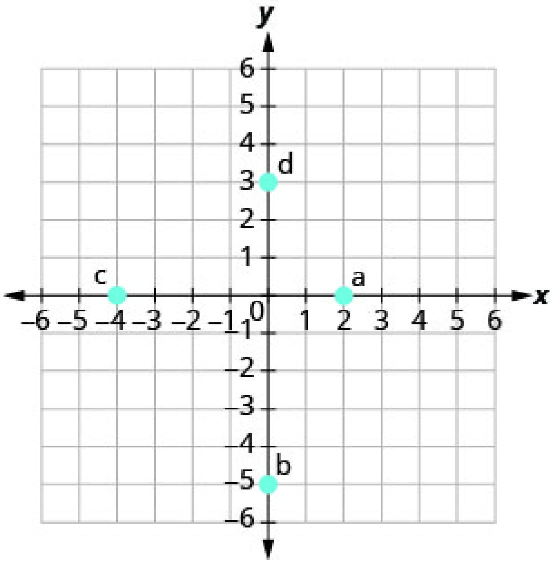
Exercise:

Problem:



Exercise:

Problem:



Solution:

ⓐ $(2,0)$ ⓑ $(0,-5)$ ⓒ (-4.0) ⓓ $(0,3)$

Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise:

Problem: $5x+y=10$

ⓐ $(5,1)$
ⓑ $(2,0)$
ⓒ $(4,-10)$

Exercise:

Problem: $y=6x-2$

ⓐ $(1,4)$
ⓑ $\left(\frac{1}{3},0\right)$
ⓒ $(6,-2)$

Solution:

a, b

Complete a Table of Solutions to a Linear Equation in Two Variables

In the following exercises, complete the table to find solutions to each linear equation.

Exercise:

Problem: $y=4x-1$

x	y	(x,y)
0		
1		
-2		

Exercise:

Problem: $y = -\frac{1}{2}x + 3$

x	y	(x, y)
0		
4		
-2		

Solution:

x	y	(x, y)
0	3	$(0, 3)$
4	1	(4, 1)
-2	4	$(-2, 4)$

Exercise:

Problem: $x + 2y = 5$

x	y	(x,y)
	0	
1		
-1		

Exercise:

Problem: $3x+2y=6$

x	y	(x,y)
0		
	0	
-2		

Solution:

x	y	(x,y)
0	-3	$(0,-3)$
2	0	$(2,0)$

-2	-6	$(-2, -6)$

Find Solutions to a Linear Equation in Two Variables

In the following exercises, find three solutions to each linear equation.
Exercise:

Problem: $x + y = 3$

Exercise:

Problem: $x + y = -4$

Solution:

Answers will vary.

Exercise:

Problem: $y = 3x + 1$

Exercise:

Problem: $y = -x - 1$

Solution:

Answers will vary.

Graphing Linear Equations

Recognize the Relation Between the Solutions of an Equation and its Graph

In the following exercises, for each ordered pair, decide:

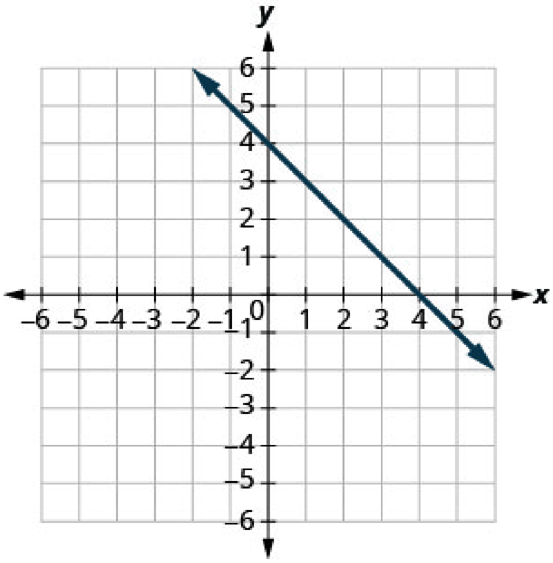
ⓐ Is the ordered pair a solution to the equation?
ⓑ Is the point on the line?

Exercise:

Problem: $y=-x+4$

$(0,4)$ $(-1,3)$

$(2,2)$ $(-2,6)$

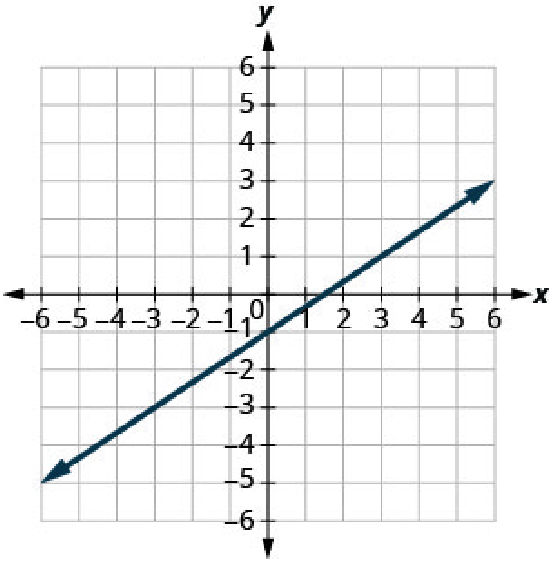


Exercise:

Problem: $y=\frac{2}{3}x-1$

$(0,-1)$ (3, 1)

$(-3,-3)$ (6, 4)



Solution:

ⓐ yes; yes ⓑ yes; no

Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

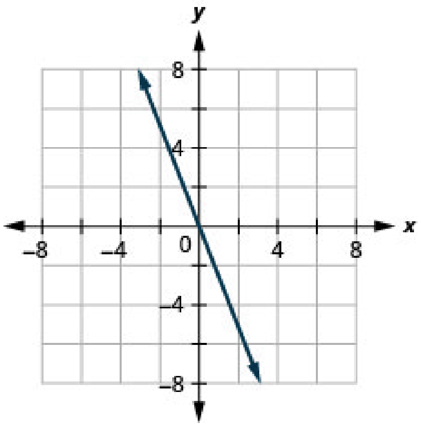
Exercise:

Problem: $y = 4x - 3$

Exercise:

Problem: $y = -3x$

Solution:



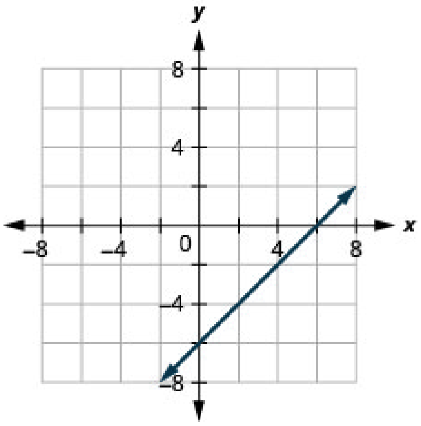
Exercise:

Problem: $y = \frac{1}{2}x + 3$

Exercise:

Problem: $x - y = 6$

Solution:



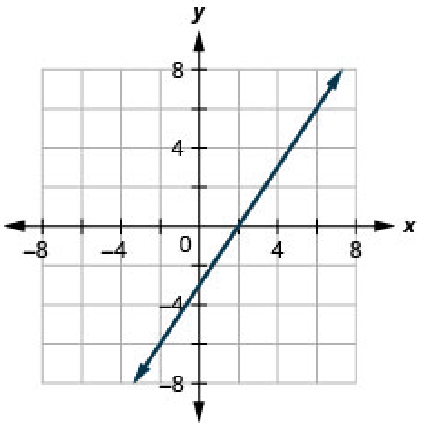
Exercise:

Problem: $2x + y = 7$

Exercise:

Problem: $3x - 2y = 6$

Solution:



Graph Vertical and Horizontal lines

In the following exercises, graph each equation.

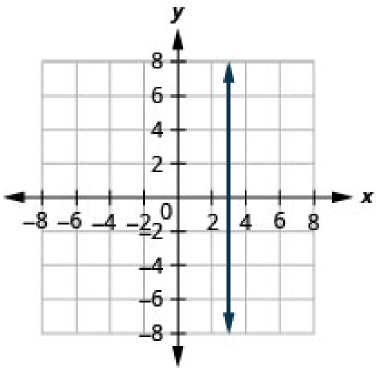
Exercise:

Problem: $y = -2$

Exercise:

Problem: $x = 3$

Solution:



In the following exercises, graph each pair of equations in the same rectangular coordinate system.

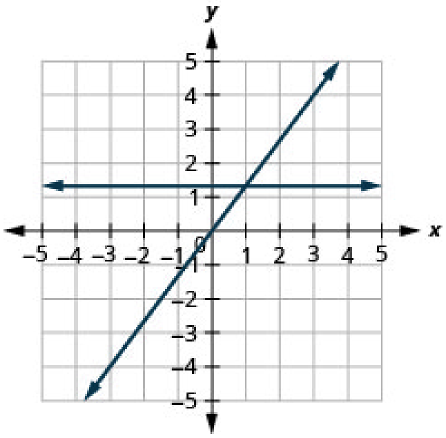
Exercise:

Problem: $y = -2x$ and $y = -2$

Exercise:

Problem: $y = \frac{4}{3}x$ and $y = \frac{4}{3}$

Solution:



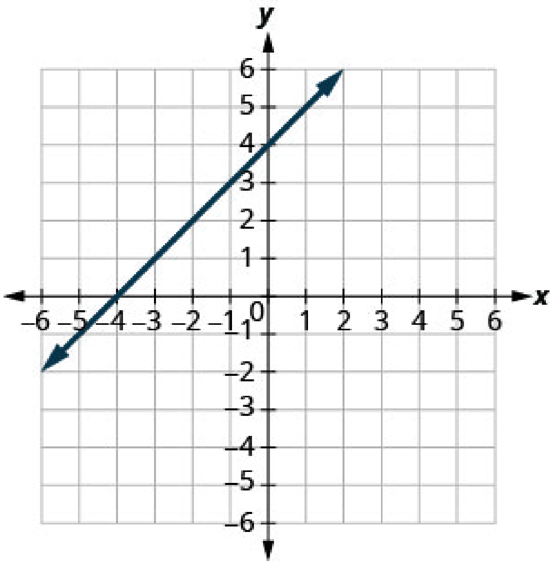
Graphing with Intercepts

Identify the *x*- and *y*-Intercepts on a Graph

In the following exercises, find the *x*- and *y*-intercepts.

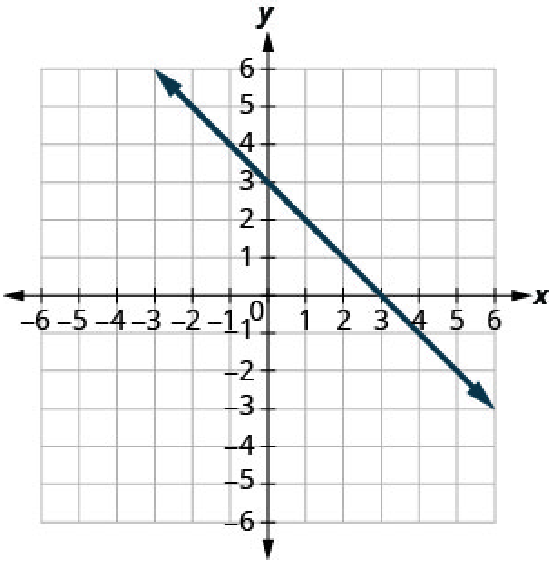
Exercise:

Problem:



Exercise:

Problem:



Solution:

$(3,0),(0,3)$

Find the *x*- and *y*-Intercepts from an Equation of a Line

In the following exercises, find the intercepts of each equation.

Exercise:

Problem: $x+y=5$

Exercise:

Problem: $x-y=-1$

Solution:

$(-1,0),(0,1)$

Exercise:

Problem: $x+2y=6$

Exercise:

Problem: $2x+3y=12$

Solution:

$(6,0),(0,4)$

Exercise:

Problem: $y=\frac{3}{4}x-12$

Exercise:

Problem: $y=3x$

Solution:

$(0,0)$

Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

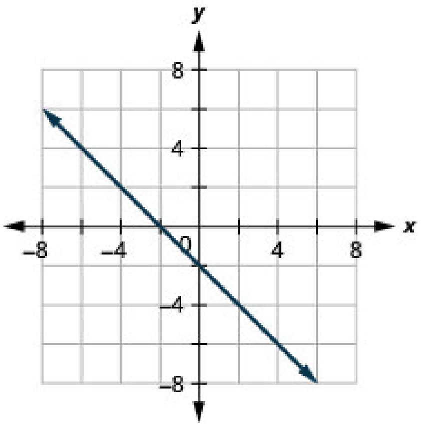
Exercise:

Problem: $-x+3y=3$

Exercise:

Problem: $x + y = -2$

Solution:



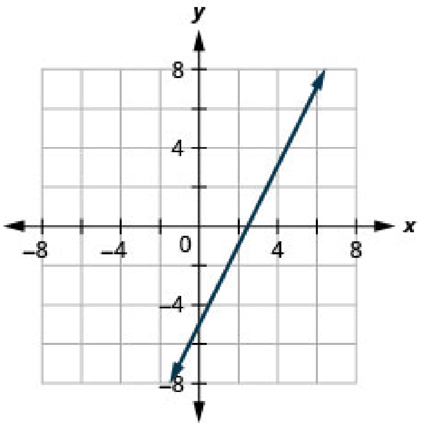
Exercise:

Problem: $x - y = 4$

Exercise:

Problem: $2x - y = 5$

Solution:



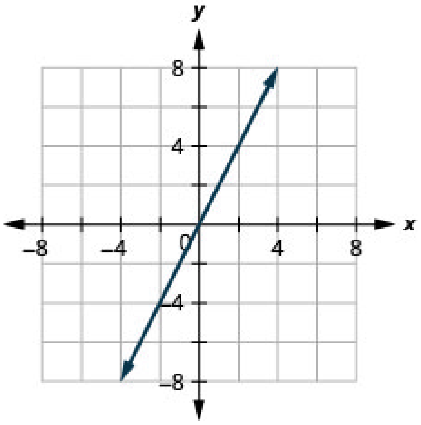
Exercise:

Problem: $2x - 4y = 8$

Exercise:

Problem: $y = 2x$

Solution:



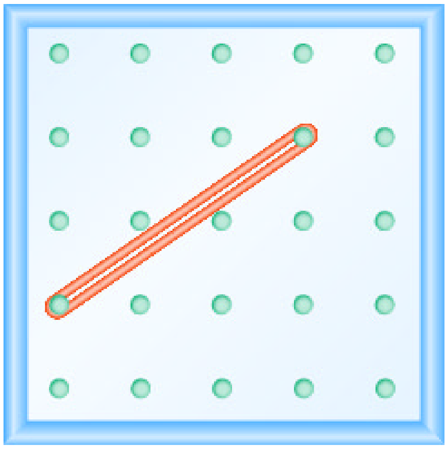
Slope of a Line

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

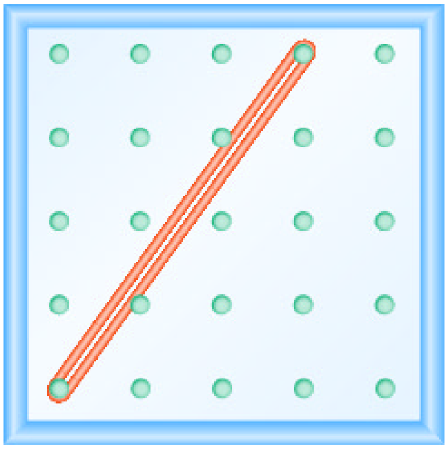
Exercise:

Problem:



Exercise:

Problem:

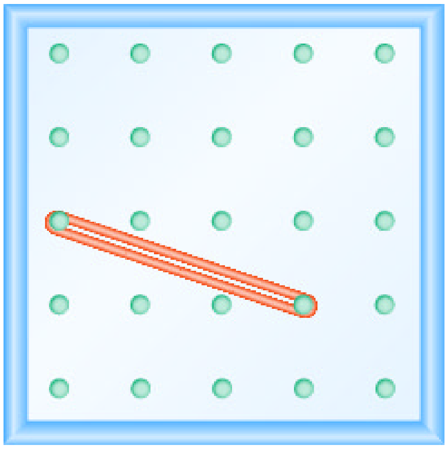


Solution:

$\frac{4}{3}$

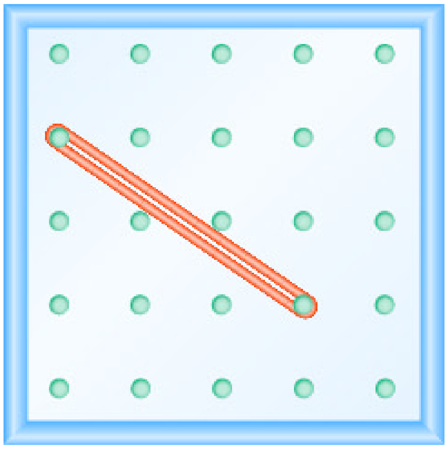
Exercise:

Problem:



Exercise:

Problem:



Solution:

$-\frac{2}{3}$

In the following exercises, model each slope. Draw a picture to show your results.

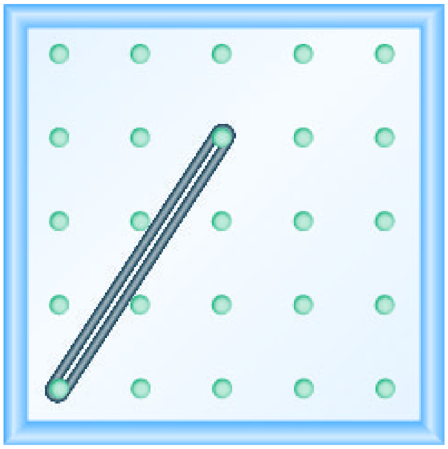
Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{3}{2}$

Solution:



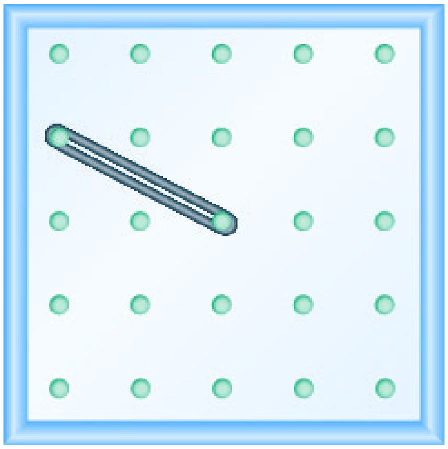
Exercise:

Problem: $-\frac{2}{3}$

Exercise:

Problem: $-\frac{1}{2}$

Solution:

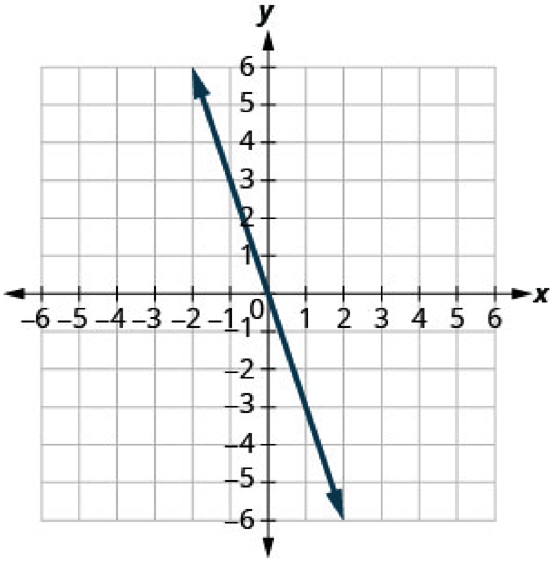


Use $m=\frac{\text{rise}}{\text{run}}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

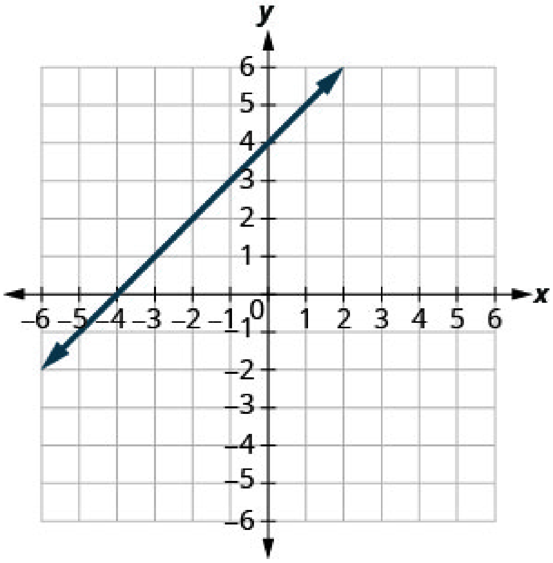
Exercise:

Problem:



Exercise:

Problem:

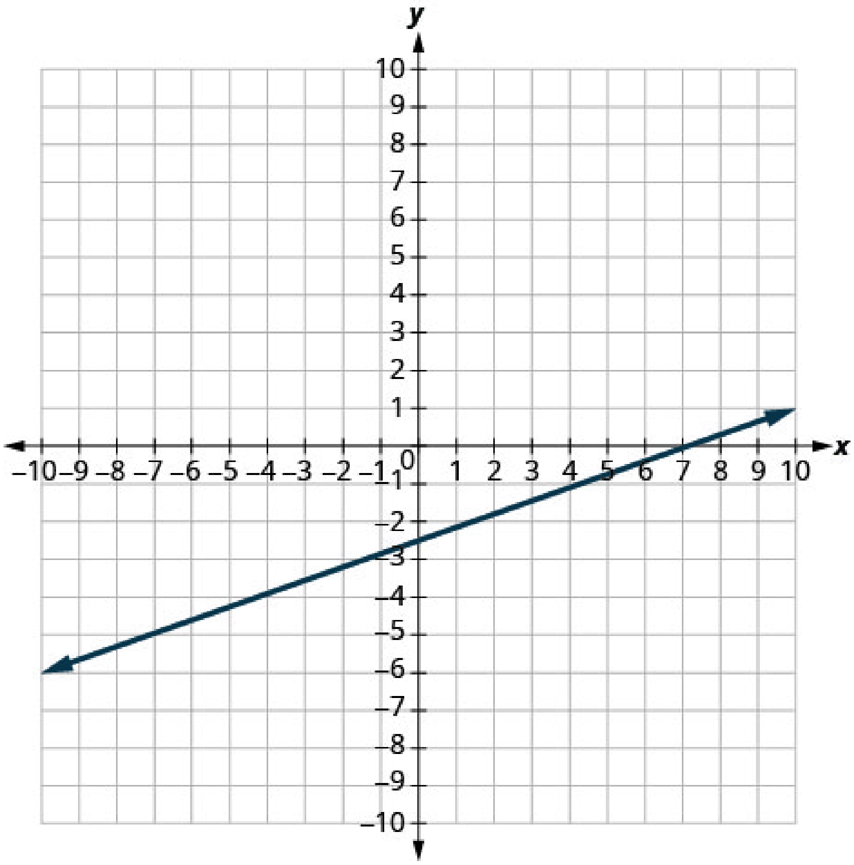


Solution:

1

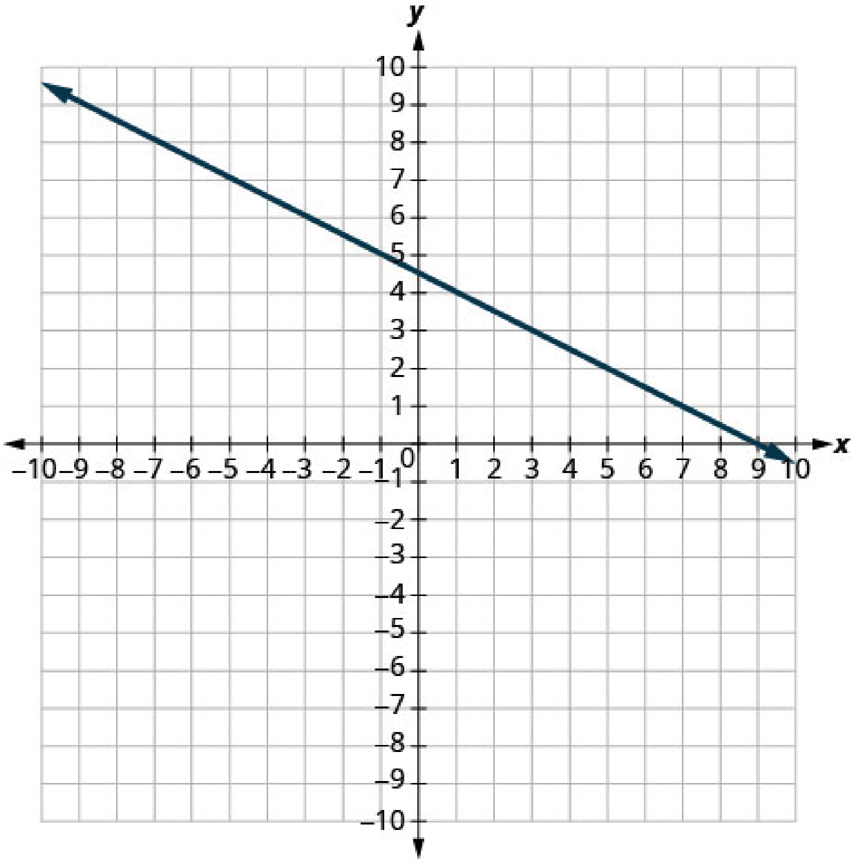
Exercise:

Problem:



Exercise:

Problem:



Solution:

$-\frac{1}{2}$

Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise:

Problem: $y = 2$

Exercise:

Problem: $x = 5$

Solution:

undefined

Exercise:

Problem: $x = -3$

Exercise:

Problem: $y = -1$

Solution:

0

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise:

Problem: $(-1, -1), (0, 5)$

Exercise:

Problem: $(3, 5), (4, -1)$

Solution:

-6

Exercise:

Problem: $(-5, -2), (3, 2)$

Exercise:

Problem: $(2, 1), (4, 6)$

Solution:

$\frac{5}{2}$

Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

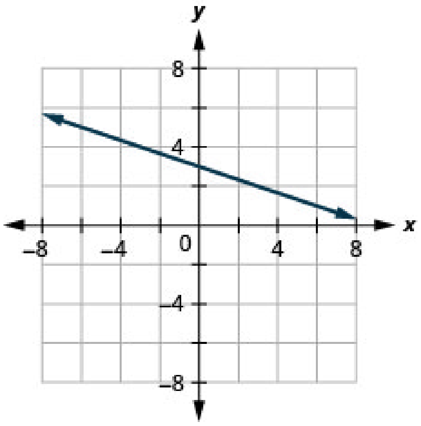
Exercise:

Problem: $(2, -2)$; $m = \frac{5}{2}$

Exercise:

Problem: $(-3, 4)$; $m = -\frac{1}{3}$

Solution:



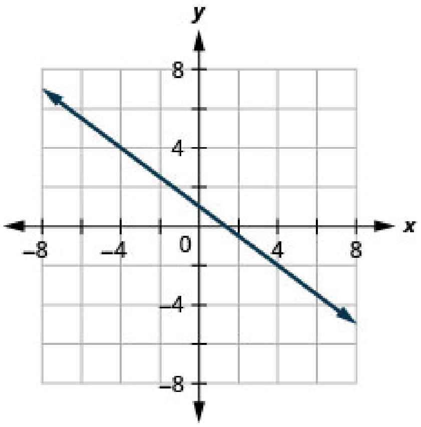
Exercise:

Problem: x-intercept -4; $m = 3$

Exercise:

Problem: y-intercept 1; $m = -\frac{3}{4}$

Solution:



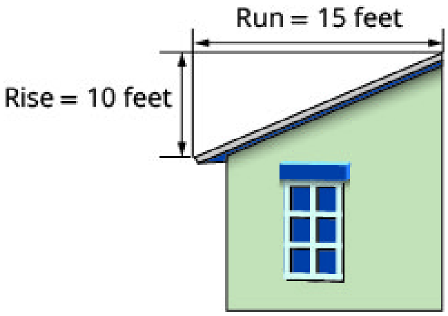
Solve Slope Applications

In the following exercises, solve these slope applications.

Exercise:

Problem:

The roof pictured below has a rise of 10 feet and a run of 15 feet. What is its slope?



Exercise:

Problem: A mountain road rises 50 feet for a 500-foot run. What is its slope?

Solution:

$\frac{1}{10}$

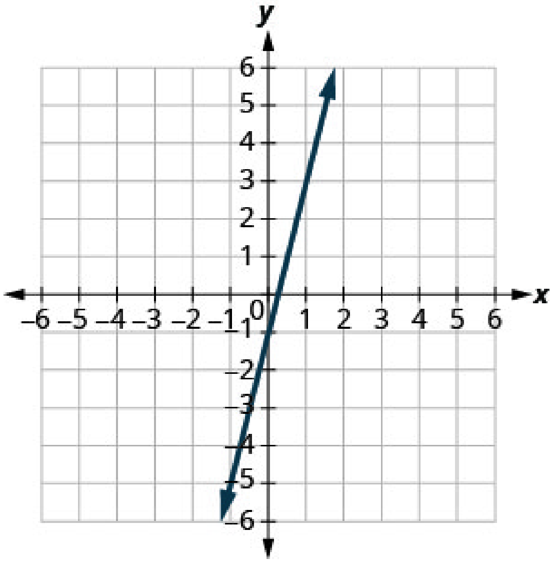
Intercept Form of an Equation of a Line

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise:

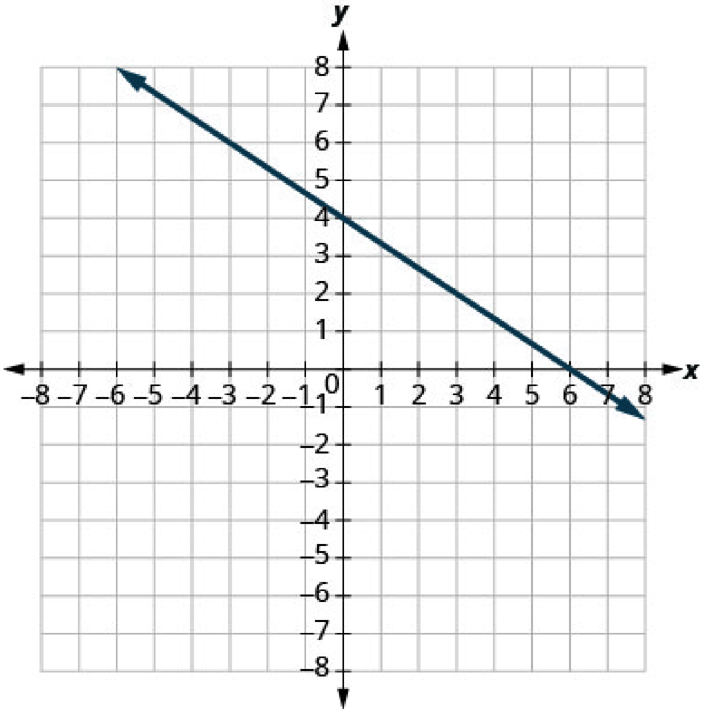
Problem:



$y = 4x - 1$

Exercise:

Problem:



$y=-\frac{2}{3}x+4$

Solution:

slope $m=-\frac{2}{3}$ and *y*-intercept $(0,4)$

Identify the Slope and y-Intercept from an Equation of a Line

In the following exercises, identify the slope and *y*-intercept of each line.

Exercise:

Problem: $y=-4x+9$

Exercise:

Problem: $y=\frac{5}{3}x-6$

Solution:

$\frac{5}{3}$; $(0,-6)$

Exercise:

Problem: $5x + y = 10$

Exercise:

Problem: $4x - 5y = 8$

Solution:

$\frac{4}{5}; \left(0, -\frac{8}{5}\right)$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and *y*-intercept.

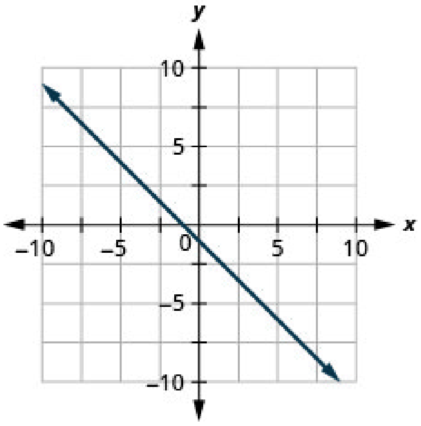
Exercise:

Problem: $y = 2x + 3$

Exercise:

Problem: $y = -x - 1$

Solution:



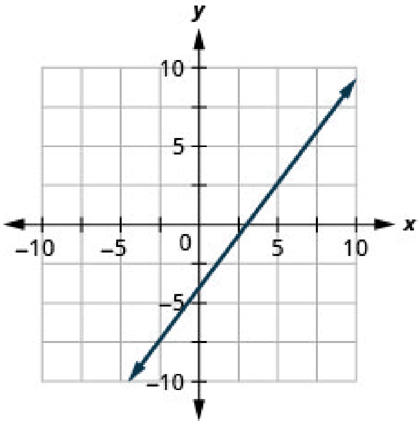
Exercise:

Problem: $y = -\frac{2}{5}x + 3$

Exercise:

Problem: $4x - 3y = 12$

Solution:



In the following exercises, determine the most convenient method to graph each line.

Exercise:

Problem: $x = 5$

Exercise:

Problem: $y = -3$

Solution:

horizontal line

Exercise:

Problem: $2x + y = 5$

Exercise:

Problem: $x - y = 2$

Solution:

intercepts

Exercise:

Problem: $y = x + 2$

Exercise:

Problem: $y = \frac{3}{4}x - 1$

Solution:

plotting points

Graph and Interpret Applications of Slope–Intercept

Exercise:

Problem:

Katherine is a private chef. The equation $C = 6.5m + 42$ models the relation between her weekly cost, *C*, in dollars and the number of meals, *m*, that she serves.

ⓐ Find Katherine's cost for a week when she serves no meals.
ⓑ Find the cost for a week when she serves 14 meals.
ⓒ Interpret the slope and *C*-intercept of the equation.
ⓓ Graph the equation.

Exercise:

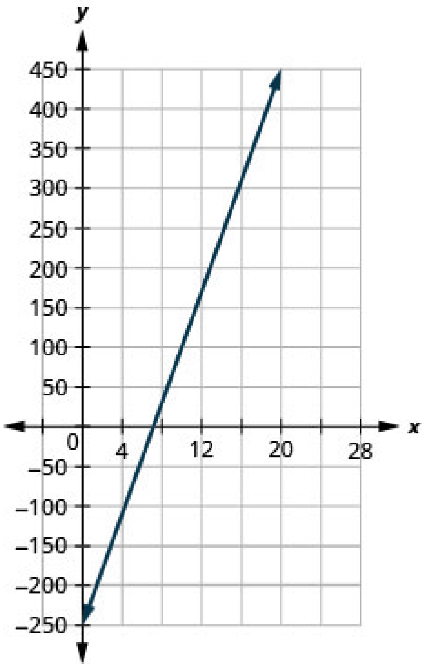
Problem:

Marjorie teaches piano. The equation $P = 35h - 250$ models the relation between her weekly profit, *P*, in dollars and the number of student lessons, *s*, that she teaches.

ⓐ Find Marjorie's profit for a week when she teaches no student lessons.
ⓑ Find the profit for a week when she teaches 20 student lessons.
ⓒ Interpret the slope and *P*–intercept of the equation.
ⓓ Graph the equation.

Solution:

ⓐ −$250 ⓑ $450 ⓒ The slope, 35, means that Marjorie's weekly profit, *P*, increases by $35 for each additional student lesson she teaches. The *P*–intercept means that when the number of lessons is 0, Marjorie loses $250.
ⓓ



Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise:

Problem: $4x - 3y = -1;\ y = \frac{4}{3}x - 3$

Exercise:

Problem: $2x - y = 8;\ x - 2y = 4$

Solution:

not parallel

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise:

Problem: $y = 5x - 1; 10x + 2y = 0$

Exercise:

Problem: $3x - 2y = 5; 2x + 3y = 6$

Solution:

perpendicular

Find the Equation of a Line

Find an Equation of the Line Given the Slope and *y*-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

Exercise:

Problem: slope $\frac{1}{3}$ and y-intercept $(0, -6)$

Exercise:

Problem: slope -5 and y-intercept $(0, -3)$

Solution:

$y = -5x - 3$

Exercise:

Problem: slope 0 and y-intercept $(0, 4)$

Exercise:

Problem: slope -2 and y-intercept $(0, 0)$

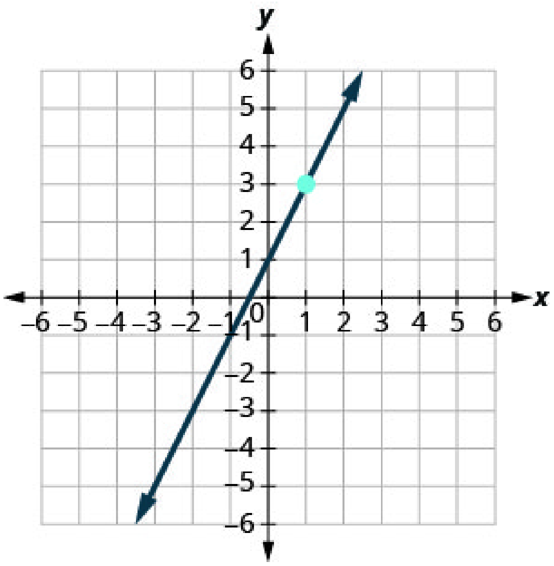
Solution:

$y = -2x$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

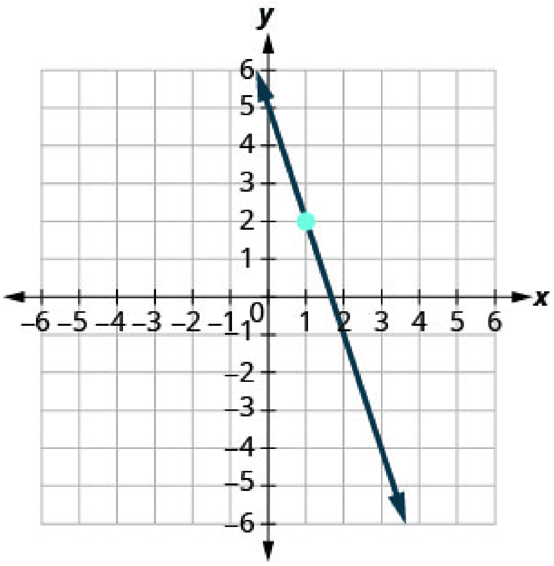
Exercise:

Problem:



Exercise:

Problem:

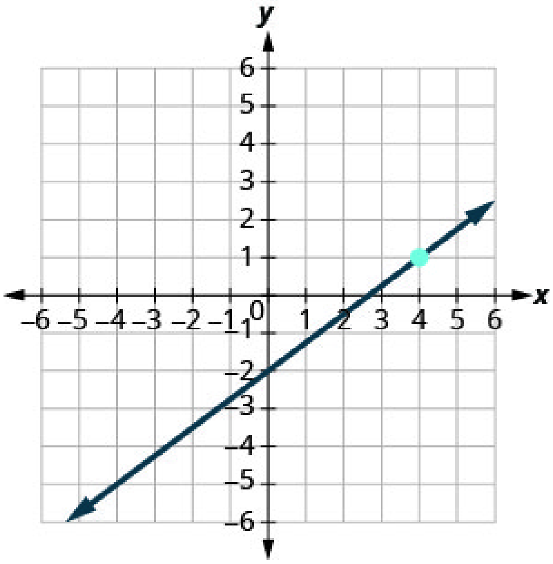


Solution:

$y = -3x + 5$

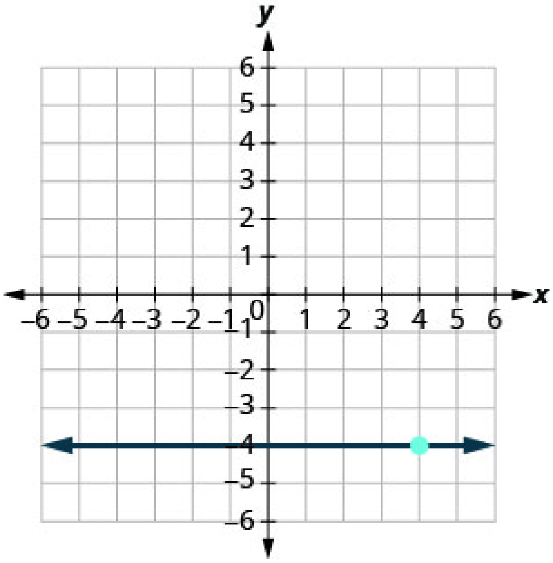
Exercise:

Problem:



Exercise:

Problem:



Solution:

$y = -4$

Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

Exercise:

Problem: $m = -\frac{1}{4}$, point $(-8, 3)$

Exercise:

Problem: $m = \frac{3}{5}$, point $(10, 6)$

Solution:

$y = \frac{3}{5}x$

Exercise:

Problem: Horizontal line containing $(-2, 7)$

Exercise:

Problem: $m = -2$, point $(-1, -3)$

Solution:

$y = -2x - 5$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

Exercise:

Problem: $(2, 10)$ and $(-2, -2)$

Exercise:

Problem: $(7,1)$ and $(5,0)$

Solution:

$y=\frac{1}{2}x-\frac{5}{2}$

Exercise:

Problem: $(3,8)$ and $(3,-4)$.

Exercise:

Problem: $(5,2)$ and $(-1,2)$

Solution:

$y=2$

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise:

Problem: line $y=-3x+6$, point $(1,-5)$

Exercise:

Problem: line $2x+5y=-10$, point $(10,4)$

Solution:

$y=-\frac{2}{5}x+8$

Exercise:

Problem: line $x=4$, point $(-2,-1)$

Exercise:

Problem: line $y = -5$, point $(-4, 3)$

Solution:

$y = 3$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise:

Problem: line $y = -\frac{4}{5}x + 2$, point $(8, 9)$

Exercise:

Problem: line $2x - 3y = 9$, point $(-4, 0)$

Solution:

$y = -\frac{3}{2}x - 6$

Exercise:

Problem: line $y = 3$, point $(-1, -3)$

Exercise:

Problem: line $x = -5$ point $(2, 1)$

Solution:

$y = 1$

Practice Test

Exercise:

Problem: Plot each point in a rectangular coordinate system.

ⓐ $(2, 5)$
ⓑ $(-1, -3)$
ⓒ $(0, 2)$
ⓓ $\left(-4, \frac{3}{2}\right)$
ⓔ $(5, 0)$

Exercise:

Problem:

Which of the given ordered pairs are solutions to the equation $3x - y = 6$?

ⓐ $(3, 3)$
ⓑ $(2, 0)$
ⓒ $(4, -6)$

Solution:

ⓐ yes ⓑ yes ⓒ no

Exercise:

Problem: Find three solutions to the linear equation $y = -2x - 4$.

Exercise:

Problem: Find the *x*- and *y*-intercepts of the equation $4x - 3y = 12$.

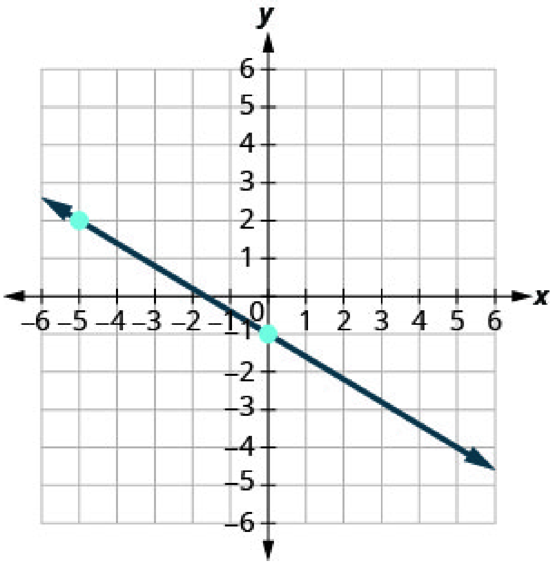
Solution:

$(3, 0), (0, -4)$

Find the slope of each line shown.

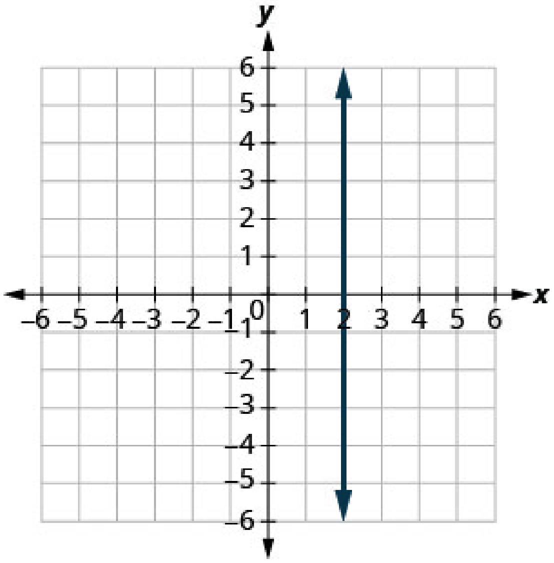
Exercise:

Problem:



Exercise:

Problem:

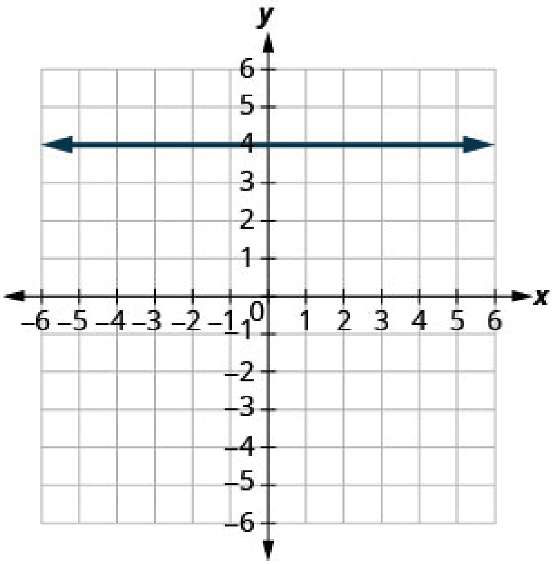


Solution:

undefined

Exercise:

Problem:



Exercise:

Problem: Find the slope of the line between the points $(5, 2)$ and $(-1, -4)$.

Solution:

1

Exercise:

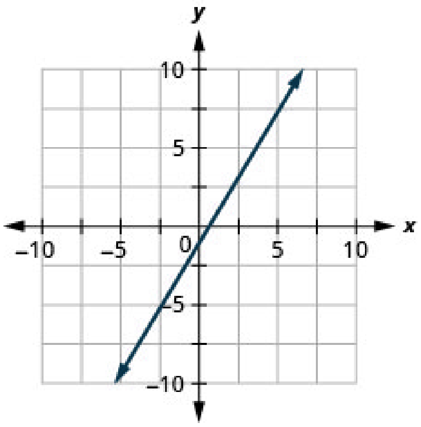
Problem: Graph the line with slope $\frac{1}{2}$ containing the point $(-3, -4)$.

Graph the line for each of the following equations.

Exercise:

Problem: $y = \frac{5}{3}x - 1$

Solution:



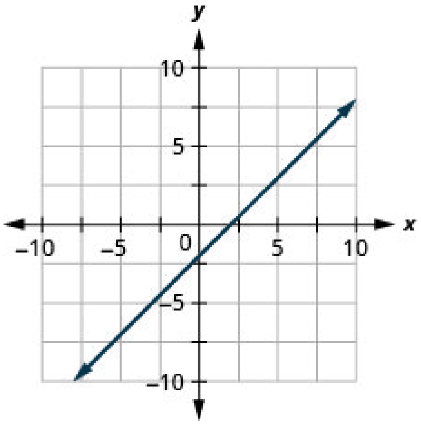
Exercise:

Problem: $y = -x$

Exercise:

Problem: $x - y = 2$

Solution:



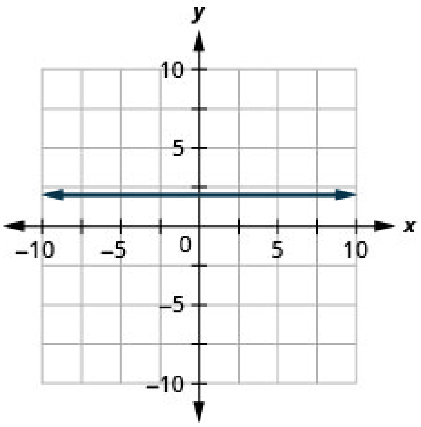
Exercise:

Problem: $4x + 2y = -8$

Exercise:

Problem: $y = 2$

Solution:



Exercise:

Problem: $x = -3$

Find the equation of each line. Write the equation in slope–intercept form.

Exercise:

Problem: slope $-\frac{3}{4}$ and *y*-intercept $(0, -2)$

Solution:

$y = -\frac{3}{4}x - 2$

Exercise:

Problem: $m = 2$, point $(-3, -1)$

Exercise:

Problem: containing $(10, 1)$ and $(6, -1)$

Solution:

$y = \frac{1}{2}x - 4$

Exercise:

Problem: parallel to the line $y = -\frac{2}{3}x - 1$, containing the point $(-3, 8)$

Exercise:

Problem: perpendicular to the line $y = \frac{5}{4}x + 2$, containing the point $(-10, 3)$

Solution:

$y = -\frac{4}{5}x - 5$

Glossary

boundary line

The line with equation $Ax + By = C$ that separates the region where $Ax + By > C$ from the region where $Ax + By < C$.

linear inequality

An inequality that can be written in one of the following forms:

Equation:

$$Ax + By > C \qquad Ax + By \geq C \qquad Ax + By < C \qquad Ax + By \leq C$$

where A and B are not both zero.

solution of a linear inequality

An ordered pair (x, y) is a solution to a linear inequality the inequality is true when we substitute the values of *x* and *y*.

Introduction class="introduction"

Architects use polynomials to design curved shapes such as this suspension bridge, the Silver Jubilee bridge in Halton, England.

We have seen that the graphs of linear equations are straight lines. Graphs of other types of equations, called polynomial equations, are curves, like the outline of this suspension bridge. Architects use polynomials to design the shape of a bridge like this and to draw the blueprints for it. Engineers use polynomials to calculate the stress on the bridge's supports to ensure they are strong enough for the intended load. In this chapter, you will explore operations with and properties of polynomials.

Add and Subtract Polynomials

By the end of this section, you will be able to:

- Identify polynomials, monomials, binomials, and trinomials
- Determine the degree of polynomials
- Add and subtract monomials
- Add and subtract polynomials
- Evaluate a polynomial for a given value

Note:
Before you get started, take this readiness quiz.

1. Simplify: $8x+3x$.
   If you missed this problem, review [link].
2. Subtract: $(5n+8)-(2n-1)$.
   If you missed this problem, review [link].
3. Write in expanded form: a^5.
   If you missed this problem, review [link].

Identify Polynomials, Monomials, Binomials and Trinomials

You have learned that a *term* is a constant or the product of a constant and one or more variables. When it is of the form ax^m, where a is a constant and m is a whole number, it is called a monomial. Some examples of monomial are $8, -2x^2, 4y^3, \text{ and } 11z^7$.

Note:
Monomials
A **monomial** is a term of the form ax^m, where a is a constant and m is a positive whole number.

A monomial, or two or more monomials combined by addition or subtraction, is a polynomial. Some polynomials have special names, based on the number of terms. A monomial is a polynomial with exactly one term. A binomial has exactly two terms, and a trinomial has exactly three terms. There are no special names for polynomials with more than three terms.

Note:
Polynomials
polynomial—A monomial, or two or more monomials combined by addition or subtraction, is a polynomial.

- **monomial**—A polynomial with exactly one term is called a monomial.
- **binomial**—A polynomial with exactly two terms is called a binomial.
- **trinomial**—A polynomial with exactly three terms is called a trinomial.

Here are some examples of polynomials.

Polynomial	$b+1$	$4y^2-7y+2$	$4x^4+x^3+8x^2-9x+1$	
Monomial	14	$8y^2$	$-9x^3y^5$	$-$
Binomial	$a+7$	$4b-5$	y^2-16	$3a$
Trinomial	$x^2-7x+12$	$9y^2+2y-8$	$6m^4-m^3+8m$	z^4

Notice that every monomial, binomial, and trinomial is also a polynomial. They are just special members of the "family" of polynomials and so they have special names. We use the words *monomial*, *binomial*, and *trinomial* when referring to these special polynomials and just call all the rest *polynomials*.

Example:
Exercise:

Problem: **Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial.

ⓐ $4y^2-8y-6$
ⓑ $-5a^4b^2$
ⓒ $2x^5-5x^3-9x^2+3x+4$
ⓓ $13-5m^3$
ⓔ q

Solution:
Solution

	Polynomial	Number of terms	Type
ⓐ	$4y^2-8y-6$	3	Trinomial
ⓑ	$-5a^4b^2$	1	Monomial
ⓒ	$2x^5-5x^3-9x^2+3x+4$	5	Polynomial
ⓓ	$13-5m^3$	2	Binomial
ⓔ	q	1	Monomial

Note:
Exercise:

TRY IT :: 5.1

Problem: Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial:

ⓐ $5b$ ⓑ $8y^3-7y^2-y-3$ ⓒ $-3x^2-5x+9$ ⓓ $81-4a^2$ ⓔ $-5x^6$

Solution:

ⓐ monomial ⓑ polynomial ⓒ trinomial ⓓ binomial ⓔ monomial

Note:
Exercise:

TRY IT :: 5.2

Problem: Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial:

ⓐ $27z^3 - 8$ ⓑ $12m^3 - 5m^2 - 2m$ ⓒ $\frac{5}{6}$ ⓓ $8x^4 - 7x^2 - 6x - 5$ ⓔ $-n^4$

Solution:

ⓐ binomial ⓑ trinomial ⓒ monomial ⓓ polynomial ⓔ monomial

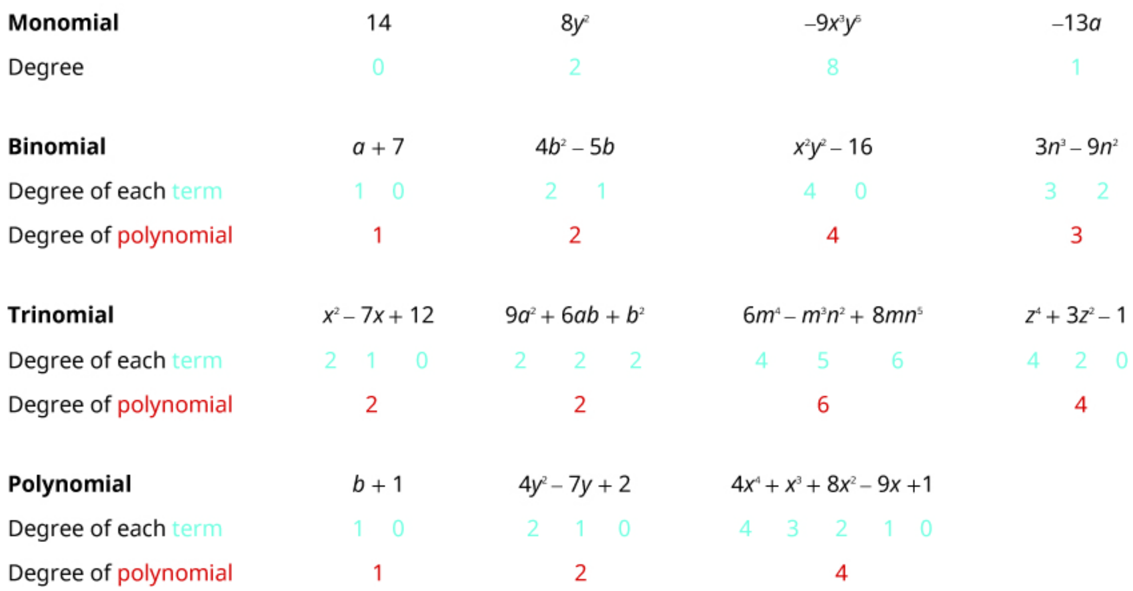
Determine the Degree of Polynomials

The degree of a polynomial and the degree of its terms are determined by the exponents of the variable.

A monomial that has no variable, just a constant, is a special case. The degree of a constant is 0—it has no variable.

Note:
Degree of a Polynomial
The **degree of a term** is the sum of the exponents of its variables.
The **degree of a constant** is 0.
The **degree of a polynomial** is the highest degree of all its terms.

Let’s see how this works by looking at several polynomials. We’ll take it step by step, starting with monomials, and then progressing to polynomials with more terms.



Monomial	14	$8y^2$	$-9x^3y^5$	$-13a$
Degree	0	2	8	1
Binomial	$a+7$	$4b^2-5b$	x^2y^2-16	$3n^3-9n^2$
Degree of each term	1 0	2 1	4 0	3 2
Degree of polynomial	1	2	4	3
Trinomial	$x^2-7x+12$	$9a^2+6ab+b^2$	$6m^4-m^3n^2+8mn^5$	z^4+3z^2-1
Degree of each term	2 1 0	2 2 2	4 5 6	4 2 0
Degree of polynomial	2	2	6	4
Polynomial	$b+1$	$4y^2-7y+2$	$4x^4+x^3+8x^2-9x+1$	
Degree of each term	1 0	2 1 0	4 3 2 1 0	
Degree of polynomial	1	2	4	

A polynomial is in **standard form** when the terms of a polynomial are written in descending order of degrees. Get in the habit of writing the term with the highest degree first.

Example:
Exercise:

Problem: **Find the degree of the following polynomials.

ⓐ $10y$
ⓑ $4x^3-7x+5$
ⓒ -15
ⓓ $-8b^2+9b-2$
ⓔ $8xy^2+2y$

Solution:
Solution

ⓐ The exponent of y is one. $y=y^1$	$10y$ The degree is 1.
ⓑ The highest degree of all the terms is 3.	$4x^3-7x+5$ The degree is 3.
ⓒ The degree of a constant is 0.	-15 The degree is 0.
ⓓ The highest degree of all the terms is 2.	$-8b^2+9b-2$ The degree is 2.

ⓔ The highest degree of all the terms is 3.	$8xy^2 + 2y$ The degree is 3.

Note:
Exercise:

TRY IT :: 5.3
Problem: Find the degree of the following polynomials:

ⓐ $-15b$ ⓑ $10z^4 + 4z^2 - 5$ ⓒ $12c^5d^4 + 9c^3d^9 - 7$ ⓓ $3x^2y - 4x$ ⓔ -9

Solution:

ⓐ 1 ⓑ 4 ⓒ 12 ⓓ 3 ⓔ 0

Note:
Exercise:

TRY IT :: 5.4
Problem: Find the degree of the following polynomials:

ⓐ 52 ⓑ $a^4b - 17a^4$ ⓒ $5x + 6y + 2z$ ⓓ $3x^2 - 5x + 7$ ⓔ $-a^3$

Solution:

ⓐ 0 ⓑ 5 ⓒ 1 ⓓ 2 ⓔ 3

Add and Subtract Monomials

You have learned how to simplify expressions by combining like terms. Remember, like terms must have the same variables with the same exponent. Since monomials are terms, adding and subtracting monomials is the same as combining like terms. If the monomials are like terms, we just combine them by adding or subtracting the coefficient.

Example:
Exercise:

Problem: **Add: $25y^2 + 15y^2$.

Solution:
Solution

	$25y^2 + 15y^2$
Combine like terms.	$40y^2$

Note:
Exercise:

TRY IT :: 5.5
Problem: Add: $12q^2 + 9q^2$.

Solution:

$21q^2$

Note:
Exercise:

TRY IT :: 5.6
Problem: Add: $-15c^2 + 8c^2$.

Solution:

$-7c^2$

Example:
Exercise:

Problem: **Subtract: $16p - (-7p)$.

Solution:
Solution

	$16p - (-7p)$
Combine like terms.	$23p$

Note:
Exercise:

TRY IT :: 5.7

Problem: Subtract: $8m - (-5m)$.

Solution:

$13m$

Note:
Exercise:

TRY IT :: 5.8

Problem: Subtract: $-15z^3 - \left(-5z^3\right)$.

Solution:

$-10z^3$

Remember that like terms must have the same variables with the same exponents.

Example:
Exercise:

Problem: **Simplify: $c^2 + 7d^2 - 6c^2$.

Solution:
Solution

	$c^2 + 7d^2 - 6c^2$
Combine like terms.	$-5c^2 + 7d^2$

Note:
Exercise:

TRY IT :: 5.9

Problem: Add: $8y^2 + 3z^2 - 3y^2$.

Solution:

$5y^2 + 3z^2$

Note:
Exercise:

TRY IT :: 5.10
Problem: Add: $3m^2 + n^2 - 7m^2$.

Solution:

$-4m^2 + n^2$

Example:
Exercise:

Problem: **Simplify: $u^2v + 5u^2 - 3v^2$.

Solution:
Solution

	$u^2v + 5u^2 - 3v^2$
There are no like terms to combine.	$u^2v + 5u^2 - 3v^2$

Note:
Exercise:

TRY IT :: 5.11
Problem: Simplify: $m^2n^2 - 8m^2 + 4n^2$.

Solution:

There are no like terms to combine.

Note:
Exercise:

TRY IT :: 5.12
Problem: Simplify: $pq^2 - 6p - 5q^2$.

Solution:

There are no like terms to combine.

Add and Subtract Polynomials

We can think of adding and subtracting polynomials as just adding and subtracting a series of monomials. Look for the like terms—those with the same variables and the same exponent. The Commutative Property allows us to rearrange the terms to put like terms together.

Example:
Exercise:

Problem: **Find the sum: $\left(5y^2 - 3y + 15\right) + \left(3y^2 - 4y - 11\right)$.

Solution:
Solution

Identify like terms.	$(\underline{\underline{5y^2}} - \underline{3y} + \underline{1}5) + (\underline{\underline{3y^2}} - \underline{4y} - \underline{1}1)$
Rearrange to get the like terms together.	$\underline{\underline{5y^2 + 3y^2}} - \underline{3y - 4y} + \underline{1}5 - \underline{1}1$
Combine like terms.	$8y^2 - 7y + 4$

Note:
Exercise:

TRY IT :: 5.13
Problem: Find the sum: $\left(7x^2 - 4x + 5\right) + \left(x^2 - 7x + 3\right)$.

Solution:

$8x^2 - 11x + 8$

Note:

Exercise:

TRY IT :: 5.14

Problem: Find the sum: $\left(14y^2+6y-4\right)+\left(3y^2+8y+5\right)$.

Solution:

$17y^2+14y+1$

Example:
Exercise:

Problem: **Find the difference: $\left(9w^2-7w+5\right)-\left(2w^2-4\right)$.

Solution:
Solution

	$(9w^2-7w+5)-(2w^2-4)$
Distribute and identify like terms.	$\underline{\underline{9w^2}}-\underline{7w}+\underset{\sim}{5}-\underline{\underline{2w^2}}+\underset{\sim}{4}$
Rearrange the terms.	$\underline{\underline{9w^2-2w^2}}-\underline{7w}+\underset{\sim}{5}+\underset{\sim}{4}$
Combine like terms.	$7w^2-7w+9$

Note:
Exercise:

TRY IT :: 5.15

Problem: Find the difference: $\left(8x^2+3x-19\right)-\left(7x^2-14\right)$.

Solution:

x^2+3x-5

Note:
Exercise:

TRY IT :: 5.16
Problem: Find the difference: $\left(9b^2-5b-4\right)-\left(3b^2-5b-7\right)$.

Solution:

$6b^2+3$

Example:
Exercise:

Problem: **Subtract: $\left(c^2-4c+7\right)$ from $\left(7c^2-5c+3\right)$.

Solution:
Solution

	Subtract (c^2-4c+7) from $(7c^2-5c+3)$.
	$(7c^2-5c+3)-(c^2-4c+7)$
Distribute and identify like terms.	$\underline{\underline{7c^2}}-\underline{5c}+\underset{\sim}{3}-\underline{\underline{c^2}}+\underline{4c}-\underset{\sim}{7}$
Rearrange the terms.	$\underline{\underline{7c^2-c^2}}-\underline{5c+4c}+\underset{\sim}{3}-\underset{\sim}{7}$
Combine like terms.	$6c^2-c-4$

Note:
Exercise:

TRY IT :: 5.17
Problem: Subtract: $\left(5z^2-6z-2\right)$ from $\left(7z^2+6z-4\right)$.

Solution:

$2z^2 + 12z - 2$

Note:
Exercise:

TRY IT :: 5.18
Problem: Subtract: $\left(x^2 - 5x - 8\right)$ from $\left(6x^2 + 9x - 1\right)$.

Solution:

$5x^2 + 14x + 7$

Example:
Exercise:

Problem: **Find the sum: $\left(u^2 - 6uv + 5v^2\right) + \left(3u^2 + 2uv\right)$.

Solution:
Solution

	$\left(u^2 - 6uv + 5v^2\right) + \left(3u^2 + 2uv\right)$
Distribute.	$u^2 - 6uv + 5v^2 + 3u^2 + 2uv$
Rearrange the terms, to put like terms together.	$u^2 + 3u^2 - 6uv + 2uv + 5v^2$
Combine like terms.	$4u^2 - 4uv + 5v^2$

Note:
Exercise:

TRY IT :: 5.19
Problem: Find the sum: $\left(3x^2 - 4xy + 5y^2\right) + \left(2x^2 - xy\right)$.

Solution:

$5x^2 - 5xy + 5y^2$

Note:

Exercise:

TRY IT :: 5.20

Problem: Find the sum: $\left(2x^2 - 3xy - 2y^2\right) + \left(5x^2 - 3xy\right)$.

Solution:

$7x^2 - 6xy - 2y^2$

Example:
Exercise:

Problem: **Find the difference: $\left(p^2 + q^2\right) - \left(p^2 + 10pq - 2q^2\right)$.

Solution:
Solution

	$\left(p^2 + q^2\right) - \left(p^2 + 10pq - 2q^2\right)$
Distribute.	$p^2 + q^2 - p^2 - 10pq + 2q^2$
Rearrange the terms, to put like terms together.	$p^2 - p^2 - 10pq + q^2 + 2q^2$
Combine like terms.	$-10pq + 3q^2$

Note:
Exercise:

TRY IT :: 5.21

Problem: Find the difference: $\left(a^2 + b^2\right) - \left(a^2 + 5ab - 6b^2\right)$.

Solution:

$-5ab + 7b^2$

Note:
Exercise:

TRY IT :: 5.22

Problem: Find the difference: $\left(m^2 + n^2\right) - \left(m^2 - 7mn - 3n^2\right)$.

Solution:

$4n^2 + 7mn$

Example:
Exercise:

Problem: **Simplify: $\left(a^3 - a^2b\right) - \left(ab^2 + b^3\right) + \left(a^2b + ab^2\right)$.

Solution:
Solution

	$\left(a^3 - a^2b\right) - \left(ab^2 + b^3\right) + \left(a^2b + ab^2\right)$
Distribute.	$a^3 - a^2b - ab^2 - b^3 + a^2b + ab^2$
Rearrange the terms, to put like terms together.	$a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3$
Combine like terms.	$a^3 - b^3$

Note:
Exercise:

TRY IT :: 5.23
Problem: Simplify: $\left(x^3 - x^2y\right) - \left(xy^2 + y^3\right) + \left(x^2y + xy^2\right)$.

Solution:

$x^3 - y^3$

Note:
Exercise:

TRY IT :: 5.24
Problem: Simplify: $\left(p^3 - p^2q\right) + \left(pq^2 + q^3\right) - \left(p^2q + pq^2\right)$.

Solution:

$p^3 - 2p^2q + q^3$

Evaluate a Polynomial for a Given Value

We have already learned how to evaluate expressions. Since polynomials are expressions, we'll follow the same procedures to evaluate a polynomial. We will substitute the given value for the variable and then simplify using the order of operations.

Example:
Exercise:

Problem: **Evaluate $5x^2 - 8x + 4$ when

ⓐ $x = 4$
ⓑ $x = -2$
ⓒ $x = 0$

Solution:
Solution

ⓐ $x = 4$	
	$5x^2 - 8x + 4$
Substitute 4 for *x*.	$5(4)^2 - 8(4) + 4$
Simplify the exponents.	$5 \cdot 16 - 8(4) + 4$
Multiply.	$80 - 32 + 4$
Simplify.	52

ⓑ $x = -2$	
	$5x^2 - 8x + 4$

Substitute -2 for x.	$5(-2)^2 - 8(-2) + 4$
Simplify the exponents.	$5 \cdot 4 - 8(-2) + 4$
Multiply.	$20 + 16 + 4$
Simplify.	40

ⓒ $x = 0$	
	$5x^2 - 8x + 4$
Substitute 0 for x.	$5(0)^2 - 8(0) + 4$
Simplify the exponents.	$5 \cdot 0 - 8(0) + 4$
Multiply.	$0 + 0 + 4$
Simplify.	4

Note:
Exercise:

TRY IT :: 5.25

Problem: Evaluate: $3x^2 + 2x - 15$ when

ⓐ $x = 3$
ⓑ $x = -5$
ⓒ $x = 0$

Solution:

ⓐ 18 ⓑ 50 ⓒ -15

Note:
Exercise:

TRY IT :: 5.26

Problem: Evaluate: $5z^2 - z - 4$ when

ⓐ $z = -2$
ⓑ $z = 0$
ⓒ $z = 2$

Solution:

ⓐ 18 ⓑ -4 ⓒ 14

Example:
Exercise:

Problem:

**The polynomial $-16t^2 + 250$ gives the height of a ball t seconds after it is dropped from a 250 foot tall building. Find the height after $t = 2$ seconds.

Solution:
Solution

	$-16t^2 + 250$
Substitute $t = 2$.	$-16(2)^2 + 250$
Simplify.	$-16 \cdot 4 + 250$
Simplify.	$-64 + 250$
Simplify.	186
	After 2 seconds the height of the ball is 186 feet.

Note:
Exercise:

Problem:

TRY IT :: 5.27
The polynomial $-16t^2 + 250$ gives the height of a ball t seconds after it is dropped from a 250-foot tall building. Find the height after $t = 0$ seconds.

Solution:

250

Note:
Exercise:

Problem:

TRY IT :: 5.28
The polynomial $-16t^2 + 250$ gives the height of a ball t seconds after it is dropped from a 250-foot tall building. Find the height after $t = 3$ seconds.

Solution:

106

Example:
Exercise:

Problem:

**The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with side x feet and sides of height y feet. Find the cost of producing a box with $x = 4$ feet and $y = 6$ feet.

Solution:
Solution

	$6x^2 + 15xy$
Substitute $x = 4$, $y = 6$.	$6(4)^2 + 15(4)(6)$
Simplify.	$6 \cdot 16 + 15(4)(6)$
Simplify.	$96 + 360$
Simplify.	456
	The cost of producing the box is $456.

Note:
Exercise:

Problem:

TRY IT :: 5.29
The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with side x feet and sides of height y feet. Find the cost of producing a box with $x = 6$ feet and $y = 4$ feet.

Solution:

$576

Note:
Exercise:

Problem:

TRY IT :: 5.30
The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with side x feet and sides of height y feet. Find the cost of producing a box with $x = 5$ feet and $y = 8$ feet.

Solution:

$750

Note:
Access these online resources for additional instruction and practice with adding and subtracting polynomials.

- Add and Subtract Polynomials 1
- Add and Subtract Polynomials 2
- Add and Subtract Polynomial 3
- Add and Subtract Polynomial 4

Key Concepts

- **Monomials**
  - A monomial is a term of the form ax^m, where a is a constant and m is a whole number
- **Polynomials**
  - **polynomial**—A monomial, or two or more monomials combined by addition or subtraction is a polynomial.
  - **monomial**—A polynomial with exactly one term is called a monomial.

  - **binomial—**A polynomial with exactly two terms is called a binomial.
  - **trinomial—**A polynomial with exactly three terms is called a trinomial.

- **Degree of a Polynomial**

  - The **degree of a term** is the sum of the exponents of its variables.
  - The **degree of a constant** is 0.
  - The **degree of a polynomial** is the highest degree of all its terms.

Practice Makes Perfect

Identify Polynomials, Monomials, Binomials, and Trinomials

In the following exercises, determine if each of the following polynomials is a monomial, binomial, trinomial, or other polynomial.

Exercise:

ⓐ $81b^5 - 24b^3 + 1$
ⓑ $5c^3 + 11c^2 - c - 8$
ⓒ $\frac{14}{15}y + \frac{1}{7}$
ⓓ 5
Problem: ⓔ $4y + 17$

Solution:

ⓐ trinomial ⓑ polynomial ⓒ binomial ⓓ monomial ⓔ binomial

Exercise:

ⓐ $x^2 - y^2$
ⓑ $-13c^4$
ⓒ $x^2 + 5x - 7$
ⓓ $x^2y^2 - 2xy + 8$
Problem: ⓔ 19

Exercise:

ⓐ $8 - 3x$
ⓑ $z^2 - 5z - 6$
ⓒ $y^3 - 8y^2 + 2y - 16$
ⓓ $81b^5 - 24b^3 + 1$
Problem: ⓔ -18

Solution:

ⓐ binomial ⓑ trinomial ⓒ polynomial ⓓ trinomial ⓔ monomial

Exercise:

ⓐ $11y^2$
ⓑ -73
ⓒ $6x^2-3xy+4x-2y+y^2$
ⓓ $4y+17$
Problem: ⓔ $5c^3+11c^2-c-8$

Determine the Degree of Polynomials

In the following exercises, determine the degree of each polynomial.

Exercise:

ⓐ $6a^2+12a+14$
ⓑ $18xy^2z$
ⓒ $5x+2$
ⓓ $y^3-8y^2+2y-16$
Problem: ⓔ -24

Solution:

ⓐ 2 ⓑ 4 ⓒ 1 ⓓ 3 ⓔ 0

Exercise:

ⓐ $9y^3-10y^2+2y-6$
ⓑ $-12p^4$
ⓒ $a^2+9a+18$
ⓓ $20x^2y^2-10a^2b^2+30$
Problem: ⓔ 17

Exercise:

ⓐ $14-29x$
ⓑ z^2-5z-6
ⓒ $y^3-8y^2+2y-16$
ⓓ $23ab^2-14$
Problem: ⓔ -3

Solution:

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 3 ⓔ 0

Exercise:

ⓐ $62y^2$
ⓑ 15
ⓒ $6x^2-3xy+4x-2y+y^2$
ⓓ $10-9x$
Problem: ⓔ $m^4+4m^3+6m^2+4m+1$

Add and Subtract Monomials

In the following exercises, add or subtract the monomials.

Exercise:

Problem: $7\mathrm{x}^2 + 5x^2$

Solution:

$12x^2$

Exercise:

Problem: $4\mathrm{y}^3 + 6y^3$

Exercise:

Problem: $-12w + 18w$

Solution:

$6w$

Exercise:

Problem: $-3m + 9m$

Exercise:

Problem: $4\mathrm{a} - 9a$

Solution:

$-5a$

Exercise:

Problem: $-y - 5y$

Exercise:

Problem: $28x - (-12x)$

Solution:

$40x$

Exercise:

Problem: $13z - (-4z)$

Exercise:

Problem: $-5b - 17b$

Solution:

$-22b$

Exercise:

Problem: $-10x - 35x$

Exercise:

Problem: $12a + 5b - 22a$

Solution:

$-10\text{a} + 5b$

Exercise:

Problem: $14\text{x} - 3y - 13x$

Exercise:

Problem: $2a^2 + b^2 - 6a^2$

Solution:

$-4a^2 + b^2$

Exercise:

Problem: $5u^2 + 4v^2 - 6u^2$

Exercise:

Problem: $xy^2 - 5x - 5y^2$

Solution:

$xy^2 - 5x - 5y^2$

Exercise:

Problem: $pq^2 - 4p - 3q^2$

Exercise:

Problem: $a^2b - 4a - 5ab^2$

Solution:

$a^2b - 4a - 5ab^2$

Exercise:

Problem: $x^2y - 3x + 7xy^2$

Exercise:

Problem: $12\text{a} + 8b$

Solution:

$12\text{a}+8b$

Exercise:

Problem: $19\text{y}+5z$

Exercise:

Problem: Add: $4a,-3b,-8a$

Solution:

$-4a-3b$

Exercise:

Problem: Add: $4\text{x},3y,-3x$

Exercise:

Problem: Subtract $5x^6\text{from}-12x^6$.

Solution:

$-17x^6$

Exercise:

Problem: Subtract $2p^4\text{from}-7p^4$.

Add and Subtract Polynomials

In the following exercises, add or subtract the polynomials.

Exercise:

Problem: $(5y^2+12y+4)+(6y^2-8y+7)$

Solution:

$11y^2+4y+11$

Exercise:

Problem: $(4y^2+10y+3)+(8y^2-6y+5)$

Exercise:

Problem: $(x^2+6x+8)+(-4x^2+11x-9)$

Solution:

$-3x^2+17x-1$

Exercise:

Problem: $(y^2+9y+4)+(-2y^2-5y-1)$

Exercise:

Problem: $(8x^2 - 5x + 2) + (3x^2 + 3)$

Solution:

$11x^2 - 5x + 5$

Exercise:

Problem: $(7x^2 - 9x + 2) + (6x^2 - 4)$

Exercise:

Problem: $(5a^2 + 8) + (a^2 - 4a - 9)$

Solution:

$6a^2 - 4a - 1$

Exercise:

Problem: $(p^2 - 6p - 18) + (2p^2 + 11)$

Exercise:

Problem: $(4m^2 - 6m - 3) - (2m^2 + m - 7)$

Solution:

$2m^2 - 7m + 4$

Exercise:

Problem: $(3b^2 - 4b + 1) - (5b^2 - b - 2)$

Exercise:

Problem: $(a^2 + 8a + 5) - (a^2 - 3a + 2)$

Solution:

$5a + 3$

Exercise:

Problem: $(b^2 - 7b + 5) - (b^2 - 2b + 9)$

Exercise:

Problem: $(12s^2 - 15s) - (s - 9)$

Solution:

$12s^2 - 16s + 9$

Exercise:

Problem: $(10r^2 - 20r) - (r - 8)$

Exercise:

Problem: Subtract $\left(9x^2+2\right)$ from $\left(12x^2-x+6\right)$.

Solution:

$3x^2-x+4$

Exercise:

Problem: Subtract $\left(5y^2-y+12\right)$ from $\left(10y^2-8y-20\right)$.

Exercise:

Problem: Subtract $\left(7w^2-4w+2\right)$ from $\left(8w^2-w+6\right)$.

Solution:

w^2+3w+4

Exercise:

Problem: Subtract $\left(5x^2-x+12\right)$ from $\left(9x^2-6x-20\right)$.

Exercise:

Problem: Find the sum of $\left(2p^3-8\right)$ and $\left(p^2+9p+18\right)$.

Solution:

$2p^3+p^2+9p+10$

Exercise:

Problem: Find the sum of $\left(q^2+4q+13\right)$ and $\left(7q^3-3\right)$.

Exercise:

Problem: Find the sum of $\left(8a^3-8a\right)$ and $\left(a^2+6a+12\right)$.

Solution:

$8a^3+a^2-2a+12$

Exercise:

Problem: Find the sum of $\left(b^2+5b+13\right)$ and $\left(4b^3-6\right)$.

Exercise:

Problem: Find the difference of $\left(w^2+w-42\right)$ and $\left(w^2-10w+24\right)$.

Solution:

$11w - 66$

Exercise:

Problem: Find the difference of $\left(z^2 - 3z - 18\right)$ and $\left(z^2 + 5z - 20\right)$.

Exercise:

Problem: Find the difference of $\left(c^2 + 4c - 33\right)$ and $\left(c^2 - 8c + 12\right)$.

Solution:

$12c - 45$

Exercise:

Problem: Find the difference of $\left(t^2 - 5t - 15\right)$ and $\left(t^2 + 4t - 17\right)$.

Exercise:

Problem: $\left(7x^2 - 2xy + 6y^2\right) + \left(3x^2 - 5xy\right)$

Solution:

$10x^2 - 7xy + 6y^2$

Exercise:

Problem: $\left(-5x^2 - 4xy - 3y^2\right) + \left(2x^2 - 7xy\right)$

Exercise:

Problem: $\left(7m^2 + mn - 8n^2\right) + \left(3m^2 + 2mn\right)$

Solution:

$10m^2 + 3mn - 8n^2$

Exercise:

Problem: $\left(2r^2 - 3rs - 2s^2\right) + \left(5r^2 - 3rs\right)$

Exercise:

Problem: $\left(a^2 - b^2\right) - \left(a^2 + 3ab - 4b^2\right)$

Solution:

$-3ab+3b^2$

Exercise:

Problem: $\left(m^2+2n^2\right)-\left(m^2-8mn-n^2\right)$

Exercise:

Problem: $\left(u^2-v^2\right)-\left(u^2-4uv-3v^2\right)$

Solution:

$4uv+2v^2$

Exercise:

Problem: $\left(j^2-k^2\right)-\left(j^2-8jk-5k^2\right)$

Exercise:

Problem: $\left(p^3-3p^2q\right)+\left(2pq^2+4q^3\right)-\left(3p^2q+pq^2\right)$

Solution:

$p^3-6p^2q+pq^2+4q^3$

Exercise:

Problem: $\left(a^3-2a^2b\right)+\left(ab^2+b^3\right)-\left(3a^2b+4ab^2\right)$

Exercise:

Problem: $\left(x^3-x^2y\right)-\left(4xy^2-y^3\right)+\left(3x^2y-xy^2\right)$

Solution:

$x^3+2x^2y-5xy^2+y^3$

Exercise:

Problem: $\left(x^3-2x^2y\right)-\left(xy^2-3y^3\right)-\left(x^2y-4xy^2\right)$

Evaluate a Polynomial for a Given Value

In the following exercises, evaluate each polynomial for the given value.

Exercise:

Problem: Evaluate $8y^2-3y+2$ when:

ⓐ $y=5$
ⓑ $y=-2$
ⓒ $y=0$

Solution:

ⓐ 187 ⓑ 46 ⓒ 2

Exercise:

Problem: Evaluate $5y^2 - y - 7$ when:

ⓐ $y = -4$
ⓑ $y = 1$
ⓒ $y = 0$

Exercise:

Problem: Evaluate $4 - 36x$ when:

ⓐ $x = 3$
ⓑ $x = 0$
ⓒ $x = -1$

Solution:

ⓐ −104 ⓑ 4 ⓒ 40

Exercise:

Problem: Evaluate $16 - 36x^2$ when:

ⓐ $x = -1$
ⓑ $x = 0$
ⓒ $x = 2$

Exercise:

Problem:

A painter drops a brush from a platform 75 feet high. The polynomial $-16t^2 + 75$ gives the height of the brush t seconds after it was dropped. Find the height after $t = 2$ seconds.

Solution:

11

Exercise:

Problem:

A girl drops a ball off a cliff into the ocean. The polynomial $-16t^2 + 250$ gives the height of a ball t seconds after it is dropped from a 250-foot tall cliff. Find the height after $t = 2$ seconds.

Exercise:

Problem:

A manufacturer of stereo sound speakers has found that the revenue received from selling the speakers at a cost of p dollars each is given by the polynomial $-4p^2 + 420p$. Find the revenue received when $p = 60$ dollars.

Solution:

$10,800

Exercise:

Problem:

A manufacturer of the latest basketball shoes has found that the revenue received from selling the shoes at a cost of p dollars each is given by the polynomial $-4p^2 + 420p$. Find the revenue received when $p = 90$ dollars.

Everyday Math

Exercise:

Problem:

Fuel Efficiency The fuel efficiency (in miles per gallon) of a car going at a speed of x miles per hour is given by the polynomial $-\frac{1}{150}x^2 + \frac{1}{3}x$. Find the fuel efficiency when $x = 30\,\text{mph}$.

Solution:

4

Exercise:

Problem:

Stopping Distance The number of feet it takes for a car traveling at x miles per hour to stop on dry, level concrete is given by the polynomial $0.06x^2 + 1.1x$. Find the stopping distance when $x = 40\,\text{mph}$.

Exercise:

Problem:

Rental Cost The cost to rent a rug cleaner for d days is given by the polynomial $5.50d + 25$. Find the cost to rent the cleaner for 6 days.

Solution:

$58

Exercise:

Problem:

Height of Projectile The height (in feet) of an object projected upward is given by the polynomial $-16t^2 + 60t + 90$ where t represents time in seconds. Find the height after $t = 2.5$ seconds.

Exercise:

Problem:

Temperature Conversion The temperature in degrees Fahrenheit is given by the polynomial $\frac{9}{5}c + 32$ where c represents the temperature in degrees Celsius. Find the temperature in degrees Fahrenheit when $c = 65^\circ$.

Solution:

149

Writing Exercises

Exercise:

Problem: Using your own words, explain the difference between a monomial, a binomial, and a trinomial.

Exercise:

Problem:

Using your own words, explain the difference between a polynomial with five terms and a polynomial with a degree of 5.

Solution:

Answers will vary.

Exercise:

Problem: Ariana thinks the sum $6y^2 + 5y^4$ is $11y^6$. What is wrong with her reasoning?

Exercise:

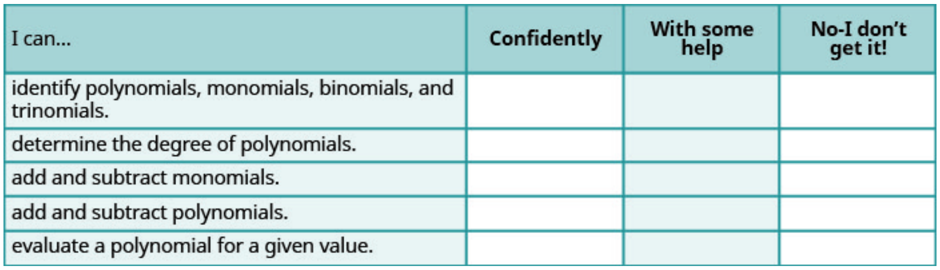
Problem: Jonathan thinks that $\frac{1}{3}$ and $\frac{1}{x}$ are both monomials. What is wrong with his reasoning?

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
identify polynomials, monomials, binomials, and trinomials.			
determine the degree of polynomials.			
add and subtract monomials.			
add and subtract polynomials.			
evaluate a polynomial for a given value.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

binomial
A binomial is a polynomial with exactly two terms.

degree of a constant
The degree of any constant is 0.

degree of a polynomial
The degree of a polynomial is the highest degree of all its terms.

degree of a term
The degree of a term is the exponent of its variable.

monomial
A monomial is a term of the form ax^m, where a is a constant and m is a whole number; a monomial has exactly one term.

polynomial
A polynomial is a monomial, or two or more monomials combined by addition or subtraction.

standard form
A polynomial is in standard form when the terms of a polynomial are written in descending order of degrees.

trinomial
A trinomial is a polynomial with exactly three terms.

Use Multiplication Properties of Exponents
By the end of this section, you will be able to:

- Simplify expressions with exponents
- Simplify expressions using the Product Property for Exponents
- Simplify expressions using the Power Property for Exponents
- Simplify expressions using the Product to a Power Property
- Simplify expressions by applying several properties
- Multiply monomials

Note:
Before you get started, take this readiness quiz.

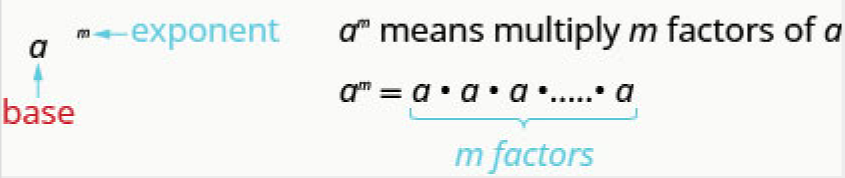
1. Simplify: $\frac{3}{4} \cdot \frac{3}{4}$.
   If you missed this problem, review [link].
2. Simplify: $(-2)(-2)(-2)$.
   If you missed this problem, review [link].

Simplify Expressions with Exponents

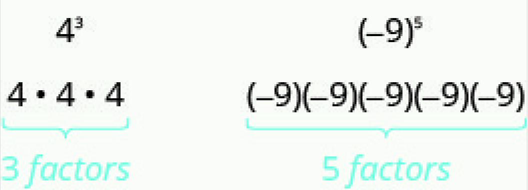
Remember that an exponent indicates repeated multiplication of the same quantity. For example, 2^4 means to multiply 2 by itself 4 times, so 2^4 means $2 \cdot 2 \cdot 2 \cdot 2$.

Let's review the vocabulary for expressions with exponents.

Note:
Exponential Notation


a
m
exponent
base
a^m means multiply m factors of a
$a^m = a \cdot a \cdot a \cdot \ldots\ldots \cdot a$
m factors

This is read a to the m^{th} power.
In the expression a^m, the *exponent* m tells us how many times we use the *base* a as a factor.



Before we begin working with variable expressions containing exponents, let's simplify a few expressions involving only numbers.

Example:
Exercise:

Problem: **Simplify: ⓐ 4^3 ⓑ 7^1 ⓒ $\left(\frac{5}{6}\right)^2$ ⓓ $(0.63)^2$.

Solution:
Solution

ⓐ	4^3
Multiply three factors of 4.	$4 \cdot 4 \cdot 4$

Simplify.	64
ⓑ	7^1
Multiply one factor of 7.	7
ⓒ	$\left(\frac{5}{6}\right)^2$
Multiply two factors.	$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$
Simplify.	$\frac{25}{36}$
ⓓ	$(0.63)^2$
Multiply two factors.	$(0.63)(0.63)$
Simplify.	0.3969

Note:
Exercise:

TRY IT :: 5.31

Problem: Simplify: ⓐ 6^3 ⓑ 15^1 ⓒ $\left(\frac{3}{7}\right)^2$ ⓓ $(0.43)^2$.

Solution:

ⓐ 216 ⓑ 15 ⓒ $\frac{9}{49}$ ⓓ 0.1849

Note:
Exercise:

TRY IT : : 5.32

Problem: Simplify: ⓐ 2^5 ⓑ 21^1 ⓒ $\left(\frac{2}{5}\right)^3$ ⓓ $(0.218)^2$.

Solution:

ⓐ 32 ⓑ 21 ⓒ $\frac{8}{125}$ ⓓ 0.047524

Example:
Exercise:

Problem: **Simplify: ⓐ $(-5)^4$ ⓑ -5^4.

Solution:
Solution

ⓐ	$(-5)^4$
Multiply four factors of -5.	$(-5)(-5)(-5)(-5)$
Simplify.	625
ⓑ	-5^4
Multiply four factors of 5.	$-(5\cdot5\cdot5\cdot5)$

Simplify.	-625

Note:
Exercise:

TRY IT :: 5.33

Problem: Simplify: ⓐ $(-3)^4$ ⓑ -3^4.

Solution:

ⓐ 81 ⓑ -81

Note:
Exercise:

TRY IT :: 5.34

Problem: Simplify: ⓐ $(-13)^2$ ⓑ -13^2.

Solution:

ⓐ 169 ⓑ -169

Notice the similarities and differences in [link]ⓐ and [link]ⓑ! Why are the answers different? As we follow the order of operations in part ⓐ the parentheses tell us to raise the (-5) to the 4^{th} power. In part ⓑ we raise just the 5 to the 4^{th} power and then take the opposite.

Simplify Expressions Using the Product Property for Exponents

You have seen that when you combine like terms by adding and subtracting, you need to have the same base with the same exponent. But when you multiply and divide, the exponents may be different, and sometimes the bases may be different, too.

We’ll derive the properties of exponents by looking for patterns in several examples.

First, we will look at an example that leads to the Product Property.

	$x^2 \quad \cdot \quad x^3$
What does this mean? How many factors altogether?	$\underbrace{\underbrace{x \cdot x}_{2\ factors} \quad \cdot \quad \underbrace{x \cdot x \cdot x}_{3\ factors}}_{5\ factors}$
So, we have	x^5
Notice that 5 is the sum of the exponents, 2 and 3.	$x^2 \cdot x^3$ is x^{2+3}, or x^5

We write:
Equation:

$$x^2 \cdot x^3$$
$$x^{2+3}$$
$$x^5$$

The base stayed the same and we added the exponents. This leads to the **Product Property for Exponents**.

Note:
Product Property for Exponents
If a is a real number, and m and n are counting numbers, then
Equation:

$$a^m \cdot a^n = a^{m+n}$$

To multiply with like bases, add the exponents.

An example with numbers helps to verify this property.
Equation:

$$\begin{aligned} 2^2 \cdot 2^3 &\overset{?}{=} 2^{2+3} \\ 4 \cdot 8 &\overset{?}{=} 2^5 \\ 32 &= 32 \checkmark \end{aligned}$$

Example:
Exercise:

Problem: **Simplify: $y^5 \cdot y^6$.

Solution:
Solution

	$y^5 \cdot y^6$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	y^{5+6}
Simplify.	y^{11}

Note:
Exercise:

TRY IT :: 5.35

Problem: Simplify: $b^9 \cdot b^8$.

Solution:

b^{17}

Note:
Exercise:

TRY IT :: 5.36

Problem: Simplify: $x^{12} \cdot x^4$.

Solution:

x^{16}

Example:
Exercise:

Problem: **Simplify: ⓐ $2^5 \cdot 2^9$ ⓑ $3 \cdot 3^4$.

Solution:
Solution

ⓐ

	$2^5 \cdot 2^9$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	2^{5+9}
Simplify.	2^{14}

ⓑ

	$3 \cdot 3^4$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	3^{1+4}
Simplify.	3^5

Note:
Exercise:

TRY IT :: 5.37

Problem: Simplify: ⓐ $5 \cdot 5^5$ ⓑ $4^9 \cdot 4^9$.

Solution:

ⓐ 5^6 ⓑ 4^{18}

Note:
Exercise:

TRY IT :: 5.38

Problem: Simplify: ⓐ $7^6 \cdot 7^8$ ⓑ $10 \cdot 10^{10}$.

Solution:

ⓐ 7^{14} ⓑ 10^{11}

Example:
Exercise:

Problem: **Simplify: ⓐ $a^7 \cdot a$ ⓑ $x^{27} \cdot x^{13}$.

Solution:
Solution

ⓐ

	$a^7 \cdot a$
Rewrite, a = a^1.	$a^7 \cdot a^1$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	a^{7+1}
Simplify.	a^8

ⓑ

	$x^{27} \cdot x^{13}$

Notice, the bases are the same, so add the exponents.	x^{27+13}
Simplify.	x^{40}

Note:
Exercise:

TRY IT :: 5.39

Problem: Simplify: ⓐ $p^5 \cdot p$ ⓑ $y^{14} \cdot y^{29}$.

Solution:

ⓐ p^6 ⓑ y^{43}

Note:
Exercise:

TRY IT :: 5.40

Problem: Simplify: ⓐ $z \cdot z^7$ ⓑ $b^{15} \cdot b^{34}$.

Solution:

ⓐ z^8 ⓑ b^{49}

We can extend the Product Property for Exponents to more than two factors.

Example:
Exercise:

Problem: **Simplify: $d^4 \cdot d^5 \cdot d^2$.

Solution:
Solution

	$d^4 \cdot d^5 \cdot d^2$
Add the exponents, since bases are the same.	d^{4+5+2}
Simplify.	d^{11}

Note:
Exercise:

TRY IT :: 5.41
Problem: Simplify: $x^6 \cdot x^4 \cdot x^8$.

Solution:

x^{18}

Note:
Exercise:

TRY IT :: 5.42

Problem: Simplify: $b^5 \cdot b^9 \cdot b^5$.

Solution:

b^{19}

Simplify Expressions Using the Power Property for Exponents

Now let’s look at an exponential expression that contains a power raised to a power. See if you can discover a general property.

	$(x^2)^3$
What does this mean? How many factors altogether?	$\begin{array}{c} x^2 \cdot x^2 \cdot x^2 \\ \underbrace{\underbrace{x \cdot x}_{2\ factors} \cdot \underbrace{x \cdot x}_{2\ factors} \cdot \underbrace{x \cdot x}_{2\ factors}}_{6\ factors} \end{array}$

So we have	x^6
Notice that 6 is the product of the exponents, 2 and 3.	$(x^2)^3$ is $x^{2 \cdot 3}$ or x^6

We write:
Equation:

$$\left(x^2\right)^3$$
$$x^{2 \cdot 3}$$
$$x^6$$

We multiplied the exponents. This leads to the **Power Property for Exponents.**

Note:
Power Property for Exponents
If a is a real number, and m and n are whole numbers, then
Equation:

$$(a^m)^n = a^{m \cdot n}$$

To raise a power to a power, multiply the exponents.

An example with numbers helps to verify this property.
Equation:

$$\begin{aligned}\left(3^2\right)^3 &\overset{?}{=} 3^{2\cdot3}\\ (9)^3 &\overset{?}{=} 3^6\\ 729 &= 729\,\checkmark\end{aligned}$$

Example:
Exercise:

Problem: **Simplify: ⓐ $\left(y^5\right)^9$ ⓑ $\left(4^4\right)^7$.

Solution:
Solution

ⓐ

	$\left(y^5\right)^9$
Use the power property, $(a^m)^n = a^{m\cdot n}$.	$y^{5\cdot9}$
Simplify.	y^{45}

ⓑ

	$(4^4)^7$
Use the power property.	$4^{4\cdot 7}$
Simplify.	4^{28}

Note:
Exercise:

TRY IT :: 5.43

Problem: Simplify: ⓐ $(b^7)^5$ ⓑ $(5^4)^3$.

Solution:

ⓐ b^{35} ⓑ 5^{12}

Note:
Exercise:

TRY IT :: 5.44

Problem: Simplify: ⓐ $(z^6)^9$ ⓑ $(3^7)^7$.

Solution:

ⓐ z^{54} ⓑ 3^{49}

Simplify Expressions Using the Product to a Power Property

We will now look at an expression containing a product that is raised to a power. Can you find this pattern?

	$(2x)^3$
What does this mean?	$2x \cdot 2x \cdot 2x$
We group the like factors together.	$2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$
How many factors of 2 and of x?	$2^3 \cdot x^3$

Notice that each factor was raised to the power and $(2x)^3$ is $2^3 \cdot x^3$.

We write:	$(2x)^3$
	$2^3 \cdot x^3$

The exponent applies to each of the factors! This leads to the **Product to a Power Property for Exponents.**

Note:
Product to a Power Property for Exponents

If a and b are real numbers and m is a whole number, then
Equation:

$$(ab)^m = a^m b^m$$

To raise a product to a power, raise each factor to that power.

An example with numbers helps to verify this property:
Equation:

$$\begin{aligned}(2\cdot 3)^2 &\overset{?}{=} 2^2\cdot 3^2\\ 6^2 &\overset{?}{=} 4\cdot 9\\ 36 &= 36\,\checkmark\end{aligned}$$

Example:
Exercise:

Problem: **Simplify: ⓐ $(-9d)^2$ ⓑ $(3mn)^3$.

Solution:
Solution

ⓐ

	$(-9d)^2$

Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(-9)^2 d^2$
Simplify.	$81d^2$

ⓑ

	$(3mn)^3$
Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(3)^3 m^3 n^3$
Simplify.	$27m^3n^3$

Note:
Exercise:

TRY IT :: 5.45

Problem: Simplify: ⓐ $(-12y)^2$ ⓑ $(2wx)^5$.

Solution:

ⓐ $144y^2$ ⓑ $32w^5x^5$

Note:
Exercise:

TRY IT :: 5.46

Problem: Simplify: ⓐ $(5wx)^3$ ⓑ $(-3y)^3$.

Solution:

ⓐ $125w^3x^3$ ⓑ $-27y^3$

Simplify Expressions by Applying Several Properties

We now have three properties for multiplying expressions with exponents. Let’s summarize them and then we’ll do some examples that use more than one of the properties.

Note:
Properties of Exponents
If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n \quad = \quad a^{m+n}$
Power Property	$(a^m)^n \quad = \quad a^{m \cdot n}$

Product to a Power	$(ab)^m \quad = \quad a^m b^m$

All exponent properties hold true for any real numbers m and n. Right now, we only use whole number exponents.

Example:
Exercise:

Problem: **Simplify: ⓐ $\left(y^3\right)^6\left(y^5\right)^4$ ⓑ $\left(-6x^4y^5\right)^2$.

Solution:
Solution

ⓐ	$\left(y^3\right)^6\left(y^5\right)^4$
Use the Power Property.	$y^{18} \cdot y^{20}$
Add the exponents.	y^{38}
ⓑ	$\left(-6x^4y^5\right)^2$
Use the Product to a Power Property.	$(-6)^2\left(x^4\right)^2\left(y^5\right)^2$
Use the Power Property.	$(-6)^2$
Simplify.	

	$36x^8y^{10}$

Note:
Exercise:

TRY IT :: 5.47

Problem: Simplify: ⓐ $(a^4)^5(a^7)^4$ ⓑ $\left(-2c^4d^2\right)^3$.

Solution:

ⓐ a^{48} ⓑ $-8c^{12}d^6$

Note:
Exercise:

TRY IT :: 5.48

Problem: Simplify: ⓐ $\left(-3x^6y^7\right)^4$ ⓑ $(q^4)^5(q^3)^3$.

Solution:

ⓐ $81x^{24}y^{28}$ ⓑ q^{29}

Example:
Exercise:

Problem: **Simplify: ⓐ $(5m)^2(3m^3)$ ⓑ $(3x^2y)^4(2xy^2)^3$.

Solution:
Solution

ⓐ	$(5m)^2(3m^3)$
Raise $5m$ to the second power.	$5^2m^2 \cdot 3m^3$
Simplify.	$25m^2 \cdot 3m^3$
Use the Commutative Property.	$25 \cdot 3 \cdot m^2 \cdot m^3$
Multiply the constants and add the exponents.	$75m^5$
ⓑ	$(3x^2y)^4(2xy^2)^3$
Use the Product to a Power Property.	$(3^4x^8y^4)(2^3x^3y^6)$
Simplify.	$(81x^8y^4)(8x^3y^6)$
Use the Commutative Property.	$81 \cdot 8 \cdot x^8 \cdot x^3 \cdot y^4 \cdot y^6$
Multiply the constants and add the exponents.	$648x^{11}y^{10}$

Note:

Exercise:

TRY IT :: 5.49

Problem: Simplify: ⓐ $(5n)^2(3n^{10})$ ⓑ $(c^4d^2)^5(3cd^5)^4$.

Solution:

ⓐ $75n^{12}$ ⓑ $81c^{24}d^{30}$

Note:
Exercise:

TRY IT :: 5.50

Problem: Simplify: ⓐ $(a^3b^2)^6(4ab^3)^4$ ⓑ $(2x)^3(5x^7)$.

Solution:

ⓐ $256a^{22}b^{24}$ ⓑ $40x^{10}$

Multiply Monomials

Since a monomial is an algebraic expression, we can use the properties of exponents to multiply monomials.

Example:
Exercise:

Problem: **Multiply: $\left(3x^2\right)\left(-4x^3\right)$.

Solution:
Solution

	$\left(3x^2\right)\left(-4x^3\right)$
Use the Commutative Property to rearrange the terms.	$3\cdot(-4)\cdot x^2\cdot x^3$
Multiply.	$-12x^5$

Note:
Exercise:

TRY IT :: 5.51
Problem: Multiply: $\left(5y^7\right)\left(-7y^4\right)$.

Solution:

$-35y^{11}$

Note:
Exercise:

TRY IT :: 5.52
Problem: Multiply: $\left(-6b^4\right)\left(-9b^5\right)$.

Solution:

$54b^9$

Example:
Exercise:

Problem: **Multiply: $\left(\frac{5}{6}x^3y\right)\left(12xy^2\right)$.

Solution:
Solution

	$\left(\frac{5}{6}x^3y\right)\left(12xy^2\right)$
Use the Commutative Property to rearrange the terms.	$\frac{5}{6} \cdot 12 \cdot x^3 \cdot x \cdot y \cdot y^2$
Multiply.	$10x^4y^3$

Note:
Exercise:

TRY IT :: 5.53
Problem: Multiply: $\left(\frac{2}{5}a^4b^3\right)\left(15ab^3\right)$.

Solution:

$6a^5b^6$

Note:
Exercise:

TRY IT :: 5.54

Problem: Multiply: $\left(\frac{2}{3}r^5s\right)\left(12r^6s^7\right)$.

Solution:

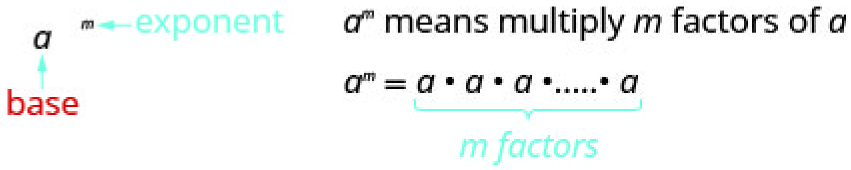
$8r^{11}s^8$

Note:
Access these online resources for additional instruction and practice with using multiplication properties of exponents:

- Multiplication Properties of Exponents

Key Concepts

- **Exponential Notation**


a
m
exponent
base
a^m means multiply m factors of a
$a^m = a \cdot a \cdot a \cdot \ldots \cdot a$
m factors

- **Properties of Exponents**

- If a, b are real numbers and m, n are whole numbers, then

$$\begin{array}{lrcl} \textbf{Product Property} & a^m \cdot a^n & = & a^{m+n} \\ \textbf{Power Property} & (a^m)^n & = & a^{m \cdot n} \\ \textbf{Product to a Power} & (ab)^m & = & a^m b^m \end{array}$$

Practice Makes Perfect

Simplify Expressions with Exponents

In the following exercises, simplify each expression with exponents.

Exercise:

Problem:
ⓐ 3^5
ⓑ 9^1
ⓒ $\left(\frac{1}{3}\right)^2$
ⓓ $(0.2)^4$

Exercise:

Problem:
ⓐ 10^4
ⓑ 17^1
ⓒ $\left(\frac{2}{9}\right)^2$
ⓓ $(0.5)^3$

Solution:

ⓐ 10,000 ⓑ 17 ⓒ $\frac{4}{81}$ ⓓ 0.125

Exercise:

ⓐ 2^6
ⓑ 14^1
ⓒ $\left(\frac{2}{5}\right)^3$
Problem: ⓓ $(0.7)^2$

Exercise:

ⓐ 8^3
ⓑ 8^1
ⓒ $\left(\frac{3}{4}\right)^3$
Problem: ⓓ $(0.4)^3$

Solution:

ⓐ 512 ⓑ 8 ⓒ $\frac{27}{64}$
ⓓ 0.064

Exercise:

ⓐ $(-6)^4$
Problem: ⓑ -6^4

Exercise:

ⓐ $(-2)^6$
Problem: ⓑ -2^6

Solution:

ⓐ 64 ⓑ -64

Exercise:

Problem: ⓐ $-\left(\frac{1}{4}\right)^4$ ⓑ $\left(-\frac{1}{4}\right)^4$

Exercise:

Problem: ⓐ $-\left(\frac{2}{3}\right)^2$ ⓑ $\left(-\frac{2}{3}\right)^2$

Solution:

ⓐ $-\frac{4}{9}$
ⓑ $\frac{4}{9}$

Exercise:

Problem: ⓐ -0.5^2 ⓑ $(-0.5)^2$

Exercise:

Problem: ⓐ -0.1^4 ⓑ $(-0.1)^4$

Solution:

ⓐ -0.0001 ⓑ 0.0001

Simplify Expressions Using the Product Property for Exponents

In the following exercises, simplify each expression using the Product Property for Exponents.

Exercise:

Problem: $d^3 \cdot d^6$

Exercise:

Problem: $x^4 \cdot x^2$

Solution:

x^6

Exercise:

Problem: $n^{19} \cdot n^{12}$

Exercise:

Problem: $q^{27} \cdot q^{15}$

Solution:

q^{42}

Exercise:

Problem: ⓐ $4^5 \cdot 4^9$ ⓑ $8^9 \cdot 8$

Exercise:

Problem: ⓐ $3^{10} \cdot 3^6$ ⓑ $5 \cdot 5^4$

Solution:

ⓐ 3^{16} ⓑ 5^5

Exercise:

Problem: ⓐ $y \cdot y^3$ ⓑ $z^{25} \cdot z^8$

Exercise:

Problem: ⓐ $w^5 \cdot w$ ⓑ $u^{41} \cdot u^{53}$

Solution:

ⓐ w^6 ⓑ u^{94}

Exercise:

Problem: $w \cdot w^2 \cdot w^3$

Exercise:

Problem: $y \cdot y^3 \cdot y^5$

Solution:

y^9

Exercise:

Problem: $a^4 \cdot a^3 \cdot a^9$

Exercise:

Problem: $c^5 \cdot c^{11} \cdot c^2$

Solution:

c^{18}

Exercise:

Problem: $m^x \cdot m^3$

Exercise:

Problem: $n^y \cdot n^2$

Solution:

n^{y+2}

Exercise:

Problem: $y^a \cdot y^b$

Exercise:

Problem: $x^p \cdot x^q$

Solution:

x^{p+q}

Simplify Expressions Using the Power Property for Exponents

In the following exercises, simplify each expression using the Power Property for Exponents.

Exercise:

Problem: ⓐ $(m^4)^2$ ⓑ $(10^3)^6$

Exercise:

Problem: ⓐ $(b^2)^7$ ⓑ $(3^8)^2$

Solution:

ⓐ b^{14} ⓑ 3^{16}

Exercise:

Problem: ⓐ $\left(y^3\right)^x$ ⓑ $(5^x)^y$

Exercise:

Problem: ⓐ $\left(x^2\right)^y$ ⓑ $(7^a)^b$

Solution:

ⓐ x^{2y} ⓑ 7^{ab}

Simplify Expressions Using the Product to a Power Property

In the following exercises, simplify each expression using the Product to a Power Property.

Exercise:

Problem: ⓐ $(6a)^2$ ⓑ $(3xy)^2$

Exercise:

Problem: ⓐ $(5x)^2$ ⓑ $(4ab)^2$

Solution:

ⓐ $25x^2$ ⓑ $16a^2b^2$

Exercise:

Problem: ⓐ $(-4m)^3$ ⓑ $(5ab)^3$

Exercise:

Problem: ⓐ $(-7n)^3$ ⓑ $(3xyz)^4$

Solution:

ⓐ $-343n^3$ ⓑ $81x^4y^4z^4$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify each expression.

Exercise:

Problem: ⓐ $\left(y^2\right)^4 \cdot \left(y^3\right)^2$ ⓑ $(10a^2b)^3$

Exercise:

Problem: ⓐ $\left(w^4\right)^3 \cdot \left(w^5\right)^2$ ⓑ $(2xy^4)^5$

Solution:

ⓐ w^{22} ⓑ $32x^5y^{20}$

Exercise:

Problem: ⓐ $(-2r^3s^2)^4$ ⓑ $\left(m^5\right)^3 \cdot \left(m^9\right)^4$

Exercise:

Problem: ⓐ $(-10q^2p^4)^3$ ⓑ $\left(n^3\right)^{10} \cdot \left(n^5\right)^2$

Solution:

ⓐ $-1000q^6p^{12}$ ⓑ n^{40}

Exercise:

ⓐ $(3x)^2(5x)$

Problem: ⓑ $(5t^2)^3(3t)^2$

Exercise:

ⓐ $(2y)^3(6y)$

Problem: ⓑ $(10k^4)^3(5k^6)^2$

Solution:

ⓐ $48y^4$ ⓑ $25{,}000k^{24}$

Exercise:

ⓐ $(5a)^2(2a)^3$

Problem: ⓑ $(\frac{1}{2}y^2)^3(\frac{2}{3}y)^2$

Exercise:

ⓐ $(4b)^2(3b)^3$

Problem: ⓑ $(\frac{1}{2}j^2)^5(\frac{2}{5}j^3)^2$

Solution:

ⓐ $432b^5$ ⓑ $\frac{1}{200}j^{16}$

Exercise:

Problem: ⓐ $\left(\frac{2}{5}x^2y\right)^3$ ⓑ $\left(\frac{8}{9}xy^4\right)^2$

Exercise:

Problem: ⓐ $(2r^2)^3(4r)^2$ ⓑ $(3x^3)^3(x^5)^4$

Solution:

ⓐ $128r^8$ ⓑ $\frac{1}{200}j^{16}$

Exercise:

Problem: ⓐ $(m^2n)^2(2mn^5)^4$ ⓑ $(3pq^4)^2(6p^6q)^2$

Multiply Monomials

In the following exercises, multiply the monomials.

Exercise:

Problem: $\left(6y^7\right)\left(-3y^4\right)$

Solution:

$-18y^{11}$

Exercise:

Problem: $\left(-10x^5\right)\left(-3x^3\right)$

Exercise:

Problem: $(-8u^6)(-9u)$

Solution:

$72u^7$

Exercise:

Problem: $(-6c^4)(-12c)$

Exercise:

Problem: $(\frac{1}{5}f^8)(20f^3)$

Solution:

$4f^{11}$

Exercise:

Problem: $(\frac{1}{4}d^5)(36d^2)$

Exercise:

Problem: $(4a^3b)(9a^2b^6)$

Solution:

$36a^5b^7$

Exercise:

Problem: $(6m^4n^3)(7mn^5)$

Exercise:

Problem: $\left(\frac{4}{7}rs^2\right)\left(14rs^3\right)$

Solution:

$8r^2s^5$

Exercise:

Problem: $\left(\frac{5}{8}x^3y\right)\left(24x^5y\right)$

Exercise:

Problem: $\left(\frac{2}{3}x^2y\right)\left(\frac{3}{4}xy^2\right)$

Solution:

$\frac{1}{2}x^3y^3$

Exercise:

Problem: $\left(\frac{3}{5}m^3n^2\right)\left(\frac{5}{9}m^2n^3\right)$

Mixed Practice

In the following exercises, simplify each expression.

Exercise:

Problem: $\left(x^2\right)^4 \cdot \left(x^3\right)^2$

Solution:

x^{14}

Exercise:

Problem: $\left(y^4\right)^3 \cdot \left(y^5\right)^2$

Exercise:

Problem: $\left(a^2\right)^6 \cdot \left(a^3\right)^8$

Solution:

a^{36}

Exercise:

Problem: $\left(b^7\right)^5 \cdot \left(b^2\right)^6$

Exercise:

Problem: $(2m^6)^3$

Solution:

$8m^{18}$

Exercise:

Problem: $(3y^2)^4$

Exercise:

Problem: $(10x^2y)^3$

Solution:

$1000x^6y^3$

Exercise:

Problem: $(2mn^4)^5$

Exercise:

Problem: $(-2a^3b^2)^4$

Solution:

$16a^{12}b^8$

Exercise:

Problem: $(-10u^2v^4)^3$

Exercise:

Problem: $\left(\frac{2}{3}x^2y\right)^3$

Solution:

$\frac{8}{27}x^6y^3$

Exercise:

Problem: $\left(\frac{7}{9}pq^4\right)^2$

Exercise:

Problem: $(8a^3)^2(2a)^4$

Solution:

$1024a^{10}$

Exercise:

Problem: $\left(5r^{2}\right)^{3}\left(3r\right)^{2}$

Exercise:

Problem: $\left(10p^{4}\right)^{3}\left(5p^{6}\right)^{2}$

Solution:

$25000p^{24}$

Exercise:

Problem: $\left(4x^{3}\right)^{3}\left(2x^{5}\right)^{4}$

Exercise:

Problem: $\left(\frac{1}{2}x^{2}y^{3}\right)^{4}\left(4x^{5}y^{3}\right)^{2}$

Solution:

$x^{18}y^{18}$

Exercise:

Problem: $\left(\frac{1}{3}m^{3}n^{2}\right)^{4}\left(9m^{8}n^{3}\right)^{2}$

Exercise:

Problem: $\left(3m^{2}n\right)^{2}\left(2mn^{5}\right)^{4}$

Solution:

$144m^{8}n^{22}$

Exercise:

Problem: $(2pq^4)^3(5p^6q)^2$

Everyday Math

Exercise:

Problem:

Email Kate emails a flyer to ten of her friends and tells them to forward it to ten of their friends, who forward it to ten of their friends, and so on. The number of people who receive the email on the second round is 10^2, on the third round is 10^3, as shown in the table below. How many people will receive the email on the sixth round? Simplify the expression to show the number of people who receive the email.

Round	**Number of people**
1	10
2	10^2
3	10^3
…	…
6	?

Solution:

1,000,000

Exercise:

Problem:

Salary Jamal’s boss gives him a 3% raise every year on his birthday. This means that each year, Jamal’s salary is 1.03 times his last year’s salary. If his original salary was $35,000, his salary after 1 year was $\$35{,}000\,(1.03)$, after 2 years was $\$35{,}000(1.03)^2$, after 3 years was $\$35{,}000(1.03)^3$, as shown in the table below. What will Jamal’s salary be after 10 years? Simplify the expression, to show Jamal’s salary in dollars.

Year	**Salary**
1	$\$35{,}000\,(1.03)$
2	$\$35{,}000(1.03)^2$
3	$\$35{,}000(1.03)^3$
…	…
10	?

Exercise:

Problem:

Clearance A department store is clearing out merchandise in order to make room for new inventory. The plan is to mark down items by 30% each week. This means that each week the cost of an item is 70% of the previous week's cost. If the original cost of a sofa was $1,000, the cost for the first week would be $\$1,000\,(0.70)$ and the cost of the item during the second week would be $\$1,000(0.70)^2$. Complete the table shown below. What will be the cost of the sofa during the fifth week? Simplify the expression, to show the cost in dollars.

Week	**Cost**
1	$\$1,000\,(0.70)$
2	$\$1,000(0.70)^2$
3	
…	…
8	?

Solution:

$168.07

Exercise:

Problem:

Depreciation Once a new car is driven away from the dealer, it begins to lose value. Each year, a car loses 10% of its value. This means that each year the value of a car is 90% of the previous year's value. If a new car was purchased for $20,000, the value at the end of the first year would be $\$20{,}000\,(0.90)$ and the value of the car after the end of the second year would be $\$20{,}000(0.90)^2$. Complete the table shown below. What will be the value of the car at the end of the eighth year? Simplify the expression, to show the value in dollars.

Week	**Cost**
1	$\$20{,}000\,(0.90)$
2	$\$20{,}000(0.90)^2$
3	
4	…
5	?

Writing Exercises

Exercise:

Problem:

Use the Product Property for Exponents to explain why $x \cdot x = x^2$.

Solution:

Answers will vary.

Exercise:

Problem: Explain why $-5^3 = (-5)^3$ but $-5^4 \neq (-5)^4$.

Exercise:

Problem: Jorge thinks $\left(\frac{1}{2}\right)^2$ is 1. What is wrong with his reasoning?

Solution:

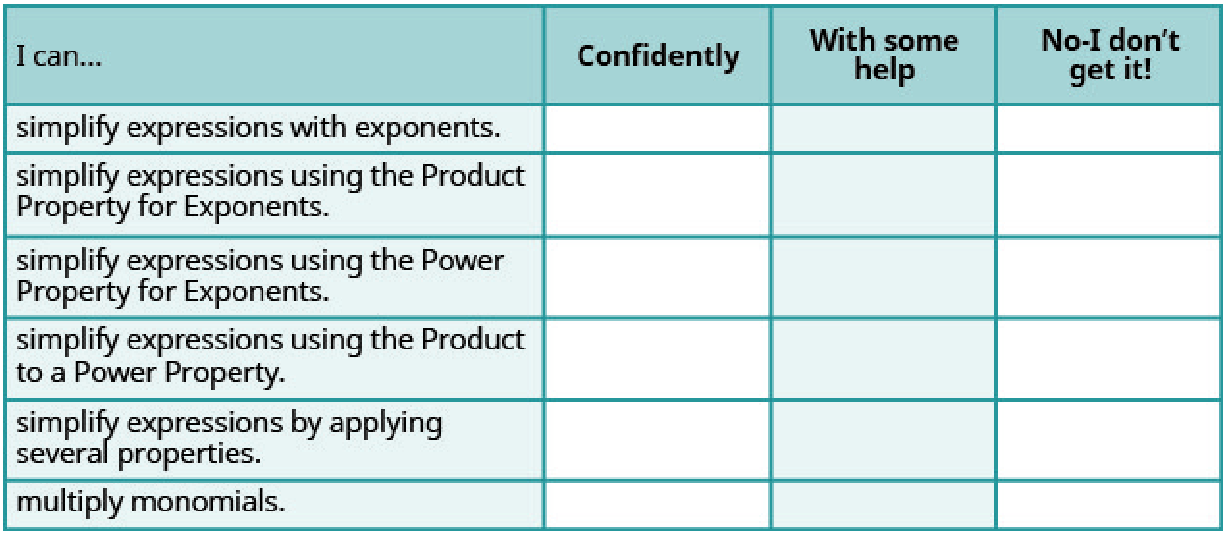
Answers will vary.

Exercise:

Problem: Explain why $x^3 \cdot x^5$ is x^8, and not x^{15}.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
simplify expressions with exponents.			
simplify expressions using the Product Property for Exponents.			
simplify expressions using the Power Property for Exponents.			
simplify expressions using the Product to a Power Property.			
simplify expressions by applying several properties.			
multiply monomials.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

Multiply Polynomials
By the end of this section, you will be able to:

- Multiply a polynomial by a monomial
- Multiply a binomial by a binomial
- Multiply a trinomial by a binomial

Note:
Before you get started, take this readiness quiz.

1. Distribute: $2\left(x+3\right)$.
   If you missed this problem, review [link].
2. Combine like terms: $x^2+9x+7x+63$.
   If you missed this problem, review [link].

Multiply a Polynomial by a Monomial

We have used the Distributive Property to simplify expressions like $2(x-3)$. You multiplied both terms in the parentheses, x and 3, by 2, to get $2x-6$. With this chapter's new vocabulary, you can say you were multiplying a binomial, $x-3$, by a monomial, 2.

Multiplying a binomial by a monomial is nothing new for you! Here's an example:

Example:
Exercise:

Problem: **Multiply: $4\left(x+3\right)$.

Solution:

Solution

	$4(x+3)$
Distribute.	$4 \cdot x + 4 \cdot 3$
Simplify.	$4x + 12$

Note:
Exercise:

TRY IT :: 5.55

Problem: Multiply: $5\left(x+7\right)$.

Solution:

$5x+35$

Note:
Exercise:

TRY IT :: 5.56
Problem: Multiply: $3\left(y+13\right)$.

Solution:

$3y+39$

Example:
Exercise:

Problem: **Multiply: $y\left(y-2\right)$.

Solution:
Solution

	$y(y-2)$
Distribute.	$y\cdot y-y\cdot 2$
Simplify.	y^2-2y

Note:
Exercise:

TRY IT :: 5.57

Problem: Multiply: $x\left(x-7\right)$.

Solution:

x^2-7x

Note:
Exercise:

TRY IT :: 5.58

Problem: Multiply: $d\left(d-11\right)$.

Solution:

d^2-11d

Example:
Exercise:

Problem: **Multiply: $7x(2x+y)$.

Solution:
Solution

	$7x(2x+y)$
Distribute.	$7x \cdot 2x + 7x \cdot y$
Simplify.	$14x^2 + 7xy$

Note:
Exercise:

TRY IT :: 5.59

Problem: Multiply: $5x(x+4y)$.

Solution:

$5x^2 + 20xy$

Note:
Exercise:

TRY IT :: 5.60

Problem: Multiply: $2p(6p+r)$.

Solution:

$12p^2 + 2pr$

Example:
Exercise:

Problem: **Multiply: $-2y\left(4y^2+3y-5\right)$.

Solution:
Solution

	$-2y(4y^2+3y-5)$
Distribute.	$-2y\cdot 4y^2+(-2y)\cdot 3y-(-2y)\cdot 5$
Simplify.	$-8y^3-6y^2+10y$

Note:
Exercise:

TRY IT :: 5.61
Problem: Multiply: $-3y\left(5y^2+8y-7\right)$.

Solution:

$-15y^3 - 24y^2 + 21y$

Note:
Exercise:

TRY IT :: 5.62
Problem: Multiply: $4x^2(2x^2 - 3x + 5)$.

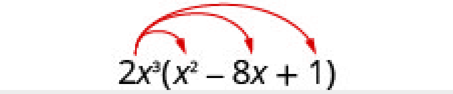
Solution:

$8x^4 - 12x^3 + 20x^2$

Example:
Exercise:

Problem: **Multiply: $2x^3(x^2 - 8x + 1)$.

Solution:
Solution

Distribute.	$2x^3 \cdot x^2 + (2x^3) \cdot (-8x) + (2x^3) \cdot 1$
Simplify.	$2x^5 - 16x^4 + 2x^3$

Note:
Exercise:

TRY IT :: 5.63

Problem: Multiply: $4x\left(3x^2 - 5x + 3\right)$.

Solution:

$12x^3 - 20x^2 + 12x$

Note:
Exercise:

TRY IT :: 5.64

Problem: Multiply: $-6a^3(3a^2 - 2a + 6)$.

Solution:

$-18a^5 + 12a^4 - 36a^3$

Example:
Exercise:

Problem: **Multiply: $(x+3)p$.

Solution:
Solution

The monomial is the second factor.	$(x+3)p$
Distribute.	$x \cdot p + 3 \cdot p$
Simplify.	$xp + 3p$

Note:
Exercise:

TRY IT :: 5.65
Problem: Multiply: $(x+8)p$.

Solution:

$xp + 8p$

Note:
Exercise:

TRY IT :: 5.66

Problem: Multiply: $(a+4)p$.

Solution:

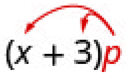
$ap+4p$

Multiply a Binomial by a Binomial

Just like there are different ways to represent multiplication of numbers, there are several methods that can be used to multiply a binomial times a binomial. We will start by using the Distributive Property.

Multiply a Binomial by a Binomial Using the Distributive Property

Look at [link], where we multiplied a binomial by a monomial.

	
We distributed the *p* to get:	$xp+3p$
What if we have $(x+7)$ instead of *p*?	

	$(x+3)(x+7)$
Distribute $(x + 7)$.	$x(x+7)+3(x+7)$
Distribute again.	$x^2+7x+3x+21$
Combine like terms.	$x^2+10x+21$

Notice that before combining like terms, you had four terms. You multiplied the two terms of the first binomial by the two terms of the second binomial—four multiplications.

Example:
Exercise:

Problem: **Multiply: $(y+5)\,(y+8)$.

Solution:
Solution

	$(y+5)(y+8)$

Distribute (*y* + 8).	$y(y+8)+5(y+8)$
Distribute again	$y^2+8y+5y+40$
Combine like terms.	$y^2+13y+40$

Note:
Exercise:

TRY IT :: 5.67

Problem: Multiply: $(x+8)(x+9)$.

Solution:

$x^2+17x+72$

Note:
Exercise:

TRY IT :: 5.68

Problem: Multiply: $(5x+9)(4x+3)$.

Solution:

$20x^2+51x+27$

Example:
Exercise:

Problem: **Multiply: $(2y+5)\,(3y+4)$.

Solution:
Solution

	$(2y+5)(3y+4)$
Distribute (3*y* + 4).	$2y(3y+4)+5(3y+4)$
Distribute again	$6y^2+8y+15y+20$
Combine like terms.	$6y^2+23y+20$

Note:
Exercise:

TRY IT :: 5.69
Problem: Multiply: $(3b+5)\,(4b+6)$.

Solution:

$12b^2 + 38b + 30$

Note:
Exercise:

TRY IT :: 5.70
Problem: Multiply: $(a + 10)(a + 7)$.

Solution:

$a^2 + 17a + 70$

Example:
Exercise:

Problem: **Multiply: $(4y + 3)(2y - 5)$.

Solution:
Solution

	$(4y + 3)(2y - 5)$
Distribute.	$4y(2y - 5) + 3(2y - 5)$

Distribute again.	$8y^2 - 20y + 6y - 15$
Combine like terms.	$8y^2 - 14y - 15$

Note:
Exercise:

TRY IT :: 5.71

Problem: Multiply: $(5y + 2)(6y - 3)$.

Solution:

$30y^2 - 3y - 6$

Note:
Exercise:

TRY IT :: 5.72

Problem: Multiply: $(3c + 4)(5c - 2)$.

Solution:

$15c^2 + 14c - 8$

Example:

Exercise:

Problem: **Multiply: $(x-2)(x-y)$.

Solution:
Solution

	$(x-2)(x-y)$
Distribute.	$x(x-y)-2(x-y)$
Distribute again.	$x^2-xy-2x+2y$
There are no like terms to combine.	

Note:
Exercise:

TRY IT :: 5.73
Problem: Multiply: $(a+7)(a-b)$.

Solution:

$a^2 - ab + 7a - 7b$

Note:
Exercise:

TRY IT :: 5.74

Problem: Multiply: $(x+5)(x-y)$.

Solution:

$x^2 - xy + 5x - 5y$

Multiply a Binomial by a Binomial Using the FOIL Method

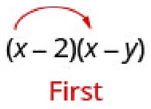
Remember that when you multiply a binomial by a binomial you get four terms. Sometimes you can combine like terms to get a trinomial, but sometimes, like in [link], there are no like terms to combine.

Let’s look at the last example again and pay particular attention to how we got the four terms.
Equation:

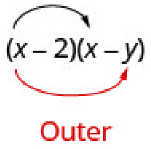
$$(x-2)(x-y)$$
$$x^2 - xy - 2x + 2y$$

Where did the first term, x^2, come from?

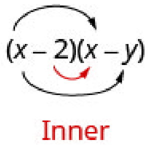
It is the product of x and x, the *first* terms in $(x - 2)$ and $(x - y)$.



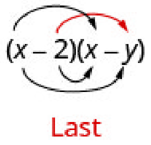
The next term, $-xy$, is the product of x and $-y$, the two *outer* terms.



The third term, $-2x$, is the product of -2 and x, the two *inner* terms.



And the last term, $+2y$, came from multiplying the two *last* terms, -2 and $-y$.



We abbreviate “First, Outer, Inner, Last” as FOIL. The letters stand for ‘**F**irst, **O**uter, **I**nner, **L**ast’. The word FOIL is easy to remember and ensures we find all four products.

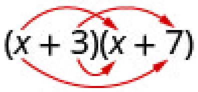
Equation:

$$(x - 2)(x - y)$$

$$\underset{F}{x^2} - \underset{O}{xy} - \underset{I}{2x} + \underset{L}{2y}$$

Let’s look at $(x + 3)\,(x + 7)$.

Distibutive Property	FOIL

$(x+3)(x+7)$	
$x(x+7)+3(x+7)$	
$\underset{F}{x^2}+\underset{O}{7x}+\underset{I}{3x}+\underset{L}{21}$	$\underset{F}{x^2}+\underset{O}{7x}+\underset{I}{3x}+\underset{L}{21}$
$x^2+10x+21$	$x^2+10x+21$

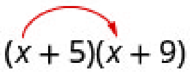
Notice how the terms in third line fit the FOIL pattern.

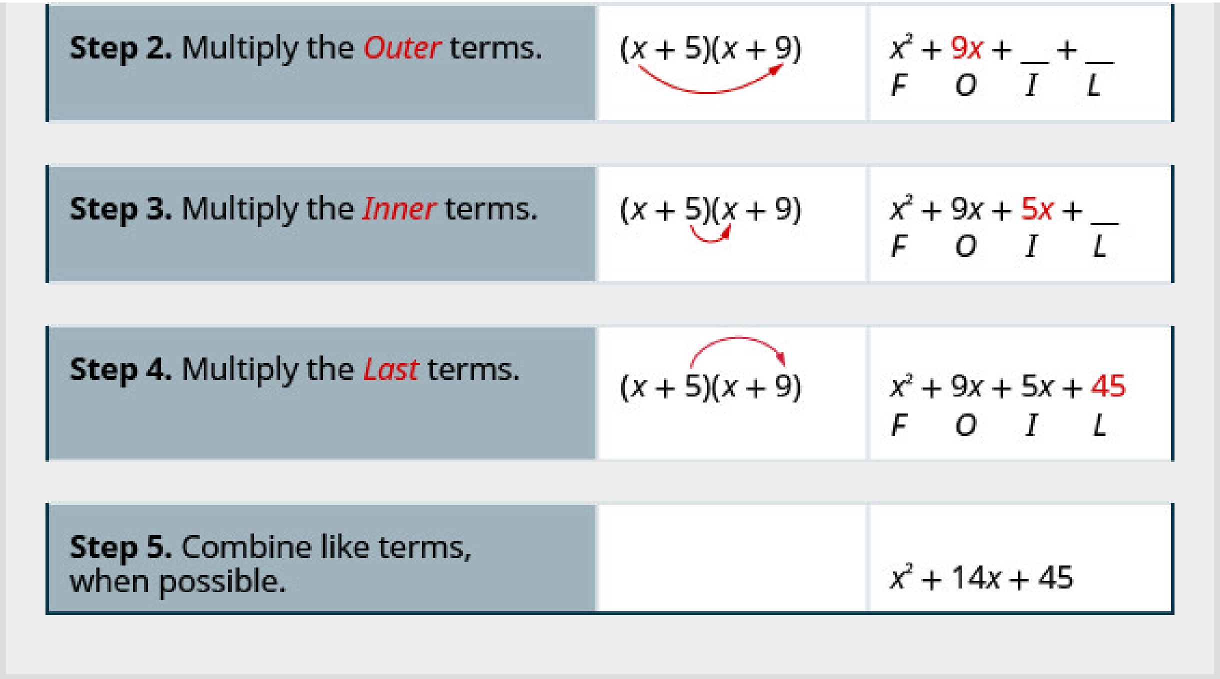
Now we will do an example where we use the FOIL pattern to multiply two binomials.

Example:
How to Multiply a Binomial by a Binomial using the FOIL Method
Exercise:

Problem: **Multiply using the FOIL method: $(x+5)(x+9)$.

Solution:
Solution

Step 1. Multiply the *First* terms.	$(x+5)(x+9)$ 	$\underset{F}{x^2}+\underset{O}{\_\_}+\underset{I}{\_\_}+\underset{L}{\_\_}$



Step 2. Multiply the *Outer* terms.	$(x+5)(x+9)$	$x^2 + 9x + \_ + \_$ F O I L
Step 3. Multiply the *Inner* terms.	$(x+5)(x+9)$	$x^2 + 9x + 5x + \_$ F O I L
Step 4. Multiply the *Last* terms.	$(x+5)(x+9)$	$x^2 + 9x + 5x + 45$ F O I L
Step 5. Combine like terms, when possible.		$x^2 + 14x + 45$

Note:
Exercise:

TRY IT :: 5.75

Problem: Multiply using the FOIL method: $(x+6)(x+8)$.

Solution:

$x^2 + 14x + 48$

Note:
Exercise:

TRY IT :: 5.76

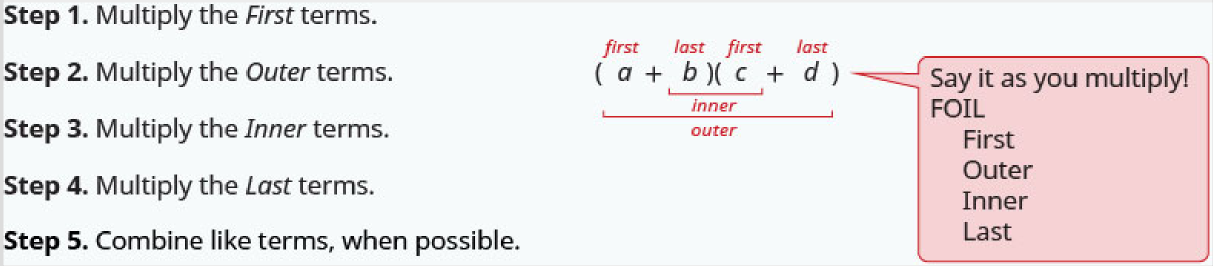
Problem: Multiply using the FOIL method: $(y+17)(y+3)$.

Solution:

$y^2 + 20y + 51$

We summarize the steps of the FOIL method below. The FOIL method only applies to multiplying binomials, not other polynomials!

Note:
Multiply two binomials using the FOIL method



Step 1. Multiply the *First* terms.

Step 2. Multiply the *Outer* terms.

Step 3. Multiply the *Inner* terms.

Step 4. Multiply the *Last* terms.

Step 5. Combine like terms, when possible.

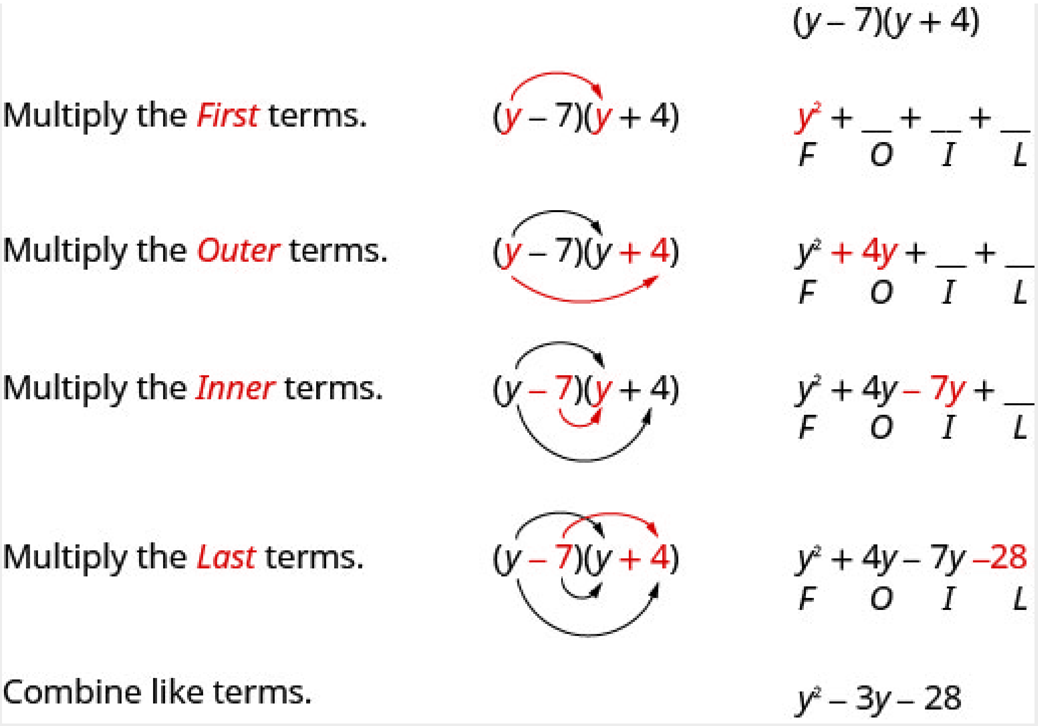
Say it as you multiply!
FOIL
First
Outer
Inner
Last

When you multiply by the FOIL method, drawing the lines will help your brain focus on the pattern and make it easier to apply.

Example:
Exercise:

Problem: **Multiply: $(y - 7)\,(y + 4)$.

Solution:
Solution



		$(y-7)(y+4)$
Multiply the *First* terms.	$(y-7)(y+4)$	$y^2 + \_\_ + \_\_ + \_\_$ F O I L
Multiply the *Outer* terms.	$(y-7)(y+4)$	$y^2 + 4y + \_\_ + \_\_$ F O I L
Multiply the *Inner* terms.	$(y-7)(y+4)$	$y^2 + 4y - 7y + \_\_$ F O I L
Multiply the *Last* terms.	$(y-7)(y+4)$	$y^2 + 4y - 7y - 28$ F O I L
Combine like terms.		$y^2 - 3y - 28$

Note:
Exercise:

TRY IT :: 5.77

Problem: Multiply: $(x-7)(x+5)$.

Solution:

$x^2 - 2x - 35$

Note:
Exercise:

TRY IT :: 5.78

Problem: Multiply: $(b-3)(b+6)$.

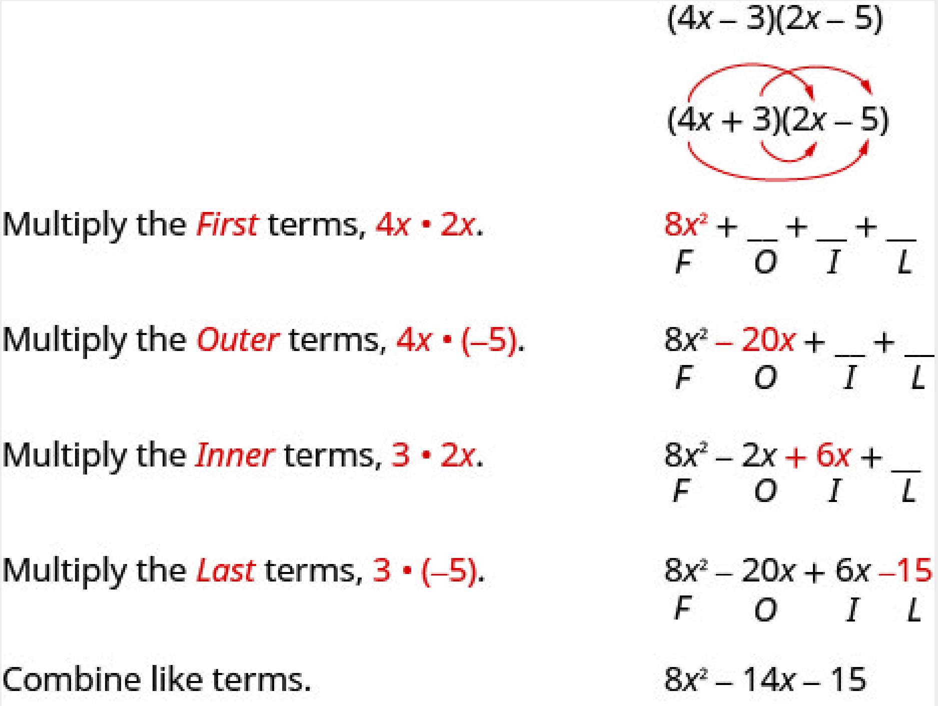
Solution:

$b^2+3b-18$

Example:
Exercise:

Problem: **Multiply: $(4x+3)(2x-5)$.

Solution:
Solution



	$(4x-3)(2x-5)$
	$(4x+3)(2x-5)$
Multiply the *First* terms, $4x \cdot 2x$.	$\underset{F}{8x^2} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Outer* terms, $4x \cdot (-5)$.	$\underset{F}{8x^2} \underset{O}{-20x} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Inner* terms, $3 \cdot 2x$.	$\underset{F}{8x^2} \underset{O}{-2x} \underset{I}{+6x} + \underset{L}{\_\_}$
Multiply the *Last* terms, $3 \cdot (-5)$.	$\underset{F}{8x^2} \underset{O}{-20x} \underset{I}{+6x} \underset{L}{-15}$
Combine like terms.	$8x^2-14x-15$

Note:
Exercise:

TRY IT :: 5.79
Problem: Multiply: $(3x+7)\,(5x-2)$.

Solution:

$15x^2+29x-14$

Note:
Exercise:

TRY IT :: 5.80
Problem: Multiply: $(4y+5)\,(4y-10)$.

Solution:

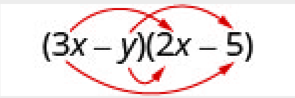
$16y^2-20y-50$

The final products in the last four examples were trinomials because we could combine the two middle terms. This is not always the case.

Example:
Exercise:

Problem: **Multiply: $(3x-y)(2x-5)$.

Solution:
Solution

	$(3x-y)(2x-5)$
	
Multiply the *First*.	$\underset{F}{6x^2} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Outer*.	$\underset{F}{6x^2} \underset{O}{-15x} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Inner*.	$\underset{F}{6x^2} \underset{O}{-15x} \underset{I}{-2xy} + \underset{L}{\_\_}$
Multiply the *Last*.	$\underset{F}{6x^2} \underset{O}{-15x} \underset{I}{-2xy} \underset{L}{+5y}$
Combine like terms—there are none.	$6x^2-15x-2xy+5y$

Note:
Exercise:

TRY IT :: 5.81

Problem: Multiply: $(10c-d)(c-6)$.

Solution:

$10c^2-60c-cd+6d$

Note:
Exercise:

TRY IT :: 5.82

Problem: Multiply: $(7x-y)(2x-5)$.

Solution:

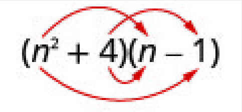
$14x^2-35x-2xy+5y$

Be careful of the exponents in the next example.

Example:
Exercise:

Problem: **Multiply: $\left(n^2+4\right)(n-1)$.

Solution:
Solution

	$(n^2+4)(n-1)$
	 $(n^2+4)(n-1)$
Multiply the *First*.	$\underset{F}{n^3} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Outer*.	$\underset{F}{n^3} \underset{O}{-\,n^2} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Inner*.	$\underset{F}{n^3} \underset{O}{-\,n^2} \underset{I}{+\,4n} + \underset{L}{\_\_}$
Multiply the *Last*.	$\underset{F}{n^3} \underset{O}{-\,n^2} \underset{I}{+\,4n} \underset{L}{-\,4}$
Combine like terms—there are none.	n^3-n^2+4n-4

Note:
Exercise:

TRY IT :: 5.83

Problem: Multiply: $\left(x^2+6\right)(x-8)$.

Solution:

$x^3-8x^2+6x-48$

Note:
Exercise:

TRY IT :: 5.84

Problem: Multiply: $\left(y^2+7\right)(y-9)$.

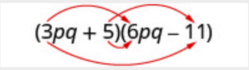
Solution:

$y^3-9y^2+7y-63$

Example:
Exercise:

Problem: **Multiply: $(3pq+5)(6pq-11)$.

Solution:
Solution

	$(3pq+5)(6pq-11)$	
Multiply the *First*.	$\underset{F}{18p^2q^2} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$	
Multiply the *Outer*.	$\underset{F}{18p^2q^2} \underset{O}{- 33pq} + \underset{I}{\_\_} + \underset{L}{\_\_}$	
Multiply the *Inner*.	$\underset{F}{18p^2q^2} \underset{O}{- 33pq} \underset{I}{+ 30pq} + \underset{L}{\_\_}$	
Multiply the *Last*.	$\underset{F}{18p^2q^2} \underset{O}{- 33pq} \underset{I}{+ 30pq} \underset{L}{- 55}$	
Combine like terms—there are none.	$18p^2q^2 - 3pq - 55$	

Note:
Exercise:

TRY IT :: 5.85

Problem: Multiply: $(2ab+5)\,(4ab-4)$.

Solution:

$8a^2b^2 + 12ab - 20$

Note:

Exercise:

TRY IT :: 5.86

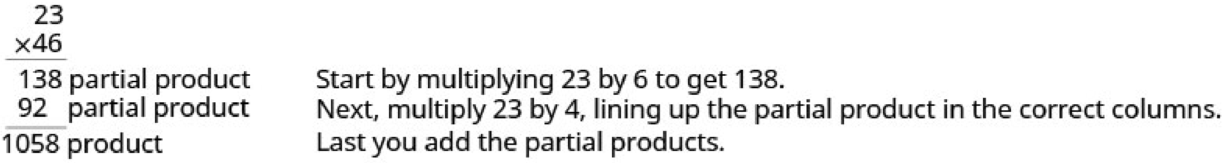
Problem: Multiply: $(2xy+3)(4xy-5)$.

Solution:

$8x^2y^2+2xy-15$

Multiply a Binomial by a Binomial Using the Vertical Method

The FOIL method is usually the quickest method for multiplying two binomials, but it *only* works for binomials. You can use the Distributive Property to find the product of any two polynomials. Another method that works for all polynomials is the Vertical Method. It is very much like the method you use to multiply whole numbers. Look carefully at this example of multiplying two-digit numbers.



23 ×46	
138 partial product	Start by multiplying 23 by 6 to get 138.
92 partial product	Next, multiply 23 by 4, lining up the partial product in the correct columns.
1058 product	Last you add the partial products.

Now we’ll apply this same method to multiply two binomials.

Example:
Exercise:

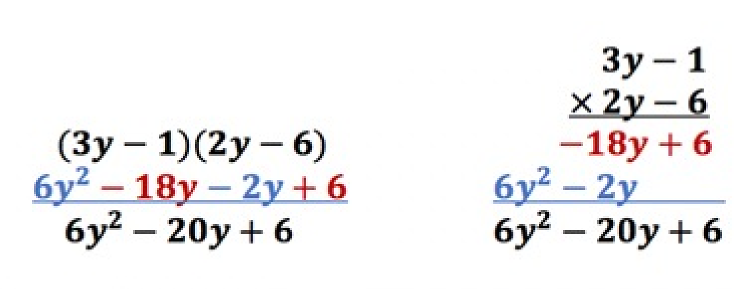
Problem: **Multiply using the Vertical Method: $(3y-1)(2y-6)$.

Solution:
Solution

It does not matter which binomial goes on the top.

Multiply $3y - 1$ by -6.	$3y - 1$	
Multiply $3y - 1$ by 2y.	$\underline{\times\ 2y - 6}$	
	$-18y + 6$	partial product
	$\underline{6y^2 - 2y}$	partial product
Add like terms.	$6y^2 - 20y + 6$	product

Notice the partial products are the same as the terms in the FOIL method.



Note:
Exercise:

TRY IT :: 5.87

Problem: Multiply using the Vertical Method: $(5m - 7)(3m - 6)$.

Solution:

$15m^2 - 51m + 42$

Note:
Exercise:

TRY IT :: 5.88

Problem: Multiply using the Vertical Method: $(6b - 5)\,(7b - 3)$.

Solution:

$42b^2 - 53b + 15$

We have now used three methods for multiplying binomials. Be sure to practice each method, and try to decide which one you prefer. The methods are listed here all together, to help you remember them.

Note:
Multiplying Two Binomials
To multiply binomials, use the:

- Distributive Property
- FOIL Method
- Vertical Method

Remember, FOIL only works when multiplying two binomials.

Multiply a Trinomial by a Binomial

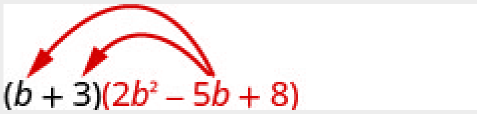
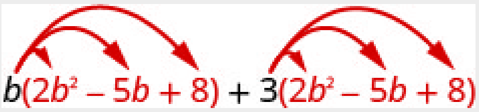
We have multiplied monomials by monomials, monomials by polynomials, and binomials by binomials. Now we’re ready to multiply a trinomial by a binomial. Remember, FOIL will not work in this case, but we can use either the Distributive Property or the Vertical Method. We first look at an example using the Distributive Property.

Example:
Exercise:

Problem:

**Multiply using the Distributive Property: $(b+3)(2b^2-5b+8)$.

Solution:
Solution

	 $(b+3)(2b^2-5b+8)$
Distribute.	 $b(2b^2-5b+8)+3(2b^2-5b+8)$
Multiply.	$2b^3-5b^2+8b+6b^2-15b+24$
Combine like terms.	$2b^3+b^2-7b+24$

Note:
Exercise:

Problem:

TRY IT :: 5.89
Multiply using the Distributive Property: $(y-3)(y^2-5y+2)$.

Solution:

$y^3-8y^2+17y-6$

Note:
Exercise:

Problem:

TRY IT :: 5.90
Multiply using the Distributive Property: $(x+4)(2x^2-3x+5)$.

Solution:

$2x^3+5x^2-7x+20$

Now let’s do this same multiplication using the Vertical Method.

Example:
Exercise:

Problem:

**Multiply using the Vertical Method: $(b+3)(2b^2-5b+8)$.

Solution:
Solution

It is easier to put the polynomial with fewer terms on the bottom because we get fewer partial products this way.

Multiply $(2b^2 - 5b + 8)$ by 3.	$\begin{array}{r} 2b^2-5b+8 \\ \times \quad b+3 \\ \hline 6b^2-15b+24 \end{array}$
	$\underline{2b^3-5b^2+8b}$
Multiply $(2b^2 - 5b + 8)$ by b.	$2b^3+b^2-7b+24$
Add like terms.	

Note:
Exercise:

TRY IT :: 5.91

Problem: Multiply using the Vertical Method: $(y-3)(y^2-5y+2)$.

Solution:

$y^3-8y^2+17y-6$

Note:
Exercise:

Problem:

TRY IT :: 5.92
Multiply using the Vertical Method: $(x+4)(2x^2-3x+5)$.

Solution:

$2x^3+5x^2-7x+20$

We have now seen two methods you can use to multiply a trinomial by a binomial. After you practice each method, you’ll probably find you prefer one way over the other. We list both methods are listed here, for easy reference.

Note:
Multiplying a Trinomial by a Binomial
To multiply a trinomial by a binomial, use the:

- Distributive Property
- Vertical Method

Note:
Access these online resources for additional instruction and practice with multiplying polynomials:

- Multiplying Exponents 1
- Multiplying Exponents 2
- Multiplying Exponents 3

Key Concepts

- **FOIL Method for Multiplying Two Binomials**—To multiply two binomials:

  Multiply the**First**terms.
  Multiply the**Outer**terms.
  Multiply the**Inner**terms.
  Multiply the**Last**terms.
- **Multiplying Two Binomials**—To multiply binomials, use the:
  - Distributive Property ([link])
  - FOIL Method ([link])
  - Vertical Method ([link])
- **Multiplying a Trinomial by a Binomial**—To multiply a trinomial by a binomial, use the:
  - Distributive Property ([link])
  - Vertical Method ([link])

Practice Makes Perfect

Multiply a Polynomial by a Monomial

In the following exercises, multiply.

Exercise:

Problem: $4(w+10)$

Solution:

$4w+40$

Exercise:

Problem: $6(b+8)$

Exercise:

Problem: $-3(a+7)$

Solution:

$-3a-21$

Exercise:

Problem: $-5(p+9)$

Exercise:

Problem: $2(x-7)$

Solution:

$2x-14$

Exercise:

Problem: $7(y-4)$

Exercise:

Problem: $-3(k-4)$

Solution:

$-3k+12$

Exercise:

Problem: $-8(j-5)$

Exercise:

Problem: $q(q+5)$

Solution:

q^2+5q

Exercise:

Problem: $k(k+7)$

Exercise:

Problem: $-b(b+9)$

Solution:

$-b^2-9b$

Exercise:

Problem: $-y(y+3)$

Exercise:

Problem: $-x(x-10)$

Solution:

$-x^2+10x$

Exercise:

Problem: $-p(p-15)$

Exercise:

Problem: $6r(4r+s)$

Solution:

$24r^2 + 6rs$

Exercise:

Problem: $5c(9c + d)$

Exercise:

Problem: $12x(x - 10)$

Solution:

$12x^2 - 120x$

Exercise:

Problem: $9m(m - 11)$

Exercise:

Problem: $-9a(3a + 5)$

Solution:

$-27a^2 - 45a$

Exercise:

Problem: $-4p(2p + 7)$

Exercise:

Problem: $3(p^2 + 10p + 25)$

Solution:

$3p^2 + 30p + 75$

Exercise:

Problem: $6(y^2 + 8y + 16)$

Exercise:

Problem: $-8x(x^2 + 2x - 15)$

Solution:

$-8x^3 - 16x^2 + 120x$

Exercise:

Problem: $-5t(t^2 + 3t - 18)$

Exercise:

Problem: $5q^3 \left(q^3 - 2q + 6\right)$

Solution:

$5q^6 - 10q^4 + 30q^3$

Exercise:

Problem: $4x^3 \left(x^4 - 3x + 7\right)$

Exercise:

Problem: $-8y(y^2 + 2y - 15)$

Solution:

$-8y^3 - 16y^2 + 120y$

Exercise:

Problem: $-5m(m^2+3m-18)$

Exercise:

Problem: $5q^3(q^2-2q+6)$

Solution:

$5q^5-10q^4+30q^3$

Exercise:

Problem: $9r^3(r^2-3r+5)$

Exercise:

Problem: $-4z^2(3z^2+12z-1)$

Solution:

$-12z^4-48z^3+4z^2$

Exercise:

Problem: $-3x^2(7x^2+10x-1)$

Exercise:

Problem: $(2m-9)m$

Solution:

$2m^2-9m$

Exercise:

Problem: $(8j-1)j$

Exercise:

Problem: $(w-6)\cdot 8$

Solution:

$8w-48$

Exercise:

Problem: $(k-4)\cdot 5$

Exercise:

Problem: $4(x+10)$

Solution:

$4x+40$

Exercise:

Problem: $6(y+8)$

Exercise:

Problem: $15(r-24)$

Solution:

$15r-360$

Exercise:

Problem: $12(v-30)$

Exercise:

Problem: $-3(m+11)$

Solution:

$-3m-33$

Exercise:

Problem: $-4(p+15)$

Exercise:

Problem: $-8(z-5)$

Solution:

$-8z+40$

Exercise:

Problem: $-3(x-9)$

Exercise:

Problem: $u(u+5)$

Solution:

u^2+5u

Exercise:

Problem: $q(q+7)$

Exercise:

Problem: $n(n^2-3n)$

Solution:

$n^3 - 3n^2$

Exercise:

Problem: $s(s^2 - 6s)$

Exercise:

Problem: $6x(4x + y)$

Solution:

$24x^2 + 6xy$

Exercise:

Problem: $5a(9a + b)$

Exercise:

Problem: $5p(11p - 5q)$

Solution:

$55p^2 - 25pq$

Exercise:

Problem: $12u(3u - 4v)$

Exercise:

Problem: $3(v^2 + 10v + 25)$

Solution:

$3v^2 + 30v + 75$

Exercise:

Problem: $6(x^2 + 8x + 16)$

Exercise:

Problem: $2n(4n^2 - 4n + 1)$

Solution:

$8n^3 - 8n^2 + 2n$

Exercise:

Problem: $3r(2r^2 - 6r + 2)$

Exercise:

Problem: $-8y(y^2 + 2y - 15)$

Solution:

$-8y^3 - 16y^2 + 120y$

Exercise:

Problem: $-5m(m^2 + 3m - 18)$

Exercise:

Problem: $5q^3(q^2 - 2q + 6)$

Solution:

$5q^5 - 10q^4 + 30q^3$

Exercise:

Problem: $9r^3(r^2 - 3r + 5)$

Exercise:

Problem: $-4z^2(3z^2 + 12z - 1)$

Solution:

$-12z^4 - 48z^3 + 4z^2$

Exercise:

Problem: $-3x^2(7x^2 + 10x - 1)$

Exercise:

Problem: $(2y - 9)y$

Solution:

$18y^2 - 9y$

Exercise:

Problem: $(8b - 1)b$

Multiply a Binomial by a Binomial

In the following exercises, multiply the following binomials using: ⓐ the Distributive Property ⓑ the FOIL method ⓒ the Vertical Method.

Exercise:

Problem: $(w + 5)(w + 7)$

Solution:

$w^2 + 12w + 35$

Exercise:

Problem: $(y+9)(y+3)$

Exercise:

Problem: $(p+11)(p-4)$

Solution:

$p^2+7p-44$

Exercise:

Problem: $(q+4)(q-8)$

In the following exercises, multiply the binomials. Use any method.

Exercise:

Problem: $(x+8)(x+3)$

Solution:

$x^2+11x+24$

Exercise:

Problem: $(y+7)(y+4)$

Exercise:

Problem: $(y-6)(y-2)$

Solution:

$y^2-8y+12$

Exercise:

Problem: $(x-7)(x-2)$

Exercise:

Problem: $(w-4)(w+7)$

Solution:

$w^2+3w-28$

Exercise:

Problem: $(q-5)(q+8)$

Exercise:

Problem: $(p+12)(p-5)$

Solution:

$p^2+7p-60$

Exercise:

Problem: $(m+11)(m-4)$

Exercise:

Problem: $(6p+5)(p+1)$

Solution:

$6p^2+11p+5$

Exercise:

Problem: $(7m+1)(m+3)$

Exercise:

Problem: $(2t-9)(10t+1)$

Solution:

$20t^2-88t-9$

Exercise:

Problem: $(3r-8)(11r+1)$

Exercise:

Problem: $(5x-y)\,(3x-6)$

Solution:

$15x^2-3xy-30x+6y$

Exercise:

Problem: $(10a-b)\,(3a-4)$

Exercise:

Problem: $(a+b)(2a+3b)$

Solution:

$2a^2+5ab+3b^2$

Exercise:

Problem: $(r+s)(3r+2s)$

Exercise:

Problem: $(4z-y)(z-6)$

Solution:

$4z^2 - 24z - zy + 6y$

Exercise:

Problem: $(5x - y)(x - 4)$

Exercise:

Problem: $(x^2 + 3)(x + 2)$

Solution:

$x^3 + 2x^2 + 3x + 6$

Exercise:

Problem: $(y^2 - 4)(y + 3)$

Exercise:

Problem: $(x^2 + 8)(x^2 - 5)$

Solution:

$x^4 + 3x^2 - 40$

Exercise:

Problem: $(y^2 - 7)(y^2 - 4)$

Exercise:

Problem: $(5ab - 1)(2ab + 3)$

Solution:

$10a^2b^2+13ab-3$

Exercise:

Problem: $(2xy+3)(3xy+2)$

Exercise:

Problem: $(6pq-3)(4pq-5)$

Solution:

$24p^2q^2-42pq+15$

Exercise:

Problem: $(3rs-7)(3rs-4)$

Multiply a Trinomial by a Binomial

In the following exercises, multiply using ⓐ the Distributive Property ⓑ the Vertical Method.

Exercise:

Problem: $(x+5)(x^2+4x+3)$

Solution:

$x^3+9x^2+23x+15$

Exercise:

Problem: $(u+4)(u^2+3u+2)$

Exercise:

Problem: $(y+8)(4y^2+y-7)$

Solution:

$4y^3 + 33y^2 + y - 56$

Exercise:

Problem: $(a + 10)(3a^2 + a - 5)$

In the following exercises, multiply. Use either method.

Exercise:

Problem: $(w - 7)(w^2 - 9w + 10)$

Solution:

$w^3 - 16w^2 + 73w - 70$

Exercise:

Problem: $(p - 4)(p^2 - 6p + 9)$

Exercise:

Problem: $(3q + 1)(q^2 - 4q - 5)$

Solution:

$3q^3 - 11q^2 - 19q - 5$

Exercise:

Problem: $(6r + 1)(r^2 - 7r - 9)$

Mixed Practice

Exercise:

Problem: $(10y - 6) + (4y - 7)$

Solution:

$14y - 13$

Exercise:

Problem: $(15p - 4) + (3p - 5)$

Exercise:

Problem: $\left(x^2 - 4x - 34\right) - \left(x^2 + 7x - 6\right)$

Solution:

$-11x - 28$

Exercise:

Problem: $\left(j^2 - 8j - 27\right) - \left(j^2 + 2j - 12\right)$

Exercise:

Problem: $5q(3q^2 - 6q + 11)$

Solution:

$15q^3 - 30q^2 + 55q$

Exercise:

Problem: $8t(2t^2 - 5t + 6)$

Exercise:

Problem: $(s - 7)\,(s + 9)$

Solution:

$s^2 + 2s - 63$

Exercise:

Problem: $(x-5)(x+13)$

Exercise:

Problem: $\left(y^2-2y\right)(y+1)$

Solution:

$y^3 - y^2 - 2y$

Exercise:

Problem: $\left(a^2-3a\right)(4a+5)$

Exercise:

Problem: $(3n-4)\left(n^2+n-7\right)$

Solution:

$3n^3 - n^2 - 25n + 28$

Exercise:

Problem: $(6k-1)\left(k^2+2k-4\right)$

Exercise:

Problem: $(7p+10)(7p-10)$

Solution:

$49p^2 - 100$

Exercise:

Problem: $(3y+8)\,(3y-8)$

Exercise:

Problem: $(4m^2-3m-7)m^2$

Solution:

$4m^4-3m^3-7m^2$

Exercise:

Problem: $(15c^2-4c+5)c^4$

Exercise:

Problem: $(5a+7b)\,(5a+7b)$

Solution:

$25a^2+70ab+49b^2$

Exercise:

Problem: $(3x-11y)\,(3x-11y)$

Exercise:

Problem: $(4y+12z)\,(4y-12z)$

Solution:

$16y^2-144z^2$

Everyday Math

Exercise:

Problem:

Mental math You can use binomial multiplication to multiply numbers without a calculator. Say you need to multiply 13 times 15. Think of 13 as $10+3$ and 15 as $10+5$.

ⓐ Multiply $(10+3)(10+5)$ by the FOIL method.
ⓑ Multiply $13\cdot15$ without using a calculator.
ⓒ Which way is easier for you? Why?

Exercise:

Problem:

Mental math You can use binomial multiplication to multiply numbers without a calculator. Say you need to multiply 18 times 17. Think of 18 as $20-2$ and 17 as $20-3$.

ⓐ Multiply $(20-2)(20-3)$ by the FOIL method.
ⓑ Multiply $18\cdot17$ without using a calculator.
ⓒ Which way is easier for you? Why?

Solution:

ⓐ 306 ⓑ 306 ⓒ Answers will vary.

Writing Exercises

Exercise:

Problem:

Which method do you prefer to use when multiplying two binomials: the Distributive Property, the FOIL method, or the Vertical Method? Why?

Exercise:

Problem:

Which method do you prefer to use when multiplying a trinomial by a binomial: the Distributive Property or the Vertical Method? Why?

Solution:

Answers will vary.

Exercise:

Problem: Multiply the following:

$(x+2)(x-2)$
$(y+7)(y-7)$
$(w+5)(w-5)$

Explain the pattern that you see in your answers.

Exercise:

Problem: Multiply the following:

$(m-3)(m+3)$
$(n-10)(n+10)$
$(p-8)(p+8)$

Explain the pattern that you see in your answers.

Solution:

Answers may vary.

Exercise:

Problem: Multiply the following:

$(p+3)(p+3)$
$(q+6)(q+6)$
$(r+1)(r+1)$

Explain the pattern that you see in your answers.

Exercise:

Problem: Multiply the following:

$(x-4)(x-4)$
$(y-1)(y-1)$
$(z-7)(z-7)$

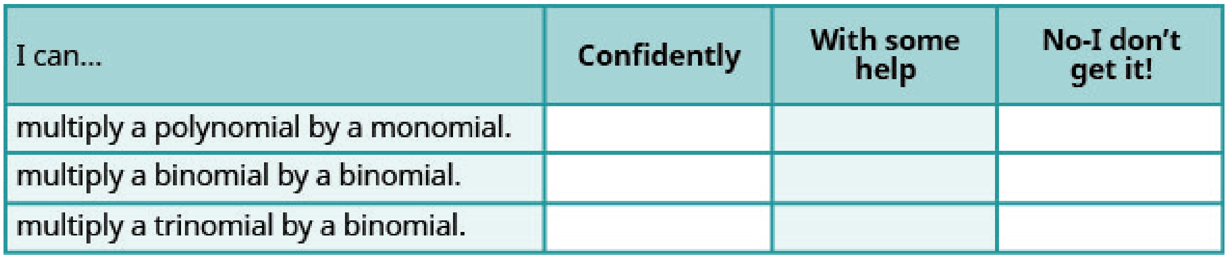
Explain the pattern that you see in your answers.

Solution:

Answers may vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
multiply a polynomial by a monomial.			
multiply a binomial by a binomial.			
multiply a trinomial by a binomial.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Special Products
By the end of this section, you will be able to:

- Square a binomial using the Binomial Squares Pattern
- Multiply conjugates using the Product of Conjugates Pattern
- Recognize and use the appropriate special product pattern

Note:
Before you get started, take this readiness quiz.

1. Simplify: ⓐ 9^2 ⓑ $(-9)^2$ ⓒ -9^2.
   If you missed this problem, review [link].

Square a Binomial Using the Binomial Squares Pattern

Mathematicians like to look for patterns that will make their work easier. A good example of this is squaring binomials. While you can always get the product by writing the binomial twice and using the methods of the last section, there is less work to do if you learn to use a pattern.

Let's start by looking at $(x+9)^2$.	
What does this mean?	$(x+9)^2$
It means to multiply $(x+9)$ by itself.	$(x+9)(x+9)$
Then, using FOIL, we get:	$x^2+9x+9x+81$
Combining like terms gives:	$x^2+18x+81$

Here's another one:	$(y-7)^2$
Multiply $(y-7)$ by itself.	$(y-7)(y-7)$

Using FOIL, we get:	$y^2 - 7y - 7y + 49$
And combining like terms:	$y^2 - 14y + 49$

And one more:	$(2x+3)^2$
Multiply.	$(2x+3)(2x+3)$
Use FOIL:	$4x^2 + 6x + 6x + 9$
Combine like terms.	$4x^2 + 12x + 9$

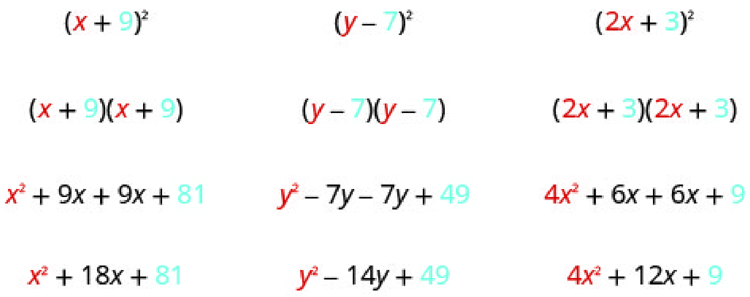
Look at these results. Do you see any patterns?

What about the number of terms? In each example we squared a binomial and the result was a trinomial.

Equation:

$$(a+b)^2 = \_\_\_\_ + \_\_\_\_ + \_\_\_\_$$

Now look at the ***first term*** in each result. Where did it come from?



$(x+9)^2$	$(y-7)^2$	$(2x+3)^2$
$(x+9)(x+9)$	$(y-7)(y-7)$	$(2x+3)(2x+3)$
$x^2 + 9x + 9x + 81$	$y^2 - 7y - 7y + 49$	$4x^2 + 6x + 6x + 9$
$x^2 + 18x + 81$	$y^2 - 14y + 49$	$4x^2 + 12x + 9$

The first term is the product of the first terms of each binomial. Since the binomials are identical, it is just the square of the first term!

Equation:

$$(a+b)^2 = a^2 + \_\_\_\_ + \_\_\_\_$$

To get the ***first term*** of the product, ***square the first term***.

Where did the ***last term*** come from? Look at the examples and find the pattern.

The last term is the product of the last terms, which is the square of the last term.
Equation:

$$(a+b)^2 = \_\_\_\_ + \_\_\_\_ + b^2$$

To get the ***last term*** *of the product,* ***square the last term****.*

Finally, look at the ***middle term***. Notice it came from adding the “outer” and the “inner” terms—which are both the same! So the middle term is double the product of the two terms of the binomial.
Equation:

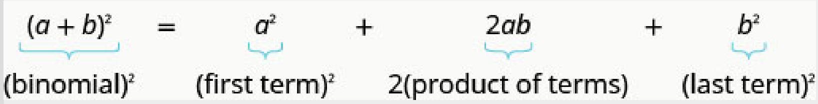
$$\begin{aligned}(a+b)^2 &= \_\_\_\_ + 2ab + \_\_\_\_\\(a-b)^2 &= \_\_\_\_ - 2ab + \_\_\_\_\end{aligned}$$

To get the ***middle term*** *of the product,* ***multiply the terms and double their product****.*

Putting it all together:

Note:
Binomial Squares Pattern
If a and b are real numbers,
Equation:

$$\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2\\(a-b)^2 &= a^2 - 2ab + b^2\end{aligned}$$



$$\underbrace{(a+b)^2}_{(\text{binomial})^2} = \underbrace{a^2}_{(\text{first term})^2} + \underbrace{2ab}_{2(\text{product of terms})} + \underbrace{b^2}_{(\text{last term})^2}$$

To square a binomial:

- square the first term
- square the last term
- double their product

A number example helps verify the pattern.

	$(10+4)^2$
Square the first term.	$10^2 + \_\_\_ +$
Square the last term.	$10^2 + \_\_\_ + 4^2$
Double their product.	$10^2 + 2 \cdot 10 \cdot 4 + 4^2$
Simplify.	$100 + 80 + 16$
Simplify.	196

To multiply $(10+4)^2$ usually you’d follow the Order of Operations.
Equation:

$$\begin{gathered}(10+4)^2\\(14)^2\\196\end{gathered}$$

The pattern works!

Example:
Exercise:

Problem: **Multiply: $(x+5)^2$.

Solution:
Solution

	$\left(\overset{a+b}{x+5}\right)^2$
Square the first term.	$\overset{a^2}{x^2} + \overset{2ab}{\_\_\_\_} + \overset{b^2}{\_\_\_\_}$

Square the last term.	$a^2 + \quad 2ab \quad + b^2$ $x^2 + \underline{\quad\quad} + 5^2$
Double the product.	$a^2 + 2 \cdot a \cdot b + b^2$ $x^2 + 2 \cdot x \cdot 5 + 5^2$
Simplify.	$x^2 + 10x + 25$

Note:
Exercise:

TRY IT :: 5.93

Problem: Multiply: $(x + 9)^2$.

Solution:

$x^2 + 18x + 81$

Note:
Exercise:

TRY IT :: 5.94

Problem: Multiply: $(y + 11)^2$.

Solution:

$y^2 + 22y + 121$

Example:
Exercise:

Problem: **Multiply: $(y - 3)^2$.

Solution:

Solution

	$(\overset{a}{y} - \overset{b}{3})^2$
Square the first term.	$\overset{a^2}{y^2} - \overset{2ab}{\_\_\_\_} + \overset{b^2}{\_\_\_\_}$
Square the last term.	$\overset{a^2}{y^2} - \overset{2ab}{\_\_\_\_} + \overset{b^2}{3^2}$
Double the product.	$\overset{a^2}{y^2} - \overset{2}{2} \cdot \overset{a}{y} \cdot \overset{b}{3} + \overset{b^2}{3^2}$
Simplify.	$y^2 - 6y + 9$

Note:
Exercise:

TRY IT :: 5.95

Problem: Multiply: $(x - 9)^2$.

Solution:

$x^2 - 18x + 81$

Note:
Exercise:

TRY IT :: 5.96

Problem: Multiply: $(p-13)^2$.

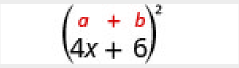
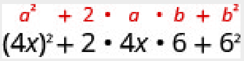
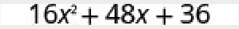
Solution:

$p^2-26p+169$

Example:
Exercise:

Problem: **Multiply: $(4x+6)^2$.

Solution:
Solution

	 $(\overset{a}{4x} + \overset{b}{6})^2$
Use the pattern.	 $\overset{a^2}{(4x)^2} + \overset{2}{2} \cdot \overset{a}{4x} \cdot \overset{b}{6} + \overset{b^2}{6^2}$
Simplify.	 $16x^2+48x+36$

Note:
Exercise:

TRY IT :: 5.97

Problem: Multiply: $(6x+3)^2$.

Solution:

$36x^2 + 36x + 9$

Note:
Exercise:

TRY IT :: 5.98

Problem: Multiply: $(4x+9)^2$.

Solution:

$16x^2 + 72x + 81$

Example:
Exercise:

Problem: **Multiply: $(2x-3y)^2$.

Solution:
Solution

	$\left(\overset{a}{2x} \overset{}{-} \overset{b}{3y}\right)^2$
Use the pattern.	$\overset{a^2}{(2x)^2} - \overset{2}{2} \cdot \overset{a}{2x} \cdot \overset{b}{3y} + \overset{b^2}{(3y)^2}$
Simplify.	$4x^2 - 12xy + 9y^2$

Note:

Exercise:

TRY IT :: 5.99

Problem: Multiply: $(2c-d)^2$.

Solution:

$4c^2-4cd+d^2$

Note:
Exercise:

TRY IT :: 5.100

Problem: Multiply: $(4x-5y)^2$.

Solution:

$16x^2-40xy+25y^2$

Example:
Exercise:

Problem: **Multiply: $\left(4u^3+1\right)^2$.

Solution:
Solution

	$\left(\overset{a}{4u^3}\overset{+}{+}\overset{b}{1}\right)^2$
Use the pattern.	$\overset{a^2}{(4u^3)^2}+2\cdot\overset{a}{4u^3}\cdot\overset{b}{1}+\overset{b^2}{(1)^2}$
Simplify.	

$16u^6 + 8u^3 + 1$

Note:
Exercise:

TRY IT :: 5.101

Problem: Multiply: $\left(2x^2 + 1\right)^2$.

Solution:

$4x^4 + 4x^2 + 1$

Note:
Exercise:

TRY IT :: 5.102

Problem: Multiply: $\left(3y^3 + 2\right)^2$.

Solution:

$9y^6 + 12y^3 + 4$

Multiply Conjugates Using the Product of Conjugates Pattern

We just saw a pattern for squaring binomials that we can use to make multiplying some binomials easier. Similarly, there is a pattern for another product of binomials. But before we get to it, we need to introduce some vocabulary.

What do you notice about these pairs of binomials?
Equation:

$$(x - 9)(x + 9) \qquad (y - 8)(y + 8) \qquad (2x - 5)(2x + 5)$$

Look at the first term of each binomial in each pair.

$(x - 9)(x + 9) \qquad (y - 8)(y + 8) \qquad (2x - 5)(2x + 5)$

Notice the first terms are the same in each pair.

Look at the last terms of each binomial in each pair.

$$(x-9)(x+9) \qquad (y-8)(y+8) \qquad (2x-5)(2x+5)$$

Notice the last terms are the same in each pair.

Notice how each pair has one sum and one difference.

$$\left(\underset{\underset{\text{Difference}}{\uparrow}}{x-9}\right)\left(\underset{\underset{\text{Sum}}{\uparrow}}{x+9}\right) \qquad \left(\underset{\underset{\text{Difference}}{\uparrow}}{y-8}\right)\left(\underset{\underset{\text{Sum}}{\uparrow}}{y+8}\right) \qquad \left(\underset{\underset{\text{Difference}}{\uparrow}}{2x-5}\right)\left(\underset{\underset{\text{Sum}}{\uparrow}}{2x+5}\right)$$

A pair of binomials that each have the same first term and the same last term, but one is a sum and one is a difference has a special name. It is called a *conjugate pair* and is of the form $(a-b),(a+b)$.

Note:
Conjugate Pair
A **conjugate pair** is two binomials of the form
Equation:

$$(a-b),(a+b).$$

The pair of binomials each have the same first term and the same last term, but one binomial is a sum and the other is a difference.

There is a nice pattern for finding the product of conjugates. You could, of course, simply FOIL to get the product, but using the pattern makes your work easier.

Let’s look for the pattern by using FOIL to multiply some conjugate pairs.
Equation:

$$\begin{array}{ccc} (x-9)\,(x+9) & (y-8)\,(y+8) & (2x-5)\,(2x+5) \\ x^2+9x-9x-81 & y^2+8y-8y-64 & 4x^2+10x-10x-25 \\ x^2-81 & y^2-64 & 4x^2-25 \end{array}$$

$$\begin{array}{ccc} (x+9)(x-9) & (y-8)(y+8) & (2x-5)(2x+5) \\ x^2-9x+9x-81 & y^2+8y-8y-64 & 4x^2+10x-10x-25 \\ x^2-81 & y^2-64 & 4x^2-25 \end{array}$$

Each **first term** is the product of the first terms of the binomials, and since they are identical it is the square of the first term.

Equation:

$$(a+b)(a-b) = a^2 - \underline{\qquad}$$

To get the **first term, square the first term**.

The **last term** came from multiplying the last terms, the square of the last term.

Equation:

$$(a+b)(a-b) = a^2 - b^2$$

To get the **last term, square the last term**.

What do you observe about the products?

The product of the two binomials is also a binomial! Most of the products resulting from FOIL have been trinomials.

Why is there no middle term? Notice the two middle terms you get from FOIL combine to 0 in every case, the result of one addition and one subtraction.

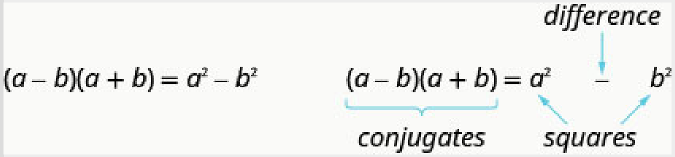
The product of conjugates is always of the form $a^2 - b^2$. This is called a difference of squares.

This leads to the pattern:

Note:

Product of Conjugates Pattern

If a and b are real numbers,



$$(a-b)(a+b) = a^2 - b^2 \qquad \underbrace{(a-b)(a+b)}_{\text{conjugates}} = a^2 \overset{\text{difference}}{-} b^2$$

The product is called a difference of squares.

To multiply conjugates, square the first term, square the last term, and write the product as a difference of squares.

Let’s test this pattern with a numerical example.

	$(10-2)(10+2)$
It is the product of conjudgates, so the result will be the difference of two squares.	____ – ____
Square the first term.	$10^2 -$ ____
Square the last term.	$10^2 - 2^2$
Simplify.	$100 - 4$
Simplify.	96
What do you get using the order of operations?	
	$(10-2)(10+2)$ $(8)(12)$ 96

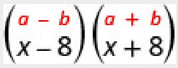
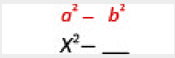
Notice, the result is the same!

Example:
Exercise:

Problem: **Multiply: $(x-8)\,(x+8)$.

Solution:
Solution

First, recognize this as a product of conjugates. The binomials have the same first terms, and the same last terms, and one binomial is a sum and the other is a difference.

It fits the pattern.	 $\left(\overset{a}{x} \overset{-}{-} \overset{b}{8}\right)\left(\overset{a}{x} \overset{+}{+} \overset{b}{8}\right)$
Square the first term, *x*.	 $\overset{a^2}{x^2} - \overset{b^2}{\underline{\quad}}$

Square the last term, 8.	$\overset{a^2}{x^2} - \overset{b^2}{8^2}$
The product is a difference of squares.	$\overset{a^2}{x^2} - \overset{b^2}{64}$

Note:
Exercise:

TRY IT :: 5.103
Problem: Multiply: $(x-5)(x+5)$.

Solution:

$x^2 - 25$

Note:
Exercise:

TRY IT :: 5.104
Problem: Multiply: $(w-3)(w+3)$.

Solution:

$w^2 - 9$

Example:
Exercise:

Problem: **Multiply: $(2x+5)(2x-5)$.

Solution:
Solution

Are the binomials conjugates?

It is the product of conjugates.	$\overset{a\ +\ b}{(2x+5)}\ \overset{a\ -\ b}{(2x-5)}$
Square the first term, 2*x*.	$\overset{a^2\ -\ b^2}{(2x)^2 - \_\_}$
Square the last term, 5.	$\overset{a^2\ -\ b^2}{(2x)^2 - 5^2}$
Simplify. The product is a difference of squares.	$\overset{a^2\ -\ b^2}{4x^2 - 25}$

Note:
Exercise:

TRY IT :: 5.105

Problem: Multiply: $(6x+5)(6x-5)$.

Solution:

$36x^2 - 25$

Note:
Exercise:

TRY IT :: 5.106

Problem: Multiply: $(2x+7)(2x-7)$.

Solution:

$4x^2 - 49$

The binomials in the next example may look backwards – the variable is in the second term. But the two binomials are still conjugates, so we use the same pattern to multiply them.

Example:
Exercise:

Problem: **Find the product: $(3+5x)\,(3-5x)$.

Solution:
Solution

It is the product of conjugates.	$\left(\overset{a}{3}\overset{-}{+}\overset{b}{5x}\right)\left(\overset{a}{3}\overset{+}{-}\overset{b}{5x}\right)$
Use the pattern.	$\overset{a^2}{3^2}\overset{-}{-}\overset{b^2}{(5x)^2}$
Simplify.	$9-25x^2$

Note:
Exercise:

TRY IT :: 5.107
Problem: Multiply: $(7+4x)\,(7-4x)$.

Solution:

$49-16x^2$

Note:

Exercise:

TRY IT :: 5.108

Problem: Multiply: $(9-2y)(9+2y)$.

Solution:

$81-4y^2$

Now we’ll multiply conjugates that have two variables.

Example:
Exercise:

Problem: **Find the product: $(5m-9n)(5m+9n)$.

Solution:
Solution

This fits the pattern.	$(\overset{a}{5m}\overset{-}{-}\overset{b}{9n})(\overset{a}{5m}\overset{+}{+}\overset{b}{9n})$
Use the pattern.	$\overset{a^2}{(5m)^2}\overset{-}{-}\overset{b^2}{(9n)^2}$
Simplify.	$25m^2-81n^2$

Note:
Exercise:

TRY IT :: 5.109

Problem: Find the product: $(4p-7q)(4p+7q)$.

Solution:

$16p^2-49q^2$

Note:
Exercise:

TRY IT :: 5.110

Problem: Find the product: $(3x-y)(3x+y)$.

Solution:

$9x^2-y^2$

Example:
Exercise:

Problem: **Find the product: $(cd-8)(cd+8)$.

Solution:
Solution

This fits the pattern.	$\left(\overset{a}{cd}\overset{-}{-}\overset{b}{8}\right)\left(\overset{a}{cd}\overset{+}{+}\overset{b}{8}\right)$
Use the pattern.	$\overset{a^2}{(cd)^2}\overset{-}{-}\overset{b^2}{(8)^2}$
Simplify.	c^2d^2-64

Note:
Exercise:

TRY IT :: 5.111

Problem: Find the product: $(xy-6)(xy+6)$.

Solution:

x^2y^2-36

Note:
Exercise:

TRY IT :: 5.112

Problem: Find the product: $(ab-9)(ab+9)$.

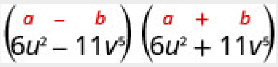
Solution:

a^2b^2-81

Example:
Exercise:

Problem: **Find the product: $\left(6u^2-11v^5\right)\left(6u^2+11v^5\right)$.

Solution:
Solution

This fits the pattern.	 $\left(\overset{a}{6u^2}\overset{-}{-}\overset{b}{11v^5}\right)\left(\overset{a}{6u^2}\overset{+}{+}\overset{b}{11v^5}\right)$

Use the pattern.	$\overset{a^2}{(6u^2)^2} \overset{-}{-} \overset{b^2}{(11v^5)^2}$
Simplify.	$36u^4 - 121v^{10}$

Note:
Exercise:

TRY IT :: 5.113

Problem: Find the product: $\left(3x^2 - 4y^3\right)\left(3x^2 + 4y^3\right)$.

Solution:

$9x^4 - 16y^6$

Note:
Exercise:

TRY IT :: 5.114

Problem: Find the product: $\left(2m^2 - 5n^3\right)\left(2m^2 + 5n^3\right)$.

Solution:

$4m^4 - 25n^6$

Recognize and Use the Appropriate Special Product Pattern

We just developed special product patterns for Binomial Squares and for the Product of Conjugates. The products look similar, so it is important to recognize when it is appropriate to use each of these patterns and to notice how they differ. Look at the two patterns together and note their similarities and differences.

Note:
Comparing the Special Product Patterns

Binomial Squares	**Product of Conjugates**
$(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)(a+b) = a^2 - b^2$
$(a-b)^2 = a^2 - 2ab + b^2$	
- Squaring a binomial	- Multiplying conjugates
- Product is a **trinomial**	- Product is a **binomial**
- Inner and outer terms with FOIL are **the same.**	- Inner and outer terms with FOIL are **opposites.**
- Middle term is **double the product** of the terms.	- There is **no** middle term.

Example:
Exercise:

Problem: **Choose the appropriate pattern and use it to find the product:

ⓐ $(2x-3)(2x+3)$ ⓑ $(8x-5)^2$ ⓒ $(6m+7)^2$ ⓓ $(5x-6)(6x+5)$

Solution:
Solution

ⓐ $(2x-3)(2x+3)$ These are conjugates. They have the same first numbers, and the same last numbers, and one binomial is a sum and the other is a difference. It fits the Product of Conjugates pattern.

This fits the pattern.	$\left(\overset{a}{2x} - \overset{b}{3}\right)\left(\overset{a}{2x} + \overset{b}{3}\right)$
Use the pattern.	$\overset{a^2}{(2x)^2} - \overset{b^2}{3^2}$

Simplify.	$4x^2-9$

ⓑ $(8x-5)^2$ We are asked to square a binomial. It fits the **binomial squares** pattern.

	$\left(\overset{a}{8x} \overset{-}{-} \overset{b}{5}\right)^2$
Use the pattern.	$\overset{a^2}{(8x)^2} \overset{-}{-} \overset{2ab}{2\cdot 8x\cdot 5} \overset{+}{+} \overset{b^2}{5^2}$
Simplify.	$64x^2-80x+25$

ⓒ $(6m+7)^2$ Again, we will square a binomial so we use the **binomial squares** pattern.

	$\left(\overset{a}{6m} \overset{+}{+} \overset{b}{7}\right)^2$
Use the pattern.	$\overset{a^2}{(6m)^2} \overset{+}{+} \overset{2ab}{2\cdot 6m\cdot 7} \overset{+}{+} \overset{b^2}{7^2}$
Simplify.	$36m^2+84m+49$

ⓓ $(5x-6)(6x+5)$ This product does not fit the patterns, so we will use FOIL.

	$(5x-6)(6x+5)$
Use FOIL.	$30x^2+25x-36x-30$
Simplify.	$30x^2-11x-30$

Note:
Exercise:

TRY IT :: 5.115

Problem: Choose the appropriate pattern and use it to find the product:

ⓐ $(9b-2)(2b+9)$ ⓑ $(9p-4)^2$ ⓒ $(7y+1)^2$ ⓓ $(4r-3)(4r+3)$

Solution:

ⓐ FOIL; $18b^2+77b-18$ ⓑ Binomial Squares; $81p^2-72p+16$ ⓒ Binomial Squares; $49y^2+14y+1$ ⓓ Product of Conjugates; $16r^2-9$

Note:
Exercise:

TRY IT :: 5.116

Problem: Choose the appropriate pattern and use it to find the product:

ⓐ $(6x+7)^2$ ⓑ $(3x-4)(3x+4)$ ⓒ $(2x-5)(5x-2)$ ⓓ $(6n-1)^2$

Solution:

ⓐ Binomial Squares; $36x^2+84x+49$ ⓑ Product of Conjugates; $9x^2-16$ ⓒ FOIL; $10x^2-29x+10$ ⓓ Binomial Squares; $36n^2-12n+1$

Note:
Access these online resources for additional instruction and practice with special products:

- Special Products

Key Concepts

- **Binomial Squares Pattern**
  - If a, b are real numbers,

$$\underbrace{(a+b)^2}_{(\text{binomial})^2} = \underbrace{a^2}_{(\text{first term})^2} + \underbrace{2ab}_{2(\text{product of terms})} + \underbrace{b^2}_{(\text{last term})^2}$$

  - $(a+b)^2 = a^2 + 2ab + b^2$
  - $(a-b)^2 = a^2 - 2ab + b^2$
  - To square a binomial: square the first term, square the last term, double their product.

- **Product of Conjugates Pattern**
  - If a, b are real numbers,

$$\underbrace{(a-b)(a+b)}_{\text{conjugates}} = a^2 \overset{\overset{\text{difference}}{\downarrow}}{\underset{\nwarrow \; \text{squares} \; \nearrow}{-}} b^2$$

  - $(a-b)(a+b) = a^2 - b^2$
  - The product is called a difference of squares.

- **To multiply conjugates:**
  - **square the first term square the last term** write it as a difference of squares

Practice Makes Perfect

Square a Binomial Using the Binomial Squares Pattern

In the following exercises, square each binomial using the Binomial Squares Pattern.

Exercise:

Problem: $(w+4)^2$

Exercise:

Problem: $(q+12)^2$

Solution:

$q^2 + 24q + 144$

Exercise:

Problem: $\left(y + \frac{1}{4}\right)^2$

Exercise:

Problem: $\left(x + \frac{2}{3}\right)^2$

Solution:

$x^2 + \frac{4}{3}x + \frac{4}{9}$

Exercise:

Problem: $(b - 7)^2$

Exercise:

Problem: $(y - 6)^2$

Solution:

$y^2 - 12y + 36$

Exercise:

Problem: $(m - 15)^2$

Exercise:

Problem: $(p - 13)^2$

Solution:

$p^2 - 26p + 169$

Exercise:

Problem: $(3d + 1)^2$

Exercise:

Problem: $(4a + 10)^2$

Solution:

$16a^2 + 80a + 100$

Exercise:

Problem: $(2q + \frac{1}{3})^2$

Exercise:

Problem: $\left(3z + \frac{1}{5}\right)^2$

Solution:

$9z^2 + \frac{6}{5}z + \frac{1}{25}$

Exercise:

Problem: $(3x - y)^2$

Exercise:

Problem: $(2y - 3z)^2$

Solution:

$4y^2 - 12yz + 9z^2$

Exercise:

Problem: $\left(\frac{1}{5}x - \frac{1}{7}y\right)^2$

Exercise:

Problem: $\left(\frac{1}{8}x - \frac{1}{9}y\right)^2$

Solution:

$\frac{1}{64}x^2 - \frac{1}{36}xy + \frac{1}{81}y^2$

Exercise:

Problem: $(3x^2 + 2)^2$

Exercise:

Problem: $(5u^2 + 9)^2$

Solution:

$25u^4 + 90u^2 + 81$

Exercise:

Problem: $(4y^3 - 2)^2$

Exercise:

Problem: $(8p^3 - 3)^2$

Solution:

$64p^6 - 48p^3 + 9$

Multiply Conjugates Using the Product of Conjugates Pattern

In the following exercises, multiply each pair of conjugates using the Product of Conjugates Pattern.

Exercise:

Problem: $(m - 7)(m + 7)$

Exercise:

Problem: $(c - 5)(c + 5)$

Solution:

$c^2 - 25$

Exercise:

Problem: $\left(x + \frac{3}{4}\right)\left(x - \frac{3}{4}\right)$

Exercise:

Problem: $\left(b + \frac{6}{7}\right)\left(b - \frac{6}{7}\right)$

Solution:

$b^2 - \frac{36}{49}$

Exercise:

Problem: $(5k + 6)(5k - 6)$

Exercise:

Problem: $(8j+4)(8j-4)$

Solution:

$64j^2-16$

Exercise:

Problem: $(11k+4)(11k-4)$

Exercise:

Problem: $(9c+5)(9c-5)$

Solution:

$81c^2-25$

Exercise:

Problem: $(11-b)(11+b)$

Exercise:

Problem: $(13-q)(13+q)$

Solution:

$169-q^2$

Exercise:

Problem: $(5-3x)(5+3x)$

Exercise:

Problem: $(4-6y)(4+6y)$

Solution:

$16-36y^2$

Exercise:

Problem: $(9c-2d)(9c+2d)$

Exercise:

Problem: $(7w+10x)(7w-10x)$

Solution:

$49w^2-100x^2$

Exercise:

Problem: $\left(m+\frac{2}{3}n\right)\left(m-\frac{2}{3}n\right)$

Exercise:

Problem: $\left(p+\frac{4}{5}q\right)\left(p-\frac{4}{5}q\right)$

Solution:

$p^2-\frac{16}{25}q^2$

Exercise:

Problem: $(ab-4)(ab+4)$

Exercise:

Problem: $(xy-9)(xy+9)$

Solution:

x^2y^2-81

Exercise:

Problem: $(uv-\frac{3}{5})(uv+\frac{3}{5})$

Exercise:

Problem: $(rs-\frac{2}{7})(rs+\frac{2}{7})$

Solution:

$r^2s^2-\frac{4}{49}$

Exercise:

Problem: $(2x^2-3y^4)(2x^2+3y^4)$

Exercise:

Problem: $(6m^3-4n^5)(6m^3+4n^5)$

Solution:

$36m^6-16n^{10}$

Exercise:

Problem: $(12p^3-11q^2)(12p^3+11q^2)$

Exercise:

Problem: $(15m^2-8n^4)(15m^2+8n^4)$

Solution:

$225m^4-64n^8$

Recognize and Use the Appropriate Special Product Pattern

In the following exercises, find each product.

Exercise:

Problem:
ⓐ $(p-3)\,(p+3)$
ⓑ $(t-9)^2$
ⓒ $(m+n)^2$
ⓓ $(2x+y)(x-2y)$

Exercise:

Problem:

ⓐ $(2r+12)^2$
ⓑ $(3p+8)(3p-8)$
ⓒ $(7a+b)(a-7b)$
ⓓ $(k-6)^2$

Solution:

ⓐ $4r^2+48r+144$ ⓑ $9p^2-64$ ⓒ $7a^2-48ab-7b^2$ ⓓ $k^2-12k+36$

Exercise:

Problem:

ⓐ $(a^5 - 7b)^2$
ⓑ $(x^2 + 8y)(8x - y^2)$
ⓒ $(r^6 + s^6)(r^6 - s^6)$
ⓓ $(y^4 + 2z)^2$

Exercise:

Problem:

ⓐ $(x^5 + y^5)(x^5 - y^5)$
ⓑ $(m^3 - 8n)^2$
ⓒ $(9p + 8q)^2$
ⓓ $(r^2 - s^3)(r^3 + s^2)$

Solution:

ⓐ $x^{10} - y^{10}$ ⓑ $m^6 - 16m^3n + 64n^2$ ⓒ $81p^2 + 144pq + 64q^2$ ⓓ $r^5 + r^2s^2 - r^3s^3 - s^5$

Everyday Math

Exercise:

Problem:

Mental math You can use the product of conjugates pattern to multiply numbers without a calculator. Say you need to multiply 47 times 53. Think of 47 as $50 - 3$ and 53 as $50 + 3$.

ⓐ Multiply $(50 - 3)(50 + 3)$ by using the product of conjugates pattern, $(a - b)\,(a + b) = a^2 - b^2$.
ⓑ Multiply $47 \cdot 53$ without using a calculator.
ⓒ Which way is easier for you? Why?

Exercise:

Problem:

Mental math You can use the binomial squares pattern to multiply numbers without a calculator. Say you need to square 65. Think of 65 as $60 + 5$.

ⓐ Multiply $(60 + 5)^2$ by using the binomial squares pattern, $(a + b)^2 = a^2 + 2ab + b^2$.
ⓑ Square 65 without using a calculator.
ⓒ Which way is easier for you? Why?

Solution:

ⓐ 4,225 ⓑ 4,225 ⓒ Answers will vary.

Writing Exercises

Exercise:

Problem: How do you decide which pattern to use?

Exercise:

Problem: Why does $(a+b)^2$ result in a trinomial, but $(a-b)(a+b)$ result in a binomial?

Solution:

Answers will vary.

Exercise:

Problem: Marta did the following work on her homework paper:
Equation:

$$\begin{array}{c}(3-y)^2 \\ 3^2-y^2 \\ 9-y^2\end{array}$$

Explain what is wrong with Marta's work.

Exercise:

Problem:

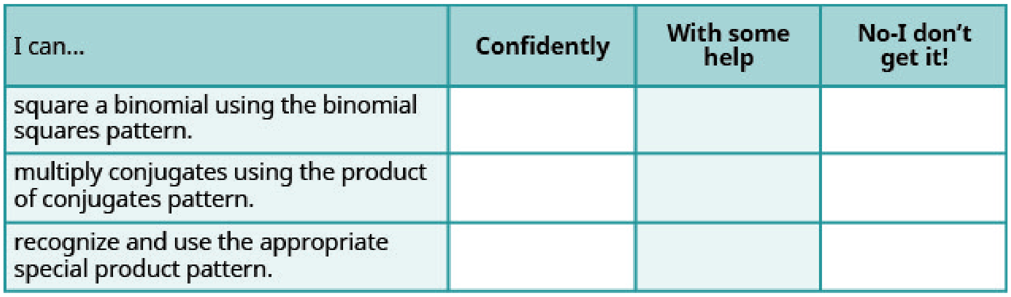
Use the order of operations to show that $(3+5)^2$ is 64, and then use that numerical example to explain why $(a+b)^2 \neq a^2+b^2$.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
square a binomial using the binomial squares pattern.			
multiply conjugates using the product of conjugates pattern.			
recognize and use the appropriate special product pattern.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

conjugate pair

> A conjugate pair is two binomials of the form $(a-b),(a+b)$; the pair of binomials each have the same first term and the same last term, but one binomial is a sum and the other is a difference.

Divide Monomials
By the end of this section, you will be able to:

- Simplify expressions using the Quotient Property for Exponents
- Simplify expressions with zero exponents
- Simplify expressions using the quotient to a Power Property
- Simplify expressions by applying several properties
- Divide monomials

Note:
Before you get started, take this readiness quiz.

1. Simplify: $\frac{8}{24}$.
   If you missed this problem, review [link].
2. Simplify: $\left(2m^3\right)^5$.
   If you missed this problem, review [link].
3. Simplify: $\frac{12x}{12y}$.
   If you missed this problem, review [link].

Simplify Expressions Using the Quotient Property for Exponents

Earlier in this chapter, we developed the properties of exponents for multiplication. We summarize these properties below.

Note:
Summary of Exponent Properties for Multiplication
If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$

Product to a Power	$(ab)^m = a^m b^m$

Now we will look at the exponent properties for division. A quick memory refresher may help before we get started. You have learned to simplify fractions by dividing out common factors from the numerator and denominator using the Equivalent Fractions Property. This property will also help you work with algebraic fractions—which are also quotients.

Note:
Equivalent Fractions Property
If $a, b,$ and c are whole numbers where $b \neq 0, c \neq 0,$
Equation:

$$\text{then} \quad \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

As before, we'll try to discover a property by looking at some examples.

Consider	$\frac{x^5}{x^2}$	and	$\frac{x^2}{x^3}$
What do they mean?	$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$		$\frac{x \cdot x}{x \cdot x \cdot x}$
Use the Equivalent Fractions Property.	$\frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}}$		$\frac{\cancel{x} \cdot \cancel{x} \cdot 1}{\cancel{x} \cdot \cancel{x} \cdot x}$
Simplify.	x^3		$\frac{1}{x}$

Notice, in each case the bases were the same and we subtracted exponents.

When the larger exponent was in the numerator, we were left with factors in the numerator.

When the larger exponent was in the denominator, we were left with factors in the denominator—notice the numerator of 1.

We write:
Equation:

$$\begin{array}{cc} \frac{x^5}{x^2} & \frac{x^2}{x^3} \\ x^{5-2} & \frac{1}{x^{3-2}} \\ x^3 & \frac{1}{x} \end{array}$$

This leads to the *Quotient Property for Exponents*.

Note:
Quotient Property for Exponents
If a is a real number, $a \neq 0$, and m and n are whole numbers, then
Equation:

$$\frac{a^m}{a^n} = a^{m-n}, m > n \text{ and } \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$$

A couple of examples with numbers may help to verify this property.
Equation:

$$\begin{array}{ccc} \frac{3^4}{3^2} & = & 3^{4-2} \\ \frac{81}{9} & = & 3^2 \\ 9 & = & 9\checkmark \end{array} \qquad \begin{array}{ccc} \frac{5^2}{5^3} & = & \frac{1}{5^{3-2}} \\ \frac{25}{125} & = & \frac{1}{5^1} \\ \frac{1}{5} & = & \frac{1}{5}\checkmark \end{array}$$

Example:
Exercise:

Problem: **Simplify: ⓐ $\frac{x^9}{x^7}$ ⓑ $\frac{3^{10}}{3^2}$.

Solution:
Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

ⓐ

Since 9 > 7, there are more factors of *x* in the numerator.	$\frac{x^9}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$.	x^{9-7}
Simplify.	x^2

ⓑ

Since 10 > 2, there are more factors of *x* in the numerator.	$\frac{3^{10}}{3^2}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$.	3^{10-2}
Simplify.	

	3^8

Notice that when the larger exponent is in the numerator, we are left with factors in the numerator.

Note:
Exercise:

TRY IT :: 5.117

Problem: Simplify: ⓐ $\frac{x^{15}}{x^{10}}$ ⓑ $\frac{6^{14}}{6^5}$.

Solution:

ⓐ x^5 ⓑ 6^9

Note:
Exercise:

TRY IT :: 5.118

Problem: Simplify: ⓐ $\frac{y^{43}}{y^{37}}$ ⓑ $\frac{10^{15}}{10^7}$.

Solution:

ⓐ y^6 ⓑ 10^8

Example:
Exercise:

Problem: **Simplify: ⓐ $\frac{b^8}{b^{12}}$ ⓑ $\frac{7^3}{7^5}$.

Solution:

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

ⓐ

Since 12 > 8, there are more factors of *b* in the denominator.	$\frac{b^8}{b^{12}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{b^{12-8}}$
Simplify.	$\frac{1}{b^4}$

ⓑ

Since 5 > 3, there are more factors of 3 in the denominator.	$\frac{7^3}{7^5}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{7^{5-3}}$
Simplify.	$\frac{1}{7^2}$

Simplify.	$\frac{1}{49}$

Notice that when the larger exponent is in the denominator, we are left with factors in the denominator.

Note:
Exercise:

TRY IT :: 5.119

Problem: Simplify: ⓐ $\frac{x^{18}}{x^{22}}$ ⓑ $\frac{12^{15}}{12^{30}}$.

Solution:

ⓐ $\frac{1}{x^4}$ ⓑ $\frac{1}{12^{15}}$

Note:
Exercise:

TRY IT :: 5.120

Problem: Simplify: ⓐ $\frac{m^7}{m^{15}}$ ⓑ $\frac{9^8}{9^{19}}$.

Solution:

ⓐ $\frac{1}{m^8}$ ⓑ $\frac{1}{9^{11}}$

Notice the difference in the two previous examples:

- If we start with more factors in the numerator, we will end up with factors in the numerator.

- If we start with more factors in the denominator, we will end up with factors in the denominator.

The first step in simplifying an expression using the Quotient Property for Exponents is to determine whether the exponent is larger in the numerator or the denominator.

Example:
Exercise:

Problem: **Simplify: ⓐ $\frac{a^5}{a^9}$ ⓑ $\frac{x^{11}}{x^7}$.

Solution:
Solution

ⓐ Is the exponent of a larger in the numerator or denominator? Since 9 > 5, there are more a's in the denominator and so we will end up with factors in the denominator.

	$\frac{a^5}{a^9}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{a^{9-5}}$
Simplify.	$\frac{1}{a^4}$

ⓑ Notice there are more factors of x in the numerator, since 11 > 7. So we will end up with factors in the numerator.

	$\frac{x^{11}}{x^{7}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	x^{11-7}
Simplify.	x^{4}

Note:
Exercise:

TRY IT :: 5.121

Problem: Simplify: ⓐ $\frac{b^{19}}{b^{11}}$ ⓑ $\frac{z^5}{z^{11}}$.

Solution:

ⓐ b^8 ⓑ $\frac{1}{z^6}$

Note:
Exercise:

TRY IT :: 5.122

Problem: Simplify: ⓐ $\frac{p^9}{p^{17}}$ ⓑ $\frac{w^{13}}{w^9}$.

Solution:

ⓐ $\frac{1}{p^8}$ ⓑ w^4

Simplify Expressions with an Exponent of Zero

A special case of the Quotient Property is when the exponents of the numerator and denominator are equal, such as an expression like $\frac{a^m}{a^m}$. From your earlier work with fractions, you know that:

Equation:

$$\frac{2}{2} = 1 \qquad \frac{17}{17} = 1 \qquad \frac{-43}{-43} = 1$$

In words, a number divided by itself is 1. So, $\frac{x}{x} = 1$, for any x $(x \neq 0)$, since any number divided by itself is 1.

The Quotient Property for Exponents shows us how to simplify $\frac{a^m}{a^n}$ when $m > n$ and when $n < m$ by subtracting exponents. What if $m = n$?

Consider $\frac{8}{8}$, which we know is 1.

	$\frac{8}{8} = 1$
Write 8 as 2^3.	$\frac{2^3}{2^3} = 1$
Subtract exponents.	$2^{3-3} = 1$
Simplify.	$2^0 = 1$

Now we will simplify $\frac{a^m}{a^m}$ in two ways to lead us to the definition of the zero exponent. In general, for $a \neq 0$:

$$\begin{array}{ll} \dfrac{a^m}{a^m} & \dfrac{a^m}{a^m} \\ a^{m-m} & \dfrac{\overbrace{\cancel{a}\cdot\cancel{a}\cdot\ldots\cdot\cancel{a}}^{m\text{ factors}}}{\underbrace{\cancel{a}\cdot\cancel{a}\cdot\ldots\cdot\cancel{a}}_{m\text{ factors}}} \\ a^0 & 1 \end{array}$$

We see $\frac{a^m}{a^m}$ simplifies to a^0 and to 1. So $a^0 = 1$.

Note:
Zero Exponent
If a is a non-zero number, then $a^0 = 1$.
Any nonzero number raised to the zero power is 1.

In this text, we assume any variable that we raise to the zero power is not zero.

Example:
Exercise:

Problem: **Simplify: ⓐ 9^0 ⓑ n^0.

Solution:
Solution

The definition says any non-zero number raised to the zero power is 1.

ⓐ Use the definition of the zero exponent.	9^0 1

ⓑ Use the definition of the zero exponent.	n^0 1

Note:
Exercise:

TRY IT :: 5.123

Problem: Simplify: ⓐ 15^0 ⓑ m^0.

Solution:

ⓐ 1 ⓑ 1

Note:
Exercise:

TRY IT :: 5.124

Problem: Simplify: ⓐ k^0 ⓑ 29^0.

Solution:

ⓐ 1 ⓑ 1

Now that we have defined the zero exponent, we can expand all the Properties of Exponents to include whole number exponents.

What about raising an expression to the zero power? Let's look at $(2x)^0$. We can use the product to a power rule to rewrite this expression.

	$(2x)^0$
Use the product to a power rule.	2^0x^0
Use the zero exponent property.	$1 \cdot 1$
Simplify.	1

This tells us that any nonzero expression raised to the zero power is one.

Example:
Exercise:

Problem: **Simplify: ⓐ $(5b)^0$ ⓑ $\left(-4a^2b\right)^0$.

Solution:
Solution

ⓐ	$(5b)^0$
Use the definition of the zero exponent.	1
ⓑ	$\left(-4a^2b\right)^0$
Use the definition of the zero exponent.	1

Note:
Exercise:

TRY IT :: 5.125

Problem: Simplify: ⓐ $(11z)^0$ ⓑ $\left(-11pq^3\right)^0$.

Solution:

ⓐ 1 ⓑ 1

Note:
Exercise:

TRY IT :: 5.126

Problem: Simplify: ⓐ $(-6d)^0$ ⓑ $\left(-8m^2n^3\right)^0$.

Solution:

ⓐ 1 ⓑ 1

Simplify Expressions Using the Quotient to a Power Property

Now we will look at an example that will lead us to the Quotient to a Power Property.

	$\left(\frac{x}{y}\right)^3$
This means:	$\frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y}$
Multiply the fractions.	$\frac{x \cdot x \cdot x}{y \cdot y \cdot y}$
Write with exponents.	$\frac{x^3}{y^3}$

Notice that the exponent applies to both the numerator and the denominator.

We write:	$\left(\frac{x}{y}\right)^3$
	$\frac{x^3}{y^3}$

This leads to the *Quotient to a Power Property for Exponents*.

Note:
Quotient to a Power Property for Exponents
If a and b are real numbers, $b \neq 0$, and m is a counting number, then
Equation:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

To raise a fraction to a power, raise the numerator and denominator to that power.

An example with numbers may help you understand this property:
Equation:

$$\begin{array}{rcl} \left(\frac{2}{3}\right)^3 & = & \frac{2^3}{3^3} \\ \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} & = & \frac{8}{27} \\ \frac{8}{27} & = & \frac{8}{27}\checkmark \end{array}$$

Example:
Exercise:

Problem: **Simplify: ⓐ $\left(\frac{3}{7}\right)^2$ ⓑ $\left(\frac{b}{3}\right)^4$ ⓒ $\left(\frac{k}{j}\right)^3$.

Solution:
Solution

ⓐ

	$\left(\frac{3}{7}\right)^2$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	$\frac{3^2}{7^2}$
Simplify.	$\frac{9}{49}$

ⓑ

	$\left(\frac{b}{3}\right)^4$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	$\frac{b^4}{3^4}$

Simplify.	$\frac{b^4}{81}$

ⓒ

	$\left(\frac{k}{j}\right)^3$
Raise the numerator and denominator to the third power.	$\frac{k^3}{j^3}$

Note:
Exercise:

TRY IT :: 5.127

Problem: Simplify: ⓐ $\left(\frac{5}{8}\right)^2$ ⓑ $\left(\frac{p}{10}\right)^4$ ⓒ $\left(\frac{m}{n}\right)^7$.

Solution:

ⓐ $\frac{25}{64}$ ⓑ $\frac{p^4}{10{,}000}$ ⓒ $\frac{m^7}{n^7}$

Note:
Exercise:

TRY IT :: 5.128

Problem: Simplify: ⓐ $\left(\frac{1}{3}\right)^3$ ⓑ $\left(\frac{-2}{q}\right)^3$ ⓒ $\left(\frac{w}{x}\right)^4$.

Solution:

ⓐ $\frac{1}{27}$ ⓑ $\frac{-8}{q^3}$ ⓒ $\frac{w^4}{x^4}$

Simplify Expressions by Applying Several Properties

We’ll now summarize all the properties of exponents so they are all together to refer to as we simplify expressions using several properties. Notice that they are now defined for whole number exponents.

Note:
Summary of Exponent Properties
If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$
Quotient Property	$\frac{a^m}{b^m} = a^{m-n}, a \neq 0, m > n$ $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, a \neq 0, n > m$
Zero Exponent Definition	$a^o = 1, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Example:
Exercise:

Problem: **Simplify: $\frac{(y^4)^2}{y^6}$.

Solution:
Solution

	$\frac{(y^4)^2}{y^6}$
Multiply the exponents in the numerator.	$\frac{y^8}{y^6}$
Subtract the exponents.	y^2

Note:
Exercise:

TRY IT :: 5.129

Problem: Simplify: $\frac{(m^5)^4}{m^7}$.

Solution:

m^{13}

Note:

Exercise:

TRY IT :: 5.130

Problem: Simplify: $\frac{\left(k^2\right)^6}{k^7}$.

Solution:

k^5

Example:
Exercise:

Problem: **Simplify: $\frac{b^{12}}{\left(b^2\right)^6}$.

Solution:
Solution

	$\frac{b^{12}}{\left(b^2\right)^6}$
Multiply the exponents in the numerator.	$\frac{b^{12}}{b^{12}}$
Subtract the exponents.	b^0
Simplify.	1

Note:
Exercise:

TRY IT :: 5.131

Problem: Simplify: $\frac{n^{12}}{(n^3)^4}$.

Solution:

1

Note:
Exercise:

TRY IT :: 5.132

Problem: Simplify: $\frac{x^{15}}{(x^3)^5}$.

Solution:

1

Example:
Exercise:

Problem: **Simplify: $\left(\frac{y^9}{y^4}\right)^2$.

Solution:
Solution

	$\left(\frac{y^9}{y^4}\right)^2$
Remember parentheses come before exponents.	$\left(y^5\right)^2$

Notice the bases are the same, so we can simplify inside the parentheses. Subtract the exponents.	
Multiply the exponents.	y^{10}

Note:
Exercise:

TRY IT :: 5.133

Problem: Simplify: $\left(\frac{r^5}{r^3}\right)^4$.

Solution:

r^8

Note:
Exercise:

TRY IT :: 5.134

Problem: Simplify: $\left(\frac{v^6}{v^4}\right)^3$.

Solution:

v^6

Example:
Exercise:

Problem: **Simplify: $\left(\frac{j^2}{k^3}\right)^4$.

Solution:
Solution

Here we cannot simplify inside the parentheses first, since the bases are not the same.

	$\left(\frac{j^2}{k^3}\right)^4$
Raise the numberator and denominator to the third power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	
Use the Powert Property and simplify.	$\frac{j^8}{k^{12}}$

Note:
Exercise:

TRY IT :: 5.135

Problem: Simplify: $\left(\frac{a^3}{b^2}\right)^4$.

Solution:

$\frac{a^{12}}{b^8}$

Note:
Exercise:

TRY IT :: 5.136

Problem: Simplify: $\left(\frac{q^7}{r^5}\right)^3$.

Solution:

$\frac{q^{21}}{r^{15}}$

Example:
Exercise:

Problem: **Simplify: $\left(\frac{2m^2}{5n}\right)^4$.

Solution:
Solution

	$\left(\frac{2m^2}{5n}\right)^4$
Raise the numberator and denominator to the fourth power, using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	$\frac{(2m^2)^4}{(5n)^4}$
Raise each factor to the fourth power.	$\frac{(2m^2)^4}{(5n)^4}$
Use the Power Property and simplify.	$\frac{16m^8}{625n^4}$

Note:

Exercise:

TRY IT :: 5.137

Problem: Simplify: $\left(\frac{7x^3}{9y}\right)^2$.

Solution:

$\frac{49x^6}{81y^2}$

Note:

Exercise:

TRY IT :: 5.138

Problem: Simplify: $\left(\frac{3x^4}{7y}\right)^2$.

Solution:

$\frac{9x^8}{49y^2}$

Example:

Exercise:

Problem: **Simplify: $\frac{(x^3)^4(x^2)^5}{(x^6)^5}$.

Solution:

Solution

	$\frac{(x^3)^4(x^2)^5}{(x^6)^5}$

Use the Power Property, $(a^m)^n = a^{m \cdot n}$.	$\frac{(x^{12})(x^{10})}{(x^{30})}$
Add the exponents in the numerator.	$\frac{x^{22}}{x^{30}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{x^8}$

Note:
Exercise:

TRY IT :: 5.139

Problem: Simplify: $\frac{(a^2)^3(a^2)^4}{(a^4)^5}$.

Solution:

$\frac{1}{a^6}$

Note:
Exercise:

TRY IT :: 5.140

Problem: Simplify: $\frac{(p^3)^4(p^5)^3}{(p^7)^6}$.

Solution:

$\frac{1}{p^{15}}$

Example:
Exercise:

Problem: **Simplify: $\frac{\left(10p^3\right)^2}{(5p)^3(2p^5)^4}$.

Solution:
Solution

	$\frac{\left(10p^3\right)^2}{(5p)^3(2p^5)^4}$
Use the Product to a Power Property, $(ab)^m = a^m b^m$.	$\frac{(10)^2\left(p^3\right)^2}{(5)^3(p)^3(2)^4(p^5)^4}$
Use the Power Property, $(a^m)^n = a^{m\cdot n}$.	$\frac{100p^6}{125p^3\cdot 16p^{20}}$
Add the exponents in the denominator.	$\frac{100p^6}{125\cdot 16p^{23}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{100}{125\cdot 16p^{17}}$
Simplify.	$\frac{1}{20p^{17}}$

Note:
Exercise:

TRY IT :: 5.141

Problem: Simplify: $\frac{\left(3r^3\right)^2\left(r^3\right)^7}{(r^3)^3}$.

Solution:

$9r^{18}$

Note:
Exercise:

TRY IT :: 5.142

Problem: Simplify: $\frac{\left(2x^4\right)^5}{\left(4x^3\right)^2\left(x^3\right)^5}$.

Solution:

$\frac{2}{x}$

Divide Monomials

You have now been introduced to all the properties of exponents and used them to simplify expressions. Next, you’ll see how to use these properties to divide monomials. Later, you’ll use them to divide polynomials.

Example:
Exercise:

Problem: **Find the quotient: $56x^7 \div 8x^3$.

Solution:
Solution

	$56x^7 \div 8x^3$
Rewrite as a fraction.	$\frac{56x^7}{8x^3}$
Use fraction multiplication.	$\frac{56}{8} \cdot \frac{x^7}{x^3}$
Simplify and use the Quotient Property.	$7x^4$

Note:
Exercise:

TRY IT :: 5.143

Problem: Find the quotient: $42y^9 \div 6y^3$.

Solution:

$7y^6$

Note:
Exercise:

TRY IT :: 5.144

Problem: Find the quotient: $48z^8 \div 8z^2$.

Solution:

$6z^6$

Example:
Exercise:

Problem: **Find the quotient: $\frac{45a^2b^3}{-5ab^5}$.

Solution:
Solution

	$\frac{45a^2b^3}{-5ab^5}$
Use fraction multiplication.	$\frac{45}{-5} \cdot \frac{a^2}{a} \cdot \frac{b^3}{b^5}$
Simplify and use the Quotient Property.	$-9 \cdot a \cdot \frac{1}{b^2}$
Multiply.	$-\frac{9a}{b^2}$

Note:
Exercise:

TRY IT :: 5.145

Problem: Find the quotient: $\frac{-72a^7b^3}{8a^{12}b^4}$.

Solution:

$-\frac{9}{a^5b}$

Note:
Exercise:

TRY IT :: 5.146

Problem: Find the quotient: $\frac{-63c^8d^3}{7c^{12}d^2}$.

Solution:

$\frac{-9d}{c^4}$

Example:
Exercise:

Problem: **Find the quotient: $\frac{24a^5b^3}{48ab^4}$.

Solution:
Solution

	$\frac{24a^5b^3}{48ab^4}$
Use fraction multiplication.	$\frac{24}{48}\cdot\frac{a^5}{a}\cdot\frac{b^3}{b^4}$
Simplify and use the Quotient Property.	$\frac{1}{2}\cdot a^4\cdot\frac{1}{b}$
Multiply.	$\frac{a^4}{2b}$

Note:
Exercise:

TRY IT :: 5.147

Problem: Find the quotient: $\frac{16a^7b^6}{24ab^8}$.

Solution:

$\frac{2a^6}{3b^2}$

Note:
Exercise:

TRY IT :: 5.148

Problem: Find the quotient: $\frac{27p^4q^7}{-45p^{12}q}$.

Solution:

$-\frac{3q^6}{5p^8}$

Once you become familiar with the process and have practiced it step by step several times, you may be able to simplify a fraction in one step.

Example:
Exercise:

Problem: **Find the quotient: $\frac{14x^7y^{12}}{21x^{11}y^6}$.

Solution:
Solution

Be very careful to simplify $\frac{14}{21}$ by dividing out a common factor, and to simplify the variables by subtracting their exponents.

	$\frac{14x^7y^{12}}{21x^{11}y^6}$
Simplify and use the Quotient Property.	$\frac{2y^6}{3x^4}$

Note:

Exercise:

TRY IT :: 5.149

Problem: Find the quotient: $\frac{28x^5y^{14}}{49x^9y^{12}}$.

Solution:

$\frac{4y^2}{7x^4}$

Note:
Exercise:

TRY IT :: 5.150

Problem: Find the quotient: $\frac{30m^5n^{11}}{48m^{10}n^{14}}$.

Solution:

$\frac{5}{8m^5n^3}$

In all examples so far, there was no work to do in the numerator or denominator before simplifying the fraction. In the next example, we'll first find the product of two monomials in the numerator before we simplify the fraction. This follows the order of operations. Remember, a fraction bar is a grouping symbol.

Example:
Exercise:

Problem: **Find the quotient: $\frac{\left(6x^2y^3\right)\left(5x^3y^2\right)}{\left(3x^4y^5\right)}$.

Solution:
Solution

	$\frac{\left(6x^2y^3\right)\left(5x^3y^2\right)}{\left(3x^4y^5\right)}$
Simplify the numerator.	$\frac{30x^5y^5}{3x^4y^5}$
Simplify.	$10x$

Note:
Exercise:

TRY IT :: 5.151

Problem: Find the quotient: $\frac{\left(6a^4b^5\right)\left(4a^2b^5\right)}{12a^5b^8}$.

Solution:

$2ab^2$

Note:
Exercise:

TRY IT :: 5.152

Problem: Find the quotient: $\frac{\left(-12x^6y^9\right)\left(-4x^5y^8\right)}{-12x^{10}y^{12}}$.

Solution:

$-4xy^5$

Note:
Access these online resources for additional instruction and practice with dividing monomials:

- Rational Expressions

- Dividing Monomials
- Dividing Monomials 2

Key Concepts

- **Quotient Property for Exponents:**
  - If a is a real number, $a \neq 0$, and m, n are whole numbers, then: $\frac{a^m}{a^n} = a^{m-n}, m > n \text{ and } \frac{a^m}{a^n} = \frac{1}{a^{m-n}}, n > m$
- **Zero Exponent**
  - If a is a non-zero number, then $a^0 = 1$.
- **Quotient to a Power Property for Exponents**:
  - If a and b are real numbers, $b \neq 0$, and m is a counting number, then: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  - To raise a fraction to a power, raise the numerator and denominator to that power.
- **Summary of Exponent Properties**
  - If a, b are real numbers and m, n are whole numbers, then

$$\begin{array}{lrcl}
\textbf{Product Property} & a^m \cdot a^n & = & a^{m+n} \\
\textbf{Power Property} & (a^m)^n & = & a^{m \cdot n} \\
\textbf{Product to a Power} & (ab)^m & = & a^m b^m \\
\textbf{Quotient Property} & \frac{a^m}{b^m} & = & a^{m-n}, a \neq 0, m > n \\
 & \frac{a^m}{a^n} & = & \frac{1}{a^{n-m}}, a \neq 0, n > m \\
\textbf{Zero Exponent Definition} & a^o & = & 1, a \neq 0 \\
\textbf{Quotient to a Power Property} & \left(\frac{a}{b}\right)^m & = & \frac{a^m}{b^m}, b \neq 0
\end{array}$$

Practice Makes Perfect

Simplify Expressions Using the Quotient Property for Exponents

In the following exercises, simplify.

Exercise:

Problem: ⓐ $\frac{x^{18}}{x^3}$ ⓑ $\frac{5^{12}}{5^3}$

Exercise:

Problem: ⓐ $\frac{y^{20}}{y^{10}}$ ⓑ $\frac{7^{16}}{7^2}$

Solution:

ⓐ y^{10} ⓑ 7^{14}

Exercise:

Problem: ⓐ $\frac{p^{21}}{p^7}$ ⓑ $\frac{4^{16}}{4^4}$

Exercise:

Problem: ⓐ $\frac{u^{24}}{u^3}$ ⓑ $\frac{9^{15}}{9^5}$

Solution:

ⓐ u^{21} ⓑ 9^{10}

Exercise:

Problem: ⓐ $\frac{q^{18}}{q^{36}}$ ⓑ $\frac{10^2}{10^3}$

Exercise:

Problem: ⓐ $\frac{t^{10}}{t^{40}}$ ⓑ $\frac{8^3}{8^5}$

Solution:

ⓐ $\frac{1}{t^{30}}$ ⓑ $\frac{1}{64}$

Exercise:

Problem: ⓐ $\frac{b}{b^9}$ ⓑ $\frac{4}{4^6}$

Exercise:

Problem: ⓐ $\frac{x}{x^7}$ ⓑ $\frac{10}{10^3}$

Solution:

ⓐ $\frac{1}{x^6}$ ⓑ $\frac{1}{100}$

Simplify Expressions with Zero Exponents

In the following exercises, simplify.

Exercise:

Problem: ⓐ 20^0 ⓑ b^0

Exercise:

Problem: ⓐ 13^0 ⓑ k^0

Solution:

ⓐ 1 ⓑ 1

Exercise:

Problem: ⓐ -27^0 ⓑ $-\left(27^0\right)$

Exercise:

Problem: ⓐ -15^0 ⓑ $-\left(15^0\right)$

Solution:

ⓐ -1 ⓑ -1

Exercise:

Problem: ⓐ $(25x)^0$
ⓑ $25x^0$

Exercise:

Problem: ⓐ $(6y)^0$
ⓑ $6y^0$

Solution:

ⓐ 1 ⓑ 6

Exercise:

Problem: ⓐ $(12x)^0$
ⓑ $\left(-56p^4q^3\right)^0$

Exercise:

Problem: ⓐ $7y^0(17y)^0$
ⓑ $\left(-93c^7d^{15}\right)^0$

Solution:

ⓐ 7 ⓑ 1

Exercise:

Problem: ⓐ $12n^0 - 18m^0$
ⓑ $(12n)^0 - (18m)^0$

Exercise:

Problem: ⓐ $15r^0 - 22s^0$
ⓑ $(15r)^0 - (22s)^0$

Solution:

ⓐ -7 ⓑ 0

Simplify Expressions Using the Quotient to a Power Property

In the following exercises, simplify.

Exercise:

Problem: ⓐ $\left(\frac{3}{4}\right)^3$ ⓑ $\left(\frac{p}{2}\right)^5$ ⓒ $\left(\frac{x}{y}\right)^6$

Exercise:

Problem: ⓐ $\left(\frac{2}{5}\right)^2$ ⓑ $\left(\frac{x}{3}\right)^4$ ⓒ $\left(\frac{a}{b}\right)^5$

Solution:

ⓐ $\frac{4}{25}$ ⓑ $\frac{x^4}{81}$ ⓒ $\frac{a^5}{b^5}$

Exercise:

Problem: ⓐ $\left(\frac{a}{3b}\right)^4$ ⓑ $\left(\frac{5}{4m}\right)^2$

Exercise:

Problem: ⓐ $\left(\frac{x}{2y}\right)^3$ ⓑ $\left(\frac{10}{3q}\right)^4$

Solution:

ⓐ $\frac{x^3}{8y^3}$ ⓑ $\frac{10{,}000}{81q^4}$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify.

Exercise:

Problem: $\frac{\left(a^2\right)^3}{a^4}$

Exercise:

Problem: $\frac{\left(p^3\right)^4}{p^5}$

Solution:

p^7

Exercise:

Problem: $\frac{\left(y^3\right)^4}{y^{10}}$

Exercise:

Problem: $\frac{\left(x^4\right)^5}{x^{15}}$

Solution:

x^5

Exercise:

Problem: $\frac{u^6}{\left(u^3\right)^2}$

Exercise:

Problem: $\frac{v^{20}}{\left(v^4\right)^5}$

Solution:

1

Exercise:

Problem: $\frac{m^{12}}{\left(m^8\right)^3}$

Exercise:

Problem: $\frac{n^8}{\left(n^6\right)^4}$

Solution:

$\frac{1}{n^{16}}$

Exercise:

Problem: $\left(\frac{p^9}{p^3}\right)^5$

Exercise:

Problem: $\left(\frac{q^8}{q^2}\right)^3$

Solution:

q^{18}

Exercise:

Problem: $\left(\frac{r^2}{r^6}\right)^3$

Exercise:

Problem: $\left(\frac{m^4}{m^7}\right)^4$

Solution:

$\frac{1}{m^{12}}$

Exercise:

Problem: $\left(\frac{p}{r^{11}}\right)^2$

Exercise:

Problem: $\left(\frac{a}{b^6}\right)^3$

Solution:

$\frac{a^3}{b^{18}}$

Exercise:

Problem: $\left(\frac{w^5}{x^3}\right)^8$

Exercise:

Problem: $\left(\frac{y^4}{z^{10}}\right)^5$

Solution:

$\frac{y^{20}}{z^{50}}$

Exercise:

Problem: $\left(\frac{2j^3}{3k}\right)^4$

Exercise:

Problem: $\left(\frac{3m^5}{5n}\right)^3$

Solution:

$\frac{27m^{15}}{125n^3}$

Exercise:

Problem: $\left(\frac{3c^2}{4d^6}\right)^3$

Exercise:

Problem: $\left(\frac{5u^7}{2v^3}\right)^4$

Solution:

$\frac{625u^{28}}{16v^{12}}$

Exercise:

Problem: $\left(\frac{k^2k^8}{k^3}\right)^2$

Exercise:

Problem: $\left(\frac{j^2j^5}{j^4}\right)^3$

Solution:

j^9

Exercise:

Problem: $\frac{\left(t^2\right)^5\left(t^4\right)^2}{\left(t^3\right)^7}$

Exercise:

Problem: $\frac{\left(q^3\right)^6\left(q^2\right)^3}{\left(q^4\right)^8}$

Solution:

$\frac{1}{q^8}$

Exercise:

Problem: $\frac{\left(-2p^2\right)^4\left(3p^4\right)^2}{\left(-6p^3\right)^2}$

Exercise:

Problem: $\frac{\left(-2k^3\right)^2\left(6k^2\right)^4}{\left(9k^4\right)^2}$

Solution:

$64k^6$

Exercise:

Problem: $\frac{\left(-4m^3\right)^2\left(5m^4\right)^3}{\left(-10m^6\right)^3}$

Exercise:

Problem: $\frac{\left(-10n^2\right)^3\left(4n^5\right)^2}{\left(2n^8\right)^2}$

Solution:

$-4{,}000$

Divide Monomials

In the following exercises, divide the monomials.

Exercise:

Problem: $56b^8 \div 7b^2$

Exercise:

Problem: $63v^{10} \div 9v^2$

Solution:

$7v^8$

Exercise:

Problem: $-88y^{15} \div 8y^3$

Exercise:

Problem: $-72u^{12} \div 12u^4$

Solution:

$-6u^8$

Exercise:

Problem: $\frac{45a^6b^8}{-15a^{10}b^2}$

Exercise:

Problem: $\frac{54x^9y^3}{-18x^6y^{15}}$

Solution:

$-\frac{3x^3}{y^{12}}$

Exercise:

Problem: $\frac{15r^4s^9}{18r^9s^2}$

Exercise:

Problem: $\frac{20m^8n^4}{30m^5n^9}$

Solution:

$\frac{-2m^3}{3n^5}$

Exercise:

Problem: $\frac{18a^4b^8}{-27a^9b^5}$

Exercise:

Problem: $\frac{45x^5y^9}{-60x^8y^6}$

Solution:

$\frac{-3y^3}{4x^3}$

Exercise:

Problem: $\frac{64q^{11}r^9s^3}{48q^6r^8s^5}$

Exercise:

Problem: $\frac{65a^{10}b^8c^5}{42a^7b^6c^8}$

Solution:

$\frac{65a^3b^2}{42c^3}$

Exercise:

Problem: $\frac{\left(10m^5n^4\right)\left(5m^3n^6\right)}{25m^7n^5}$

Exercise:

Problem: $\frac{\left(-18p^4q^7\right)\left(-6p^3q^8\right)}{-36p^{12}q^{10}}$

Solution:

$\frac{-3q^5}{p^5}$

Exercise:

Problem: $\frac{\left(6a^4b^3\right)\left(4ab^5\right)}{(12a^2b)(a^3b)}$

Exercise:

Problem: $\frac{\left(4u^2v^5\right)\left(15u^3v\right)}{(12u^3v)(u^4v)}$

Solution:

$\frac{5v^4}{u^2}$

Mixed Practice

Exercise:

Problem:
ⓐ $24a^5 + 2a^5$
ⓑ $24a^5 - 2a^5$
ⓒ $24a^5 \cdot 2a^5$
ⓓ $24a^5 \div 2a^5$

Exercise:

Problem: ⓐ $15n^{10} + 3n^{10}$
ⓑ $15n^{10} - 3n^{10}$
ⓒ $15n^{10} \cdot 3n^{10}$
ⓓ $15n^{10} \div 3n^{10}$

Solution:

ⓐ $18n^{10}$
ⓑ $12n^{10}$
ⓒ $45n^{20}$
ⓓ 5

Exercise:

Problem: ⓐ $p^4 \cdot p^6$
ⓑ $\left(p^4\right)^6$

Exercise:

Problem: ⓐ $q^5 \cdot q^3$
ⓑ $\left(q^5\right)^3$

Solution:

ⓐ q^8
ⓑ q^{15}

Exercise:

Problem: ⓐ $\frac{y^3}{y}$
ⓑ $\frac{y}{y^3}$

Exercise:

ⓐ $\frac{z^6}{z^5}$

Problem: ⓑ $\frac{z^5}{z^6}$

Solution:

ⓐ z ⓑ $\frac{1}{z}$

Exercise:

Problem: $\left(8x^5\right)(9x) \div 6x^3$

Exercise:

Problem: $(4y)\left(12y^7\right) \div 8y^2$

Solution:

$6y^6$

Exercise:

Problem: $\frac{27a^7}{3a^3} + \frac{54a^9}{9a^5}$

Exercise:

Problem: $\frac{32c^{11}}{4c^5} + \frac{42c^9}{6c^3}$

Solution:

$15c^6$

Exercise:

Problem: $\frac{32y^5}{8y^2} - \frac{60y^{10}}{5y^7}$

Exercise:

Problem: $\frac{48x^6}{6x^4} - \frac{35x^9}{7x^7}$

Solution:

$3x^2$

Exercise:

Problem: $\frac{63r^6s^3}{9r^4s^2} - \frac{72r^2s^2}{6s}$

Exercise:

Problem: $\frac{56y^4z^5}{7y^3z^3} - \frac{45y^2z^2}{5y}$

Solution:

$-yz^2$

Everyday Math

Exercise:

Problem:

Memory One megabyte is approximately 10^6 bytes. One gigabyte is approximately 10^9 bytes. How many megabytes are in one gigabyte?

Exercise:

Problem:

Memory One gigabyte is approximately 10^9 bytes. One terabyte is approximately 10^{12} bytes. How many gigabytes are in one terabyte?

Solution:

10^3

Writing Exercises

Exercise:

Problem:

Jennifer thinks the quotient $\frac{a^{24}}{a^6}$ simplifies to a^4. What is wrong with her reasoning?

Exercise:

Problem:

Maurice simplifies the quotient $\frac{d^7}{d}$ by writing $\frac{\cancel{d}^7}{\cancel{d}} = 7$. What is wrong with his reasoning?

Solution:

Answers will vary.

Exercise:

Problem:

When Drake simplified -3^0 and $(-3)^0$ he got the same answer. Explain how using the Order of Operations correctly gives different answers.

Exercise:

Problem:

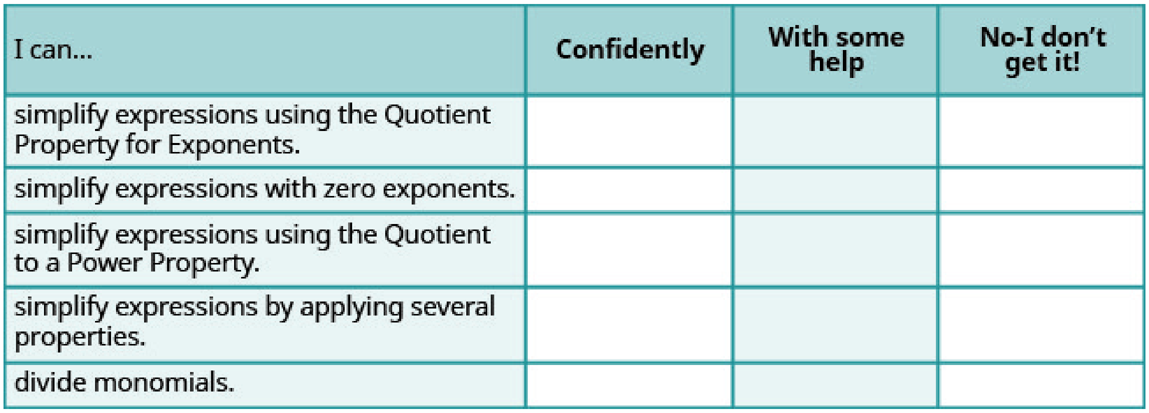
Robert thinks x^0 simplifies to 0. What would you say to convince Robert he is wrong?

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
simplify expressions using the Quotient Property for Exponents.			
simplify expressions with zero exponents.			
simplify expressions using the Quotient to a Power Property.			
simplify expressions by applying several properties.			
divide monomials.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Divide Polynomials
By the end of this section, you will be able to:

- Divide a polynomial by a monomial
- Divide a polynomial by a binomial

Note:
Before you get started, take this readiness quiz.

1. Add: $\frac{3}{d}+\frac{x}{d}$.
   If you missed this problem, review [link].
2. Simplify: $\frac{30xy^3}{5xy}$.
   If you missed this problem, review [link].
3. Combine like terms: $8a^2+12a+1+3a^2-5a+4$.
   If you missed this problem, review [link].

Divide a Polynomial by a Monomial

In the last section, you learned how to divide a monomial by a monomial. As you continue to build up your knowledge of polynomials the next procedure is to divide a polynomial of two or more terms by a monomial.

The method we'll use to divide a polynomial by a monomial is based on the properties of fraction addition. So we'll start with an example to review fraction addition.

The sum,	$\frac{y}{5}+\frac{2}{5}$,

simplifies to	$\frac{y+2}{5}$.

Now we will do this in reverse to split a single fraction into separate fractions.

We'll state the fraction addition property here just as you learned it and in reverse.

Note:
Fraction Addition
If $a, b,$ and c are numbers where $c \neq 0$, then
Equation:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \text{and} \quad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

We use the form on the left to add fractions and we use the form on the right to divide a polynomial by a monomial.

For example,	$\frac{y+2}{5}$
can be written	$\frac{y}{5} + \frac{2}{5}$.

We use this form of fraction addition to divide polynomials by monomials.

Note:
Division of a Polynomial by a Monomial
To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example:
Exercise:

Problem: **Find the quotient: $\frac{7y^2+21}{7}$.

Solution:
Solution

	$\frac{7y^2+21}{7}$
Divide each term of the numerator by the denominator.	$\frac{7y^2}{7}+\frac{21}{7}$
Simplify each fraction.	y^2+3

Note:
Exercise:

TRY IT :: 5.153
Problem: Find the quotient: $\frac{8z^2+24}{4}$.

Solution:

$2z^2 + 6$

Note:
Exercise:

TRY IT :: 5.154

Problem: Find the quotient: $\frac{18z^2 - 27}{9}$.

Solution:

$2z^2 - 3$

Remember that division can be represented as a fraction. When you are asked to divide a polynomial by a monomial and it is not already in fraction form, write a fraction with the polynomial in the numerator and the monomial in the denominator.

Example:
Exercise:

Problem: **Find the quotient: $\left(18x^3 - 36x^2\right) \div 6x$.

Solution:
Solution

	$\left(18x^3 - 36x^2\right) \div 6x$
Rewrite as a fraction.	$\frac{18x^3-36x^2}{6x}$
Divide each term of the numerator by the denominator.	$\frac{18x^3}{6x} - \frac{36x^2}{6x}$
Simplify.	$3x^2 - 6x$

Note:
Exercise:

TRY IT :: 5.155

Problem: Find the quotient: $\left(27b^3 - 33b^2\right) \div 3b$.

Solution:

$9b^2 - 11b$

Note:
Exercise:

TRY IT :: 5.156

Problem: Find the quotient: $\left(25y^3 - 55y^2\right) \div 5y$.

Solution:

$5y^2 - 11y$

When we divide by a negative, we must be extra careful with the signs.

Example:
Exercise:

Problem: **Find the quotient: $\frac{12d^2-16d}{-4}$.

Solution:
Solution

	$\frac{12d^2-16d}{-4}$
Divide each term of the numerator by the denominator.	$\frac{12d^2}{-4}-\frac{16d}{-4}$
Simplify. Remember, subtracting a negative is like adding a positive!	$-3d^2+4d$

Note:
Exercise:

TRY IT :: 5.157

Problem: Find the quotient: $\frac{25y^2-15y}{-5}$.

Solution:

$-5y^2 + 3y$

Note:
Exercise:

TRY IT :: 5.158

Problem: Find the quotient: $\frac{42b^2-18b}{-6}$.

Solution:

$-7b^2 + 3b$

Example:
Exercise:

Problem: **Find the quotient: $\frac{105y^5+75y^3}{5y^2}$.

Solution:
Solution

	$\frac{105y^5+75y^3}{5y^2}$
Separate the terms.	$\frac{105y^5}{5y^2} + \frac{75y^3}{5y^2}$
Simplify.	$21y^3 + 15y$

Note:
Exercise:

TRY IT :: 5.159

Problem: Find the quotient: $\frac{60d^7+24d^5}{4d^3}$.

Solution:

$15d^4 + 6d^2$

Note:
Exercise:

TRY IT :: 5.160

Problem: Find the quotient: $\frac{216p^7-48p^5}{6p^3}$.

Solution:

$36p^4 - 8p^2$

Example:
Exercise:

Problem: **Find the quotient: $\left(15x^3y - 35xy^2\right) \div (-5xy)$.

Solution:
Solution

	$\left(15x^3y - 35xy^2\right) \div (-5xy)$
Rewrite as a fraction.	$\frac{15x^3y-35xy^2}{-5xy}$
Separate the terms.	$\frac{15x^3y}{-5xy} - \frac{35xy^2}{-5xy}$
Simplify.	$-3x^2 + 7y$

Note:
Exercise:

TRY IT :: 5.161

Problem: Find the quotient: $\left(32a^2b - 16ab^2\right) \div (-8ab)$.

Solution:

$-4a + 2b$

Note:
Exercise:

TRY IT :: 5.162

Problem: Find the quotient: $\left(-48a^8b^4 - 36a^6b^5\right) \div \left(-6a^3b^3\right)$.

Solution:

$8a^5b + 6a^3b^2$

Example:
Exercise:

Problem: **Find the quotient: $\frac{36x^3y^2+27x^2y^2-9x^2y^3}{9x^2y}$.

Solution:
Solution

	$\frac{36x^3y^2+27x^2y^2-9x^2y^3}{9x^2y}$
Separate the terms.	$\frac{36x^3y^2}{9x^2y}+\frac{27x^2y^2}{9x^2y}-\frac{9x^2y^3}{9x^2y}$
Simplify.	$4xy+3y-y^2$

Note:
Exercise:

TRY IT :: 5.163

Problem: Find the quotient: $\frac{40x^3y^2+24x^2y^2-16x^2y^3}{8x^2y}$.

Solution:

$5xy+3y-2y^2$

Note:
Exercise:

TRY IT :: 5.164

Problem: Find the quotient: $\frac{35a^4b^2+14a^4b^3-42a^2b^4}{7a^2b^2}$.

Solution:

$5a^2 + 2a^2b - 6b^2$

Example:
Exercise:

Problem: **Find the quotient: $\frac{10x^2+5x-20}{5x}$.

Solution:
Solution

	$\frac{10x^2+5x-20}{5x}$
Separate the terms.	$\frac{10x^2}{5x} + \frac{5x}{5x} - \frac{20}{5x}$
Simplify.	$2x + 1 - \frac{4}{x}$

Note:
Exercise:

TRY IT :: 5.165

Problem: Find the quotient: $\frac{18c^2+6c-9}{6c}$.

Solution:

$3c+1-\frac{3}{2c}$

Note:
Exercise:

TRY IT :: 5.166

Problem: Find the quotient: $\frac{10d^2-5d-2}{5d}$.

Solution:

$2d-1-\frac{2}{5d}$

Divide a Polynomial by a Binomial

To divide a polynomial by a binomial, we follow a procedure very similar to long division of numbers. So let’s look carefully the steps we take when we divide a 3-digit number, 875, by a 2-digit number, 25.

We write the long division	$25\overline{)875}$
We divide the first two digits, 87, by 25.	$\begin{array}{r}3\\25\overline{)875}\end{array}$
We multiply 3 times 25 and write the product under the 87.	$\begin{array}{r}3\\25\overline{)875}\\\underline{75}\end{array}$
Now we subtract 75 from 87.	$\begin{array}{r}3\\25\overline{)875}\\\underline{-75}\\12\end{array}$
Then we bring down the third digit of the dividend, 5.	$\begin{array}{r}3\\25\overline{)875}\\\underline{-75}\\125\end{array}$
Repeat the process, dividing 25 into 125.	$\begin{array}{r}35\\25\overline{)875}\\\underline{-75}\\125\\\underline{-125}\end{array}$

We check division by multiplying the quotient by the divisor.

If we did the division correctly, the product should equal the dividend.
Equation:

$$35 \cdot 25$$
$$875 \checkmark$$

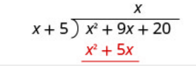
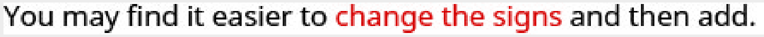
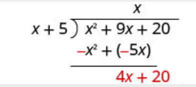
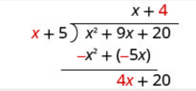
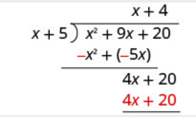
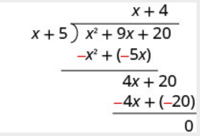
Now we will divide a trinomial by a binomial. As you read through the example, notice how similar the steps are to the numerical example above.

Example:
Exercise:

Problem: **Find the quotient: $\left(x^2+9x+20\right) \div (x+5)$.

Solution:
Solution

	$(x^2+9x+20) \div (x+5)$
Write it as a long division problem.	
Be sure the dividend is in standard form.	$x+5\overline{)\,x^2+9x+20}$
Divide x^2 by x. It may help to ask yourself, "What do I need to multiply x by to get x^2?"	
Put the answer, x, in the quotient over the x term.	$\begin{array}{r} x \qquad\quad \\ x+5\overline{)\,x^2+9x+20} \end{array}$

Multiply x times $x + 5$. Line up the like terms under the dividend.	
Subtract $x^2 + 5x$ from $x^2 + 9x$.	
 You may find it easier to change the signs and then add. Then bring down the last term, 20.	
Divide $4x$ by x. It may help to ask yourself, "What do I need to multiply x by to get $4x$?"	
Put the answer, 4, in the quotient over the constant term.	
Multiply 4 times $x + 5$.	 $x+4$ $x+5\,)\,x^2+9x+20$ $-x^2+(-5x)$ $4x+20$ $4x+20$
Subtract $4x + 20$ from $4x + 20$.	 $x+4$ $x+5\,)\,x^2+9x+20$ $-x^2+(-5x)$ $4x+20$ $-4x+(-20)$ 0
Check:	
Multiply the quotient by the divisor.	
$(x + 4)(x + 5)$	
You should get the dividend.	

$x^2 + 9x + 20$✓	

Note:
Exercise:

TRY IT :: 5.167

Problem: Find the quotient: $\left(y^2 + 10y + 21\right) \div (y + 3)$.

Solution:

$y + 7$

Note:
Exercise:

TRY IT :: 5.168

Problem: Find the quotient: $\left(m^2 + 9m + 20\right) \div (m + 4)$.

Solution:

$m + 5$

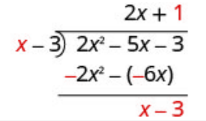
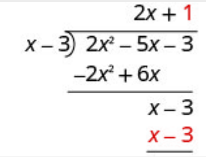
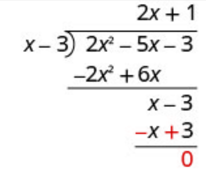
When the divisor has subtraction sign, we must be extra careful when we multiply the partial quotient and then subtract. It may be safer to show that we change the signs and then add.

Example:
Exercise:

Problem: **Find the quotient: $(2x^2-5x-3)\div(x-3)$.

Solution:
Solution

	$(2x^2-5x-3)\div(x-3)$
Write it as a long division problem.	
Be sure the dividend is in standard form.	$x-3\overline{)2x^2-5x-3}$
Divide $2x^2$ by x. Put the answer, $2x$, in the quotient over the x term.	$\begin{array}{r} 2x \\ x-3\overline{)2x^2-5x-3} \end{array}$
Multiply $2x$ times $x-3$. Line up the like terms under the dividend.	$\begin{array}{r} 2x \\ x-3\overline{)2x^2-5x-3} \\ \underline{2x^2-6x} \end{array}$
Subtract $2x^2-6x$ from $2x^2-5x$. Change the signs and then add. Then bring down the last term.	$\begin{array}{r} 2x \\ x-3\overline{)\;2x^2-5x-3} \\ \underline{-2x^2+6x} \\ x-3 \end{array}$
Divide x by x.	

Put the answer, 1, in the quotient over the constant term.	
Multiply 1 times $x - 3$.	
Subtract $x - 3$ from $x - 3$ by changing the signs and adding.	
To check, multiply $(x - 3)(2x + 1)$.	
The result should be $2x^2 - 5x - 3$.	

Note:
Exercise:

TRY IT :: 5.169

Problem: Find the quotient: $\left(2x^2 - 3x - 20\right) \div (x - 4)$.

Solution:

$2x + 5$

Note:
Exercise:

TRY IT :: 5.170

Problem: Find the quotient: $\left(3x^2 - 16x - 12\right) \div (x - 6)$.

Solution:

$3x + 2$

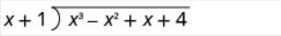
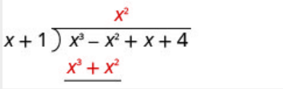
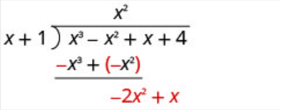
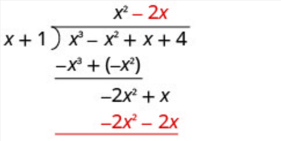
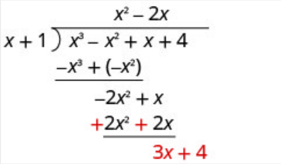
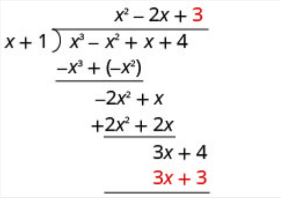
When we divided 875 by 25, we had no remainder. But sometimes division of numbers does leave a remainder. The same is true when we divide polynomials. In [link], we'll have a division that leaves a remainder. We write the remainder as a fraction with the divisor as the denominator.

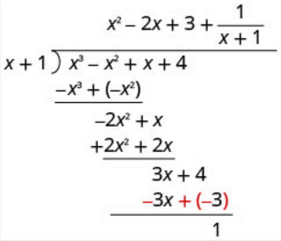
Example:
Exercise:

Problem: **Find the quotient: $\left(x^3 - x^2 + x + 4\right) \div (x + 1)$.

Solution:
Solution

	$(x^3 - x^2 + x + 4) \div (x + 1)$
Write it as a long division problem.	

Be sure the dividend is in standard form.	
Divide x^3 by x. Put the answer, x^2, in the quotient over the x^2 term. Multiply x^2 times $x + 1$. Line up the like terms under the dividend.	
Subtract $x^3 + x^2$ from $x^3 − x^2$ by changing the signs and adding. Then bring down the next term.	
Divide $−2x^2$ by x. Put the answer, $−2x$, in the quotient over the x term. Multiply $−2x$ times $x + 1$. Line up the like terms under the dividend.	
Subtract $−2x^2 − 2x$ from $−2x^2 + x$ by changing the signs and adding. Then bring down the last term.	
Divide $3x$ by x. Put the answer, 3, in the quotient over the constant term. Multiply 3 times $x + 1$. Line up the like terms under the dividend.	
Subtract $3x + 3$ from $3x + 4$ by changing the signs and adding.	

Write the remainder as a fraction with the divisor as the denominator.	 $x^2-2x+3+\frac{1}{x+1}$ $x+1\overline{)\,x^3-x^2+x+4}$ $-x^3+(-x^2)$ $-2x^2+x$ $+2x^2+2x$ $3x+4$ $-3x+(-3)$ 1
To check, multiply $(x+1)\left(x^2-2x+3+\frac{1}{x+1}\right)$. The result should be x^3-x^2+x+4.	

Note:
Exercise:

TRY IT :: 5.171

Problem: Find the quotient: $\left(x^3+5x^2+8x+6\right)\div(x+2)$.

Solution:

$x^2+3x+2+\frac{2}{x+2}$

Note:
Exercise:

TRY IT :: 5.172

Problem: Find the quotient: $\left(2x^3+8x^2+x-8\right)\div(x+1)$.

Solution:

$2x^2+6x-5-\frac{3}{x+1}$

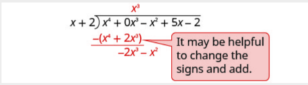
Look back at the dividends in [link], [link], and [link]. The terms were written in descending order of degrees, and there were no missing degrees. The dividend in [link] will be $x^4 - x^2 + 5x - 2$. It is missing an x^3 term. We will add in $0x^3$ as a placeholder.

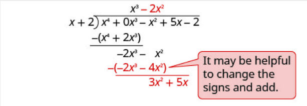
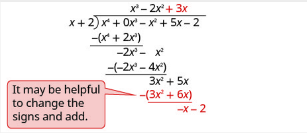
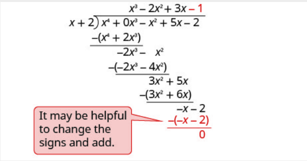
Example:
Exercise:

Problem: **Find the quotient: $\left(x^4 - x^2 + 5x - 2\right) \div (x + 2)$.

Solution:
Solution

Notice that there is no x^3 term in the dividend. We will add $0x^3$ as a placeholder.

	$(x^4 - x^2 + 5x - 2) \div (x + 2)$
Write it as a long division problem. Be sure the dividend is in standard form with placeholders for missing terms.	$x + 2 \overline{)\, x^4 - 0x^3 - x^2 + 5x - 2}$
Divide x^4 by x. Put the answer, x^3, in the quotient over the x^3 term. Multiply x^3 times $x + 2$. Line up the like	 x^3 $x + 2 \overline{)\, x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ It may be helpful to change the signs and add.

terms. Subtract and then bring down the next term.	
Divide $-2x^3$ by x. Put the answer, $-2x^2$, in the quotient over the x^2 term. Multiply $-2x^2$ times $x + 1$. Line up the like terms. Subtract and bring down the next term.	 $x^3 - 2x^2$ $x+2\overline{)x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ $-(-2x^3 - 4x^2)$ $3x^2 + 5x$It may be helpful to change the signs and add.
Divide $3x^2$ by x. Put the answer, $3x$, in the quotient over the x term. Multiply $3x$ times $x + 1$. Line up the like terms. Subtract and bring down the next term.	 $x^3 - 2x^2 + 3x$ $x+2\overline{)x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ $-(-2x^3 - 4x^2)$ $3x^2 + 5x$ $-(3x^2 + 6x)$ $-x - 2$It may be helpful to change the signs and add.
Divide $-x$ by x. Put the answer, -1, in the quotient over the constant term. Multiply -1 times $x + 1$. Line up the like terms. Change the signs, add.	 $x^3 - 2x^2 + 3x - 1$ $x+2\overline{)x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ $-(-2x^3 - 4x^2)$ $3x^2 + 5x$ $-(3x^2 + 6x)$ $-x - 2$ $-(-x - 2)$ 0It may be helpful to change the signs and add.
To check, multiply $(x+2)\left(x^3 - 2x^2 + 3x - 1\right)$.	
The result should be $x^4 - x^2 + 5x - 2$.	

Note:
Exercise:

TRY IT :: 5.173

Problem: Find the quotient: $\left(x^3+3x+14\right)\div(x+2)$.

Solution:

x^2-2x+7

Note:
Exercise:

TRY IT :: 5.174

Problem: Find the quotient: $\left(x^4-3x^3-1000\right)\div(x+5)$.

Solution:

$x^3-8x^2+40x-200$

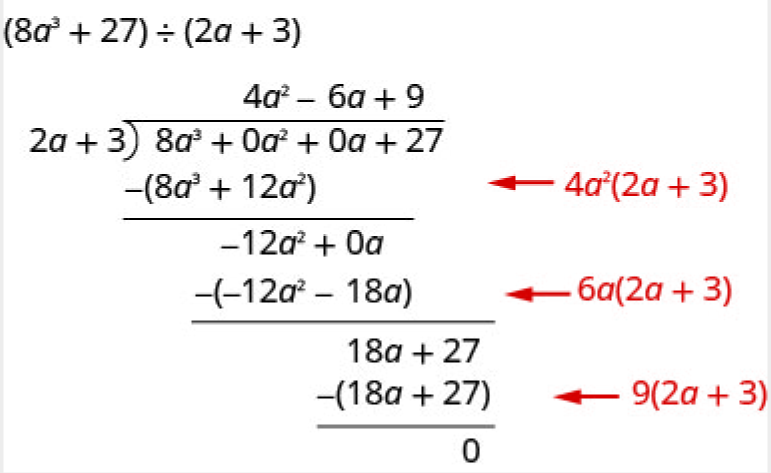
In [link], we will divide by $2a-3$. As we divide we will have to consider the constants as well as the variables.

Example:
Exercise:

Problem: **Find the quotient: $\left(8a^3+27\right)\div(2a+3)$.

Solution:
Solution

This time we will show the division all in one step. We need to add two placeholders in order to divide.



To check, multiply $(2a+3)\left(4a^2-6a+9\right)$.

The result should be $8a^3+27$.

Note:
Exercise:

TRY IT :: 5.175

Problem: Find the quotient: $\left(x^3-64\right)\div(x-4)$.

Solution:

$x^2+4x+16$

Note:
Exercise:

TRY IT :: 5.176

Problem: Find the quotient: $\left(125x^3 - 8\right) \div (5x - 2)$.

Solution:

$25x^2 + 10x + 4$

Note:

Access these online resources for additional instruction and practice with dividing polynomials:

- Divide a Polynomial by a Monomial
- Divide a Polynomial by a Monomial 2
- Divide Polynomial by Binomial

Key Concepts

- **Fraction Addition**
    - If $a, b,$ and c are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$
- **Division of a Polynomial by a Monomial**
    - To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Practice Makes Perfect

In the following exercises, divide each polynomial by the monomial.

Exercise:

Problem: $\frac{45y+36}{9}$

Exercise:

Problem: $\frac{30b+75}{5}$

Solution:

$6b+15$

Exercise:

Problem: $\frac{8d^2-4d}{2}$

Exercise:

Problem: $\frac{42x^2-14x}{7}$

Solution:

$6x^2-2x$

Exercise:

Problem: $\left(16y^2-20y\right)\div 4y$

Exercise:

Problem: $\left(55w^2-10w\right)\div 5w$

Solution:

$11w-2$

Exercise:

Problem: $\left(9n^4+6n^3\right)\div 3n$

Exercise:

Problem: $\left(8x^3+6x^2\right)\div 2x$

Solution:

$4x^2+3x$

Exercise:

Problem: $\frac{18y^2-12y}{-6}$

Exercise:

Problem: $\frac{20b^2-12b}{-4}$

Solution:

$-5b^2+3b$

Exercise:

Problem: $\frac{35a^4+65a^2}{-5}$

Exercise:

Problem: $\frac{51m^4+72m^3}{-3}$

Solution:

$-17m^4-24m^3$

Exercise:

Problem: $\frac{310y^4-200y^3}{5y^2}$

Exercise:

Problem: $\frac{412z^8-48z^5}{4z^3}$

Solution:

$103z^5 - 12z^2$

Exercise:

Problem: $\frac{46x^3+38x^2}{2x^2}$

Exercise:

Problem: $\frac{51y^4+42y^2}{3y^2}$

Solution:

$17y^2 + 14$

Exercise:

Problem: $\left(24p^2 - 33p\right) \div (-3p)$

Exercise:

Problem: $\left(35x^4 - 21x\right) \div (-7x)$

Solution:

$-5x^3 + 3$

Exercise:

Problem: $\left(63m^4 - 42m^3\right) \div \left(-7m^2\right)$

Exercise:

Problem: $\left(48y^4 - 24y^3\right) \div \left(-8y^2\right)$

Solution:

$-6y^2 + 3y$

Exercise:

Problem: $\left(63a^2b^3 + 72ab^4\right) \div (9ab)$

Exercise:

Problem: $\left(45x^3y^4 + 60xy^2\right) \div (5xy)$

Solution:

$9x^2y^3 + 12y$

Exercise:

Problem: $\frac{52p^5q^4+36p^4q^3-64p^3q^2}{4p^2q}$

Exercise:

Problem: $\frac{49c^2d^2-70c^3d^3-35c^2d^4}{7cd^2}$

Solution:

$7c - 10c^2d - 5cd^2$

Exercise:

Problem: $\frac{66x^3y^2-110x^2y^3-44x^4y^3}{11x^2y^2}$

Exercise:

Problem: $\frac{72r^5s^2+132r^4s^3-96r^3s^5}{12r^2s^2}$

Solution:

$6r^3+11r^2s-8rs^3$

Exercise:

Problem: $\frac{4w^2+2w-5}{2w}$

Exercise:

Problem: $\frac{12q^2+3q-1}{3q}$

Solution:

$4q+1-\frac{1}{3q}$

Exercise:

Problem: $\frac{10x^2+5x-4}{-5x}$

Exercise:

Problem: $\frac{20y^2+12y-1}{-4y}$

Solution:

$-5y-3+\frac{1}{4y}$

Exercise:

Problem: $\frac{36p^3+18p^2-12p}{6p^2}$

Exercise:

Problem: $\frac{63a^3-108a^2+99a}{9a^2}$

Solution:

$7a-12+\frac{11}{a}$

Divide a Polynomial by a Binomial

In the following exercises, divide each polynomial by the binomial.

Exercise:

Problem: $\left(y^2+7y+12\right)\div(y+3)$

Exercise:

Problem: $\left(d^2+8d+12\right)\div(d+2)$

Solution:

$d+6$

Exercise:

Problem: $\left(x^2-3x-10\right)\div(x+2)$

Exercise:

Problem: $\left(a^2-2a-35\right)\div(a+5)$

Solution:

$a-7$

Exercise:

Problem: $\left(t^2-12t+36\right)\div(t-6)$

Exercise:

Problem: $\left(x^2-14x+49\right)\div(x-7)$

Solution:

$x-7$

Exercise:

Problem: $\left(6m^2-19m-20\right)\div(m-4)$

Exercise:

Problem: $\left(4x^2-17x-15\right)\div(x-5)$

Solution:

$4x+3$

Exercise:

Problem: $\left(q^2+2q+20\right)\div(q+6)$

Exercise:

Problem: $\left(p^2+11p+16\right)\div(p+8)$

Solution:

$p+3-\frac{8}{p+8}$

Exercise:

Problem: $\left(y^2 - 3y - 15\right) \div (y - 8)$

Exercise:

Problem: $\left(x^2 + 2x - 30\right) \div (x - 5)$

Solution:

$x + 7 + \frac{5}{x-5}$

Exercise:

Problem: $\left(3b^3 + b^2 + 2\right) \div (b + 1)$

Exercise:

Problem: $\left(2n^3 - 10n + 24\right) \div (n + 3)$

Solution:

$2n^2 - 6n + 8$

Exercise:

Problem: $\left(2y^3 - 6y - 36\right) \div (y - 3)$

Exercise:

Problem: $\left(7q^3 - 5q - 2\right) \div (q - 1)$

Solution:

$7q^2 + 7q + 2$

Exercise:

Problem: $\left(z^3+1\right)\div(z+1)$

Exercise:

Problem: $\left(m^3+1000\right)\div(m+10)$

Solution:

$m^2-10m+100$

Exercise:

Problem: $\left(a^3-125\right)\div(a-5)$

Exercise:

Problem: $\left(x^3-216\right)\div(x-6)$

Solution:

$x^2+6x+36$

Exercise:

Problem: $\left(64x^3-27\right)\div(4x-3)$

Exercise:

Problem: $\left(125y^3-64\right)\div(5y-4)$

Solution:

$25y^2+20x+16$

Everyday Math

Exercise:

Problem:

Average cost Pictures Plus produces digital albums. The company’s average cost (in dollars) to make x albums is given by the expression $\frac{7x+500}{x}$.

ⓐ Find the quotient by dividing the numerator by the denominator.
ⓑ What will the average cost (in dollars) be to produce 20 albums?

Exercise:

Problem:

Handshakes At a company meeting, every employee shakes hands with every other employee. The number of handshakes is given by the expression $\frac{n^2-n}{2}$, where n represents the number of employees. How many handshakes will there be if there are 10 employees at the meeting?

Solution:

45

Writing Exercises

Exercise:

Problem:

James divides $48y+6$ by 6 this way: $\frac{48y+\cancel{6}}{\cancel{6}}=48y$. What is wrong with his reasoning?

Exercise:

Problem:

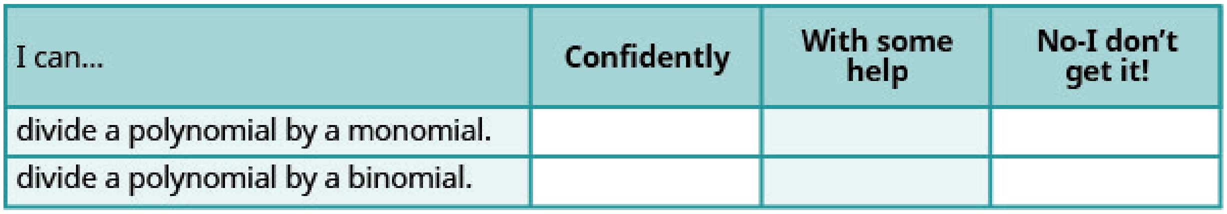
Divide $\frac{10x^2+x-12}{2x}$ and explain with words how you get each term of the quotient.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
divide a polynomial by a monomial.			
divide a polynomial by a binomial.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

Integer Exponents and Scientific Notation
By the end of this section, you will be able to:

- Use the definition of a negative exponent
- Simplify expressions with integer exponents
- Convert from decimal notation to scientific notation
- Convert scientific notation to decimal form
- Multiply and divide using scientific notation

Note:
Before you get started, take this readiness quiz.

1. What is the place value of the 6 in the number $64,891$?
   If you missed this problem, review [link].
2. Name the decimal: 0.0012.
   If you missed this problem, review [link].
3. Subtract: $5-(-3)$.
   If you missed this problem, review [link].

Use the Definition of a Negative Exponent

We saw that the Quotient Property for Exponents introduced earlier in this chapter, has two forms depending on whether the exponent is larger in the numerator or the denominator.

Note:
Quotient Property for Exponents
If a is a real number, $a \neq 0$, and m and n are whole numbers, then
Equation:

$$\frac{a^m}{a^n}=a^{m-n}, m>n \quad \text{and} \quad \frac{a^m}{a^n}=\frac{1}{a^{n-m}}, n>m$$

What if we just subtract exponents regardless of which is larger?

Let’s consider $\frac{x^2}{x^5}$.

We subtract the exponent in the denominator from the exponent in the numerator.
Equation:

$$\frac{x^2}{x^5}$$
$$x^{2-5}$$
$$x^{-3}$$

We can also simplify $\frac{x^2}{x^5}$ by dividing out common factors:

$$\frac{\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x}$$

$$\frac{1}{x^3}$$

This implies that $x^{-3} = \frac{1}{x^3}$ and it leads us to the definition of a *negative exponent*.

Note:
Negative Exponent
If n is an integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$.

The negative exponent tells us we can re-write the expression by taking the reciprocal of the base and then changing the sign of the exponent.

Any expression that has negative exponents is not considered to be in simplest form. We will use the definition of a negative exponent and other properties of exponents to write the expression with only positive exponents.

For example, if after simplifying an expression we end up with the expression x^{-3}, we will take one more step and write $\frac{1}{x^3}$. The answer is considered to be in simplest form when it has only positive exponents.

Example:
Exercise:

Problem: **Simplify: ⓐ 4^{-2} ⓑ 10^{-3}.

Solution:
Solution

ⓐ	4^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{4^2}$
Simplify.	$\frac{1}{16}$
ⓑ	10^{-3}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{10^3}$

Simplify.	$\frac{1}{1000}$

Note:
Exercise:

TRY IT :: 5.177

Problem: Simplify: ⓐ 2^{-3} ⓑ 10^{-7}.

Solution:

ⓐ $\frac{1}{8}$ ⓑ $\frac{1}{10^7}$

Note:
Exercise:

TRY IT :: 5.178

Problem: Simplify: ⓐ 3^{-2} ⓑ 10^{-4}.

Solution:

ⓐ $\frac{1}{9}$ ⓑ $\frac{1}{10{,}000}$

In [link] we raised an integer to a negative exponent. What happens when we raise a fraction to a negative exponent? We’ll start by looking at what happens to a fraction whose numerator is one and whose denominator is an integer raised to a negative exponent.

	$\frac{1}{a^{-n}}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$.	$\frac{1}{\frac{1}{a^n}}$
Simplify the complex fraction.	$1\cdot\frac{a^n}{1}$
Multiply.	a^n

This leads to the Property of Negative Exponents.

Note:
Property of Negative Exponents
If n is an integer and $a \neq 0$, then $\frac{1}{a^{-n}} = a^n$.

Example:
Exercise:

Problem: **Simplify: ⓐ $\frac{1}{y^{-4}}$ ⓑ $\frac{1}{3^{-2}}$.

Solution:
Solution

ⓐ	$\frac{1}{y^{-4}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}} = a^n$.	y^4
ⓑ	$\frac{1}{3^{-2}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}} = a^n$.	3^2
Simplify.	9

Note:
Exercise:

TRY IT :: 5.179
Problem: Simplify: ⓐ $\frac{1}{p^{-8}}$ ⓑ $\frac{1}{4^{-3}}$.

Solution:

ⓐ p^8 ⓑ 64

Note:
Exercise:

TRY IT :: 5.180
Problem: Simplify: ⓐ $\frac{1}{q^{-7}}$ ⓑ $\frac{1}{2^{-4}}$.

Solution:

ⓐ q^7 ⓑ 16

Suppose now we have a fraction raised to a negative exponent. Let's use our definition of negative exponents to lead us to a new property.

	$\left(\frac{3}{4}\right)^{-2}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$.	$\frac{1}{\left(\frac{3}{4}\right)^2}$
Simplify the denominator.	$\frac{1}{\frac{9}{16}}$
Simplify the complex fraction.	$\frac{16}{9}$
But we know that $\frac{16}{9}$ is $\left(\frac{4}{3}\right)^2$.	
This tells us that:	$\left(\frac{3}{4}\right)^{-2}=\left(\frac{4}{3}\right)^2$

To get from the original fraction raised to a negative exponent to the final result, we took the reciprocal of the base—the fraction—and changed the sign of the exponent.

This leads us to the *Quotient to a Negative Power Property*.

Note:
Quotient to a Negative Exponent Property
If a and b are real numbers, $a\neq 0, b\neq 0$, and n is an integer, then $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n$.

Example:
Exercise:

Problem: **Simplify: ⓐ $\left(\frac{5}{7}\right)^{-2}$ ⓑ $\left(-\frac{2x}{y}\right)^{-3}$.

Solution:
Solution

ⓐ	$\left(\frac{5}{7}\right)^{-2}$
Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$.	
Take the reciprocal of the fraction and change the sign of the exponent.	$\left(\frac{7}{5}\right)^{2}$
Simplify.	$\frac{49}{25}$
ⓑ	$\left(-\frac{2x}{y}\right)^{-3}$
Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$.	
Take the reciprocal of the fraction and change the sign of the exponent.	$\left(-\frac{y}{2x}\right)^{3}$
Simplify.	$-\frac{y^3}{8x^3}$

Note:
Exercise:

TRY IT :: 5.181

Problem: Simplify: ⓐ $\left(\frac{2}{3}\right)^{-4}$ ⓑ $\left(-\frac{6m}{n}\right)^{-2}$.

Solution:

ⓐ $\frac{81}{16}$ ⓑ $\frac{n^2}{36m^2}$

Note:
Exercise:

TRY IT :: 5.182

Problem: Simplify: ⓐ $\left(\frac{3}{5}\right)^{-3}$ ⓑ $\left(-\frac{a}{2b}\right)^{-4}$.

Solution:

ⓐ $\frac{125}{27}$ ⓑ $\frac{16b^4}{a^4}$

When simplifying an expression with exponents, we must be careful to correctly identify the base.

Example:
Exercise:

Problem: **Simplify: ⓐ $(-3)^{-2}$ ⓑ -3^{-2} ⓒ $\left(-\frac{1}{3}\right)^{-2}$ ⓓ $-\left(\frac{1}{3}\right)^{-2}$.

Solution:
Solution

ⓐ Here the exponent applies to the base -3.	$(-3)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{(-3)^{-2}}$
Simplify.	$\frac{1}{9}$
ⓑ The expression -3^{-2} means "find the opposite of 3^{-2}." Here the exponent applies to the base $\left(-\frac{1}{3}\right)$.	-3^{-2}
Rewrite as a product with -1.	$-1 \cdot 3^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \frac{1}{3^2}$
Simplify.	$-\frac{1}{9}$
ⓒ Here the exponent applies to the base $\left(-\frac{1}{3}\right)$.	$\left(-\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\left(-\frac{3}{1}\right)^{2}$
Simplify.	9
ⓓ The expression $-\left(\frac{1}{3}\right)^{-2}$ means "find the opposite of $\left(\frac{1}{3}\right)^{-2}$." Here the exponent applies to the base $\left(\frac{1}{3}\right)$.	
Rewrite as a product with -1.	$-1 \cdot \left(\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \left(\frac{3}{1}\right)^{2}$
Simplify.	-9

Note:
Exercise:

TRY IT :: 5.183

Problem: Simplify: ⓐ $(-5)^{-2}$ ⓑ -5^{-2} ⓒ $\left(-\frac{1}{5}\right)^{-2}$ ⓓ $-\left(\frac{1}{5}\right)^{-2}$.

Solution:

ⓐ $\frac{1}{25}$ ⓑ $-\frac{1}{25}$ ⓒ 25 ⓓ -25

Note:
Exercise:

TRY IT :: 5.184

Problem: Simplify: ⓐ $(-7)^{-2}$ ⓑ -7^{-2}, ⓒ $\left(-\frac{1}{7}\right)^{-2}$ ⓓ $-\left(\frac{1}{7}\right)^{-2}$.

Solution:

ⓐ $\frac{1}{49}$ ⓑ $-\frac{1}{49}$ ⓒ 49 ⓓ -49

We must be careful to follow the Order of Operations. In the next example, parts (a) and (b) look similar, but the results are different.

Example:
Exercise:

Problem: **Simplify: ⓐ $4 \cdot 2^{-1}$ ⓑ $(4 \cdot 2)^{-1}$.

Solution:
Solution

ⓐ Do exponents before multiplication.	$4 \cdot 2^{-1}$
Use $a^{-n} = \frac{1}{a^n}$.	$4 \cdot \frac{1}{2^1}$
Simplify.	2
ⓑ	$(4 \cdot 2)^{-1}$
Simplify inside the parentheses first.	$(8)^{-1}$

Use $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{8^1}$
Simplify.	$\frac{1}{8}$

Note:
Exercise:

TRY IT :: 5.185

Problem: Simplify: ⓐ $6 \cdot 3^{-1}$ ⓑ $(6 \cdot 3)^{-1}$.

Solution:

ⓐ 2 ⓑ $\frac{1}{18}$

Note:
Exercise:

TRY IT :: 5.186

Problem: Simplify: ⓐ $8 \cdot 2^{-2}$ ⓑ $(8 \cdot 2)^{-2}$.

Solution:

ⓐ 2 ⓑ $\frac{1}{256}$

When a variable is raised to a negative exponent, we apply the definition the same way we did with numbers. We will assume all variables are non-zero.

Example:
Exercise:

Problem: **Simplify: ⓐ x^{-6} ⓑ $\left(u^4\right)^{-3}$.

Solution:
Solution

ⓐ

	x^{-6}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{x^6}$

ⓑ

	$\left(u^4\right)^{-3}$
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{\left(u^4\right)^3}$
Simplify.	$\frac{1}{u^{12}}$

Note:
Exercise:

TRY IT :: 5.187

Problem: Simplify: ⓐ y^{-7} ⓑ $\left(z^3\right)^{-5}$.

Solution:

ⓐ $\frac{1}{y^7}$ ⓑ $\frac{1}{z^{15}}$

Note:
Exercise:

TRY IT :: 5.188

Problem: Simplify: ⓐ p^{-9} ⓑ $\left(q^4\right)^{-6}$.

Solution:

ⓐ $\frac{1}{p^9}$ ⓑ $\frac{1}{q^{24}}$

When there is a product and an exponent we have to be careful to apply the exponent to the correct quantity. According to the Order of Operations, we simplify expressions in parentheses before applying exponents. We'll see how this works in the next example.

Example:
Exercise:

Problem: **Simplify: ⓐ $5y^{-1}$ ⓑ $(5y)^{-1}$ ⓒ $(-5y)^{-1}$.

Solution:
Solution

ⓐ

$5y^{-1}$

Notice the exponent applies to just the base y. $5 \cdot \frac{1}{y^1}$

Take the reciprocal of y and change the sign of the exponent.

Simplify. $\frac{5}{y}$

ⓑ

$(5y)^{-1}$

Here the parentheses make the exponent apply to the base $5y$. $\frac{1}{(5y)^1}$

Take the reciprocal of $5y$ and change the sign of the exponent.

Simplify. $\frac{1}{5y}$

ⓒ

$(-5y)^{-1}$

The base here is $-5y$. $\frac{1}{(-5y)^1}$

Take the reciprocal of $-5y$ and change the sign of the exponent.

Simplify. $\frac{1}{-5y}$

Use $\frac{a}{-b} = -\frac{a}{b}$. $-\frac{1}{5y}$

Note:
Exercise:

TRY IT :: 5.189

Problem: Simplify: ⓐ $8p^{-1}$ ⓑ $(8p)^{-1}$ ⓒ $(-8p)^{-1}$.

Solution:

ⓐ $\frac{8}{p}$ ⓑ $\frac{1}{8p}$ ⓒ $-\frac{1}{8p}$

Note:
Exercise:

TRY IT :: 5.190

Problem: Simplify: ⓐ $11q^{-1}$ ⓑ $(11q)^{-1}$ ⓒ $(-11q)^{-1}$.

Solution:

ⓐ $\frac{11}{q}$ ⓑ $\frac{1}{11q}$ ⓒ $-\frac{1}{11q}$

With negative exponents, the Quotient Rule needs only one form $\frac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$. When the exponent in the denominator is larger than the exponent in the numerator, the exponent of the quotient will be negative.

Simplify Expressions with Integer Exponents

All of the exponent properties we developed earlier in the chapter with whole number exponents apply to integer exponents, too. We restate them here for reference.

Note:
Summary of Exponent Properties
If a and b are real numbers, and m and n are integers, then
Equation:

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Zero Exponent Property	$a^0 = 1, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0$
Properties of Negative Exponents	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$
Quotient to a Negative Exponent	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Example:
Exercise:

Problem: **Simplify: ⓐ $x^{-4} \cdot x^6$ ⓑ $y^{-6} \cdot y^4$ ⓒ $z^{-5} \cdot z^{-3}$.

Solution:
Solution

ⓐ

	$x^{-4} \cdot x^6$
Use the Product Property, $a^m \cdot a^n = a^{m+n}$.	x^{-4+6}
Simplify.	x^2

ⓑ

	$y^{-6} \cdot y^4$
Notice the same bases, so add the exponents.	y^{-6+4}
Simplify.	y^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{y^2}$

ⓒ	
	$z^{-5} \cdot z^{-3}$
Add the exponents, since the bases are the same.	z^{-5-3}
Simplify.	z^{-8}
Take the reciprocal and change the sign of the exponent, using the definition of a negative exponent.	$\frac{1}{z^{8}}$

Note:
Exercise:

TRY IT :: 5.191
Problem: Simplify: ⓐ $x^{-3} \cdot x^{7}$ ⓑ $y^{-7} \cdot y^{2}$ ⓒ $z^{-4} \cdot z^{-5}$.

Solution:

ⓐ x^{4} ⓑ $\frac{1}{y^{5}}$ ⓒ $\frac{1}{z^{9}}$

Note:
Exercise:

TRY IT :: 5.192
Problem: Simplify: ⓐ $a^{-1} \cdot a^{6}$ ⓑ $b^{-8} \cdot b^{4}$ ⓒ $c^{-8} \cdot c^{-7}$.

Solution:

ⓐ a^{5} ⓑ $\frac{1}{b^{4}}$ ⓒ $\frac{1}{c^{15}}$

In the next two examples, we’ll start by using the Commutative Property to group the same variables together. This makes it easier to identify the like bases before using the Product Property.

Example:
Exercise:

Problem: **Simplify: $\left(m^{4}n^{-3}\right)\left(m^{-5}n^{-2}\right)$.

Solution:
Solution

	$\left(m^4n^{-3}\right)\left(m^{-5}n^{-2}\right)$
Use the Commutative Property to get like bases together.	$m^4m^{-5}\cdot n^{-2}n^{-3}$
Add the exponents for each base.	$m^{-1}\cdot n^{-5}$
Take reciprocals and change the signs of the exponents.	$\frac{1}{m^1}\cdot\frac{1}{n^5}$
Simplify.	$\frac{1}{mn^5}$

Note:
Exercise:

TRY IT :: 5.193

Problem: Simplify: $\left(p^6q^{-2}\right)\left(p^{-9}q^{-1}\right)$.

Solution:

$\frac{1}{p^3q^3}$

Note:
Exercise:

TRY IT :: 5.194

Problem: Simplify: $\left(r^5s^{-3}\right)\left(r^{-7}s^{-5}\right)$.

Solution:

$\frac{1}{r^2s^8}$

If the monomials have numerical coefficients, we multiply the coefficients, just like we did earlier.

Example:
Exercise:

Problem: **Simplify: $\left(2x^{-6}y^8\right)\left(-5x^5y^{-3}\right)$.

Solution:
Solution

	$\left(2x^{-6}y^{8}\right)\left(-5x^{5}y^{-3}\right)$
Rewrite with the like bases together.	$2\left(-5\right)\cdot\left(x^{-6}x^{5}\right)\cdot\left(y^{8}y^{-3}\right)$
Multiply the coefficients and add the exponents of each variable.	$-10\cdot x^{-1}\cdot y^{5}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$.	$-10\cdot\frac{1}{x^{1}}\cdot y^{5}$
Simplify.	$\frac{-10y^{5}}{x}$

Note:
Exercise:

TRY IT :: 5.195
Problem: Simplify: $\left(3u^{-5}v^{7}\right)\left(-4u^{4}v^{-2}\right)$.

Solution:

$-\frac{12v^{5}}{u}$

Note:
Exercise:

TRY IT :: 5.196
Problem: Simplify: $\left(-6c^{-6}d^{4}\right)\left(-5c^{-2}d^{-1}\right)$.

Solution:

$\frac{30d^{3}}{c^{8}}$

In the next two examples, we’ll use the Power Property and the Product to a Power Property.

Example:
Exercise:

Problem: **Simplify: $\left(6k^{3}\right)^{-2}$.

Solution:
Solution

	$\left(6k^3\right)^{-2}$
Use the Product to a Power Property, $(ab)^m = a^m b^m$.	$(6)^{-2}\left(k^3\right)^{-2}$
Use the Power Property, $(a^m)^n = a^{m \cdot n}$.	$6^{-2}k^{-6}$
Use the Definition of a Negative Exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{6^2} \cdot \frac{1}{k^6}$
Simplify.	$\frac{1}{36k^6}$

Note:
Exercise:

TRY IT :: 5.197
Problem: Simplify: $\left(-4x^4\right)^{-2}$.

Solution:

$\frac{1}{16x^8}$

Note:
Exercise:

TRY IT :: 5.198
Problem: Simplify: $\left(2b^3\right)^{-4}$.

Solution:

$\frac{1}{16b^{12}}$

Example:
Exercise:

Problem: **Simplify: $\left(5x^{-3}\right)^2$.

Solution:
Solution

	$\left(5x^{-3}\right)^2$
Use the Product to a Power Property, $(ab)^m = a^m b^m$.	$5^2\left(x^{-3}\right)^2$
Simplify 5^2 and multiply the exponents of x using the Power Property, $(a^m)^n = a^{m \cdot n}$.	$25 \cdot x^{-6}$
Rewrite x^{-6} by using the Definition of a Negative Exponent, $a^{-n} = \frac{1}{a^n}$.	$25 \cdot \frac{1}{x^6}$
Simplify.	$\frac{25}{x^6}$

Note:
Exercise:

TRY IT :: 5.199
Problem: Simplify: $\left(8a^{-4}\right)^2$.

Solution:

$\frac{64}{a^8}$

Note:
Exercise:

TRY IT :: 5.200
Problem: Simplify: $\left(2c^{-4}\right)^3$.

Solution:

$\frac{8}{c^{12}}$

To simplify a fraction, we use the Quotient Property and subtract the exponents.

Example:
Exercise:

Problem: **Simplify: $\frac{r^5}{r^{-4}}$.

Solution:

Solution

	$\frac{r^5}{r^{-4}}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$.	$r^{5-(-4)}$
Simplify.	r^9

Note:
Exercise:

TRY IT :: 5.201
Problem: Simplify: $\frac{x^8}{x^{-3}}$.

Solution:

x^{11}

Note:
Exercise:

TRY IT :: 5.202
Problem: Simplify: $\frac{y^8}{y^{-6}}$.

Solution:

y^{14}

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on. Consider the numbers 4,000 and 0.004. We know that 4,000 means $4 \times 1{,}000$ and 0.004 means $4 \times \frac{1}{1{,}000}$.

If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:
Equation:

$$\begin{array}{ll} 4{,}000 & 0.004 \\ 4 \times 1{,}000 & 4 \times \frac{1}{1{,}000} \\ 4 \times 10^3 & 4 \times \frac{1}{10^3} \\ & 4 \times 10^{-3} \end{array}$$

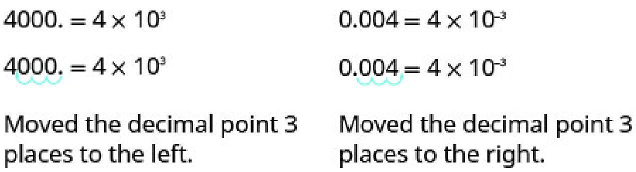
When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in *scientific notation*.

Note:
Scientific Notation
A number is expressed in **scientific notation** when it is of the form
Equation:

$$a \times 10^n \text{ where } 1 \leq a < 10 \text{ and } n \text{ is an integer}$$

It is customary in scientific notation to use as the × multiplication sign, even though we avoid using this sign elsewhere in algebra.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.



$4000. = 4 \times 10^3$ $\quad$ $0.004 = 4 \times 10^{-3}$

$4000. = 4 \times 10^3$ $\quad$ $0.004 = 4 \times 10^{-3}$

Moved the decimal point 3 places to the left.

Moved the decimal point 3 places to the right.

In both cases, the decimal was moved 3 places to get the first factor between 1 and 10.

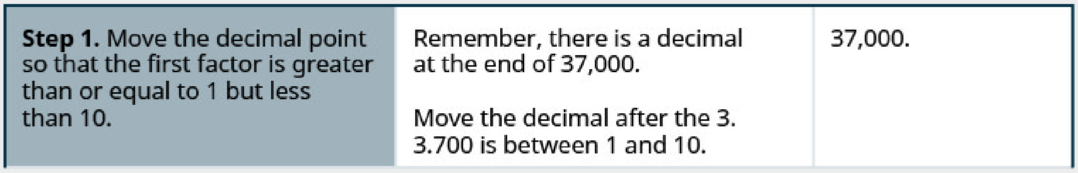
The power of 10 is positive when the number is larger than 1: $4{,}000 = 4 \times 10^3$

The power of 10 is negative when the number is between 0 and 1: $0.004 = 4 \times 10^{-3}$

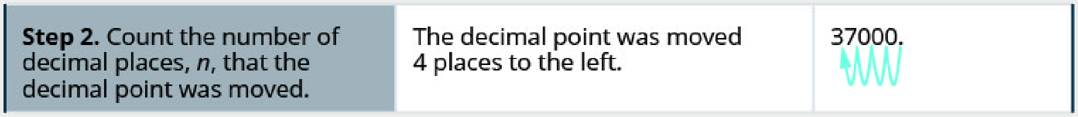
Example:
How to Convert from Decimal Notation to Scientific Notation
Exercise:

Problem: **Write in scientific notation: 37,000.

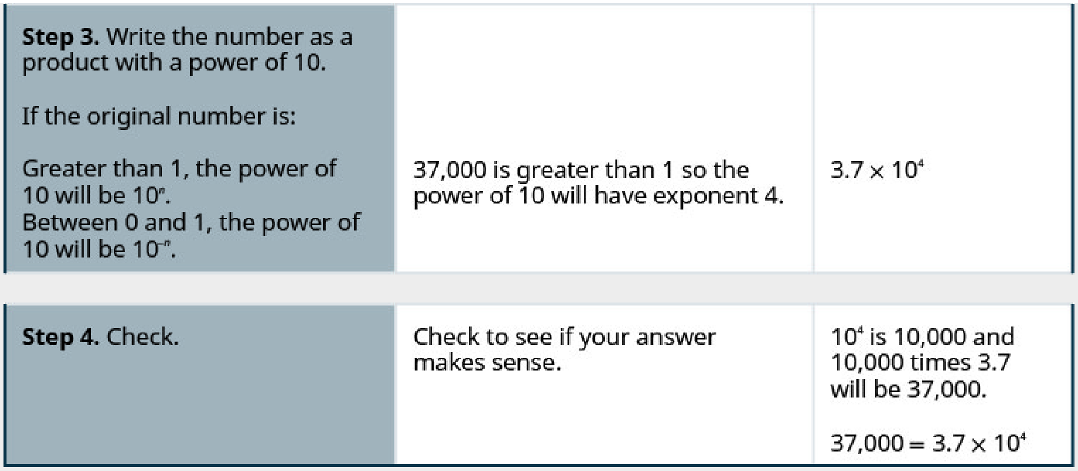
Solution:
Solution



Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.	Remember, there is a decimal at the end of 37,000. Move the decimal after the 3. 3.700 is between 1 and 10.	37,000.



Step 2. Count the number of decimal places, *n*, that the decimal point was moved.	The decimal point was moved 4 places to the left.	37000.



Step 3. Write the number as a product with a power of 10. If the original number is: Greater than 1, the power of 10 will be 10^n. Between 0 and 1, the power of 10 will be 10^{-n}.	37,000 is greater than 1 so the power of 10 will have exponent 4.	3.7×10^4
Step 4. Check.	Check to see if your answer makes sense.	10^4 is 10,000 and 10,000 times 3.7 will be 37,000. $37{,}000 = 3.7 \times 10^4$

Note:
Exercise:

TRY IT :: 5.203

Problem: Write in scientific notation: 96,000.

Solution:

9.6×10^4

Note:
Exercise:

TRY IT :: 5.204

Problem: Write in scientific notation: 48,300.

Solution:

4.83×10^4

Note:
Convert from decimal notation to scientific notation

Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
Count the number of decimal places, n, that the decimal point was moved.
Write the number as a product with a power of 10. If the original number is:
  - greater than 1, the power of 10 will be 10^n.

  - between 0 and 1, the power of 10 will be 10^{-n}.

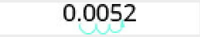
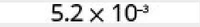
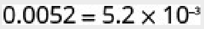
Check.

Example:
Exercise:

Problem: **Write in scientific notation: 0.0052.

Solution:
Solution

The original number, 0.0052, is between 0 and 1 so we will have a negative power of 10.

	0.0052
Move the decimal point to get 5.2, a number between 1 and 10.	 0.0052
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	 5.2×10^{-3}
Check.	
5.2×10^{-3} $5.2 \times \frac{1}{10^3}$ $5.2 \times \frac{1}{1000}$ 5.2×0.001	
0.0052	 $0.0052 = 5.2 \times 10^{-3}$

Note:
Exercise:

TRY IT :: 5.205

Problem: Write in scientific notation: 0.0078.

Solution:

7.8×10^{-3}

Note:
Exercise:

TRY IT :: 5.206

Problem: Write in scientific notation: 0.0129.

Solution:

1.29×10^{-2}

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let’s look at two numbers written in scientific notation and see.

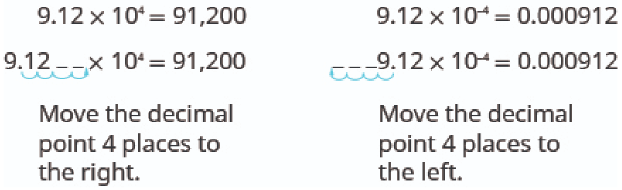
Equation:

$$\begin{array}{ccc} 9.12 \times 10^4 & \qquad & 9.12 \times 10^{-4} \\ 9.12 \times 10{,}000 & & 9.12 \times 0.0001 \\ 91{,}200 & & 0.000912 \end{array}$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

Equation:

$$9.12 \times 10^4 = 91{,}200 \qquad 9.12 \times 10^{-4} = 0.000912$$



Move the decimal point 4 places to the right.

Move the decimal point 4 places to the left.

In both cases the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example:

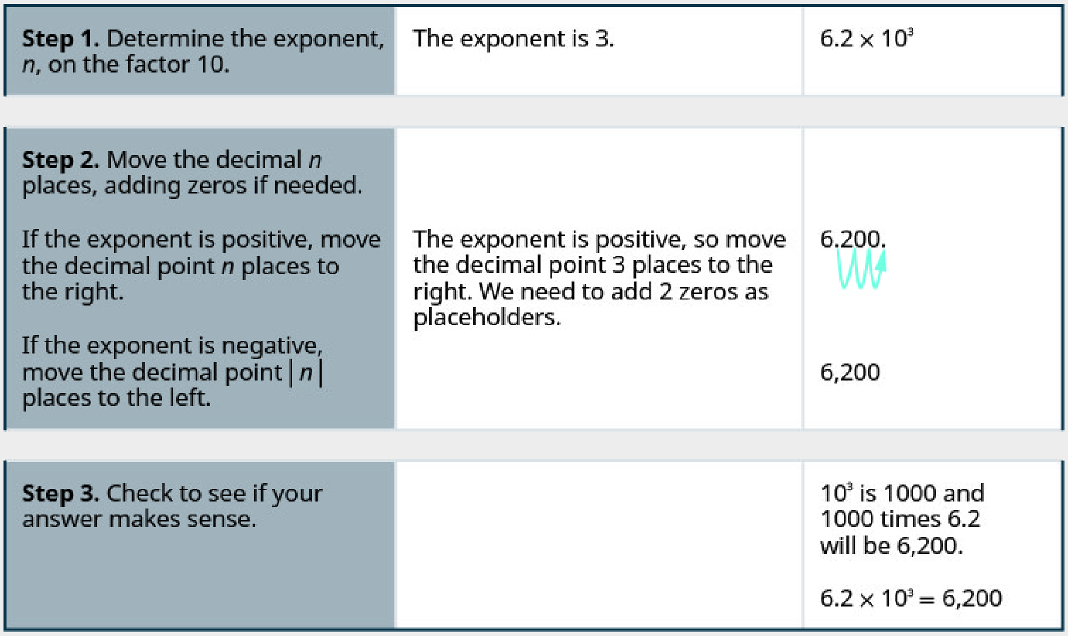
How to Convert Scientific Notation to Decimal Form

Exercise:

Problem: **Convert to decimal form: 6.2×10^3.

Solution:

Solution



Step 1. Determine the exponent, *n*, on the factor 10.	The exponent is 3.	6.2×10^3
Step 2. Move the decimal *n* places, adding zeros if needed. If the exponent is positive, move the decimal point *n* places to the right. If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.	The exponent is positive, so move the decimal point 3 places to the right. We need to add 2 zeros as placeholders.	6.200. 6,200
Step 3. Check to see if your answer makes sense.		10^3 is 1000 and 1000 times 6.2 will be 6,200. $6.2 \times 10^3 = 6{,}200$

Note:

Exercise:

TRY IT :: 5.207

Problem: Convert to decimal form: 1.3×10^3.

Solution:

1,300

Note:

Exercise:

TRY IT :: 5.208

Problem: Convert to decimal form: 9.25×10^4.

Solution:

92,500

The steps are summarized below.

Note:
Convert scientific notation to decimal form.
To convert scientific notation to decimal form:

Determine the exponent, n, on the factor 10.
Move the decimal n places, adding zeros if needed.
- If the exponent is positive, move the decimal point n places to the right.
- If the exponent is negative, move the decimal point $|n|$ places to the left.

Check.

Example:
Exercise:

Problem: **Convert to decimal form: 8.9×10^{-2}.

Solution:
Solution

	8.9×10^{-2}
Determine the exponent, *n*, on the factor 10.	The exponent is −2.
Since the exponent is negative, move the decimal point 2 places to the left.	
Add zeros as needed for placeholders.	$8.9 \times 10^{-2} = 0.089$

Note:
Exercise:

TRY IT :: 5.209

Problem: Convert to decimal form: 1.2×10^{-4}.

Solution:

0.00012

Note:
Exercise:

TRY IT :: 5.210

Problem: Convert to decimal form: 7.5×10^{-2}.

Solution:

0.075

Multiply and Divide Using Scientific Notation

Astronomers use very large numbers to describe distances in the universe and ages of stars and planets. Chemists use very small numbers to describe the size of an atom or the charge on an electron. When scientists perform calculations with very large or very small numbers, they use scientific notation. Scientific notation provides a way for the calculations to be done without writing a lot of zeros. We will see how the Properties of Exponents are used to multiply and divide numbers in scientific notation.

Example:
Exercise:

Problem: **Multiply. Write answers in decimal form: $\left(4 \times 10^5\right)\left(2 \times 10^{-7}\right)$.

Solution:
Solution

	$\left(4 \times 10^5\right)\left(2 \times 10^{-7}\right)$
Use the Commutative Property to rearrange the factors.	$4 \cdot 2 \cdot 10^5 \cdot 10^{-7}$
Multiply.	8×10^{-2}
Change to decimal form by moving the decimal two places left.	0.08

Note:
Exercise:

TRY IT :: 5.211

Problem: Multiply $\left(3 \times 10^{6}\right)\left(2 \times 10^{-8}\right)$. Write answers in decimal form.

Solution:

0.06

Note:
Exercise:

TRY IT :: 5.212

Problem: Multiply $\left(3 \times 10^{-2}\right)\left(3 \times 10^{-1}\right)$. Write answers in decimal form.

Solution:

0.009

Example:
Exercise:

Problem: **Divide. Write answers in decimal form: $\frac{9 \times 10^{3}}{3 \times 10^{-2}}$.

Solution:
Solution

	$\frac{9 \times 10^{3}}{3 \times 10^{-2}}$
Separate the factors, rewriting as the product of two fractions.	$\frac{9}{3} \times \frac{10^{3}}{10^{-2}}$
Divide.	3×10^{5}
Change to decimal form by moving the decimal five places right.	300,000

Note:
Exercise:

TRY IT :: 5.213

Problem: Divide $\frac{8 \times 10^{4}}{2 \times 10^{-1}}$. Write answers in decimal form.

Solution:

400,000

Note:
Exercise:

TRY IT :: 5.214

Problem: Divide $\frac{8 \times 10^2}{4 \times 10^{-2}}$. Write answers in decimal form.

Solution:

20,000

Note:
Access these online resources for additional instruction and practice with integer exponents and scientific notation:

- Negative Exponents
- Scientific Notation
- Scientific Notation 2

Key Concepts

- **Property of Negative Exponents**
  - If n is a positive integer and $a \neq 0$, then $\frac{1}{a^{-n}} = a^n$
- **Quotient to a Negative Exponent**
  - If a, b are real numbers, $b \neq 0$ and n is an integer , then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- **To convert a decimal to scientific notation:**

  Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
  Count the number of decimal places, n, that the decimal point was moved.
  Write the number as a product with a power of 10. If the original number is:
    - greater than 1, the power of 10 will be 10^n
    - between 0 and 1, the power of 10 will be 10^{-n}

  Check.
- **To convert scientific notation to decimal form:**

  Determine the exponent, n, on the factor 10.
  Move the n places, adding zeros if

decimal needed.

- If the exponent is positive, move the decimal point n places to the right.
- If the exponent is negative, move the decimal point $|n|$ places to the left.

Check.

Section Exercises

Practice Makes Perfect

Use the Definition of a Negative Exponent

In the following exercises, simplify.

Exercise:

Problem: ⓐ 4^{-2} ⓑ 10^{-3}

Exercise:

Problem: ⓐ 3^{-4} ⓑ 10^{-2}

Solution:

ⓐ $\frac{1}{81}$ ⓑ $\frac{1}{100}$

Exercise:

Problem: ⓐ 5^{-3} ⓑ 10^{-5}

Exercise:

Problem: ⓐ 2^{-8} ⓑ 10^{-2}

Solution:

ⓐ $\frac{1}{256}$ ⓑ $\frac{1}{100}$

Exercise:

Problem: ⓐ $\frac{1}{c^{-5}}$ ⓑ $\frac{1}{3^{-2}}$

Exercise:

ⓐ $\frac{1}{c^{-5}}$

Problem: ⓑ $\frac{1}{5^{-2}}$

Solution:

ⓐ c^5 ⓑ 25

Exercise:

ⓐ $\frac{1}{q^{-10}}$

Problem: ⓑ $\frac{1}{10^{-3}}$

Exercise:

ⓐ $\frac{1}{t^{-9}}$

Problem: ⓑ $\frac{1}{10^{-4}}$

Solution:

ⓐ t^9 ⓑ 10000

Exercise:

ⓐ $\left(\frac{5}{8}\right)^{-2}$

Problem: ⓑ $\left(-\frac{3m}{n}\right)^{-2}$

Exercise:

ⓐ $\left(\frac{3}{10}\right)^{-2}$

Problem: ⓑ $\left(-\frac{2}{cd}\right)^{-3}$

Solution:

ⓐ $\frac{100}{9}$ ⓑ $-\frac{c^3d^3}{8}$

Exercise:

ⓐ $\left(\frac{4}{9}\right)^{-3}$

Problem: ⓑ $\left(-\frac{u^2}{2v}\right)^{-5}$

Exercise:

ⓐ $\left(\frac{7}{2}\right)^{-3}$

Problem: ⓑ $\left(-\frac{3}{xy^2}\right)^{-3}$

Solution:

ⓐ $\frac{8}{343}$ ⓑ $-\frac{x^3y^6}{27}$

Exercise:

ⓐ $(-5)^{-2}$
ⓑ -5^{-2}
ⓒ $\left(-\frac{1}{5}\right)^{-2}$

Problem: ⓓ $-\left(\frac{1}{5}\right)^{-2}$

Exercise:

ⓐ $(-7)^{-2}$
ⓑ-7^{-2}
ⓒ $\left(-\frac{1}{7}\right)^{-2}$

Problem: ⓓ $-\left(\frac{1}{7}\right)^{-2}$

Solution:

ⓐ $\frac{1}{49}$ ⓑ $-\frac{1}{49}$ ⓒ 49 ⓓ -49

Exercise:

ⓐ -3^{-3}
ⓑ $\left(-\frac{1}{3}\right)^{-3}$
ⓒ $-\left(\frac{1}{3}\right)^{-3}$

Problem: ⓓ $(-3)^{-3}$

Exercise:

ⓐ -5^{-3}
ⓑ $\left(-\frac{1}{5}\right)^{-3}$
ⓒ $-\left(\frac{1}{5}\right)^{-3}$

Problem: ⓓ $(-5)^{-3}$

Solution:

ⓐ $-\frac{1}{125}$ ⓑ -125 ⓒ -125 ⓓ $-\frac{1}{125}$

Exercise:

Problem: ⓐ $3 \cdot 5^{-1}$ ⓑ $(3 \cdot 5)^{-1}$

Exercise:

Problem: ⓐ $2 \cdot 5^{-1}$ ⓑ $(2 \cdot 5)^{-1}$

Solution:

ⓐ $\frac{2}{5}$ ⓑ $\frac{1}{10}$

Exercise:

Problem: ⓐ $4 \cdot 5^{-2}$ ⓑ $(4 \cdot 5)^{-2}$

Exercise:

Problem: ⓐ $3 \cdot 4^{-2}$ ⓑ $(3 \cdot 4)^{-2}$

Solution:

ⓐ $\frac{3}{16}$ ⓑ $\frac{1}{144}$

Exercise:

Problem: ⓐ m^{-4} ⓑ $\left(x^3\right)^{-4}$

Exercise:

Problem: ⓐ b^{-5} ⓑ $\left(k^2\right)^{-5}$

Solution:

ⓐ $\frac{1}{b^5}$ ⓑ $\frac{1}{k^{10}}$

Exercise:

ⓐ p^{-10}
Problem: ⓑ $\left(q^{6}\right)^{-8}$

Exercise:

ⓐ s^{-8}
Problem: ⓑ $\left(a^{9}\right)^{-10}$

Solution:

ⓐ $\frac{1}{s^{8}}$ ⓑ $\frac{1}{a^{90}}$

Exercise:

ⓐ $7n^{-1}$
ⓑ $(7n)^{-1}$
Problem: ⓒ $(-7n)^{-1}$

Exercise:

ⓐ $6r^{-1}$
ⓑ $(6r)^{-1}$
Problem: ⓒ $(-6r)^{-1}$

Solution:

ⓐ $\frac{6}{r}$ ⓑ $\frac{1}{6r}$ ⓒ $-\frac{1}{6r}$

Exercise:

ⓐ $(3p)^{-2}$
ⓑ $3p^{-2}$
Problem: ⓒ $-3p^{-2}$

Exercise:

ⓐ $(2q)^{-4}$
ⓑ $2q^{-4}$
Problem: ⓒ $-2q^{-4}$

Solution:

ⓐ $\frac{1}{16q^{4}}$ ⓑ $\frac{2}{q^{4}}$ ⓒ $-\frac{2}{q^{4}}$

Simplify Expressions with Integer Exponents

In the following exercises, simplify.

Exercise:

ⓐ b^4b^{-8}
ⓑ $r^{-2}r^5$
Problem: ⓒ $x^{-7}x^{-3}$

Exercise:

ⓐ $s^3 \cdot s^{-7}$
ⓑ $q^{-8} \cdot q^3$
Problem: ⓒ $y^{-2} \cdot y^{-5}$

Solution:

ⓐ $\frac{1}{s^4}$ ⓑ $\frac{1}{q^5}$ ⓒ $\frac{1}{y^7}$

Exercise:

ⓐ $a^3 \cdot a^{-3}$
ⓑ $a \cdot a^3$
Problem: ⓒ $a \cdot a^{-3}$

Exercise:

ⓐ $y^5 \cdot y^{-5}$
ⓑ $y \cdot y^5$
Problem: ⓒ $y \cdot y^{-5}$

Solution:

ⓐ 1 ⓑ y^6 ⓒ $\frac{1}{y^4}$

Exercise:

Problem: $p^5 \cdot p^{-2} \cdot p^{-4}$

Exercise:

Problem: $x^4 \cdot x^{-2} \cdot x^{-3}$

Solution:

$\frac{1}{x}$

Exercise:

Problem: $\left(w^4x^{-5}\right)\left(w^{-2}x^{-4}\right)$

Exercise:

Problem: $\left(m^3n^{-3}\right)\left(m^{-5}n^{-1}\right)$

Solution:

$\frac{1}{m^2n^4}$

Exercise:

Problem: $\left(uv^{-2}\right)\left(u^{-5}v^{-3}\right)$

Exercise:

Problem: $\left(pq^{-4}\right)\left(p^{-6}q^{-3}\right)$

Solution:

$\frac{1}{p^5q^7}$

Exercise:

Problem: $\left(-6c^{-3}d^9\right)\left(2c^4d^{-5}\right)$

Exercise:

Problem: $\left(-2j^{-5}k^8\right)\left(7j^2k^{-3}\right)$

Solution:

$-\frac{14k^5}{j^3}$

Exercise:

Problem: $\left(-4r^{-2}s^{-8}\right)\left(9r^4s^3\right)$

Exercise:

Problem: $\left(-5m^4n^6\right)\left(8m^{-5}n^{-3}\right)$

Solution:

$-\frac{40n^3}{m}$

Exercise:

Problem: $\left(5x^2\right)^{-2}$

Exercise:

Problem: $\left(4y^3\right)^{-3}$

Solution:

$\frac{1}{64y^9}$

Exercise:

Problem: $\left(3z^{-3}\right)^2$

Exercise:

Problem: $\left(2p^{-5}\right)^2$

Solution:

$\frac{4}{p^{10}}$

Exercise:

Problem: $\frac{t^9}{t^{-3}}$

Exercise:

Problem: $\frac{n^5}{n^{-2}}$

Solution:

n^7

Exercise:

Problem: $\frac{x^{-7}}{x^{-3}}$

Exercise:

Problem: $\frac{y^{-5}}{y^{-10}}$

Solution:

y^5

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

Exercise:

Problem: 57,000

Exercise:

Problem: 340,000

Solution:

3.4×10^5

Exercise:

Problem: 8,750,000

Exercise:

Problem: 1,290,000

Solution:

1.29×10^6

Exercise:

Problem: 0.026

Exercise:

Problem: 0.041

Solution:

4.1×10^{-2}

Exercise:

Problem: 0.00000871

Exercise:

Problem: 0.00000103

Solution:

1.03×10^{-6}

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

Exercise:

Problem: 5.2×10^2

Exercise:

Problem: 8.3×10^2

Solution:

830

Exercise:

Problem: 7.5×10^{6}

Exercise:

Problem: 1.6×10^{10}

Solution:

16,000,000,000

Exercise:

Problem: 2.5×10^{-2}

Exercise:

Problem: 3.8×10^{-2}

Solution:

0.038

Exercise:

Problem: 4.13×10^{-5}

Exercise:

Problem: 1.93×10^{-5}

Solution:

0.0000193

Multiply and Divide Using Scientific Notation

In the following exercises, multiply. Write your answer in decimal form.

Exercise:

Problem: $\left(3 \times 10^{-5}\right)\left(3 \times 10^{9}\right)$

Exercise:

Problem: $\left(2 \times 10^{2}\right)\left(1 \times 10^{-4}\right)$

Solution:

0.02

Exercise:

Problem: $\left(7.1\ \times\ 10^{-2}\right)\left(2.4\ \times\ 10^{-4}\right)$

Exercise:

Problem: $\left(3.5\ \times\ 10^{-4}\right)\left(1.6\ \times\ 10^{-2}\right)$

Solution:

$5.6\ \times\ 10^{-6}$

In the following exercises, divide. Write your answer in decimal form.

Exercise:

Problem: $\frac{7\times 10^{-3}}{1\times 10^{-7}}$

Exercise:

Problem: $\frac{5\times 10^{-2}}{1\times 10^{-10}}$

Solution:

500,000,000

Exercise:

Problem: $\frac{6\times 10^{4}}{3\times 10^{-2}}$

Exercise:

Problem: $\frac{8\times 10^{6}}{4\times 10^{-1}}$

Solution:

20,000,000

Everyday Math

Exercise:

Problem:

The population of the United States on July 4, 2010 was almost 310,000,000. Write the number in scientific notation.

Exercise:

Problem:

The population of the world on July 4, 2010 was more than 6,850,000,000. Write the number in scientific notation

Solution:

6.85×10^9.

Exercise:

Problem:

The average width of a human hair is 0.0018 centimeters. Write the number in scientific notation.

Exercise:

Problem:

The probability of winning the 2010 Megamillions lottery was about 0.0000000057. Write the number in scientific notation.

Solution:

5.7×10^{-9}

Exercise:

Problem:

In 2010, the number of Facebook users each day who changed their status to ‘engaged’ was 2×10^4. Convert this number to decimal form.

Exercise:

Problem:

At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$. Convert this number to decimal form.

Solution:

15,000,000,000,000

Exercise:

Problem:

The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}. Convert this number to decimal form.

Exercise:

Problem:

The width of a proton is 1×10^{-5} of the width of an atom. Convert this number to decimal form.

Solution:

0.00001

Exercise:

Problem:

Health care costs The Centers for Medicare and Medicaid projects that consumers will spend more than $4 trillion on health care by 2017.

ⓐ Write 4 trillion in decimal notation.
ⓑ Write 4 trillion in scientific notation.

Exercise:

Problem:

Coin production In 1942, the U.S. Mint produced 154,500,000 nickels. Write 154,500,000 in scientific notation.

Solution:

1.545×10^8

Exercise:

Problem:

Distance The distance between Earth and one of the brightest stars in the night star is 33.7 light years. One light year is about 6,000,000,000,000 (6 trillion), miles.

ⓐ Write the number of miles in one light year in scientific notation.
ⓑUse scientific notation to find the distance between Earth and the star in miles. Write the answer in scientific notation.

Exercise:

Problem:

Debt At the end of fiscal year 2015 the gross United States federal government debt was estimated to be approximately $18,600,000,000,000 ($18.6 trillion), according to the Federal Budget. The population of the United States was approximately 300,000,000 people at the end of fiscal year 2015.

ⓐ Write the debt in scientific notation.
ⓑ Write the population in scientific notation.
ⓒ Find the amount of debt per person by using scientific notation to divide the debt by the population. Write the answer in scientific notation.

Solution:

ⓐ 1.86×10^{13} ⓑ 3×10^8 ⓒ 6.2×10^4

Writing Exercises

Exercise:

Problem:

ⓐ Explain the meaning of the exponent in the expression 2^3.
ⓑ Explain the meaning of the exponent in the expression 2^{-3}.

Exercise:

Problem:

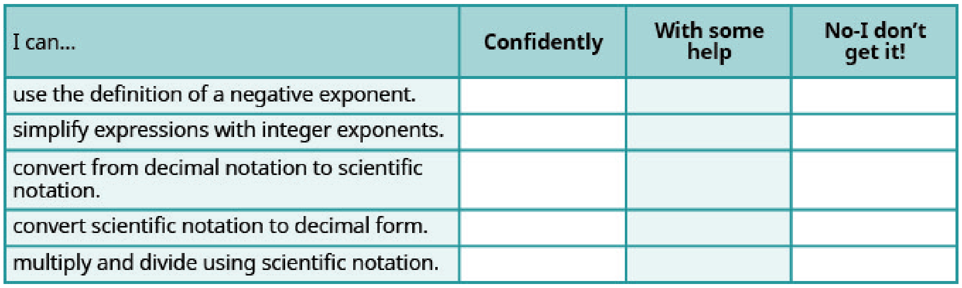
When you convert a number from decimal notation to scientific notation, how do you know if the exponent will be positive or negative?

Solution:

answers will vary

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
use the definition of a negative exponent.			
simplify expressions with integer exponents.			
convert from decimal notation to scientific notation.			
convert scientific notation to decimal form.			
multiply and divide using scientific notation.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

Glossary

negative exponent
If n is a positive integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$.

scientific notation
A number is expressed in scientific notation when it is of the form $a \times 10^n$ where $a \geq 1$ and $a < 10$ and n is an integer.

Review & Practice Test
By the end of this section, you will be prepared to take your Chapter 5 Assessment.

Chapter 5 Review Exercises

Add and Subtract Polynomials

Identify Polynomials, Monomials, Binomials and Trinomials

In the following exercises, determine if each of the following polynomials is a monomial, binomial, trinomial, or other polynomial.

Exercise:

ⓐ $11c^4 - 23c^2 + 1$
ⓑ $9p^3 + 6p^2 - p - 5$
ⓒ $\frac{3}{7}x + \frac{5}{14}$
ⓓ 10
Problem: ⓔ $2y - 12$

Exercise:

ⓐ $a^2 - b^2$
ⓑ $24d^3$
ⓒ $x^2 + 8x - 10$
ⓓ $m^2n^2 - 2mn + 6$
Problem: ⓔ $7y^3 + y^2 - 2y - 4$

Solution:

ⓐ binomial ⓑ monomial ⓒ trinomial ⓓ trinomial ⓔ other polynomial

Determine the Degree of Polynomials

In the following exercises, determine the degree of each polynomial.

Exercise:

Problem:

ⓐ $3x^2 + 9x + 10$
ⓑ $14a^2bc$
ⓒ $6y + 1$
ⓓ $n^3 - 4n^2 + 2n - 8$
ⓔ -19

Exercise:

Problem:

ⓐ $5p^3 - 8p^2 + 10p - 4$
ⓑ $-20q^4$
ⓒ $x^2 + 6x + 12$
ⓓ $23r^2s^2 - 4rs + 5$
ⓔ 100

Solution:

ⓐ 3 ⓑ 4 ⓒ 2 ⓓ 4 ⓔ 0

Add and Subtract Monomials

In the following exercises, add or subtract the monomials.

Exercise:

Problem: $5\mathrm{y}^3 + 8y^3$

Exercise:

Problem: $-14k + 19k$

Solution:

$5k$

Exercise:

Problem: $12q - (-6q)$

Exercise:

Problem: $-9c - 18c$

Solution:

$-27c$

Exercise:

Problem: $12\text{x} - 4y - 9x$

Exercise:

Problem: $3m^2 + 7n^2 - 3m^2$

Solution:

$7n^2$

Exercise:

Problem: $6x^2y - 4x + 8xy^2$

Exercise:

Problem: $13\text{a} + b$

Solution:

$13\text{a} + b$

Add and Subtract Polynomials

In the following exercises, add or subtract the polynomials.

Exercise:

Problem: $(5x^2 + 12x + 1) + (6x^2 - 8x + 3)$

Exercise:

Problem: $(9p^2 - 5p + 3) + (4p^2 - 4)$

Solution:

$13p^2 - 5p - 1$

Exercise:

Problem: $(10m^2 - 8m - 1) - (5m^2 + m - 2)$

Exercise:

Problem: $(7y^2 - 8y) - (y - 4)$

Solution:

$7y^2 - 9y + 4$

Exercise:

Problem: Subtract $\left(3s^2 + 10\right)$ from $\left(15s^2 - 2s + 8\right)$

Exercise:

Problem: Find the sum of $\left(a^2 + 6a + 9\right)$ and $\left(5a^3 - 7\right)$

Solution:

$5a^3 + a^2 + 6a + 2$

Evaluate a Polynomial for a Given Value of the Variable

In the following exercises, evaluate each polynomial for the given value.
Exercise:

Problem: Evaluate $3y^2 - y + 1$ when:

ⓐ $y = 5$
ⓑ $y = -1$
ⓒ $y = 0$

Exercise:

Problem: Evaluate $10 - 12x$ when:

ⓐ $x = 3$
ⓑ $x = 0$
ⓒ $x = -1$

Solution:

ⓐ -26 ⓑ 10 ⓒ 22

Exercise:

Problem:

Randee drops a stone off the 200 foot high cliff into the ocean. The polynomial $-16t^2 + 200$ gives the height of a stone t seconds after it is dropped from the cliff. Find the height after $t = 3$ seconds.

Exercise:

Problem:

A manufacturer of stereo sound speakers has found that the revenue received from selling the speakers at a cost of p dollars each is given by the polynomial $-4p^2 + 460p$. Find the revenue received when $p = 75$ dollars.

Solution:

12,000

Use Multiplication Properties of Exponents

Simplify Expressions with Exponents

In the following exercises, simplify.

Exercise:

Problem: 10^4

Exercise:

Problem: 17^1

Solution:

17

Exercise:

Problem: $\left(\frac{2}{9}\right)^2$

Exercise:

Problem: $(0.5)^3$

Solution:

0.125

Exercise:

Problem: $(-2)^6$

Exercise:

Problem: -2^6

Solution:

-64

Simplify Expressions Using the Product Property for Exponents

In the following exercises, simplify each expression.

Exercise:

Problem: $x^4 \cdot x^3$

Exercise:

Problem: $p^{15} \cdot p^{16}$

Solution:

p^{31}

Exercise:

Problem: $4^{10} \cdot 4^6$

Exercise:

Problem: $8 \cdot 8^5$

Solution:

8^6

Exercise:

Problem: $n \cdot n^2 \cdot n^4$

Exercise:

Problem: $y^c \cdot y^3$

Solution:

y^{c+3}

Simplify Expressions Using the Power Property for Exponents

In the following exercises, simplify each expression.

Exercise:

Problem: $(m^3)^5$

Exercise:

Problem: $(5^3)^2$

Solution:

5^6

Exercise:

Problem: $\left(y^4\right)^x$

Exercise:

Problem: $(3^r)^s$

Solution:

3^{rs}

Simplify Expressions Using the Product to a Power Property

In the following exercises, simplify each expression.

Exercise:

Problem: $(4a)^2$

Exercise:

Problem: $(-5y)^3$

Solution:

$-125y^3$

Exercise:

Problem: $(2mn)^5$

Exercise:

Problem: $(10xyz)^3$

Solution:

$1000x^3y^3z^3$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify each expression.

Exercise:

Problem: $\left(p^2\right)^5 \cdot \left(p^3\right)^6$

Exercise:

Problem: $(4a^3b^2)^3$

Solution:

$64a^9b^6$

Exercise:

Problem: $(5x)^2(7x)$

Exercise:

Problem: $(2q^3)^4(3q)^2$

Solution:

$48q^{14}$

Exercise:

Problem: $\left(\frac{1}{3}x^2\right)^2\left(\frac{1}{2}x\right)^3$

Exercise:

Problem: $\left(\frac{2}{5}m^2n\right)^3$

Solution:

$\frac{8}{125}m^6n^3$

Multiply Monomials

In the following exercises 8, multiply the monomials.

Exercise:

Problem: $\left(-15x^2\right)\left(6x^4\right)$

Exercise:

Problem: $(-9n^7)(-16n)$

Solution:

$144n^8$

Exercise:

Problem: $(7p^5q^3)(8pq^9)$

Exercise:

Problem: $\left(\frac{5}{9}ab^2\right)\left(27ab^3\right)$

Solution:

$15a^2b^5$

Multiply Polynomials

Multiply a Polynomial by a Monomial

In the following exercises, multiply.

Exercise:

Problem: $7(a+9)$

Exercise:

Problem: $-4(y + 13)$

Solution:

$-4y - 52$

Exercise:

Problem: $-5(r - 2)$

Exercise:

Problem: $p(p + 3)$

Solution:

$p^2 + 3p$

Exercise:

Problem: $-m(m + 15)$

Exercise:

Problem: $-6u(2u + 7)$

Solution:

$-12u^2 - 42u$

Exercise:

Problem: $9(b^2 + 6b + 8)$

Exercise:

Problem: $3q^2(q^2 - 7q + 6)$ 3

Solution:

$3q^4 - 21q^3 + 18q^2$

Exercise:

Problem: $(5z - 1)z$

Exercise:

Problem: $(b - 4) \cdot 11$

Solution:

$11b - 44$

Multiply a Binomial by a Binomial

In the following exercises, multiply the binomials using: ⓐ the Distributive Property, ⓑ the FOIL method, ⓒ the Vertical Method.

Exercise:

Problem: $(x - 4)(x + 10)$

Exercise:

Problem: $(6y - 7)(2y - 5)$

Solution:

ⓐ $12y^2 - 44y + 35$ ⓑ $12y^2 - 44y + 35$ ⓒ $12y^2 - 44y + 35$

In the following exercises, multiply the binomials. Use any method.

Exercise:

Problem: $(x+3)(x+9)$

Exercise:

Problem: $(y-4)(y-8)$

Solution:

$y^2-12y+32$

Exercise:

Problem: $(p-7)(p+4)$

Exercise:

Problem: $(q+16)(q-3)$

Solution:

$q^2+13q-48$

Exercise:

Problem: $(5m-8)(12m+1)$

Exercise:

Problem: $(u^2+6)(u^2-5)$

Solution:

u^4+u^2-30

Exercise:

Problem: $(9x-y)\,(6x-5)$

Exercise:

Problem: $(8mn+3)(2mn-1)$

Solution:

$16m^2n^2-2mn-3$

Multiply a Trinomial by a Binomial

In the following exercises, multiply using ⓐ the Distributive Property, ⓑ the Vertical Method.

Exercise:

Problem: $(n+1)\left(n^2+5n-2\right)$

Exercise:

Problem: $(3x-4)\left(6x^2+x-10\right)$

Solution:

ⓐ $18x^3-21x^2-34x+40$ ⓑ $18x^3-21x^2-34x+40$

In the following exercises, multiply. Use either method.

Exercise:

Problem: $(y-2)(y^2-8y+9)$

Exercise:

Problem: $(7m+1)(m^2-10m-3)$

Solution:

$7m^3-69m^2-31m-3$

Special Products

Square a Binomial Using the Binomial Squares Pattern

In the following exercises, square each binomial using the Binomial Squares Pattern.

Exercise:

Problem: $(c+11)^2$

Exercise:

Problem: $(q-15)^2$

Solution:

$q^2-30q+225$

Exercise:

Problem: $\left(x+\frac{1}{3}\right)^2$

Exercise:

Problem: $(8u+1)^2$

Solution:

$64u^2+16u+1$

Exercise:

Problem: $(3n^3-2)^2$

Exercise:

Problem: $(4a-3b)^2$

Solution:

$16a^2 - 24ab + 9b^2$

Multiply Conjugates Using the Product of Conjugates Pattern

In the following exercises, multiply each pair of conjugates using the Product of Conjugates Pattern.

Exercise:

Problem: $(s - 7)(s + 7)$

Exercise:

Problem: $\left(y + \frac{2}{5}\right)\left(y - \frac{2}{5}\right)$

Solution:

$y^2 - \frac{4}{25}$

Exercise:

Problem: $(12c + 13)(12c - 13)$

Exercise:

Problem: $(6 - r)(6 + r)$

Solution:

$36 - r^2$

Exercise:

Problem: $\left(u + \frac{3}{4}v\right)\left(u - \frac{3}{4}v\right)$

Exercise:

Problem: $(5p^4 - 4q^3)(5p^4 + 4q^3)$

Solution:

$25p^8 - 16q^6$

Recognize and Use the Appropriate Special Product Pattern

In the following exercises, find each product.

Exercise:

Problem: $(3m + 10)^2$

Exercise:

Problem: $(6a + 11)(6a - 11)$

Solution:

$36a^2 - 121$

Exercise:

Problem: $(5x + y)(x - 5y)$

Exercise:

Problem: $(c^4 + 9d)^2$

Solution:

$c^8 + 18c^4d + 81d^2$

Exercise:

Problem: $(p^5 + q^5)(p^5 - q^5)$

Exercise:

Problem: $(a^2 + 4b)(4a - b^2)$

Solution:

$4a^3 + 3a^2b - 4b^3$

Divide Monomials

Simplify Expressions Using the Quotient Property for Exponents

In the following exercises, simplify.

Exercise:

Problem: $\frac{u^{24}}{u^6}$

Exercise:

Problem: $\frac{10^{25}}{10^5}$

Solution:

10^{20}

Exercise:

Problem: $\frac{3^4}{3^6}$

Exercise:

Problem: $\frac{v^{12}}{v^{48}}$

Solution:

$\frac{1}{v^{36}}$

Exercise:

Problem: $\frac{x}{x^5}$

Exercise:

Problem: $\frac{5}{5^8}$

Solution:

$\frac{1}{5^7}$

Simplify Expressions with Zero Exponents

In the following exercises, simplify.

Exercise:

Problem: 75^0

Exercise:

Problem: x^0

Solution:

1

Exercise:

Problem: -12^0

Exercise:

Problem: $\left(-12^0\right)\left(-12\right)^0$

Solution:

1

Exercise:

Problem: $25x^0$

Exercise:

Problem: $(25x)^0$

Solution:

1

Exercise:

Problem: $19n^0 - 25m^0$

Exercise:

Problem: $(19n)^0 - (25m)^0$

Solution:

0

Simplify Expressions Using the Quotient to a Power Property

In the following exercises, simplify.

Exercise:

Problem: $\left(\frac{2}{5}\right)^3$

Exercise:

Problem: $\left(\frac{m}{3}\right)^4$

Solution:

$\frac{m^4}{81}$

Exercise:

Problem: $\left(\frac{r}{s}\right)^8$

Exercise:

Problem: $\left(\frac{x}{2y}\right)^6$

Solution:

$\frac{x^6}{64y^6}$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify.

Exercise:

Problem: $\frac{\left(x^3\right)^5}{x^9}$

Exercise:

Problem: $\frac{n^{10}}{\left(n^5\right)^2}$

Solution:

1

Exercise:

Problem: $\left(\frac{q^6}{q^8}\right)^3$

Exercise:

Problem: $\left(\frac{r^8}{r^3}\right)^4$

Solution:

r^{20}

Exercise:

Problem: $\left(\frac{c^2}{d^5}\right)^9$

Exercise:

Problem: $\left(\frac{3x^4}{2y^2}\right)^5$

Solution:

$\frac{343x^{20}}{32y^{10}}$

Exercise:

Problem: $\left(\frac{v^3v^9}{v^6}\right)^4$

Exercise:

Problem: $\frac{\left(3n^2\right)^4\left(-5n^4\right)^3}{\left(-2n^5\right)^2}$

Solution:

$-\frac{10{,}125n^{10}}{4}$

Divide Monomials

In the following exercises, divide the monomials.

Exercise:

Problem: $-65y^{14} \div 5y^2$

Exercise:

Problem: $\frac{64a^5b^9}{-16a^{10}b^3}$

Solution:

$-\frac{4b^6}{a^5}$

Exercise:

Problem: $\frac{144x^{15}y^8z^3}{18x^{10}y^2z^{12}}$

Exercise:

Problem: $\frac{\left(8p^6q^2\right)\left(9p^3q^5\right)}{16p^8q^7}$

Solution:

$\frac{9p}{2}$

Divide Polynomials

Divide a Polynomial by a Monomial

In the following exercises, divide each polynomial by the monomial.

Exercise:

Problem: $\frac{42z^2-18z}{6}$

Exercise:

Problem: $\left(35x^2-75x\right)\div 5x$

Solution:

$7x-15$

Exercise:

Problem: $\frac{81n^4+105n^2}{-3}$

Exercise:

Problem: $\frac{550p^6-300p^4}{10p^3}$

Solution:

$55p^3-30p$

Exercise:

Problem: $\left(63xy^3+56x^2y^4\right)\div(7xy)$

Exercise:

Problem: $\frac{96a^5b^2-48a^4b^3-56a^2b^4}{8ab^2}$

Solution:

$12a^4 - 6a^3b - 7ab^2$

Exercise:

Problem: $\frac{57m^2-12m+1}{-3m}$

Exercise:

Problem: $\frac{105y^5+50y^3-5y}{5y^3}$

Solution:

$21y^2 + 10 - \frac{1}{y^2}$

Divide a Polynomial by a Binomial

In the following exercises, divide each polynomial by the binomial.

Exercise:

Problem: $\left(k^2 - 2k - 99\right) \div (k + 9)$

Exercise:

Problem: $\left(v^2 - 16v + 64\right) \div (v - 8)$

Solution:

$v - 8$

Exercise:

Problem: $\left(3x^2 - 8x - 35\right) \div (x - 5)$

Exercise:

Problem: $\left(n^2 - 3n - 14\right) \div (n + 3)$

Solution:

$n-6+\frac{4}{n+3}$

Exercise:

Problem: $\left(4m^3+m-5\right)\div(m-1)$

Exercise:

Problem: $\left(u^3-8\right)\div(u-2)$

Solution:

u^2+2u+4

Integer Exponents and Scientific Notation

Use the Definition of a Negative Exponent

In the following exercises, simplify.

Exercise:

Problem: 9^{-2}

Exercise:

Problem: $(-5)^{-3}$

Solution:

$-\frac{1}{125}$

Exercise:

Problem: $3 \cdot 4^{-3}$

Exercise:

Problem: $(6u)^{-3}$

Solution:

$\frac{1}{216u^3}$

Exercise:

Problem: $\left(\frac{2}{5}\right)^{-1}$

Exercise:

Problem: $\left(\frac{3}{4}\right)^{-2}$

Solution:

$\frac{16}{9}$

Simplify Expressions with Integer Exponents

In the following exercises, simplify.

Exercise:

Problem: $p^{-2} \cdot p^8$

Exercise:

Problem: $q^{-6} \cdot q^{-5}$

Solution:

$\frac{1}{q^{11}}$

Exercise:

Problem: $\left(c^{-2}d\right)\left(c^{-3}d^{-2}\right)$

Exercise:

Problem: $\left(y^{8}\right)^{-1}$

Solution:

$\frac{1}{y^{8}}$

Exercise:

Problem: $\left(q^{-4}\right)^{-3}$

Exercise:

Problem: $\frac{a^{8}}{a^{12}}$

Solution:

$\frac{1}{a^{4}}$

Exercise:

Problem: $\frac{n^{5}}{n^{-4}}$

Exercise:

Problem: $\frac{r^{-2}}{r^{-3}}$

Solution:

r

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

Exercise:

Problem: 8,500,000

Exercise:

Problem: 0.00429

Solution:

4.29×10^{-3}

Exercise:

Problem: The thickness of a dime is about 0.053 inches.

Exercise:

Problem:

In 2015, the population of the world was about 7,200,000,000 people.

Solution:

7.2×10^{9}

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

Exercise:

Problem: 3.8×10^{5}

Exercise:

Problem: 1.5×10^{10}

Solution:

$15{,}000{,}000{,}000$

Exercise:

Problem: 9.1×10^{-7}

Exercise:

Problem: 5.5×10^{-1}

Solution:

0.55

Multiply and Divide Using Scientific Notation

In the following exercises, multiply and write your answer in decimal form.

Exercise:

Problem: $\left(2 \times 10^{5}\right)\left(4 \times 10^{-3}\right)$

Exercise:

Problem: $\left(3.5 \times 10^{-2}\right)\left(6.2 \times 10^{-1}\right)$

Solution:

0.0217

In the following exercises, divide and write your answer in decimal form.

Exercise:

Problem: $\frac{8 \times 10^5}{4 \times 10^{-1}}$

Exercise:

Problem: $\frac{9 \times 10^{-5}}{3 \times 10^2}$

Solution:

0.0000003

Chapter Practice Test

Exercise:

Problem: For the polynomial $10x^4 + 9y^2 - 1$
ⓐ Is it a monomial, binomial, or trinomial?
ⓑ What is its degree?

In the following exercises, simplify each expression.

Exercise:

Problem: $\left(12a^2 - 7a + 4\right) + \left(3a^2 + 8a - 10\right)$

Solution:

$15a^2 + a - 6$

Exercise:

Problem: $\left(9p^2 - 5p + 1\right) - \left(2p^2 - 6\right)$

Exercise:

Problem: $\left(-\frac{2}{5}\right)^{3}$

Solution:

$-\frac{8}{125}$

Exercise:

Problem: $u \cdot u^{4}$

Exercise:

Problem: $\left(4a^{3}b^{5}\right)^{2}$

Solution:

$16a^{6}b^{10}$

Exercise:

Problem: $\left(-9r^{4}s^{5}\right)\left(4rs^{7}\right)$

Exercise:

Problem: $3k\left(k^{2}-7k+13\right)$

Solution:

$3k^{3}-21k^{2}+39k$

Exercise:

Problem: $(m+6)(m+12)$

Exercise:

Problem: $\left(v-9\right)\left(9v-5\right)$

Solution:

$9v^2-86v+45$

Exercise:

Problem: $\left(4c-11\right)\left(3c-8\right)$

Exercise:

Problem: $\left(n-6\right)\left(n^2-5n+4\right)$

Solution:

$n^3-11n^2+34n-24$

Exercise:

Problem: $\left(2x-15y\right)\left(5x+7y\right)$

Exercise:

Problem: $\left(7p-5\right)\left(7p+5\right)$

Solution:

$49p^2-25$

Exercise:

Problem: $\left(9v-2\right)^2$

Exercise:

Problem: $\frac{3^8}{3^{10}}$

Solution:

$\frac{1}{9}$

Exercise:

Problem: $\left(\frac{m^4 \cdot m}{m^3}\right)^6$

Exercise:

Problem: $\left(87x^{15}y^3z^{22}\right)^0$

Solution:

1

Exercise:

Problem: $\frac{80c^8d^2}{16cd^{10}}$

Exercise:

Problem: $\frac{12x^2+42x-6}{2x}$

Solution:

$6x + 21 - \frac{3}{x}$

Exercise:

Problem: $\left(70xy^4 + 95x^3y\right) \div 5xy$

Exercise:

Problem: $\frac{64x^3-1}{4x-1}$

Solution:

$16x^2 + 4x + 1$

Exercise:

Problem: $\left(y^2 - 5y - 18\right) \div (y + 3)$

Exercise:

Problem: 5^{-2}

Solution:

$\frac{1}{25}$

Exercise:

Problem: $(4m)^{-3}$

Exercise:

Problem: $q^{-4} \cdot q^{-5}$

Solution:

$\frac{1}{q^9}$

Exercise:

Problem: $\frac{n^{-2}}{n^{-10}}$

Exercise:

Problem: Convert 83,000,000 to scientific notation.

Solution:

8.3×10^{7}

Exercise:

Problem: Convert 6.91×10^{-5} to decimal form.

In the following exercises, simplify, and write your answer in decimal form.

Exercise:

Problem: $\left(3.4 \times 10^{9}\right)\left(2.2 \times 10^{-5}\right)$

Solution:

74,800

Exercise:

Problem: $\frac{8.4 \times 10^{-3}}{4 \times 10^{3}}$

Exercise:

Problem:

A helicopter flying at an altitude of 1000 feet drops a rescue package. The polynomial $-16t^{2} + 1000$ gives the height of the package t seconds a after it was dropped. Find the height when $t = 6$ seconds.

Solution:

424 feet

Glossary

negative exponent

If n is a positive integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^{n}}$.

scientific notation

A number is expressed in scientific notation when it is of the form $a \times 10^n$ where $a \geq 1$ and $a < 10$ and n is an integer.

Introduction class="introduction"

The Sydney Harbor Bridge is one of Australia’s most photographed landmarks. It is the world’s largest steel arch bridge with the top of the bridge standing 134 meters above the harbor. Can you see why it is known by the locals as the “Coathanger” ?

Quadratic expressions may be used to model physical properties of a large bridge, the trajectory of a baseball or rocket, and revenue and profit of a business. By factoring these expressions, specific characteristics of the model can be identified. In this chapter, you will explore the process of factoring expressions and see how factoring is used to solve certain types of equations.

Greatest Common Factor and Factor by Grouping

By the end of this section, you will be able to:

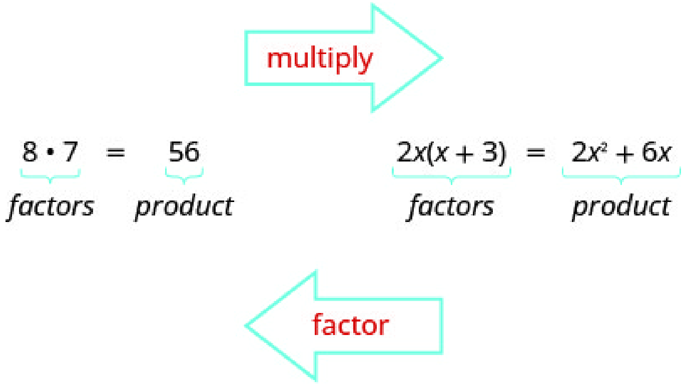
- Find the greatest common factor of two or more expressions
- Factor the greatest common factor from a polynomial
- Factor by grouping

Note:
Before you get started, take this readiness quiz.

1. Factor 56 into primes.
   If you missed this problem, review [link].
2. Find the least common multiple of 18 and 24.
   If you missed this problem, review [link].
3. Simplify $-3(6a+11)$.
   If you missed this problem, review [link].

Find the Greatest Common Factor of Two or More Expressions

Earlier we multiplied factors together to get a product. Now, we will be reversing this process; we will start with a product and then break it down into its factors. Splitting a product into factors is called **factoring**.



We have learned how to factor numbers to find the least common multiple (LCM) of two or more numbers. Now we will factor expressions and find the **greatest common factor** of two or more expressions. The method we use is similar to what we used to find the LCM.

Note:

Greatest Common Factor

The greatest common factor (GCF) of two or more expressions is the largest expression that is a factor of all the expressions.

First we'll find the GCF of two numbers.

Example:

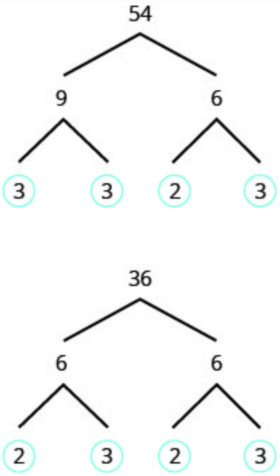
How to Find the Greatest Common Factor of Two or More Expressions

Exercise:

Problem: **Find the GCF of 54 and 36.

Solution:

Solution

Step 1. Factor each coefficient into primes. Write all variables with exponents in expanded form.	Factor 54 and 36.	 54 9 6 3 3 2 3 36 6 6 2 3 2 3
Step 2. In each column, circle the common factors.	Circle the 2, 3, and 3 that are shared by both numbers.	$36 = 2 \cdot 2 \cdot 3 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3 \cdot 3$
Step 3. Bring down the common factors that all expressions share.	Bring down the 2, 3, and 3, and then multiply.	$\text{GCF} = 2 \cdot \quad 3 \cdot 3$
Step 4. Multiply the factors.		$\text{GCF} = 18$ The GCF of 54 and 36 is 18.

Notice that, because the GCF is a factor of both numbers, 54 and 36 can be written as multiples of 18.

Equation:

$$\begin{aligned} 54 &= 18 \cdot 3 \\ 36 &= 18 \cdot 2 \end{aligned}$$

Note:
Exercise:

TRY IT : : 6.1

Problem: Find the GCF of 48 and 80.

Solution:

16

Note:
Exercise:

TRY IT : : 6.2

Problem: Find the GCF of 18 and 40.

Solution:

2

We summarize the steps we use to find the GCF below.

Note:
Find the Greatest Common Factor (GCF) of two expressions.

Factor each coefficient into primes. Write all variables with exponents in expanded form. List all factors—matching common factors in a column. In each column, circle the common factors.

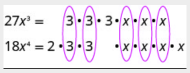
Bring down the common factors that all expressions share.
Multiply the factors.

In the first example, the GCF was a constant. In the next two examples, we will get variables in the greatest common factor.

Example:
Exercise:

Problem: **Find the greatest common factor of $27x^3$ and $18x^4$.

Solution:
Solution

Factor each coefficient into primes and write the variables with exponents in expanded form. Circle the common factors in each column.	 $27x^3 = 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x$ $18x^4 = 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x$
Bring down the common factors.	$\text{GCF} = 3 \cdot 3 \cdot x \cdot x \cdot x$
Multiply the factors.	$\text{GCF} = 9x^3$
	The GCF of $27x^3$ and $18x^4$ is $9x^3$.

Note:
Exercise:

TRY IT : : 6.3

Problem: Find the GCF: $12x^2, 18x^3$.

Solution:

$6x^2$

Note:
Exercise:

TRY IT : : 6.4

Problem: Find the GCF: $16y^2, 24y^3$.

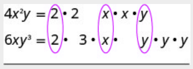
Solution:

$8y^2$

Example:
Exercise:

Problem: **Find the GCF of $4x^2y, 6xy^3$.

Solution:
Solution

Factor each coefficient into primes and write the variables with exponents in expanded form. Circle the common factors in each column.	 $4x^2y = 2 \cdot 2 \quad x \cdot x \cdot y$ $6xy^3 = 2 \cdot 3 \cdot x \cdot y \cdot y \cdot y$
Bring down the common factors.	$\text{GCF} = 2 \cdot x \cdot y$
Multiply the factors.	$\text{GCF} = 2xy$

	The GCF of $4x^2y$ and $6xy^3$ is 2 xy.

Note:
Exercise:

TRY IT : : 6.5

Problem: Find the GCF: $6ab^4$, $8a^2b$.

Solution:

$2ab$

Note:
Exercise:

TRY IT : : 6.6

Problem: Find the GCF: $9m^5n^2$, $12m^3n$.

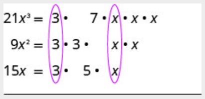
Solution:

$3m^3n$

Example:
Exercise:

Problem: **Find the GCF of: $21x^3$, $9x^2$, $15x$.

Solution:
Solution

Factor each coefficient into primes and write the variables with exponents in expanded form. Circle the common factors in each column.	 $21x^3 = 3 \cdot 7 \cdot x \cdot x \cdot x$ $9x^2 = 3 \cdot 3 \cdot x \cdot x$ $15x = 3 \cdot 5 \cdot x$
Bring down the common factors.	$\text{GCF} = 3 \cdot x$
Multiply the factors.	$\text{GCF} = 3x$
	The GCF of $21x^3$, $9x^2$ and $15x$ is $3x$.

Note:
Exercise:

TRY IT : : 6.7

Problem: Find the greatest common factor: $25m^4, 35m^3, 20m^2$.

Solution:

$5m^2$

Note:
Exercise:

TRY IT : : 6.8

Problem: Find the greatest common factor: $14x^3, 70x^2, 105x$.

Solution:

$7x$

Factor the Greatest Common Factor from a Polynomial

Just like in arithmetic, where it is sometimes useful to represent a number in factored form (for example, 12 as $2\cdot 6$ or $3\cdot 4$), in algebra, it can be useful to represent a polynomial in factored form. One way to do this is by finding the GCF of all the terms. Remember, we multiply a polynomial by a monomial as follows:

Equation:

$$\begin{array}{cc} 2\left(x+7\right) & \text{factors} \\ 2\cdot x+2\cdot 7 & \\ 2x+14 & \text{product} \end{array}$$

Now we will start with a product, like $2x+14$, and end with its factors, $2\left(x+7\right)$. To do this we apply the Distributive Property "in reverse."

We state the Distributive Property here just as you saw it in earlier chapters and "in reverse."

Note:

Distributive Property

If a,b,c are real numbers, then

Equation:

$$a\left(b+c\right)=ab+ac \qquad \text{and} \qquad ab+ac=a\left(b+c\right)$$

The form on the left is used to multiply. The form on the right is used to factor.

So how do you use the Distributive Property to factor a polynomial? You just find the GCF of all the terms and write the polynomial as a product!

Example:

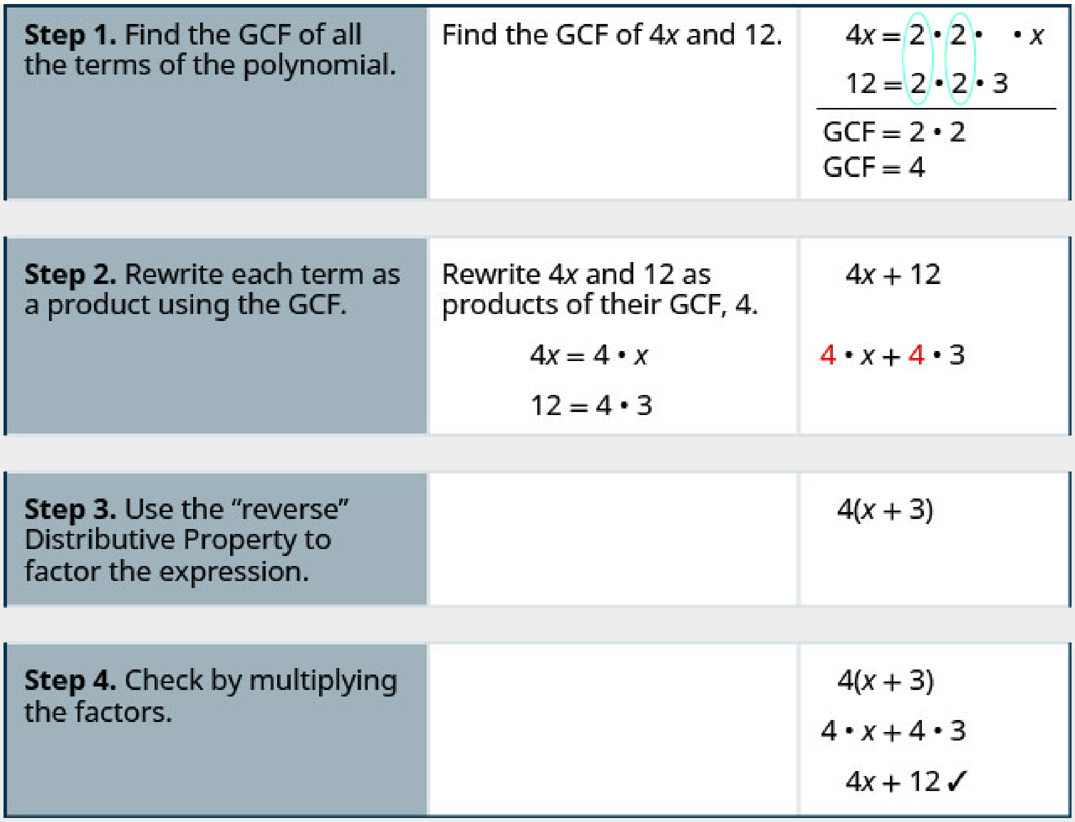
How to Factor the Greatest Common Factor from a Polynomial

Exercise:

Problem: **Factor: $4x+12$.

Solution:

Solution



Step 1. Find the GCF of all the terms of the polynomial.	Find the GCF of $4x$ and 12.	$4x = 2 \cdot 2 \cdot \quad \cdot x$ $12 = 2 \cdot 2 \cdot 3$ $\text{GCF} = 2 \cdot 2$ $\text{GCF} = 4$
Step 2. Rewrite each term as a product using the GCF.	Rewrite $4x$ and 12 as products of their GCF, 4. $4x = 4 \cdot x$ $12 = 4 \cdot 3$	$4x + 12$ $4 \cdot x + 4 \cdot 3$
Step 3. Use the “reverse” Distributive Property to factor the expression.		$4(x + 3)$
Step 4. Check by multiplying the factors.		$4(x + 3)$ $4 \cdot x + 4 \cdot 3$ $4x + 12$ ✓

Note:
Exercise:

TRY IT : : 6.9

Problem: Factor: $6a + 24$.

Solution:

$6\left(a + 4\right)$

Note:
Exercise:

TRY IT : : 6.10

Problem: Factor: $2b + 14$.

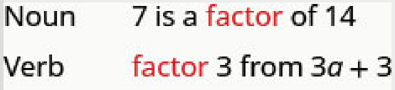
Solution:

$2\,(b+7)$

Note:
Factor the greatest common factor from a polynomial.

Find the GCF of all the terms of the polynomial.
Rewrite each term as a product using the GCF.
Use the “reverse” Distributive Property to factor the expression.
Check by multiplying the factors.

Note:
Factor as a Noun and a Verb
We use “factor” as both a noun and a verb.

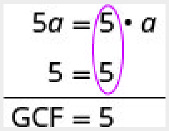


Noun	7 is a factor of 14
Verb	factor 3 from $3a+3$

Example:
Exercise:

Problem: **Factor: $5a+5$.

Solution:
Solution

Find the GCF of 5*a* and 5.	 $5a = 5 \cdot a$ $5 = 5$ $\text{GCF} = 5$

	$5a + 5$
Rewrite each term as a product using the GCF.	$5 \cdot a + 5 \cdot 1$
Use the Distributive Property "in reverse" to factor the GCF.	$5(a + 1)$
Check by mulitplying the factors to get the orginal polynomial.	
$5(a + 1)$	
$5 \cdot a + 5 \cdot 1$	
$5a + 5$✓	

Note:
Exercise:

TRY IT : : 6.11
Problem: Factor: $14x + 14$.

Solution:

$14\,(x + 1)$

Note:
Exercise:

TRY IT : : 6.12
Problem: Factor: $12p + 12$.

Solution:

$12\,(p+1)$

The expressions in the next example have several factors in common. Remember to write the GCF as the product of all the common factors.

Example:
Exercise:

Problem: **Factor: $12x-60$.

Solution:
Solution

Find the GCF of 12*x* and 60.	$12x = 2\cdot 2\cdot 3\cdot x$ $60 = 2\cdot 2\cdot 3\cdot 5$ $\text{GCF} = 2\cdot 2\cdot 3$ $\text{GCF} = 12$
	$12x-60$
Rewrite each term as a product using the GCF.	$12\cdot x-12\cdot 5$
Factor the GCF.	$12(x-5)$

Check by mulitplying the factors.	
$12(x-5)$	
$12 \cdot x - 12 \cdot 5$	
$12x - 60$✓	

Note:
Exercise:

TRY IT : : 6.13
Problem: Factor: $18u - 36$.

Solution:

$18\left(u-2\right)$

Note:
Exercise:

TRY IT : : 6.14
Problem: Factor: $30y - 60$.

Solution:

$30\left(y-2\right)$

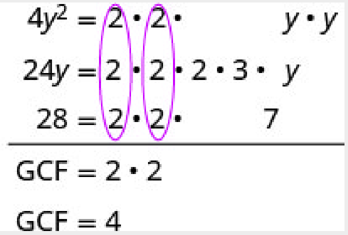
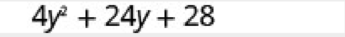
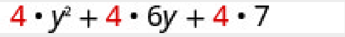
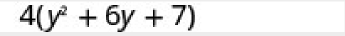
Now we’ll factor the greatest common factor from a trinomial. We start by finding the GCF of all three terms.

Example:
Exercise:

Problem: **Factor: $4y^2 + 24y + 28$.

Solution:
Solution

We start by finding the GCF of all three terms.

Find the GCF of $4y^2$, $24y$ and 28.	 $4y^2 = 2 \cdot 2 \cdot y \cdot y$ $24y = 2 \cdot 2 \cdot 2 \cdot 3 \cdot y$ $28 = 2 \cdot 2 \cdot 7$ $\text{GCF} = 2 \cdot 2$ $\text{GCF} = 4$
	 $4y^2 + 24y + 28$
Rewrite each term as a product using the GCF.	 $4 \cdot y^2 + 4 \cdot 6y + 4 \cdot 7$
Factor the GCF.	 $4(y^2 + 6y + 7)$
Check by mulitplying.	
$4\left(y^2 + 6y + 7\right)$	
$4 \cdot y^2 + 4 \cdot 6y + 4 \cdot 7$	
$4y^2 + 24y + 28$✓	

Note:
Exercise:

TRY IT : : 6.15
Problem: Factor: $5x^2 - 25x + 15$.

Solution:

$5\left(x^2 - 5x + 3\right)$

Note:
Exercise:

TRY IT : : 6.16
Problem: Factor: $3y^2 - 12y + 27$.

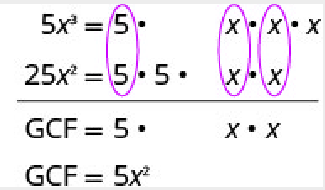
Solution:

$3\left(y^2 - 4y + 9\right)$

Example:
Exercise:

Problem: **Factor: $5x^3 - 25x^2$.

Solution:
Solution

Find the GCF of $5x^3$ and $25x^2$.	 5x³ = 5 • x • x • x 25x² = 5 • 5 • x • x GCF = 5 • x • x GCF = 5x²

	$5x^3 - 25x^2$
Rewrite each term.	$5x^2 \cdot x - 5x^2 \cdot 5$
Factor the GCF.	$5x^2(x - 5)$
Check.	
$5x^2(x-5)$	
$5x^2 \cdot x - 5x^2 \cdot 5$	
$5x^3 - 25x^2$✓	

Note:
Exercise:

TRY IT : : 6.17

Problem: Factor: $2x^3 + 12x^2$.

Solution:

$2x^2\left(x+6\right)$

Note:
Exercise:

TRY IT : : 6.18

Problem: Factor: $6y^3 - 15y^2$.

Solution:

$3y^2\,(2y-5)$

Example:
Exercise:

Problem: **Factor: $21x^3-9x^2+15x$.

Solution:
Solution

In a previous example we found the GCF of $21x^3, 9x^2, 15x$ to be $3x$.

	$21x^3-9x^2+15x$
Rewrite each term using the GCF, 3*x*.	$3x\cdot 7x^2-3x\cdot 3x+3x\cdot 5$
Factor the GCF.	$3x(7x^2-3x+5)$
Check.	
$3x\left(7x^2-3x+5\right)$	
$3x\cdot 7x^2-3x\cdot 3x+3x\cdot 5$	
$21x^3-9x^2+15x$✓	

Note:
Exercise:

TRY IT : : 6.19

Problem: Factor: $20x^3 - 10x^2 + 14x$.

Solution:

$2x\left(10x^2 - 5x + 7\right)$

Note:
Exercise:

TRY IT : : 6.20

Problem: Factor: $24y^3 - 12y^2 - 20y$.

Solution:

$4y\left(6y^2 - 3y - 5\right)$

Example:
Exercise:

Problem: **Factor: $8m^3 - 12m^2n + 20mn^2$.

Solution:
Solution

Find the GCF of $8m^3$, $12m^2n$, $20mn^2$.	

	$8m^3 = 2 \cdot 2 \cdot 2 \cdot m \cdot m \cdot m$ $12m^2n = 2 \cdot 2 \cdot 3 \cdot m \cdot m \cdot n$ $20mn^2 = 2 \cdot 2 \cdot 5 \cdot m \cdot n \cdot n$ $\text{GCF} = 2 \cdot 2 \cdot m$ $\text{GCF} = 4m$
	$8m^3 - 12m^2n + 20mn^2$
Rewrite each term.	$4m \cdot 2m^2 - 4m \cdot 3mn + 4m \cdot 5n^2$
Factor the GCF.	$4m(2m^2 - 3mn + 5n^2)$
Check.	
$4m\left(2m^2 - 3mn + 5n^2\right)$	
$4m \cdot 2m^2 - 4m \cdot 3mn + 4m \cdot 5n^2$	
$8m^3 - 12m^2n + 20mn^2$✓	

Note:
Exercise:

TRY IT : : 6.21

Problem: Factor: $9xy^2 + 6x^2y^2 + 21y^3$.

Solution:

$3y^2\left(3x + 2x^2 + 7y\right)$

Note:
Exercise:

TRY IT : : 6.22
Problem: Factor: $3p^3 - 6p^2q + 9pq^3$.

Solution:

$3p\left(p^2 - 2pq + 3q^3\right)$

When the leading coefficient is negative, we factor the negative out as part of the GCF.

Example:
Exercise:

Problem: **Factor: $-8y - 24$.

Solution:
Solution

When the leading coefficient is negative, the GCF will be negative.

Ignoring the signs of the terms, we first find the GCF of 8*y* and 24 is 8. Since the expression −8*y* − 24 has a negative leading coefficient, we use −8 as the GCF.	$8y = 2 \cdot 2 \cdot 2 \cdot y$ $24 = 2 \cdot 2 \cdot 2 \cdot 3$ $\text{GCF} = 2 \cdot 2 \cdot 2$ $\text{GCF} = 8$
Rewrite each term using the GCF.	$-8y - 24$ $-8 \cdot y + (-8) \cdot 3$
Factor the GCF.	$-8(y + 3)$

Check.	
$-8(y+3)$	
$-8 \cdot y + (-8) \cdot 3$	
$-8y - 24$✓	

Note:
Exercise:

TRY IT : : 6.23
Problem: Factor: $-16z - 64$.

Solution:

$-16\,(z+4)$

Note:
Exercise:

TRY IT : : 6.24
Problem: Factor: $-9y - 27$.

Solution:

$-9\,(y+3)$

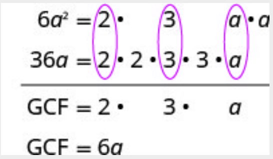
Example:
Exercise:

Problem: **Factor: $-6a^2 + 36a$.

Solution:

Solution

The leading coefficient is negative, so the GCF will be negative.?

Since the leading coefficient is negative, the GCF is negative, $-6a$.	 $6a^2 = 2 \cdot 3 \cdot a \cdot a$ $36a = 2 \cdot 2 \cdot 3 \cdot 3 \cdot a$ $\text{GCF} = 2 \cdot 3 \cdot a$ $\text{GCF} = 6a$ $-6a^2 + 36a$
Rewrite each term using the GCF.	$-6a \cdot a - (-6a) \cdot 6$
Factor the GCF.	$-6a(a - 6)$
Check.	
$-6a(a-6)$	
$-6a \cdot a + (-6a)(-6)$	
$-6a^2 + 36a$✓	

Note:
Exercise:

TRY IT : : 6.25
Problem: Factor: $-4b^2 + 16b$.

Solution:

$-4b\,(b-4)$

Note:
Exercise:

TRY IT : : 6.26

Problem: Factor: $-7a^2+21a$.

Solution:

$-7a\,(a-3)$

Example:
Exercise:

Problem: **Factor: $5q\,(q+7)-6\,(q+7)$.

Solution:
Solution

The GCF is the binomial $q+7$.

	$5q(q+7)-6(q+7)$
Factor the GCF, (q + 7).	$(q+7)(5q-6)$
Check on your own by multiplying.	

Note:
Exercise:

TRY IT : : 6.27
Problem: Factor: $4m\left(m+3\right)-7\left(m+3\right)$.

Solution:

$\left(m+3\right)\left(4m-7\right)$

Note:
Exercise:

TRY IT : : 6.28
Problem: Factor: $8n\left(n-4\right)+5\left(n-4\right)$.

Solution:

$\left(n-4\right)\left(8n+5\right)$

Factor by Grouping

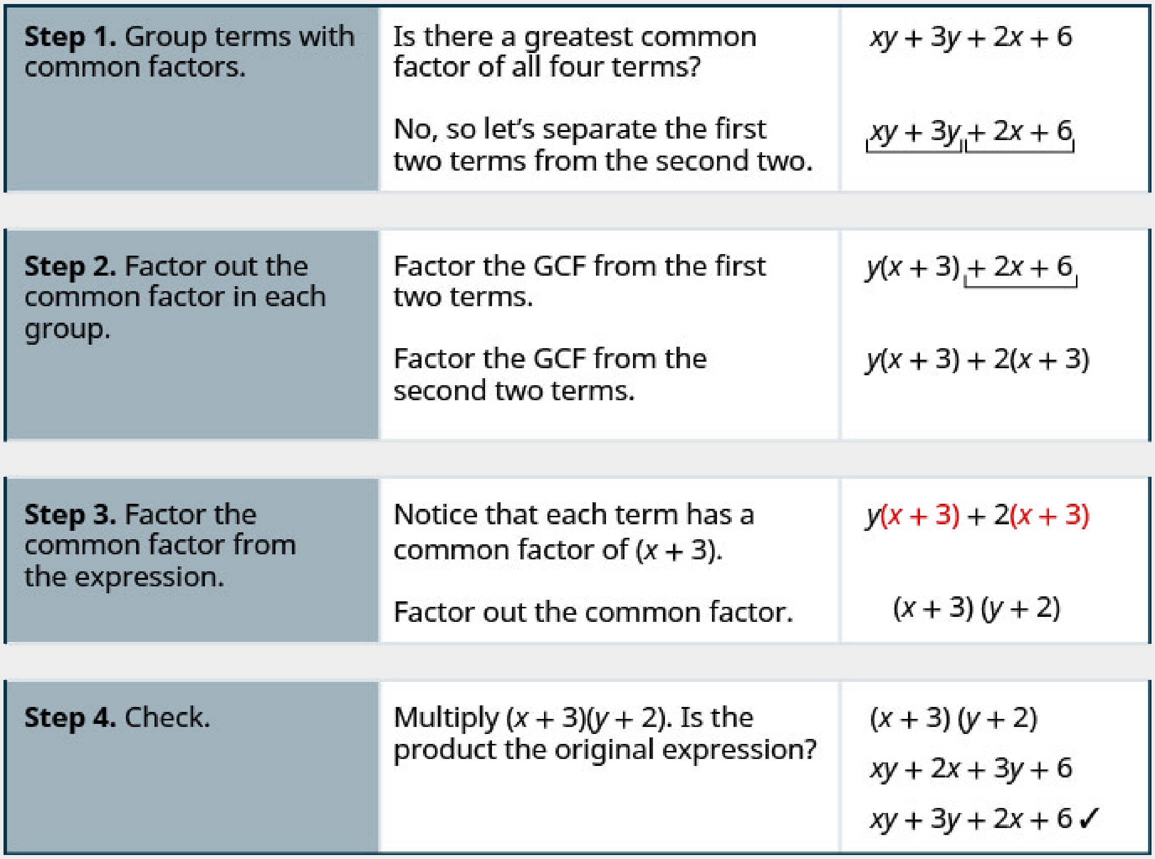
When there is no common factor of all the terms of a polynomial, look for a common factor in just some of the terms. When there are four terms, a good way to start is by separating the polynomial into two parts with two terms in each part. Then look for the GCF in each part. If the polynomial can be factored, you will find a common factor emerges from both parts.

(Not all polynomials can be factored. Just like some numbers are prime, some polynomials are prime.)

Example:
How to Factor by Grouping
Exercise:

Problem: **Factor: $xy+3y+2x+6$.

Solution:
Solution



Step 1. Group terms with common factors.	Is there a greatest common factor of all four terms? No, so let's separate the first two terms from the second two.	$xy+3y+2x+6$ $\underbrace{xy+3y}+\underbrace{2x+6}$
Step 2. Factor out the common factor in each group.	Factor the GCF from the first two terms. Factor the GCF from the second two terms.	$y(x+3)+\underbrace{2x+6}$ $y(x+3)+2(x+3)$
Step 3. Factor the common factor from the expression.	Notice that each term has a common factor of $(x+3)$. Factor out the common factor.	$y(x+3)+2(x+3)$ $(x+3)(y+2)$
Step 4. Check.	Multiply $(x+3)(y+2)$. Is the product the original expression?	$(x+3)(y+2)$ $xy+2x+3y+6$ $xy+3y+2x+6$ ✓

Note:
Exercise:

TRY IT : : 6.29

Problem: Factor: $xy+8y+3x+24$.

Solution:

$(x+8)(y+3)$

Note:
Exercise:

TRY IT : : 6.30

Problem: Factor: $ab + 7b + 8a + 56$.

Solution:

$(a + 7)(b + 8)$

Note:
Factor by grouping.

Group terms with common factors.
Factor out the common factor in each group.
Factor the common factor from the expression.
Check by multiplying the factors.

Example:
Exercise:

Problem: **Factor: $x^2 + 3x - 2x - 6$.

Solution:
Solution

There is no GCF in all four terms. $x^2 + 3x \quad -2x - 6$

Separate into two parts. $x^2 + 3x \quad -2x - 6$

Factor the GCF from both parts. Be careful with the signs when factoring the GCF from the last two terms.

$$x(x + 3) - 2(x + 3)$$
$$(x + 3)(x - 2)$$

Check on your own by multiplying.

Note:
Exercise:

TRY IT : : 6.31

Problem: Factor: $x^2 + 2x - 5x - 10$.

Solution:

$(x - 5)(x + 2)$

Note:

Exercise:

TRY IT : : 6.32

Problem: Factor: $y^2 + 4y - 7y - 28$.

Solution:

$(y + 4)(y - 7)$

Note:

Access these online resources for additional instruction and practice with greatest common factors (GFCs) and factoring by grouping.

- Greatest Common Factor (GCF)
- Factoring Out the GCF of a Binomial
- Greatest Common Factor (GCF) of Polynomials

Key Concepts

- **Finding the Greatest Common Factor (GCF):** To find the GCF of two expressions:

  Factor each coefficient into primes. Write all variables with exponents in expanded form.
  List all factors—matching common factors in a column. In each column, circle the common factors.
  Bring down the common factors that all expressions share.
  Multiply the factors as in[link].

- **Factor the Greatest Common Factor from a Polynomial:** To factor a greatest common factor from a polynomial:

  Find the GCF of all the terms of the polynomial.
  Rewrite each term as a product using the GCF.
  Use the ‘reverse’ Distributive Property to factor the expression.
  Check by multiplying the factors as in[link].

- **Factor by Grouping:** To factor a polynomial with 4 four or more terms

  Group terms with common factors.
  Factor out the common factor in each group.
  Factor the common factor from the expression.
  Check by multiplying the factors as in[link].

Practice Makes Perfect

Find the Greatest Common Factor of Two or More Expressions

In the following exercises, find the greatest common factor.

Exercise:

Problem: 8, 18

Solution:

2

Exercise:

Problem: 24, 40

Exercise:

Problem: 72, 162

Solution:

18

Exercise:

Problem: 150, 275

Exercise:

Problem: $10a$, 50

Solution:

10

Exercise:

Problem: $5b$, 30

Exercise:

Problem: $3x, 10x^2$

Solution:

x

Exercise:

Problem: $21b^2, 14b$

Exercise:

Problem: $8w^2, 24w^3$

Solution:

$8w^2$

Exercise:

Problem: $30x^2, 18x^3$

Exercise:

Problem: $10p^3q, 12pq^2$

Solution:

$2pq$

Exercise:

Problem: $8a^2b^3, 10ab^2$

Exercise:

Problem: $12m^2n^3, 30m^5n^3$

Solution:

$6m^2n^3$

Exercise:

Problem: $28x^2y^4, 42x^4y^4$

Exercise:

Problem: $10a^3, 12a^2, 14a$

Solution:

$2a$

Exercise:

Problem: $20y^3, 28y^2, 40y$

Exercise:

Problem: $35x^3, 10x^4, 5x^5$

Solution:

$5x^3$

Exercise:

Problem: $27p^2, 45p^3, 9p^4$

Factor the Greatest Common Factor from a Polynomial

In the following exercises, factor the greatest common factor from each polynomial.

Exercise:

Problem: $4x + 20$

Solution:

$4\left(x+5\right)$

Exercise:

Problem: $8y+16$

Exercise:

Problem: $6m+9$

Solution:

$3\left(2m+3\right)$

Exercise:

Problem: $14p+35$

Exercise:

Problem: $9q+9$

Solution:

$9\left(q+1\right)$

Exercise:

Problem: $7r+7$

Exercise:

Problem: $8m-8$

Solution:

$8\left(m-1\right)$

Exercise:

Problem: $4n-4$

Exercise:

Problem: $9n-63$

Solution:

$9\,(n-7)$

Exercise:

Problem: $45b-18$

Exercise:

Problem: $3x^2+6x-9$

Solution:

$3(x^2+2x-3)$

Exercise:

Problem: $4y^2+8y-4$

Exercise:

Problem: $8p^2+4p+2$

Solution:

$2(4p^2+2p+1)$

Exercise:

Problem: $10q^2+14q+20$

Exercise:

Problem: $8y^3+16y^2$

Solution:

$8y^2(y+2)$

Exercise:

Problem: $12x^3-10x$

Exercise:

Problem: $5x^3-15x^2+20x$

Solution:

$5x(x^2-3x+4)$

Exercise:

Problem: $8m^2-40m+16$

Exercise:

Problem: $12xy^2+18x^2y^2-30y^3$

Solution:

$6y^2(2x+3x^2-5y)$

Exercise:

Problem: $21pq^2+35p^2q^2-28q^3$

Exercise:

Problem: $-2x-4$

Solution:

$-2\left(x+2\right)$

Exercise:

Problem: $-3b+12$

Exercise:

Problem: $5x\left(x+1\right)+3\left(x+1\right)$

Solution:

$\left(x+1\right)\left(5x+3\right)$

Exercise:

Problem: $2x\left(x-1\right)+9\left(x-1\right)$

Exercise:

Problem: $3b\left(b-2\right)-13\left(b-2\right)$

Solution:

$\left(b-2\right)\left(3b-13\right)$

Exercise:

Problem: $6m\left(m-5\right)-7\left(m-5\right)$

Factor by Grouping

In the following exercises, factor by grouping.

Exercise:

Problem: $xy+2y+3x+6$

Solution:

$(y+3)(x+2)$

Exercise:

Problem: $mn+4n+6m+24$

Exercise:

Problem: $uv-9u+2v-18$

Solution:

$(u+2)(v-9)$

Exercise:

Problem: $pq-10p+8q-80$

Exercise:

Problem: $b^2+5b-4b-20$

Solution:

$(b-4)(b+5)$

Exercise:

Problem: $m^2 + 6m - 12m - 72$

Exercise:

Problem: $p^2 + 4p - 9p - 36$

Solution:

$(p - 9)(p + 4)$

Exercise:

Problem: $x^2 + 5x - 3x - 15$

Mixed Practice

In the following exercises, factor.

Exercise:

Problem: $-20x - 10$

Solution:

$-10\,(2x + 1)$

Exercise:

Problem: $5x^3 - x^2 + x$

Exercise:

Problem: $3x^3 - 7x^2 + 6x - 14$

Solution:

$(x^2 + 2)(3x - 7)$

Exercise:

Problem: $x^3 + x^2 - x - 1$

Exercise:

Problem: $x^2 + xy + 5x + 5y$

Solution:

$(x + y)(x + 5)$

Exercise:

Problem: $5x^3 - 3x^2 - 5x - 3$

Everyday Math

Exercise:

Problem:

Area of a rectangle The area of a rectangle with length 6 less than the width is given by the expression $w^2 - 6w$, where $w =$ width. Factor the greatest common factor from the polynomial.

Solution:

$w(w - 6)$

Exercise:

Problem:

Height of a baseball The height of a baseball *t* seconds after it is hit is given by the expression $-16t^2 + 80t + 4$. Factor the greatest common factor from the polynomial.

Writing Exercises

Exercise:

Problem: The greatest common factor of 36 and 60 is 12. Explain what this means.

Solution:

Answers will vary.

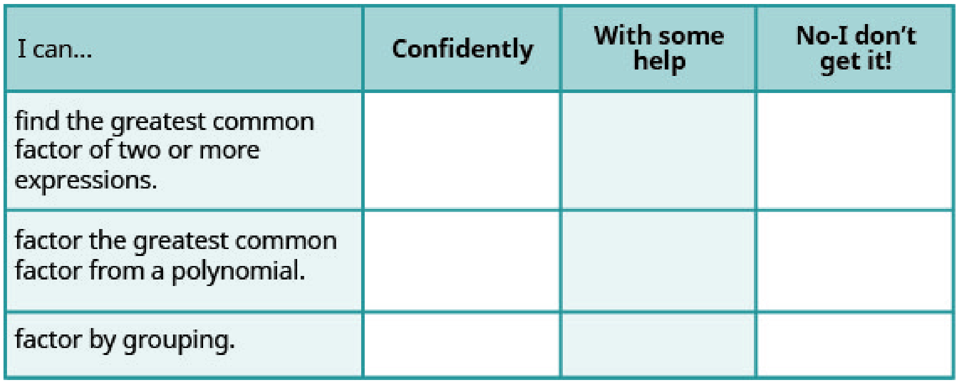
Exercise:

Problem:

What is the GCF of y^4, y^5, and y^{10}? Write a general rule that tells you how to find the GCF of y^a, y^b, andy^c.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
find the greatest common factor of two or more expressions.			
factor the greatest common factor from a polynomial.			
factor by grouping.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved your goals in this section! Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific!

…with some help. This must be addressed quickly as topics you do not master become potholes in your road to success. Math is sequential—every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is critical and you must not ignore it. You need to get help immediately or you will quickly be overwhelmed. See your instructor as soon as possible to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

factoring

Factoring is splitting a product into factors; in other words, it is the reverse process of multiplying.

greatest common factor

The greatest common factor is the largest expression that is a factor of two or more expressions is the greatest common factor (GCF).

Factor Quadratic Trinomials with Leading Coefficient 1
By the end of this section, you will be able to:

- Factor trinomials of the form $x^2 + bx + c$
- Factor trinomials of the form $x^2 + bxy + cy^2$

Note:
Before you get started, take this readiness quiz.

1. Multiply: $(x + 4)(x + 5)$.
   If you missed this problem, review [link].
2. Simplify: ⓐ $-9 + (-6)$ ⓑ $-9 + 6$.
   If you missed this problem, review [link].
3. Simplify: ⓐ $-9(6)$ ⓑ $-9(-6)$.
   If you missed this problem, review [link].
4. Simplify: ⓐ $|-5|$ ⓑ $|3|$.
   If you missed this problem, review [link].

Factor Trinomials of the Form $x^2 + bx + c$

You have already learned how to multiply binomials using FOIL. Now you’ll need to “undo” this multiplication—to start with the product and end up with the factors. Let’s look at an example of multiplying binomials to refresh your memory.

$$(x+2)(x+3) \quad \text{factors}$$
$$\quad F \quad O \quad I \quad L$$
$$\quad x^2 + 3x + 2x + 6$$
$$x^2 + 5x + 6 \quad \text{product}$$

To factor the trinomial means to start with the product, $x^2 + 5x + 6$, and end with the factors, $(x + 2)\,(x + 3)$. You need to think about where each of the terms in the trinomial came from.

The *first term* came from multiplying the first term in each binomial. So to get x^2 in the product, each binomial must start with an *x*.

Equation:

$$\begin{array}{c} x^2+5x+6 \\ (x\quad)(x\quad) \end{array}$$

The *last term* in the trinomial came from multiplying the last term in each binomial. So the last terms must multiply to 6.

What two numbers multiply to 6?

The factors of 6 could be 1 and 6, or 2 and 3. How do you know which pair to use?

Consider the *middle term*. It came from adding the outer and inner terms.

So the numbers that must have a product of 6 will need a sum of 5. We'll test both possibilities and summarize the results in [link]—the table will be very helpful when you work with numbers that can be factored in many different ways.

Factors of 6	**Sum of factors**
$1,6$	$1+6=7$
$2,3$	$2+3=5$

We see that 2 and 3 are the numbers that multiply to 6 and add to 5. So we have the factors of x^2+5x+6. They are $(x+2)(x+3)$.

Equation:

$$\begin{array}{cc} x^2+5x+6 & \text{product} \\ (x+2)(x+3) & \text{factors} \end{array}$$

You should check this by multiplying.

Looking back, we started with x^2+5x+6, which is of the form x^2+bx+c, where $b=5$ and $c=6$. We factored it into two binomials of the form $(x+m)$ and $(x+n)$.

Equation:

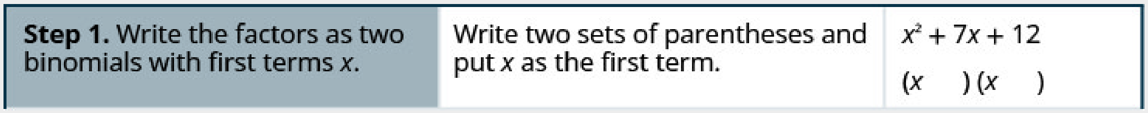
$$\begin{array}{ll} x^2+5x+6 & x^2+bx+c \\ (x+2)\,(x+3) & (x+m)\,(x+n) \end{array}$$

To get the correct factors, we found two numbers *m* and *n* whose product is *c* and sum is *b*.

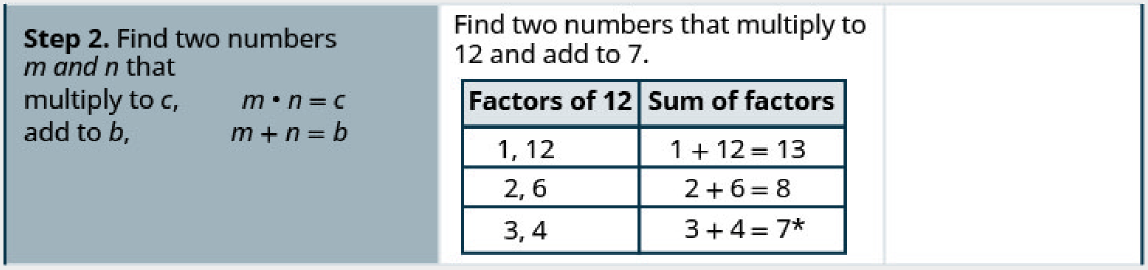
Example:
How to Factor Trinomials of the Form x^2+bx+c
Exercise:

Problem: **Factor: $x^2+7x+12$.

Solution:
Solution

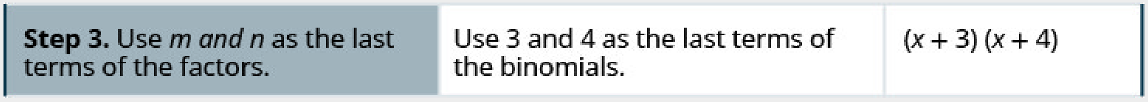


Step 1. Write the factors as two binomials with first terms *x*.	Write two sets of parentheses and put *x* as the first term.	$x^2+7x+12$ $(x \quad)(x \quad)$

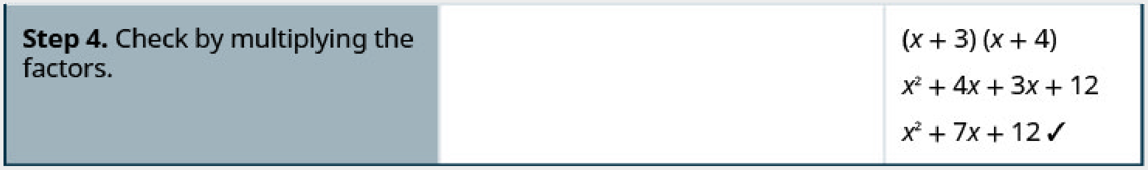


Step 2. Find two numbers *m and n* that multiply to *c*, $m\cdot n=c$ add to *b*, $m+n=b$	Find two numbers that multiply to 12 and add to 7.	

Factors of 12	Sum of factors
1, 12	$1+12=13$
2, 6	$2+6=8$
3, 4	$3+4=7$*



Step 3. Use *m and n* as the last terms of the factors.	Use 3 and 4 as the last terms of the binomials.	$(x+3)\,(x+4)$



Step 4. Check by multiplying the factors.		$(x+3)\,(x+4)$ $x^2+4x+3x+12$ $x^2+7x+12$ ✓

Note:
Exercise:

TRY IT : : 6.33

Problem: Factor: $x^2 + 6x + 8$.

Solution:

$(x + 2)\,(x + 4)$

Note:
Exercise:

TRY IT : : 6.34

Problem: Factor: $y^2 + 8y + 15$.

Solution:

$(y + 3)\,(y + 5)$

Let's summarize the steps we used to find the factors.

Note:
Factor trinomials of the form $x^2 + bx + c$.

Write the factors as two binomials with first terms*x*: $(x \quad)\,(x \quad)$.
Find two numbers*m*and*n*that Multiply to*c*,$m \cdot n = c$ Add to*b*,$m + n = b$
Use*m*and*n*as the last terms of the factors: $(x + m)(x + n)$.
Check by multiplying the factors.

Example:
Exercise:

Problem: **Factor: $u^2 + 11u + 24$.

Solution:
Solution

Notice that the variable is u, so the factors will have first terms u.

$$u^2 + 11u + 24$$

Write the factors as two binomials with first terms u. $(u \quad)(u \quad)$

Find two numbers that: multiply to 24 and add to 11.

Factors of 24	**Sum of factors**
$1, 24$	$1 + 24 = 25$
$2, 12$	$2 + 12 = 14$
$3, 8$	$3 + 8 = 11$*
$4, 6$	$4 + 6 = 10$

Use 3 and 8 as the last terms of the binomials. $(u + 3)(u + 8)$

Check.

$$(u + 3)(u + 8)$$
$$u^2 + 3u + 8u + 24$$
$$u^2 + 11u + 24 \checkmark$$

Note:
Exercise:

TRY IT : : 6.35

Problem: Factor: $q^2 + 10q + 24$.

Solution:

$(q+4)(q+6)$

Note:
Exercise:

TRY IT : : 6.36
Problem: Factor: $t^2+14t+24$.

Solution:

$(t+2)(t+12)$

Example:
Exercise:

Problem: **Factor: $y^2+17y+60$.

Solution:
Solution

$y^2+17y+60$

Write the factors as two binomials with first terms y. $(y\quad)(y\quad)$

Find two numbers that multiply to 60 and add to 17.

Factors of 60	**Sum of factors**
$1,60$	$1+60=61$

Factors of 60	**Sum of factors**
$2, 30$	$2 + 30 = 32$
$3, 20$	$3 + 20 = 23$
$4, 15$	$4 + 15 = 19$
$5, 12$	$5 + 12 = 17$*
$6, 10$	$6 + 10 = 16$

Use 5 and 12 as the last terms. $(y + 5)\,(y + 12)$

Check.

$$(y + 5)(y + 12)$$
$$(y^2 + 12y + 5y + 60)$$
$$(y^2 + 17y + 60)\checkmark$$

Note:
Exercise:

TRY IT : : 6.37

Problem: Factor: $x^2 + 19x + 60$.

Solution:

$(x + 4)\,(x + 15)$

Note:
Exercise:

TRY IT : : 6.38

Problem: Factor: $v^2 + 23v + 60$.

Solution:

$\left(v+3\right)\left(v+20\right)$

Factor Trinomials of the Form $x^2 + bx + c$ with b Negative, c Positive

In the examples so far, all terms in the trinomial were positive. What happens when there are negative terms? Well, it depends which term is negative. Let's look first at trinomials with only the middle term negative.

Remember: To get a negative sum and a positive product, the numbers must both be negative.

Again, think about FOIL and where each term in the trinomial came from. Just as before,

- the first term, x^2, comes from the product of the two first terms in each binomial factor, *x* and *y*;
- the positive last term is the product of the two last terms
- the negative middle term is the sum of the outer and inner terms.

How do you get a *positive product* and a *negative sum*? With two negative numbers.

Example:
Exercise:

Problem: **Factor: $t^2-11t+28$.

Solution:
Solution

Again, with the positive last term, 28, and the negative middle term, $-11t$, we need two negative factors. Find two numbers that multiply 28 and add to -11.

$$t^2-11t+28$$

Write the factors as two binomials with first terms t. $\left(t\quad\right)\left(t\quad\right)$

Find two numbers that: multiply to 28 and add to -11.

Factors of 28	**Sum of factors**
$-1, -28$	$-1 + (-28) = -29$
$-2, -14$	$-2 + (-14) = -16$
$-4, -7$	$-4 + (-7) = -11$*

Use $-4, -7$ as the last terms of the binomials. $(t-4)(t-7)$

Check.

$$(t-4)(t-7)$$
$$t^2 - 7t - 4t + 28$$
$$t^2 - 11t + 28 \checkmark$$

Note:
Exercise:

TRY IT : : 6.39

Problem: Factor: $u^2 - 9u + 18$.

Solution:

$(u-3)(u-6)$

Note:
Exercise:

TRY IT : : 6.40
Problem: Factor: $y^2 - 16y + 63$.

Solution:

$(y-7)(y-9)$

Factor Trinomials of the Form $x^2 + bx + c$ with *c* Negative

Now, what if the last term in the trinomial is negative? Think about FOIL. The last term is the product of the last terms in the two binomials. A negative product results from multiplying two numbers with opposite signs. You have to be very careful to choose factors to make sure you get the correct sign for the middle term, too.

Remember: To get a negative product, the numbers must have different signs.

Example:
Exercise:

Problem: **Factor: $z^2 + 4z - 5$.

Solution:
Solution

To get a negative last term, multiply one positive and one negative. We need factors of -5 that add to positive 4.

Factors of -5	Sum of factors
$1, -5$	$1 + (-5) = -4$

Factors of -5	**Sum of factors**
$-1, 5$	$-1+5=4$*

Notice: We listed both $1, -5$ and $-1, 5$ to make sure we got the sign of the middle term correct.

	z^2+4z-5
Factors will be two binomials with first terms z.	$(z \quad)(z \quad)$
Use $-1, 5$ as the last terms of the binomials.	$(z-1)(z+5)$
Check.	

$$(z-1)(z+5)$$
$$z^2+5z-1z-5$$
$$z^2+4z-5 \checkmark$$

Note:
Exercise:

TRY IT : : 6.41
Problem: Factor: $h^2+4h-12$.

Solution:

$(h-2)(h+6)$

Note:
Exercise:

TRY IT : : 6.42
Problem: Factor: k^2+k-20.

Solution:

$(k-4)(k+5)$

Let’s make a minor change to the last trinomial and see what effect it has on the factors.

Example:
Exercise:

Problem: **Factor: z^2-4z-5.

Solution:
Solution

This time, we need factors of -5 that add to -4.

Factors of -5	**Sum of factors**
$1,-5$	$1+(-5)=-4$*
$-1,5$	$-1+5=4$

	z^2-4z-5
Factors will be two binomials with first terms z.	$(z\quad)(z\quad)$
Use $1,-5$ as the last terms of the binomials.	$(z+1)(z-5)$
Check.	

$$(z+1)(z-5)$$
$$z^2-5z+1z-5$$
$$z^2-4z-5\ \checkmark$$

Notice that the factors of $z^2 - 4z - 5$ are very similar to the factors of $z^2 + 4z - 5$. It is very important to make sure you choose the factor pair that results in the correct sign of the middle term.

Note:
Exercise:

TRY IT : : 6.43
Problem: Factor: $x^2 - 4x - 12$.

Solution:

$\left(x+2\right)\left(x-6\right)$

Note:
Exercise:

TRY IT : : 6.44
Problem: Factor: $y^2 - y - 20$.

Solution:

$\left(y+4\right)\left(y-5\right)$

Example:
Exercise:

Problem: **Factor: $q^2 - 2q - 15$.

Solution:
Solution

$q^2 - 2q - 15$

Factors will be two binomials with first terms q.

$(q \quad)(q \quad)$

You can use $3, -5$ as the last terms of the binomials.

$(q+3)(q-5)$

Factors of -15	**Sum of factors**
$1, -15$	$1 + (-15) = -14$
$-1, 15$	$-1 + 15 = 14$
$3, -5$	$3 + (-5) = -2$*
$-3, 5$	$-3 + 5 = 2$

Check.

$$(q+3)(q-5)$$

$$q^2 - 5q + 3q - 15$$

$$q^2 - 2q - 15 \checkmark$$

Note:
Exercise:

TRY IT : : 6.45
Problem: Factor: $r^2 - 3r - 40$.

Solution:

$(r+5)(r-8)$

Note:
Exercise:

TRY IT : : 6.46

Problem: Factor: $s^2 - 3s - 10$.

Solution:

$(s+2)(s-5)$

Some trinomials are prime. The only way to be certain a trinomial is prime is to list all the possibilities and show that none of them work.

Example:
Exercise:

Problem: **Factor: $y^2 - 6y + 15$.

Solution:
Solution

$$y^2 - 6y + 15$$

Factors will be two binomials with first terms y.

$$(y\quad)(y\quad)$$

Factors of 15	**Sum of factors**
$-1, -15$	$-1 + (-15) = -16$
$-3, -5$	$-3 + (-5) = -8$

As shown in the table, none of the factors add to -6; therefore, the expression is prime.

Note:
Exercise:

TRY IT : : 6.47
Problem: Factor: $m^2 + 4m + 18$.

Solution:

prime

Note:
Exercise:

TRY IT : : 6.48
Problem: Factor: $n^2 - 10n + 12$.

Solution:

prime

Example:
Exercise:

Problem: **Factor: $2x + x^2 - 48$.

Solution:
Solution

$2x + x^2 - 48$

First we put the terms in decreasing degree order. $x^2 + 2x - 48$

Factors will be two binomials with first terms x. $(x \quad)(x \quad)$

As shown in the table, you can use $-6, 8$ as the last terms of the binomials.

Equation:

$$(x - 6)(x + 8)$$

Factors of -48	**Sum of factors**
$-1, 48$	$-1 + 48 = 47$
$-2, 24$ $-3, 16$ $-4, 12$ $-6, 8$	$-2 + 24 = 22$ $-3 + 16 = 13$ $-4 + 12 = 8$ $-6 + 8 = 2$

Check.

$(x - 6)(x + 8)$

$x^2 - 6q + 8q - 48$

$x^2 + 2x - 48$ ✓

Note:

Exercise:

TRY IT : : 6.49

Problem: Factor: $9m + m^2 + 18$.

Solution:

$(m + 3)(m + 6)$

Note:
Exercise:

TRY IT : : 6.50
Problem: Factor: $-7n + 12 + n^2$.

Solution:

$(n - 3)(n - 4)$

Let’s summarize the method we just developed to factor trinomials of the form $x^2 + bx + c$.

Note:
Factor trinomials.
When we factor a trinomial, we look at the signs of its terms first to determine the signs of the binomial factors.
Equation:

$$\begin{array}{c} x^2 + bx + c \\ (x + m)(x + n) \end{array}$$

When *c* is positive, *m* and *n* have the same sign.
Equation:

$$\begin{array}{cc} b \text{ positive} & b \text{ negative} \\ m, n \text{ positive} & m, n \text{ negative} \\ x^2 + 5x + 6 & x^2 - 6x + 8 \\ (x + 2)(x + 3) & (x - 4)(x - 2) \\ \text{same signs} & \text{same signs} \end{array}$$

When *c* is negative, *m* and *n* have opposite signs.

Equation:

$$\begin{array}{cc} x^2 + x - 12 & x^2 - 2x - 15 \\ (x+4)(x-3) & (x-5)(x+3) \\ \text{opposite signs} & \text{opposite signs} \end{array}$$

Notice that, in the case when *m* and *n* have opposite signs, the sign of the one with the larger absolute value matches the sign of *b*.

Factor Trinomials of the Form $x^2 + bxy + cy^2$

Sometimes you'll need to factor trinomials of the form $x^2 + bxy + cy^2$ with two variables, such as $x^2 + 12xy + 36y^2$. The first term, x^2, is the product of the first terms of the binomial factors, $x \cdot x$. The y^2 in the last term means that the second terms of the binomial factors must each contain *y*. To get the coefficients *b* and *c*, you use the same process summarized in the previous objective.

Example:
Exercise:

Problem: **Factor: $x^2 + 12xy + 36y^2$.

Solution:
Solution

$$x^2 + 12xy + 36y^2$$

Note that the first terms are x, last terms contain y.

$$(x \_ y)(x \_ y)$$

Find the numbers that multiply to 36 and add to 12.

Factors of 36	**Sum of factors**

Factors of 36	**Sum of factors**
1, 36	$1 + 36 = 37$
2, 18	$2 + 18 = 20$
3, 12	$3 + 12 = 15$
4, 9	$4 + 9 = 13$
6, 6	$6 + 6 = 12$*

Use 6 and 6 as the coefficients of the last terms. $(x + 6y)(x + 6y)$

Check your answer.

$$(x + 6y)(x + 6y)$$
$$x^2 + 6xy + 6xy + 36y^2$$
$$x^2 + 12xy + 36y^2 \checkmark$$

Note:
Exercise:

TRY IT : : 6.51

Problem: Factor: $u^2 + 11uv + 28v^2$.

Solution:

$(u + 4v)(u + 7v)$

Note:
Exercise:

TRY IT : : 6.52
Factor: $x^2 + 13xy + 42y^2$.

Problem:

Solution:

$(x + 6y)(x + 7y)$

Example:
Exercise:

Problem: **Factor: $r^2 - 8rs - 9s^2$.

Solution:
Solution

We need r in the first term of each binomial and s in the second term. The last term of the trinomial is negative, so the factors must have opposite signs.

$r^2 - 8rs - 9s^2$

Note that the first terms are r, last terms contain s. $(r \_ s)(r \_ s)$

Find the numbers that multiply to -9 and add to -8.

Factors of -9	**Sum of factors**
$1, -9$	$1 + (-9) = -8$*
$-1, 9$	$-1 + 9 = 8$
$3, -3$	$3 + (-3) = 0$

Use $1, -9$ as coefficients of the last terms. $(r+s)(r-9s)$

Check your answer.

$$(r-9s)(r+s)$$
$$r^2+rs-9rs-9s^2$$
$$r^2-8rs-9s^2 \checkmark$$

Note:
Exercise:

TRY IT : : 6.53

Problem: Factor: $a^2-11ab+10b^2$.

Solution:

$(a-b)(a-10b)$

Note:
Exercise:

TRY IT : : 6.54

Problem: Factor: $m^2-13mn+12n^2$.

Solution:

$(m-n)(m-12n)$

Example:
Exercise:

Problem: **Factor: $u^2 - 9uv - 12v^2$.

Solution:
Solution

We need u in the first term of each binomial and v in the second term. The last term of the trinomial is negative, so the factors must have opposite signs.

$$u^2 - 9uv - 12v^2$$

Note that the first terms are u, last terms contain v. $\quad (u \_ v)(u \_ v)$

Find the numbers that multiply to -12 and add to -9.

Factors of -12	**Sum of factors**
$1, -12$	$1 + (-12) = -11$
$-1, 12$	$-1 + 12 = 11$
$2, -6$	$2 + (-6) = -4$
$-2, 6$	$-2 + 6 = 4$
$3, -4$	$3 + (-4) = -1$
$-3, 4$	$-3 + 4 = 1$

Note there are no factor pairs that give us -9 as a sum. The trinomial is prime.

Note:
Exercise:

TRY IT : : 6.55

Problem: Factor: $x^2 - 7xy - 10y^2$.

Solution:

prime

Note:

Exercise:

TRY IT : : 6.56

Problem: Factor: $p^2 + 15pq + 20q^2$.

Solution:

prime

Key Concepts

- **Factor trinomials of the form $x^2 + bx + c$**

  Write the factors as two binomials with first terms$x$:$(x \quad)\,(x \quad)$.
  Find two numbers$m$and$n$thatMultiply to$c$,$m \cdot n = c$Add to$b$,$m + n = b$
  Use$m$and$n$as the last terms of the factors:$(x + m)(x + n)$.
  Check by multiplying the factors.

Practice Makes Perfect

Factor Trinomials of the Form $x^2 + bx + c$

In the following exercises, factor each trinomial of the form $x^2 + bx + c$.

Exercise:

Problem: $x^2 + 4x + 3$

Solution:

$\left(x+1\right)\left(x+3\right)$

Exercise:

Problem: y^2+8y+7

Exercise:

Problem: $m^2+12m+11$

Solution:

$\left(m+1\right)\left(m+11\right)$

Exercise:

Problem: $b^2+14b+13$

Exercise:

Problem: $a^2+9a+20$

Solution:

$\left(a+4\right)\left(a+5\right)$

Exercise:

Problem: $m^2+7m+12$

Exercise:

Problem: $p^2+11p+30$

Solution:

$\left(p+5\right)\left(p+6\right)$

Exercise:

Problem: $w^2+10x+21$

Exercise:

Problem: $n^2 + 19n + 48$

Solution:

$(n + 3)\,(n + 16)$

Exercise:

Problem: $b^2 + 14b + 48$

Exercise:

Problem: $a^2 + 25a + 100$

Solution:

$(a + 5)\,(a + 20)$

Exercise:

Problem: $u^2 + 101u + 100$

Exercise:

Problem: $x^2 - 8x + 12$

Solution:

$(x - 2)(x - 6)$

Exercise:

Problem: $q^2 - 13q + 36$

Exercise:

Problem: $y^2 - 18y + 45$

Solution:

$(y - 3)(y - 15)$

Exercise:

Problem: $m^2 - 13m + 30$

Exercise:

Problem: $x^2 - 8x + 7$

Solution:

$(x - 1)(x - 7)$

Exercise:

Problem: $y^2 - 5y + 6$

Exercise:

Problem: $p^2 + 5p - 6$

Solution:

$(p - 1)(p + 6)$

Exercise:

Problem: $n^2 + 6n - 7$

Exercise:

Problem: $y^2 - 6y - 7$

Solution:

$(y + 1)\,(y - 7)$

Exercise:

Problem: $v^2 - 2v - 3$

Exercise:

Problem: $x^2 - x - 12$

Solution:

$(x-4)(x+3)$

Exercise:

Problem: r^2-2r-8

Exercise:

Problem: $a^2-3a-28$

Solution:

$(a-7)(a+4)$

Exercise:

Problem: $b^2-13b-30$

Exercise:

Problem: $w^2-5w-36$

Solution:

$(w-9)(w+4)$

Exercise:

Problem: $t^2-3t-54$

Exercise:

Problem: x^2+x+5

Solution:

prime

Exercise:

Problem: x^2-3x-9

Exercise:

Problem: $8 - 6x + x^2$

Solution:

$(x-4)(x-2)$

Exercise:

Problem: $7x + x^2 + 6$

Exercise:

Problem: $x^2 - 12 - 11x$

Solution:

$(x-12)(x+1)$

Exercise:

Problem: $-11 - 10x + x^2$

Factor Trinomials of the Form $x^2 + bxy + cy^2$

In the following exercises, factor each trinomial of the form $x^2 + bxy + cy^2$.

Exercise:

Problem: $p^2 + 3pq + 2q^2$

Solution:

$(p+q)(p+2q)$

Exercise:

Problem: $m^2 + 6mn + 5n^2$

Exercise:

Problem: $r^2 + 15rs + 36s^2$

Solution:

$(r+3s)(r+12s)$

Exercise:

Problem: $u^2+10uv+24v^2$

Exercise:

Problem: $m^2-12mn+20n^2$

Solution:

$(m-2n)(m-10n)$

Exercise:

Problem: $p^2-16pq+63q^2$

Exercise:

Problem: $x^2-2xy-80y^2$

Solution:

$(x+8y)(x-10y)$

Exercise:

Problem: $p^2-8pq-65q^2$

Exercise:

Problem: $m^2-64mn-65n^2$

Solution:

$(m+n)(m-65n)$

Exercise:

Problem: $p^2-2pq-35q^2$

Exercise:

Problem: $a^2 + 5ab - 24b^2$

Solution:

$(a + 8b)(a - 3b)$

Exercise:

Problem: $r^2 + 3rs - 28s^2$

Exercise:

Problem: $x^2 - 3xy - 14y^2$

Solution:

prime

Exercise:

Problem: $u^2 - 8uv - 24v^2$

Exercise:

Problem: $m^2 - 5mn + 30n^2$

Solution:

prime

Exercise:

Problem: $c^2 - 7cd + 18d^2$

Mixed Practice

In the following exercises, factor each expression.

Exercise:

Problem: $u^2 - 12u + 36$

Solution:

$(u-6)(u-6)$

Exercise:

Problem: $w^2+4w-32$

Exercise:

Problem: $x^2-14x-32$

Solution:

$(x+2)(x-16)$

Exercise:

Problem: $y^2+41y+40$

Exercise:

Problem: $r^2-20rs+64s^2$

Solution:

$(r-4s)(r-16s)$

Exercise:

Problem: $x^2-16xy+64y^2$

Exercise:

Problem: $k^2+34k+120$

Solution:

$(k+4)(k+30)$

Exercise:

Problem: $m^2+29m+120$

Exercise:

Problem: $y^2 + 10y + 15$

Solution:

prime

Exercise:

Problem: $z^2 - 3z + 28$

Exercise:

Problem: $m^2 + mn - 56n^2$

Solution:

$(m + 8n)(m - 7n)$

Exercise:

Problem: $q^2 - 29qr - 96r^2$

Exercise:

Problem: $u^2 - 17uv + 30v^2$

Solution:

$(u - 15v)(u - 2v)$

Exercise:

Problem: $m^2 - 31mn + 30n^2$

Exercise:

Problem: $c^2 - 8cd + 26d^2$

Solution:

prime

Exercise:

Problem: $r^2 + 11rs + 36s^2$

Everyday Math

Exercise:

Problem:

Consecutive integers Deirdre is thinking of two consecutive integers whose product is 56. The trinomial $x^2 + x - 56$ describes how these numbers are related. Factor the trinomial.

Solution:

$(x + 8)(x - 7)$

Exercise:

Problem:

Consecutive integers Deshawn is thinking of two consecutive integers whose product is 182. The trinomial $x^2 + x - 182$ describes how these numbers are related. Factor the trinomial.

Writing Exercises

Exercise:

Problem:

Many trinomials of the form $x^2 + bx + c$ factor into the product of two binomials $(x + m)(x + n)$. Explain how you find the values of *m* and *n*.

Solution:

Answers may vary

Exercise:

Problem:

How do you determine whether to use plus or minus signs in the binomial factors of a trinomial of the form $x^2 + bx + c$ where b and c may be positive or negative numbers?

Exercise:

Problem:

Will factored $x^2 - x - 20$ as $(x + 5)(x - 4)$. Bill factored it as $(x + 4)(x - 5)$. Phil factored it as $(x - 5)(x - 4)$. Who is correct? Explain why the other two are wrong.

Solution:

Answers may vary

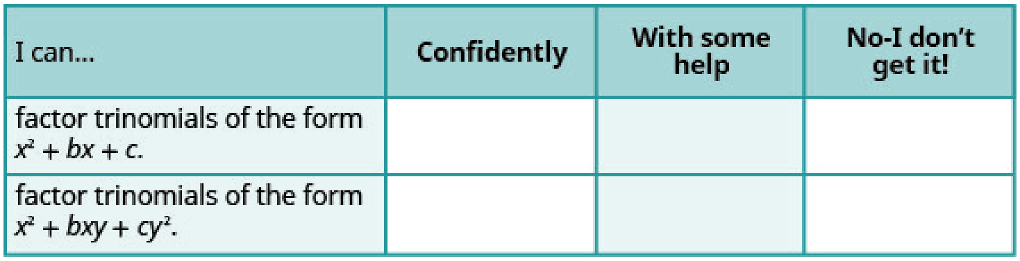
Exercise:

Problem:

Look at [link], where we factored $y^2 + 17y + 60$. We made a table listing all pairs of factors of 60 and their sums. Do you find this kind of table helpful? Why or why not?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
factor trinomials of the form $x^2 + bx + c$.			
factor trinomials of the form $x^2 + bxy + cy^2$.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

Factor Quadratic Trinomials with Leading Coefficient Other than 1

By the end of this section, you will be able to:

- Recognize a preliminary strategy to factor polynomials completely
- Factor trinomials of the form $ax^2 + bx + c$ with a GCF
- Factor trinomials using trial and error
- Factor trinomials using the ‘ac’ method

Note:
Before you get started, take this readiness quiz.

1. Find the GCF of $45p^2$ and $30p^6$.
   If you missed this problem, review [link].
2. Multiply $(3y + 4)(2y + 5)$.
   If you missed this problem, review [link].
3. Combine like terms $12x^2 + 3x + 5x + 9$.
   If you missed this problem, review [link].

Recognize a Preliminary Strategy for Factoring

Let’s summarize where we are so far with factoring polynomials. In the first two sections of this chapter, we used three methods of factoring: factoring the GCF, factoring by grouping, and factoring a trinomial by “undoing” FOIL. More methods will follow as you continue in this chapter, as well as later in your studies of algebra.

How will you know when to use each factoring method? As you learn more methods of factoring, how will you know when to apply each method and not get them confused? It will help to organize the factoring methods into a strategy that can guide you to use the correct method.

As you start to factor a polynomial, always ask first, “Is there a greatest common factor?” If there is, factor it first.

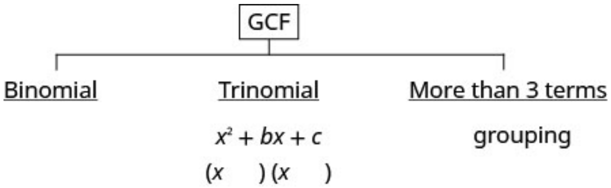
The next thing to consider is the type of polynomial. How many terms does it have? Is it a binomial? A trinomial? Or does it have more than three terms?

If it is a trinomial where the leading coefficient is one, $x^2 + bx + c$, use the “undo FOIL” method.

If it has more than three terms, try the grouping method. This is the only method to use for polynomials of more than three terms.

Some polynomials cannot be factored. They are called “prime.”

Below we summarize the methods we have so far. These are detailed in Choose a strategy to factor polynomials completely.



Note:
Choose a strategy to factor polynomials completely.

Is there a greatest common factor?	- Factor it out.
Is the polynomial a binomial, trinomial, or are there more than three terms?	- If it is a binomial, right now we have no method to factor it. - If it is a trinomial of the form $x^2 + bx + c$: Undo FOIL $(x \quad)(x \quad)$ - If it has more than three terms: Use the grouping method.
Check by multiplying the factors.	

Use the preliminary strategy to completely factor a polynomial. A polynomial is factored completely if, other than monomials, all of its factors are prime.

Example:
Exercise:

Problem: **Identify the best method to use to factor each polynomial.

ⓐ $6y^2 - 72$
ⓑ $r^2 - 10r - 24$
ⓒ $p^2 + 5p + pq + 5q$

Solution:
Solution

ⓐ

	$6y^2 - 72$
Is there a greatest common factor?	Yes, 6.
Factor out the 6.	$6\left(y^2 - 12\right)$
Is it a binomial, trinomial, or are there more than 3 terms?	Binomial, we have no method to factor binomials yet.

ⓑ

	$r^2 - 10r - 24$
Is there a greatest common factor?	No, there is no common factor.
Is it a binomial, trinomial, or are there more than three terms?	Trinomial, with leading coefficient 1, so “undo” FOIL.

ⓒ

	$p^2 + 5p + pq + 5q$
Is there a greatest common factor?	No, there is no common factor.
Is it a binomial, trinomial, or are there more than three terms?	More than three terms, so factor using grouping.

Note:
Exercise:

TRY IT : : 6.57

Problem: Identify the best method to use to factor each polynomial:

ⓐ $4y^2 + 32$
ⓑ $y^2 + 10y + 21$
ⓒ $yz + 2y + 3z + 6$

Solution:

ⓐ no method ⓑ undo using FOIL ⓒ factor with grouping

Note:
Exercise:

TRY IT : : 6.58

Problem: Identify the best method to use to factor each polynomial:

ⓐ $ab + a + 4b + 4$
ⓑ $3k^2 + 15$
ⓒ $p^2 + 9p + 8$

Solution:

ⓐ factor using grouping ⓑ no method ⓒ undo using FOIL

Factor Trinomials of the form $ax^2 + bx + c$ with a GCF

Now that we have organized what we've covered so far, we are ready to factor trinomials whose leading coefficient is not 1, trinomials of the form $ax^2 + bx + c$.

Remember to always check for a GCF first! Sometimes, after you factor the GCF, the leading coefficient of the trinomial becomes 1 and you can factor it by the methods in the last section. Let's do a few examples to see how this works.

Watch out for the signs in the next two examples.

Example:
Exercise:

Problem: **Factor completely: $2n^2 - 8n - 42$.

Solution:
Solution

Use the preliminary strategy.

Is there a greatest common factor?	$2n^2 - 8n - 42$
Yes, GCF = 2. Factor it out.	$2\left(n^2 - 4n - 21\right)$
Inside the parentheses, is it a binomial, trinomial, or are there more than three terms?	
It is a trinomial whose coefficient is 1, so undo FOIL.	$2\left(n \quad\right)\left(n \quad\right)$
Use 3 and -7 as the last terms of the binomials.	$2\left(n+3\right)\left(n-7\right)$

Factors of -21	**Sum of factors**
$1, -21$	$1 + (-21) = -20$
$3, -7$	$3 + (-7) = -4$*

Check.

$$2\left(n+3\right)\left(n-7\right)$$

$$2(n^2 - 7n + 3n - 21)$$

$$2(n^2 - 4n - 21)$$

$$2n^2 - 8n - 42 \checkmark$$

Note:
Exercise:

TRY IT : : 6.59
Problem: Factor completely: $4m^2 - 4m - 8$.

Solution:

$4\left(m+1\right)\left(m-2\right)$

Note:
Exercise:

TRY IT : : 6.60

Problem: Factor completely: $5k^2 - 15k - 50$.

Solution:

$5(k+2)(k-5)$

Example:
Exercise:

Problem: **Factor completely: $4y^2 - 36y + 56$.

Solution:
Solution

Use the preliminary strategy.

Is there a greatest common factor?	$4y^2 - 36y + 56$
Yes, GCF $= 4$. Factor it.	$4\left(y^2 - 9y + 14\right)$
Inside the parentheses, is it a binomial, trinomial, or are there more than three terms?	
It is a trinomial whose coefficient is 1. So undo FOIL.	$4(y \quad)(y \quad)$
Use a table like the one below to find two numbers that multiply to 14 and add to -9.	
Both factors of 14 must be negative.	$4(y-2)(y-7)$

Factors of 14	**Sum of factors**
$-1, -14$	$-1 + (-14) = -15$
$-2, -7$	$-2 + (-7) = -9$*

Check.

$4(y-2)(y-7)$

$4(y^2 - 7y - 2y + 14)$

$4(y^2 - 9y + 14)$

$4y^2 - 36y + 42$ ✓

Note:
Exercise:

TRY IT : : 6.61
Problem: Factor completely: $3r^2 - 9r + 6$.

Solution:

$3\left(r-1\right)\left(r-2\right)$

Note:
Exercise:

TRY IT : : 6.62
Problem: Factor completely: $2t^2 - 10t + 12$.

Solution:

$2\left(t-2\right)\left(t-3\right)$

In the next example the GCF will include a variable.

Example:
Exercise:

Problem: **Factor completely: $4u^3 + 16u^2 - 20u$.

Solution:
Solution

Use the preliminary strategy.

Is there a greatest common factor?	$4u^3+16u^2-20u$
Yes, GCF $= 4u.$ Factor it.	$4u\left(u^2+4u-5\right)$
Binomial, trinomial, or more than three terms?	
It is a trinomial. So "undo FOIL."	$4u\left(u\quad\right)\left(u\quad\right)$
Use a table like the table below to find two numbers that multiply to -5 and add to 4.	$4u\left(u-1\right)\left(u+5\right)$

Factors of -5	Sum of factors

Factors of -5	**Sum of factors**
$-1, 5$	$-1+5=4$*
$1, -5$	$1+(-5)=-4$

Check.

$$4u\left(u-1\right)\left(u+5\right)$$

$$4u(u^2+5u-u-5)$$

$$4u(u^2+4u-5)$$

$$4u^3+16u^2-20u \checkmark$$

Note:
Exercise:

TRY IT : : 6.63
Problem: Factor completely: $5x^3+15x^2-20x$.

Solution:

$5x\left(x-1\right)\left(x+4\right)$

Note:
Exercise:

TRY IT : : 6.64
Problem: Factor completely: $6y^3+18y^2-60y$.

Solution:

$6y\left(y-2\right)\left(y+5\right)$

Factor Trinomials using Trial and Error

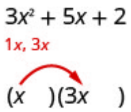
What happens when the leading coefficient is not 1 and there is no GCF? There are several methods that can be used to factor these trinomials. First we will use the Trial and Error method.

Let’s factor the trinomial $3x^2+5x+2$.

From our earlier work we expect this will factor into two binomials.
Equation:

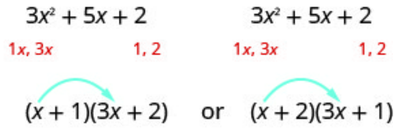
$$
\begin{array}{c}
3x^2+5x+2 \\
(\quad)(\quad)
\end{array}
$$

We know the first terms of the binomial factors will multiply to give us $3x^2$. The only factors of $3x^2$ are $1x, 3x$. We can place them in the binomials.

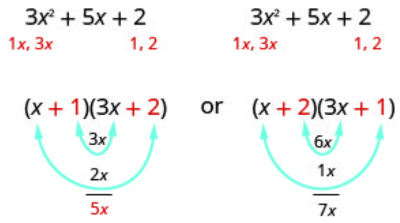


Check. Does $1x \cdot 3x = 3x^2$?

We know the last terms of the binomials will multiply to 2. Since this trinomial has all positive terms, we only need to consider positive factors. The only factors of 2 are 1 and 2. But we now have two cases to consider as it will make a difference if we write 1, 2, or 2, 1.



Which factors are correct? To decide that, we multiply the inner and outer terms.



Since the middle term of the trinomial is 5*x*, the factors in the first case will work. Let's FOIL to check.

Equation:

$$
\begin{array}{l}
(x+1)(3x+2) \\
3x^2+2x+3x+2 \\
3x^2+5x+2 \checkmark
\end{array}
$$

Our result of the factoring is:

Equation:

$$
\begin{array}{l}
3x^2+5x+2 \\
(x+1)(3x+2)
\end{array}
$$

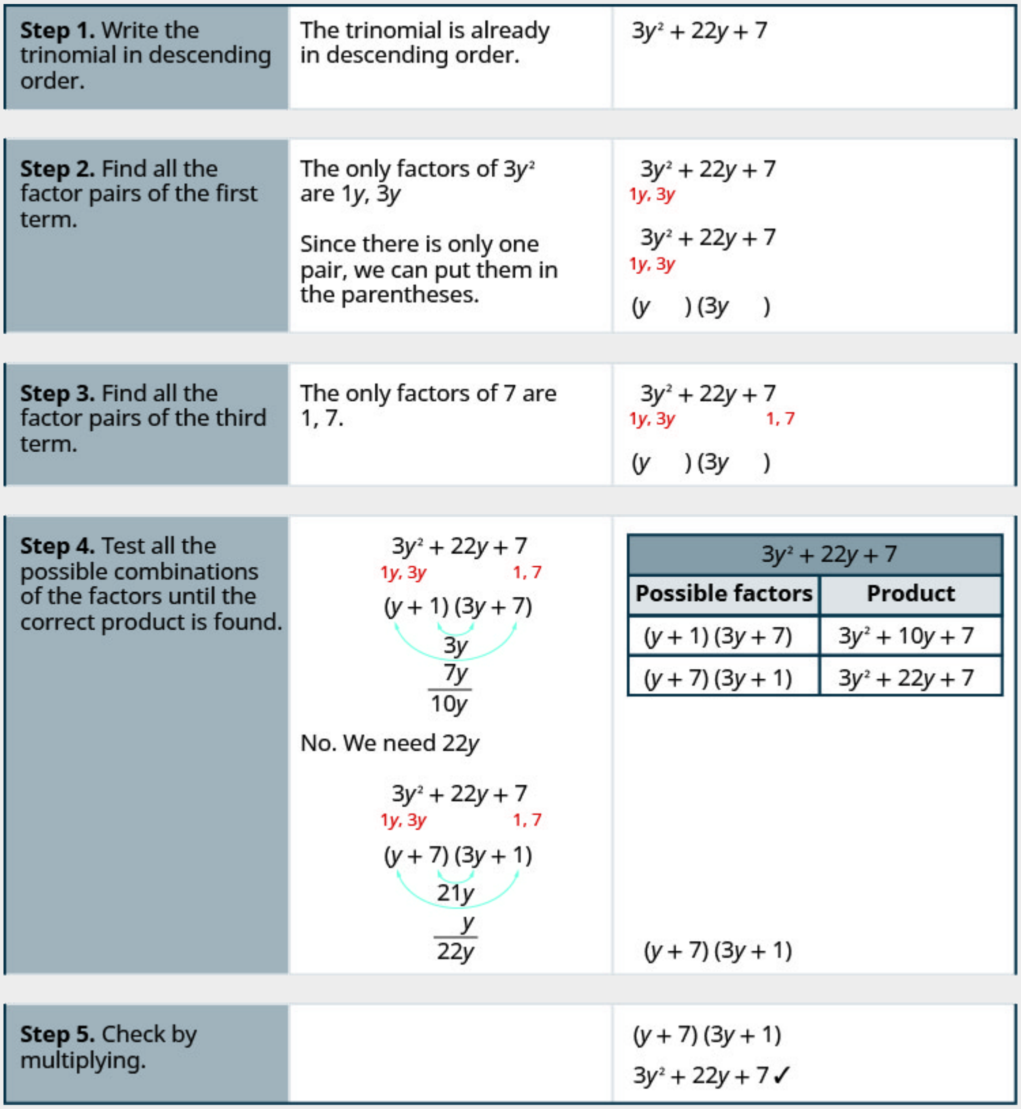
Example:

How to Factor Trinomials of the Form ax^2+bx+c Using Trial and Error

Exercise:

Problem: **Factor completely: $3y^2+22y+7$.

Solution:
Solution



<table>
<tr><td>Step 1. Write the trinomial in descending order.</td><td>The trinomial is already in descending order.</td><td>$3y^2+22y+7$</td></tr>
<tr><td>Step 2. Find all the factor pairs of the first term.</td><td>The only factors of $3y^2$ are $1y, 3y$
Since there is only one pair, we can put them in the parentheses.</td><td>$3y^2+22y+7$
$1y, 3y$
$3y^2+22y+7$
$1y, 3y$
$(y\quad)(3y\quad)$</td></tr>
<tr><td>Step 3. Find all the factor pairs of the third term.</td><td>The only factors of 7 are 1, 7.</td><td>$3y^2+22y+7$
$1y, 3y \qquad 1, 7$
$(y\quad)(3y\quad)$</td></tr>
<tr><td>Step 4. Test all the possible combinations of the factors until the correct product is found.</td><td>$3y^2+22y+7$
$1y, 3y \qquad 1, 7$
$(y+1)(3y+7)$
$3y$
$7y$
$10y$
No. We need $22y$
$3y^2+22y+7$
$1y, 3y \qquad 1, 7$
$(y+7)(3y+1)$
$21y$
y
$22y$</td><td><table><tr><th colspan="2">$3y^2+22y+7$</th></tr><tr><th>Possible factors</th><th>Product</th></tr><tr><td>$(y+1)(3y+7)$</td><td>$3y^2+10y+7$</td></tr><tr><td>$(y+7)(3y+1)$</td><td>$3y^2+22y+7$</td></tr></table>$(y+7)(3y+1)$</td></tr>
<tr><td>Step 5. Check by multiplying.</td><td></td><td>$(y+7)(3y+1)$
$3y^2+22y+7$ ✓</td></tr>
</table>

Note:
Exercise:

TRY IT : : 6.65

Problem: Factor completely: $2a^2+5a+3$.

Solution:

$(a+1)(2a+3)$

Note:
Exercise:

TRY IT : : 6.66
Problem: Factor completely: $4b^2 + 5b + 1$.

Solution:

$(b + 1)\,(4b + 1)$

Note:
Factor trinomials of the form $ax^2 + bx + c$ using trial and error.

Write the trinomial in descending order of degrees.
Find all the factor pairs of the first term.
Find all the factor pairs of the third term.
Test all the possible combinations of the factors until the correct product is found.
Check by multiplying.

When the middle term is negative and the last term is positive, the signs in the binomials must both be negative.

Example:
Exercise:

Problem: **Factor completely: $6b^2 - 13b + 5$.

Solution:
Solution

The trinomial is already in descending order.	$6b^2 - 13b + 5$
Find the factors of the first term.	$6b^2 - 13b + 5$ $1b \cdot 6b$ $2b \cdot 3b$
Find the factors of the last term. Consider the signs. Since the last term, 5 is positive its factors must both be positive or both be negative. The coefficient of the middle term is negative, so we use the negative factors.	$6b^2 - 13b + 5$ $1b \cdot 6b$ $-1, -5$ $2b \cdot 3b$

Consider all the combinations of factors.

$6b^2 - 13b + 5$	
Possible factors	**Product**
$(b-1)(6b-5)$	$6b^2 - 11b + 5$
$(b-5)(6b-1)$	$6b^2 - 31b + 5$
$(2b-1)(3b-5)$	$6b^2 - 13b + 5$ *
$(2b-5)(3b-1)$	$6b^2 - 17b + 5$

The correct factors are those whose product is the original trinomial. $(2b-1)(3b-5)$

Check by multiplying.

$$(2b-1)(3b-5)$$
$$6b^2 - 10b - 3b + 5$$
$$6b^2 - 13b + 5 \checkmark$$

Note:
Exercise:

TRY IT : : 6.67
Problem: Factor completely: $8x^2 - 14x + 3$.

Solution:

$(2x-3)(4x-1)$

Note:
Exercise:

TRY IT : : 6.68
Problem: Factor completely: $10y^2 - 37y + 7$.

Solution:

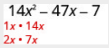
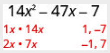
$(2y-7)(5y-1)$

When we factor an expression, we always look for a greatest common factor first. If the expression does not have a greatest common factor, there cannot be one in its factors either. This may help us eliminate some of the possible factor combinations.

Example:
Exercise:

Problem: **Factor completely: $14x^2 - 47x - 7$.

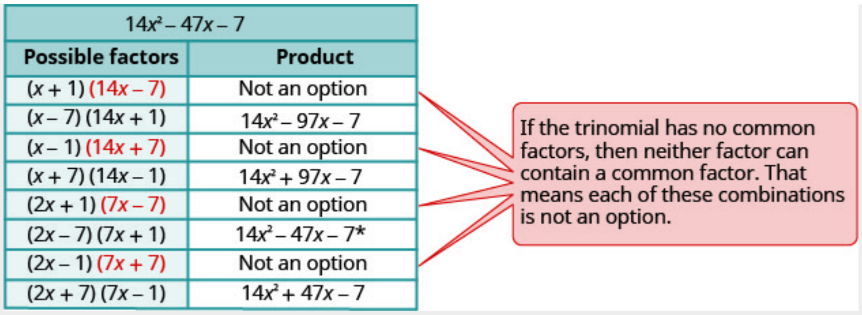
Solution:
Solution

The trinomial is already in descending order.	$14x^2 - 47x - 7$
Find the factors of the first term.	 $14x^2 - 47x - 7$ $1x \cdot 14x$ $2x \cdot 7x$
Find the factors of the last term. Consider the signs. Since it is negative, one factor must be positive and one negative.	 $14x^2 - 47x - 7$ $1x \cdot 14x$ $1, -7$ $2x \cdot 7x$ $-1, 7$

Consider all the combinations of factors. We use each pair of the factors of $14x^2$ with each pair of factors of -7.

Factors of $14x^2$	**Pair with**	**Factors of -7**
x, $14x$		$1, -7$ $-7, 1$ (reverse order)
x, $14x$		$-1, 7$ $7, -1$ (reverse order)
$2x, 7x$		$1, -7$ $-7, 1$ (reverse order)
$2x, 7x$		$-1, 7$ $7, -1$ (reverse order)

These pairings lead to the following eight combinations.



$14x^2 - 47x - 7$	
Possible factors	**Product**
$(x+1)(14x-7)$	Not an option
$(x-7)(14x+1)$	$14x^2 - 97x - 7$
$(x-1)(14x+7)$	Not an option
$(x+7)(14x-1)$	$14x^2 + 97x - 7$
$(2x+1)(7x-7)$	Not an option
$(2x-7)(7x+1)$	$14x^2 - 47x - 7$*
$(2x-1)(7x+7)$	Not an option
$(2x+7)(7x-1)$	$14x^2 + 47x - 7$

If the trinomial has no common factors, then neither factor can contain a common factor. That means each of these combinations is not an option.

The correct factors are those whose product is the original trinomial. $(2x-7)(7x+1)$

Check by multiplying.

$$(2x-7)(7x+1)$$
$$14x^2 + 2x - 49x - 7$$
$$14x^2 - 47x - 7 \checkmark$$

Note:
Exercise:

TRY IT : : 6.69

Problem: Factor completely: $8a^2 - 3a - 5$.

Solution:

$(a-1)(8a+5)$

Note:
Exercise:

TRY IT : : 6.70

Problem: Factor completely: $6b^2 - b - 15$.

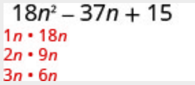
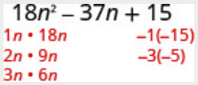
Solution:

$(2b+3)(3b-5)$

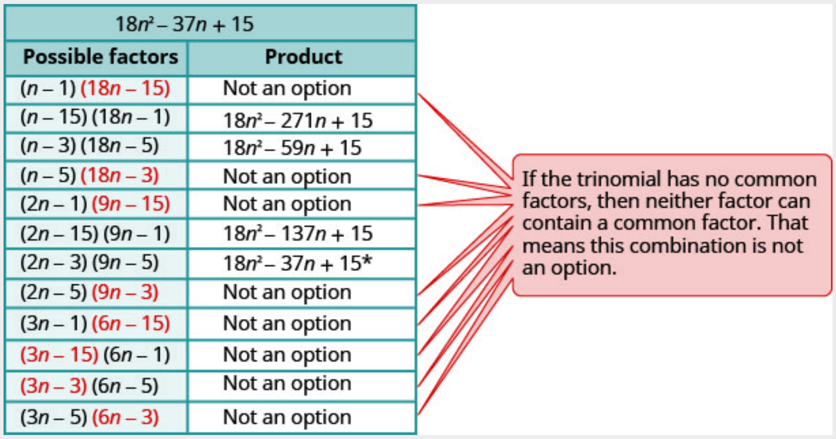
Example:
Exercise:

Problem: **Factor completely: $18n^2 - 37n + 15$.

Solution:
Solution

The trinomial is already in descending order.	$18n^2 - 37n + 15$
Find the factors of the first term.	 $18n^2 - 37n + 15$ $1n \cdot 18n$ $2n \cdot 9n$ $3n \cdot 6n$
Find the factors of the last term. Consider the signs. Since 15 is positive and the coefficient of the middle term is negative, we use the negative facotrs.	 $18n^2 - 37n + 15$ $1n \cdot 18n \quad -1(-15)$ $2n \cdot 9n \quad -3(-5)$ $3n \cdot 6n$

Consider all the combinations of factors.



$18n^2 - 37n + 15$	
Possible factors	**Product**
$(n-1)(18n-15)$	Not an option
$(n-15)(18n-1)$	$18n^2 - 271n + 15$
$(n-3)(18n-5)$	$18n^2 - 59n + 15$
$(n-5)(18n-3)$	Not an option
$(2n-1)(9n-15)$	Not an option
$(2n-15)(9n-1)$	$18n^2 - 137n + 15$
$(2n-3)(9n-5)$	$18n^2 - 37n + 15$*
$(2n-5)(9n-3)$	Not an option
$(3n-1)(6n-15)$	Not an option
$(3n-15)(6n-1)$	Not an option
$(3n-3)(6n-5)$	Not an option
$(3n-5)(6n-3)$	Not an option

If the trinomial has no common factors, then neither factor can contain a common factor. That means this combination is not an option.

The correct factors are those whose product is the original trinomial.	$(2n-3)(9n-5)$

Check by multiplying.

$$(2n-3)(9n-5)$$
$$18n^2 - 10n - 27n + 15$$
$$18n^2 - 37n + 15\ \checkmark$$

Note:
Exercise:

TRY IT : : 6.71

Problem: Factor completely: $18x^2 - 3x - 10$.

Solution:

$(3x+2)(6x-5)$

Note:

Exercise:

TRY IT : : 6.72
Problem: Factor completely: $30y^2 - 53y - 21$.

Solution:

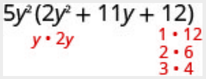
$(3y+1)(10y-21)$

Don’t forget to look for a GCF first.

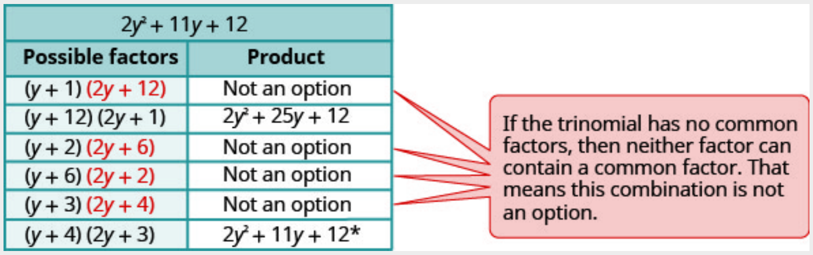
Example:
Exercise:

Problem: **Factor completely: $10y^4 + 55y^3 + 60y^2$.

Solution:
Solution

	$10y^4 + 55y^3 + 60y^2$
Notice the greatest common factor, and factor it first.	$5y^2\left(2y^2 + 11y + 12\right)$
Factor the trinomial.	 $5y^2(2y^2 + 11y + 12)$ $y \cdot 2y$ $1 \cdot 12$ $2 \cdot 6$ $3 \cdot 4$

Consider all the combinations.



$2y^2 + 11y + 12$	
Possible factors	**Product**
$(y+1)(2y+12)$	Not an option
$(y+12)(2y+1)$	$2y^2 + 25y + 12$
$(y+2)(2y+6)$	Not an option
$(y+6)(2y+2)$	Not an option
$(y+3)(2y+4)$	Not an option
$(y+4)(2y+3)$	$2y^2 + 11y + 12$*

If the trinomial has no common factors, then neither factor can contain a common factor. That means this combination is not an option.

The correct factors are those whose product is the original trinomial. Remember to include the factor $5y^2$.
Check by multiplying.

$5y^2\left(y+4\right)\left(2y+3\right)$

$$5y^2\left(y+4\right)\left(2y+3\right)$$
$$5y^2\left(2y^2+8y+3y+12\right)$$
$$10y^4+55y^3+60y^2\ \checkmark$$

Note:
Exercise:

TRY IT : : 6.73
Problem: Factor completely: $15n^3-85n^2+100n$.

Solution:

$5n\left(n-4\right)\left(3n-5\right)$

Note:
Exercise:

TRY IT : : 6.74
Problem: Factor completely: $56q^3+320q^2-96q$.

Solution:

$8q\left(q+6\right)\left(7q-2\right)$

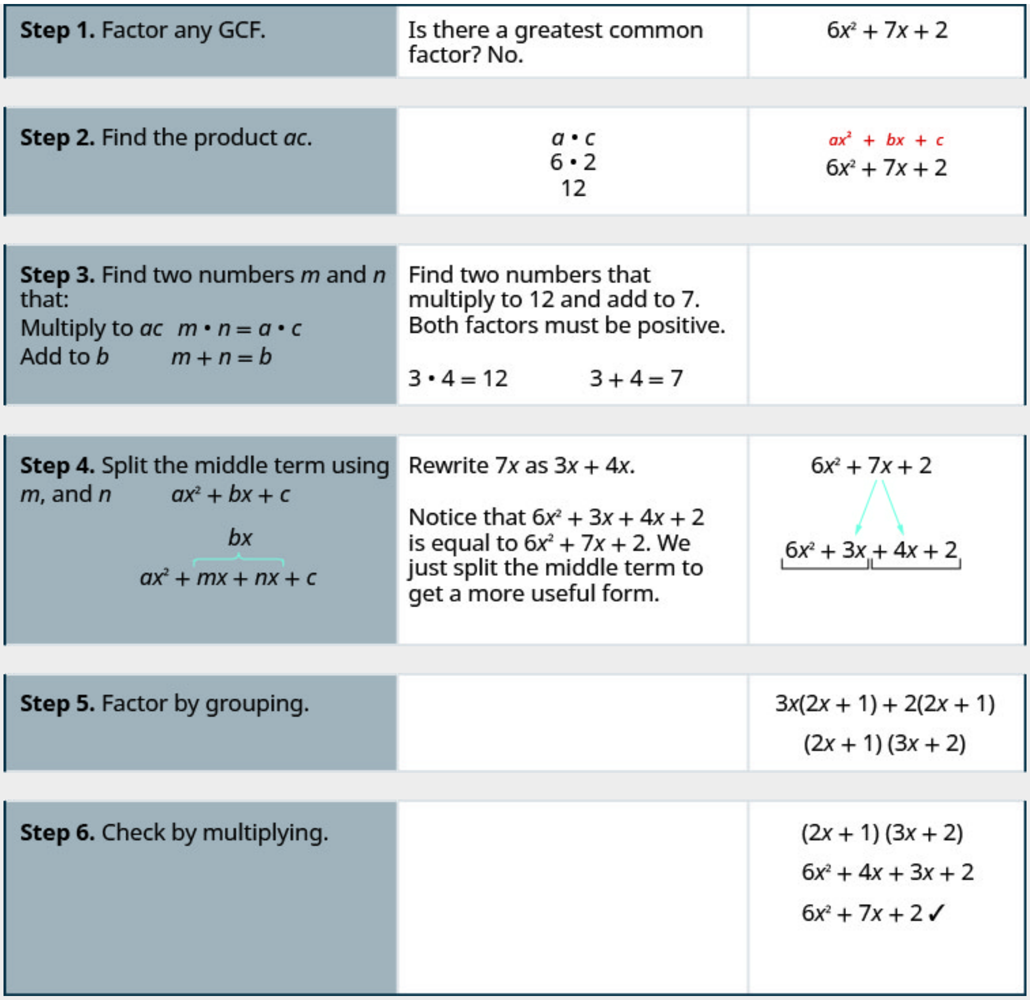
Factor Trinomials using the “ac” Method

Another way to factor trinomials of the form ax^2+bx+c is the “ac” method. (The “ac” method is sometimes called the grouping method.) The “ac” method is actually an extension of the methods you used in the last section to factor trinomials with leading coefficient one. This method is very structured (that is step-by-step), and it always works!

Example:
How to Factor Trinomials Using the “ac” Method
Exercise:

Problem: **Factor: $6x^2+7x+2$.

Solution:
Solution



Step 1. Factor any GCF.	Is there a greatest common factor? No.	$6x^2 + 7x + 2$
Step 2. Find the product *ac*.	$a \cdot c$ $6 \cdot 2$ 12	$ax^2 + bx + c$ $6x^2 + 7x + 2$
Step 3. Find two numbers *m* and *n* that: Multiply to *ac* $m \cdot n = a \cdot c$ Add to *b* $m + n = b$	Find two numbers that multiply to 12 and add to 7. Both factors must be positive. $3 \cdot 4 = 12$ $3 + 4 = 7$	
Step 4. Split the middle term using *m*, and *n* $ax^2 + bx + c$ $ax^2 + \overbrace{mx + nx}^{bx} + c$	Rewrite 7*x* as 3*x* + 4*x*. Notice that $6x^2 + 3x + 4x + 2$ is equal to $6x^2 + 7x + 2$. We just split the middle term to get a more useful form.	$6x^2 + 7x + 2$ $\underbrace{6x^2 + 3x} + \underbrace{4x + 2}$
Step 5. Factor by grouping.		$3x(2x + 1) + 2(2x + 1)$ $(2x + 1)(3x + 2)$
Step 6. Check by multiplying.		$(2x + 1)(3x + 2)$ $6x^2 + 4x + 3x + 2$ $6x^2 + 7x + 2$ ✓

Note:
Exercise:

TRY IT : : 6.75

Problem: Factor: $6x^2 + 13x + 2$.

Solution:

$(x + 2)(6x + 1)$

Note:
Exercise:

TRY IT : : 6.76

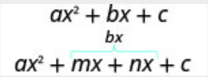
Problem: Factor: $4y^2 + 8y + 3$.

Solution:

$(2y+1)\,(2y+3)$

Note:
Factor trinomials of the form using the “ac” method.

Factor any GCF.
Find the product ac.
Find two numbers$m$and$n$that: Multiply to ac $\quad m \cdot n = a \cdot c$
Add to b $\quad m + n = b$
Split the middle term using$m$and$n$:



$$ax^2 + bx + c$$
$$ax^2 + \overbrace{mx + nx}^{bx} + c$$

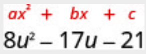
Factor by grouping.
Check by multiplying the factors.

When the third term of the trinomial is negative, the factors of the third term will have opposite signs.

Example:
Exercise:

Problem: **Factor: $8u^2 - 17u - 21$.

Solution:
Solution

Is there a greatest common factor? No.		 $ax^2 + bx + c$ $8u^2 - 17u - 21$
Find $a \cdot c$.	$a \cdot c$	
	$8(-21)$	
	-168	

Find two numbers that multiply to -168 and add to -17. The larger factor must be negative.

Factors of -168	**Sum of factors**
$1, -168$	$1 + (-168) = -167$
$2, -84$	$2 + (-84) = -82$
$3, -56$	$3 + (-56) = -53$
$4, -42$	$4 + (-42) = -38$
$6, -28$	$6 + (-28) = -22$
$7, -24$	$7 + (-24) = -17$*
$8, -21$	$8 + (-21) = -13$

Split the middle term using $7u$ and $-24u$.

$$8u^2 - \underset{\swarrow\ \searrow}{17u} - 21$$
$$8u^2 + 7u\ -24u - 21$$

Factor by grouping.

$$u\,(8u + 7) - 3\,(8u + 7)$$
$$(8u + 7)\,(u - 3)$$

Check by multiplying.

$$(8u + 7)\,(u - 3)$$
$$8u^2 - 24u + 7u - 21$$
$$8u^2 - 17u - 21\ \checkmark$$

Note:
Exercise:

TRY IT : : 6.77
Problem: Factor: $20h^2 + 13h - 15$.

Solution:

$(4h + 5)\,(5h - 3)$

Note:
Exercise:

TRY IT : : 6.78
Problem: Factor: $6g^2 + 19g - 20$.

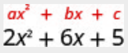
Solution:

$(g+4)(6g-5)$

Example:
Exercise:

Problem: **Factor: $2x^2+6x+5$.

Solution:
Solution

Is there a greatest common factor? No.	 ax^2+bx+c $2x^2+6x+5$
Find $a\cdot c$.	$a\cdot c$
	$2(5)$
	10

Find two numbers that multiply to 10 and add to 6.

Factors of 10	**Sum of factors**
$1, 10$	$1+10=11$
$2, 5$	$2+5=7$

There are no factors that multiply to 10 and add to 6. The polynomial is prime.

Note:
Exercise:

TRY IT : : 6.79
Problem: Factor: $10t^2+19t-15$.

Solution:

$(2t+5)(5t-3)$

Note:
Exercise:

TRY IT : : 6.80
Problem: Factor: $3u^2 + 8u + 5$.

Solution:

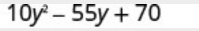
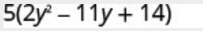
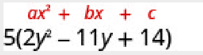
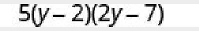
$(u + 1)(3u + 5)$

Don’t forget to look for a common factor!

Example:
Exercise:

Problem: **Factor: $10y^2 - 55y + 70$.

Solution:
Solution

Is there a greatest common factor? Yes. The GCF is 5.	 $10y^2 - 55y + 70$
Factor it. Be careful to keep the factor of 5 all the way through the solution!	 $5(2y^2 - 11y + 14)$
The trinomial inside the parentheses has a leading coefficient that is not 1.	 $ax^2 + bx + c$ $5(2y^2 - 11y + 14)$
Factor the trinomial.	 $5(y - 2)(2y - 7)$
Check by mulitplying all three factors.	
$5\left(2y^2 - 2y - 4y + 14\right)$	
$5\left(2y^2 - 11y + 14\right)$	
$10y^2 - 55y + 70$✓	

Note:
Exercise:

TRY IT : : 6.81

Problem: Factor: $16x^2 - 32x + 12$.

Solution:

$4\left(2x-3\right)\left(2x-1\right)$

Note:
Exercise:

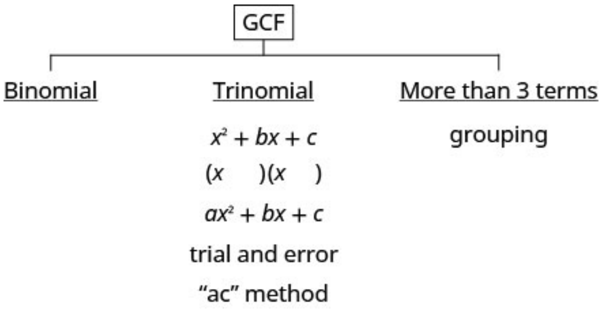
TRY IT : : 6.82

Problem: Factor: $18w^2 - 39w + 18$.

Solution:

$3\left(3w-2\right)\left(2w-3\right)$

We can now update the Preliminary Factoring Strategy, as shown in [link] and detailed in Choose a strategy to factor polynomials completely (updated), to include trinomials of the form $ax^2 + bx + c$. Remember, some polynomials are prime and so they cannot be factored.



Note:
Choose a strategy to factor polynomials completely (updated).

Is there a greatest common factor?

- Factor it.

Is the polynomial a binomial, trinomial, or are there more than three terms?

- If it is a binomial, right now we have no

method to factor it.
    - If it is a trinomial of the form x^2+bx+c
      Undo FOIL $(x\quad)(x\quad)$.
    - If it is a trinomial of the form ax^2+bx+c
      Use Trial and Error or the “ac” method.
    - If it has more than three terms
      Use the grouping method.

Check by multiplying the factors.

Note:
Access these online resources for additional instruction and practice with factoring trinomials of the form ax^2+bx+c.

- Factoring Trinomials, a is not 1

Key Concepts

- **Factor Trinomials of the Form ax^2+bx+c using Trial and Error:** See [link].

  Write the trinomial in descending order of degrees.
  Find all the factor pairs of the first term.
  Find all the factor pairs of the third term.
  Test all the possible combinations of the factors until the correct product is found.
  Check by multiplying.

- **Factor Trinomials of the Form ax^2+bx+c Using the “ac” Method:** See [link].

  Factor any GCF.
  Find the product ac.
  Find two numbers m and n that: Multiply to ac $\quad m\cdot n=a\cdot c$
  Add to b $\quad m+n=b$
  Split the middle term using m and n:

  $$\begin{array}{c} ax^2+bx+c \\ ax^2+\overbrace{mx+nx}^{bx}+c \end{array}$$

  Factor by grouping.
  Check by multiplying the factors.

- **Choose a strategy to factor polynomials completely (updated):**

  Is there a greatest common factor? Factor it.
  Is the polynomial a binomial, trinomial, or are there more than three terms?
  If it is a binomial, right now we have no method to factor it.
  If it is a trinomial of the form x^2+bx+c Undo FOIL $(x\quad)(x\quad)$.
  If it is a trinomial of the form ax^2+bx+c Use Trial and Error or the “ac” method.
  If it has more than three terms Use the grouping method.

Check by multiplying the factors.

Practice Makes Perfect

Recognize a Preliminary Strategy to Factor Polynomials Completely

In the following exercises, identify the best method to use to factor each polynomial.

Exercise:

Problem:

ⓐ $10q^2 + 50$
ⓑ $a^2 - 5a - 14$
ⓒ $uv + 2u + 3v + 6$

Solution:

ⓐ factor the GCF, binomial ⓑ Undo FOIL ⓒ factor by grouping

Exercise:

Problem:

ⓐ $n^2 + 10n + 24$
ⓑ $8u^2 + 16$
ⓒ $pq + 5p + 2q + 10$

Exercise:

Problem:

ⓐ $x^2 + 4x - 21$
ⓑ $ab + 10b + 4a + 40$
ⓒ $6c^2 + 24$

Solution:

ⓐ undo FOIL ⓑ factor by grouping ⓒ factor the GCF, binomial

Exercise:

Problem:

ⓐ $20x^2 + 100$
ⓑ $uv + 6u + 4v + 24$
ⓒ $y^2 - 8y + 15$

Factor Trinomials of the form $ax^2 + bx + c$ with a GCF

In the following exercises, factor completely.

Exercise:

Problem: $5x^2 + 35x + 30$

Solution:

$5(x+1)(x+6)$

Exercise:

Problem: $12s^2+24s+12$

Exercise:

Problem: $2z^2-2z-24$

Solution:

$2(z-4)(z+3)$

Exercise:

Problem: $3u^2-12u-36$

Exercise:

Problem: $7v^2-63v+56$

Solution:

$7(v-1)(v-8)$

Exercise:

Problem: $5w^2-30w+45$

Exercise:

Problem: p^3-8p^2-20p

Solution:

$p(p-10)(p+2)$

Exercise:

Problem: q^3-5q^2-24q

Exercise:

Problem: $3m^3-21m^2+30m$

Solution:

$3m(m-5)(m-2)$

Exercise:

Problem: $11n^3-55n^2+44n$

Exercise:

Problem: $5x^4 + 10x^3 - 75x^2$

Solution:

$5x^2(x - 3)(x + 5)$

Exercise:

Problem: $6y^4 + 12y^3 - 48y^2$

Factor Trinomials Using Trial and Error

In the following exercises, factor.

Exercise:

Problem: $2t^2 + 7t + 5$

Solution:

$(2t + 5)(t + 1)$

Exercise:

Problem: $5y^2 + 16y + 11$

Exercise:

Problem: $11x^2 + 34x + 3$

Solution:

$(11x + 1)(x + 3)$

Exercise:

Problem: $7b^2 + 50b + 7$

Exercise:

Problem: $4w^2 - 5w + 1$

Solution:

$(4w - 1)\,(w - 1)$

Exercise:

Problem: $5x^2 - 17x + 6$

Exercise:

Problem: $6p^2 - 19p + 10$

Solution:

$(3p-2)\,(2p-5)$

Exercise:

Problem: $21m^2-29m+10$

Exercise:

Problem: $4q^2-7q-2$

Solution:

$(4q+1)\,(q-2)$

Exercise:

Problem: $10y^2-53y-11$

Exercise:

Problem: $4p^2+17p-15$

Solution:

$(4p-3)\,(p+5)$

Exercise:

Problem: $6u^2+5u-14$

Exercise:

Problem: $16x^2-32x+16$

Solution:

$16\,(x-1)\,(x-1)$

Exercise:

Problem: $81a^2+153a-18$

Exercise:

Problem: $30q^3+140q^2+80q$

Solution:

$10q\,(3q+2)\,(q+4)$

Exercise:

Problem: $5y^3+30y^2-35y$

Factor Trinomials using the ‘ac’ Method

In the following exercises, factor.

Exercise:

Problem: $5n^2 + 21n + 4$

Solution:

$(5n + 1)(n + 4)$

Exercise:

Problem: $8w^2 + 25w + 3$

Exercise:

Problem: $9z^2 + 15z + 4$

Solution:

$(3z + 1)(3z + 4)$

Exercise:

Problem: $3m^2 + 26m + 48$

Exercise:

Problem: $4k^2 - 16k + 15$

Solution:

$(2k - 3)(2k - 5)$

Exercise:

Problem: $4q^2 - 9q + 5$

Exercise:

Problem: $5s^2 - 9s + 4$

Solution:

$(5s - 4)(s - 1)$

Exercise:

Problem: $4r^2 - 20r + 25$

Exercise:

Problem: $6y^2 + y - 15$

Solution:

$(3y + 5)(2y - 3)$

Exercise:

Problem: $6p^2 + p - 22$

Exercise:

Problem: $2n^2 - 27n - 45$

Solution:

$(2n + 3)(n - 15)$

Exercise:

Problem: $12z^2 - 41z - 11$

Exercise:

Problem: $3x^2 + 5x + 4$

Solution:

prime

Exercise:

Problem: $4y^2 + 15y + 6$

Exercise:

Problem: $60y^2 + 290y - 50$

Solution:

$10(6y - 1)(y + 5)$

Exercise:

Problem: $6u^2 - 46u - 16$

Exercise:

Problem: $48z^3 - 102z^2 - 45z$

Solution:

$3z(8z + 3)(2z - 5)$

Exercise:

Problem: $90n^3 + 42n^2 - 216n$

Exercise:

Problem: $16s^2 + 40s + 24$

Solution:

$8\left(2s+3\right)\left(s+1\right)$

Exercise:

Problem: $24p^2+160p+96$

Exercise:

Problem: $48y^2+12y-36$

Solution:

$12\left(4y-3\right)\left(y+1\right)$

Exercise:

Problem: $30x^2+105x-60$

Mixed Practice

In the following exercises, factor.

Exercise:

Problem: $12y^2-29y+14$

Solution:

$\left(4y-7\right)\left(3y-2\right)$

Exercise:

Problem: $12x^2+36y-24z$

Exercise:

Problem: a^2-a-20

Solution:

$\left(a-5\right)\left(a+4\right)$

Exercise:

Problem: m^2-m-12

Exercise:

Problem: $6n^2+5n-4$

Solution:

$\left(2n-1\right)\left(3n+4\right)$

Exercise:

Problem: $12y^2-37y+21$

Exercise:

Problem: $2p^2 + 4p + 3$

Solution:

prime

Exercise:

Problem: $3q^2 + 6q + 2$

Exercise:

Problem: $13z^2 + 39z - 26$

Solution:

$13\left(z^2 + 3z - 2\right)$

Exercise:

Problem: $5r^2 + 25r + 30$

Exercise:

Problem: $x^2 + 3x - 28$

Solution:

$\left(x + 7\right)\left(x - 4\right)$

Exercise:

Problem: $6u^2 + 7u - 5$

Exercise:

Problem: $3p^2 + 21p$

Solution:

$3p\left(p + 7\right)$

Exercise:

Problem: $7x^2 - 21x$

Exercise:

Problem: $6r^2 + 30r + 36$

Solution:

$6\left(r + 2\right)\left(r + 3\right)$

Exercise:

Problem: $18m^2 + 15m + 3$

Exercise:

Problem: $24n^2 + 20n + 4$

Solution:

$4\left(2n + 1\right)\left(3n + 1\right)$

Exercise:

Problem: $4a^2 + 5a + 2$

Exercise:

Problem: $x^2 + 2x - 24$

Solution:

$\left(x + 6\right)\left(x - 4\right)$

Exercise:

Problem: $2b^2 - 7b + 4$

Everyday Math

Exercise:

Problem:

Height of a toy rocket The height of a toy rocket launched with an initial speed of 80 feet per second from the balcony of an apartment building is related to the number of seconds, *t*, since it is launched by the trinomial $-16t^2 + 80t + 96$. Completely factor the trinomial.

Solution:

$-16\left(t - 6\right)\left(t + 1\right)$

Exercise:

Problem:

Height of a beach ball The height of a beach ball tossed up with an initial speed of 12 feet per second from a height of 4 feet is related to the number of seconds, *t*, since it is tossed by the trinomial $-16t^2 + 12t + 4$. Completely factor the trinomial.

Writing Exercises

Exercise:

Problem:

List, in order, all the steps you take when using the “ac” method to factor a trinomial of the form $ax^2 + bx + c$.

Solution:

Answers may vary.

Exercise:

Problem: How is the “ac” method similar to the “undo FOIL” method? How is it different?

Exercise:

Problem:

What are the questions, in order, that you ask yourself as you start to factor a polynomial? What do you need to do as a result of the answer to each question?

Solution:

Answers may vary.

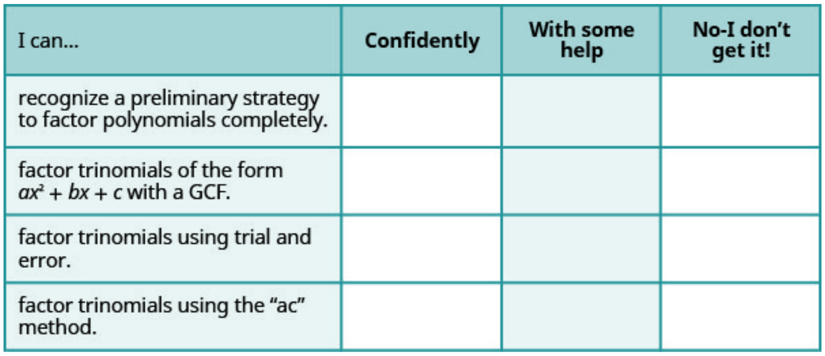
Exercise:

Problem:

On your paper draw the chart that summarizes the factoring strategy. Try to do it without looking at the book. When you are done, look back at the book to finish it or verify it.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don’t get it!
recognize a preliminary strategy to factor polynomials completely.			
factor trinomials of the form $ax^2 + bx + c$ with a GCF.			
factor trinomials using trial and error.			
factor trinomials using the “ac” method.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

prime polynomials	Polynomials that cannot be factored are prime polynomials.

Factor Special Products

By the end of this section, you will be able to:

- Factor perfect square trinomials
- Factor differences of squares
- Factor sums and differences of cubes

Note:
Before you get started, take this readiness quiz.

1. Simplify: $(12x)^2$.
   If you missed this problem, review [link].
2. Multiply: $(m+4)^2$.
   If you missed this problem, review [link].
3. Multiply: $(p-9)^2$.
   If you missed this problem, review [link].
4. Multiply: $(k+3)(k-3)$.
   If you missed this problem, review [link].

The strategy for factoring we developed in the last section will guide you as you factor most binomials, trinomials, and polynomials with more than three terms. We have seen that some binomials and trinomials result from special products—squaring binomials and multiplying conjugates. If you learn to recognize these kinds of polynomials, you can use the special products patterns to factor them much more quickly.

Factor Perfect Square Trinomials

Some trinomials are perfect squares. They result from multiplying a binomial times itself. You can square a binomial by using FOIL, but using the Binomial Squares pattern you saw in a previous chapter saves you a step. Let’s review the Binomial Squares pattern by squaring a binomial using FOIL.

$$\begin{array}{c} (3x+4)^2 \\ (3x+4)(3x+4) \\ \overset{F}{9x^2} + \overset{O}{12x} + \overset{I}{12x} + \overset{L}{16} \\ 9x^2+24x+16 \end{array}$$

The first term is the square of the first term of the binomial and the last term is the square of the last. The middle term is twice the product of the two terms of the binomial.

Equation:

$$\begin{array}{c} (3x)^2 + 2\,(3x\cdot 4) + 4^2 \\ 9x^2 + 24x + 16 \end{array}$$

The trinomial $9x^2 + 24 + 16$ is called a perfect square trinomial. It is the square of the binomial $3x+4$.

We'll repeat the Binomial Squares Pattern here to use as a reference in factoring.

Note:
Binomial Squares Pattern
If *a* and *b* are real numbers,
Equation:

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad (a-b)^2 = a^2 - 2ab + b^2$$

When you square a binomial, the product is a perfect square trinomial. In this chapter, you are learning to factor—now, you will start with a perfect square trinomial and factor it into its prime factors.

You could factor this trinomial using the methods described in the last section, since it is of the form $ax^2 + bx + c$. But if you recognize that the first and last terms are squares and the trinomial fits the **perfect square trinomials pattern**, you will save yourself a lot of work.

Here is the pattern—the reverse of the binomial squares pattern.

Note:
Perfect Square Trinomials Pattern
If *a* and *b* are real numbers,
Equation:

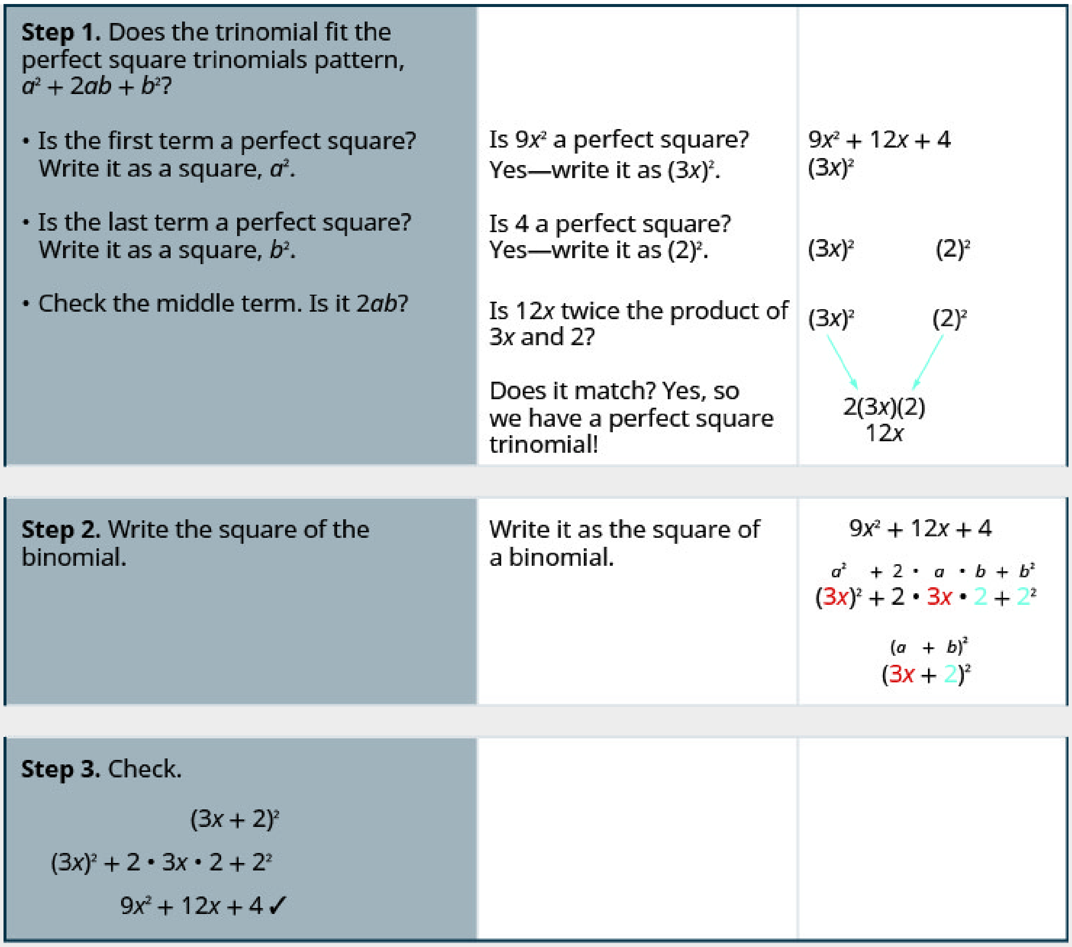
$$a^2 + 2ab + b^2 = (a+b)^2 \qquad a^2 - 2ab + b^2 = (a-b)^2$$

To make use of this pattern, you have to recognize that a given trinomial fits it. Check first to see if the leading coefficient is a perfect square, a^2. Next check that the last term is a perfect square, b^2. Then check the middle term—is it twice the product, $2ab$? If everything checks, you can easily write the factors.

Example:
How to Factor Perfect Square Trinomials
Exercise:

Problem: **Factor: $9x^2 + 12x + 4$.

Solution:
Solution



Step 1. Does the trinomial fit the perfect square trinomials pattern, $a^2 + 2ab + b^2$? • Is the first term a perfect square? Write it as a square, a^2. • Is the last term a perfect square? Write it as a square, b^2. • Check the middle term. Is it $2ab$?	Is $9x^2$ a perfect square? Yes—write it as $(3x)^2$. Is 4 a perfect square? Yes—write it as $(2)^2$. Is $12x$ twice the product of $3x$ and 2? Does it match? Yes, so we have a perfect square trinomial!	$9x^2 + 12x + 4$ $(3x)^2$ $(3x)^2 \qquad (2)^2$ $(3x)^2 \qquad (2)^2$ $2(3x)(2)$ $12x$
Step 2. Write the square of the binomial.	Write it as the square of a binomial.	$9x^2 + 12x + 4$ $\overset{a^2}{(3x)^2} + \overset{2}{2} \cdot \overset{a}{3x} \cdot \overset{b}{2} + \overset{b^2}{2^2}$ $\overset{(a+b)^2}{(3x + 2)^2}$
Step 3. Check. $(3x + 2)^2$ $(3x)^2 + 2 \cdot 3x \cdot 2 + 2^2$ $9x^2 + 12x + 4$ ✓		

Note:
Exercise:

TRY IT : : 6.83

Problem: Factor: $4x^2 + 12x + 9$.

Solution:

$(2x + 3)^2$

Note:
Exercise:

TRY IT : : 6.84

Problem: Factor: $9y^2 + 24y + 16$.

Solution:

$(3y+4)^2$

The sign of the middle term determines which pattern we will use. When the middle term is negative, we use the pattern $a^2-2ab+b^2$, which factors to $(a-b)^2$.

The steps are summarized here.

Note:
Factor perfect square trinomials.

Step 1. Does the trinomial fit the pattern?	$a^2+2ab+b^2$	$a^2-2ab+b^2$
• Is the first term a perfect square? Write it as a square.	$(a)^2$	$(a)^2$
• Is the last term a perfect square? Write it as a square.	$(a)^2 \qquad (b)^2$	$(a)^2 \qquad (b)^2$
• Check the middle term. Is it $2ab$?	$(a)^2\underset{2\cdot a\cdot b}{\searrow\ \swarrow}(b)^2$	$(a)^2\underset{2\cdot a\cdot b}{\searrow\ \swarrow}(b)^2$
Step 2. Write the square of the binomial.	$(a+b)^2$	$(a-b)^2$
Step 3. Check by multiplying.		

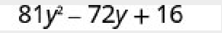
We’ll work one now where the middle term is negative.

Example:
Exercise:

Problem: **Factor: $81y^2-72y+16$.

Solution:
Solution

The first and last terms are squares. See if the middle term fits the pattern of a perfect square trinomial. The middle term is negative, so the binomial square would be $(a-b)^2$.

	 $81y^2-72y+16$

Are the first and last terms perfect squares?	$(9y)^2 \qquad (4)^2$
Check the middle term.	$(9y)^2 \qquad (4)^2$ $2(9y)(4)$ $72y$
Does is match $(a-b)^2$? Yes.	$\overset{a^2}{(9y)^2} \overset{-2}{-2} \cdot \overset{a}{9y} \cdot \overset{b}{4} \overset{+}{+} \overset{b^2}{4^2}$
Write the square of a binomial.	$(9y-4)^2$
Check by mulitplying.	
$(9y-4)^2$	
$(9y)^2 - 2 \cdot 9y \cdot 4 + 4^2$	
$81y^2 - 72y + 16\checkmark$	

Note:
Exercise:

TRY IT : : 6.85

Problem: Factor: $64y^2 - 80y + 25$.

Solution:

$(8y-5)^2$

Note:
Exercise:

TRY IT : : 6.86

Problem: Factor: $16z^2 - 72z + 81$.

Solution:

$(4z - 9)^2$

The next example will be a perfect square trinomial with two variables.

Example:
Exercise:

Problem: **Factor: $36x^2 + 84xy + 49y^2$.

Solution:
Solution

	$36x^2 + 84xy + 49y^2$
Test each term to verify the pattern.	$\overset{a^2}{(6x)^2} \overset{+\,2}{+\,2} \cdot \overset{a}{6x} \cdot \overset{b}{7y} \overset{+}{+} \overset{b^2}{(7y)^2}$
Factor.	$(6x + 7y)^2$
Check by mulitplying.	
$(6x + 7y)^2$	
$(6x)^2 + 2 \cdot 6x \cdot 7y + (7y)^2$	
$36x^2 + 84xy + 49y^2$✓	

Note:
Exercise:

TRY IT : : 6.87
Problem: Factor: $49x^2 + 84xy + 36y^2$.

Solution:

$(7x+6y)^2$

Note:
Exercise:

TRY IT : : 6.88

Problem: Factor: $64m^2+112mn+49n^2$.

Solution:

$(8m+7n)^2$

Example:
Exercise:

Problem: **Factor: $9x^2+50x+25$.

Solution:
Solution

	$9x^2+50x+25$
Are the first and last terms perfect squares?	$(3x)^2 \qquad (5)^2$
Check the middle term—is it $2ab$?	$(3x)^2 \searrow \underset{30x}{2(3x)(5)} \swarrow (5)^2$
No! $30x \neq 50x$	This does not fit the pattern!
Factor using the “ac” method.	$9x^2+50x+25$
Notice: $\underset{225}{\overset{ac}{9\cdot 25}}$ and $\begin{array}{l}5\cdot 45=225\\ 5+45=50\end{array}$	
Split the middle term.	$9x^2+5x+45x+25$
Factor by grouping.	$x\,(9x+5)+5\,(9x+5)$
	$(9x+5)\,(x+5)$

Check.

$$\begin{array}{l}(9x+5)\,(x+5)\\ 9x^2+45x+5x+25\\ 9x^2+50x+25\,\checkmark\end{array}$$

Note:
Exercise:

TRY IT : : 6.89
Problem: Factor: $16r^2 + 30rs + 9s^2$.

Solution:

$(8r + 3s)\,(2r + 3s)$

Note:
Exercise:

TRY IT : : 6.90
Problem: Factor: $9u^2 + 87u + 100$.

Solution:

$(3u + 4)\,(3u + 25)$

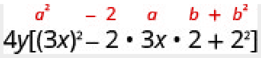
Remember the very first step in our Strategy for Factoring Polynomials? It was to ask “is there a greatest common factor?” and, if there was, you factor the GCF before going any further. Perfect square trinomials may have a GCF in all three terms and it should be factored out first. And, sometimes, once the GCF has been factored, you will recognize a perfect square trinomial.

Example:
Exercise:

Problem: **Factor: $36x^2y - 48xy + 16y$.

Solution:
Solution

	$36x^2y - 48xy + 16y$
Is there a GCF? Yes, 4*y*, so factor it out.	$4y\left(9x^2 - 12x + 4\right)$
Is this a perfect square trinomial?	

Verify the pattern.	 $4y[\overset{a^2}{(3x)^2}\overset{-2}{-2}\cdot\overset{a}{3x}\cdot\overset{b}{2}\overset{+}{+}\overset{b^2}{2^2}]$
Factor.	$4y(3x-2)^2$
Remember: Keep the factor 4*y* in the final product.	
Check.	
$4y(3x-2)^2$	
$4y\left[(3x)^2-2\cdot 3x\cdot 2+2^2\right]$	
$4y(9x)^2-12x+4$	
$36x^2y-48xy+16y$✓	

Note:
Exercise:

TRY IT : : 6.91
Problem: Factor: $8x^2y-24xy+18y$.

Solution:

$2y(2x-3)^2$

Note:
Exercise:

TRY IT : : 6.92
Problem: Factor: $27p^2q+90pq+75q$.

Solution:

$3q(3p+5)^2$

Factor Differences of Squares

The other special product you saw in the previous was the Product of Conjugates pattern. You used this to multiply two binomials that were conjugates. Here’s an example:

Equation:

$$(3x - 4)\,(3x + 4)$$
$$9x^2 - 16$$

Remember, when you multiply conjugate binomials, the middle terms of the product add to 0. All you have left is a binomial, the difference of squares.

Multiplying conjugates is the only way to get a binomial from the product of two binomials.

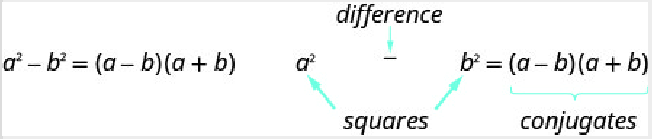
Note:
Product of Conjugates Pattern
If *a* and *b* are real numbers
Equation:

$$(a - b)\,(a + b) = a^2 - b^2$$

The product is called a difference of squares.

To factor, we will use the product pattern “in reverse” to factor the difference of squares. A **difference of squares** factors to a product of conjugates.

Note:
Difference of Squares Pattern
If *a* and *b* are real numbers,



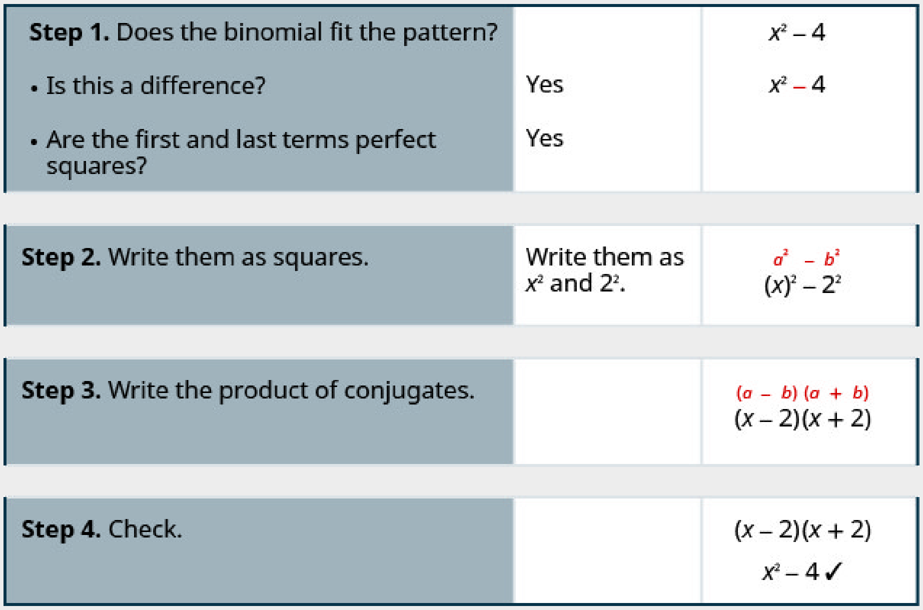
Remember, “difference” refers to subtraction. So, to use this pattern you must make sure you have a binomial in which two squares are being subtracted.

Example:
How to Factor Differences of Squares
Exercise:

Problem: **Factor: $x^2 - 4$.

Solution:

Solution



Step 1. Does the binomial fit the pattern?		$x^2 - 4$
• Is this a difference?	Yes	$x^2 - 4$
• Are the first and last terms perfect squares?	Yes	
Step 2. Write them as squares.	Write them as x^2 and 2^2.	$a^2 - b^2$ $(x)^2 - 2^2$
Step 3. Write the product of conjugates.		$(a - b)(a + b)$ $(x - 2)(x + 2)$
Step 4. Check.		$(x - 2)(x + 2)$ $x^2 - 4$ ✓

Note:
Exercise:

TRY IT : : 6.93

Problem: Factor: $h^2 - 81$.

Solution:

$(h - 9)(h + 9)$

Note:
Exercise:

TRY IT : : 6.94

Problem: Factor: $k^2 - 121$.

Solution:

$(k - 11)(k + 11)$

Note:
Factor differences of squares.

Step 1. Does the binomial fit the pattern?	$a^2 - b^2$
• Is this a difference?	$\_\_\_\_ - \_\_\_\_$
• Are the first and last terms perfect squares?	
Step 2. Write them as squares.	$(a)^2 - (b)^2$
Step 3. Write the product of conjugates.	$(a - b)(a + b)$
Step 4. Check by multiplying.	

It is important to remember that *sums of squares do not factor into a product of binomials*. There are no binomial factors that multiply together to get a sum of squares. After removing any GCF, the expression $a^2 + b^2$ is prime!

Don’t forget that 1 is a perfect square. We’ll need to use that fact in the next example.

Example:
Exercise:

Problem: **Factor: $64y^2 - 1$.

Solution:
Solution

	$64y^2 - 1$
Is this a difference? Yes.	$64y^2 - 1$
Are the first and last terms perfect squares?	
Yes - write them as squares.	$\overset{a^2}{(8y)^2} \overset{-}{-} \overset{b^2}{1^2}$
Factor as the product of conjugates.	$\overset{(a}{(8y} \overset{-}{-} \overset{b)}{1)} \overset{(a}{(8y} \overset{+}{+} \overset{b)}{1)}$
Check by multiplying.	

$(8y-1)(8y+1)$	
$64y^2-1$✓	

Note:
Exercise:

TRY IT : : 6.95
Problem: Factor: m^2-1.

Solution:

$(m-1)(m+1)$

Note:
Exercise:

TRY IT : : 6.96
Problem: Factor: $81y^2-1$.

Solution:

$(9y-1)(9y+1)$

Example:
Exercise:

Problem: **Factor: $121x^2-49y^2$.

Solution:
Solution

$$121x^2 - 49y^2$$

Is this a difference of squares? Yes. $(11x)^2 - (7y)^2$

Factor as the product of conjugates. $(11x - 7y)(11x + 7y)$

Check by multiplying.

$$(11x - 7y)(11x + 7y)$$
$$121x^2 - 49y^2 \checkmark$$

Note:
Exercise:

TRY IT : : 6.97
Problem: Factor: $196m^2 - 25n^2$.

Solution:

$(14m - 5n)(14m + 5n)$

Note:
Exercise:

TRY IT : : 6.98
Problem: Factor: $144p^2 - 9q^2$.

Solution:

$(12p - 3q)(12p + 3q)$

The binomial in the next example may look “backwards,” but it’s still the difference of squares.

Example:
Exercise:

Problem: **Factor: $100 - h^2$.

Solution:
Solution

$$100 - h^2$$

Is this a difference of squares? Yes. $(10)^2 - (h)^2$

Factor as the product of conjugates. $(10 - h)(10 + h)$

Check by multiplying.

$$(10 - h)(10 + h)$$
$$100 - h^2 \checkmark$$

Be careful not to rewrite the original expression as $h^2 - 100$.

Factor $h^2 - 100$ on your own and then notice how the result differs from $(10 - h)(10 + h)$.

Note:
Exercise:

TRY IT : : 6.99
Problem: Factor: $144 - x^2$.

Solution:

$(12 - x)(12 + x)$

Note:
Exercise:

TRY IT : : 6.100
Problem: Factor: $169 - p^2$.

Solution:

$(13 - p)(13 + p)$

To completely factor the binomial in the next example, we’ll factor a difference of squares twice!

Example:
Exercise:

Problem: **Factor: $x^4 - y^4$.

Solution:
Solution

$$x^4 - y^4$$

Is this a difference of squares? Yes.

$$\left(x^2\right)^2 - \left(y^2\right)^2$$

Factor it as the product of conjugates.

$$\left(x^2 - y^2\right)\left(x^2 + y^2\right)$$

Notice the first binomial is also a difference of squares!

$$\left((x)^2 - (y)^2\right)\left(x^2 + y^2\right)$$

Factor it as the product of conjugates. The last factor, the sum of squares, cannot be factored.

$$(x - y)(x + y)\left(x^2 + y^2\right)$$

Check by multiplying.

$$(x - y)(x + y)\left(x^2 + y^2\right)$$
$$[(x - y)(x + y)]\left(x^2 + y^2\right)$$
$$\left(x^2 - y^2\right)\left(x^2 + y^2\right)$$
$$x^4 - y^4 \checkmark$$

Note:
Exercise:

TRY IT : : 6.101

Problem: Factor: $a^4 - b^4$.

Solution:

$\left(a^2 + b^2\right)(a + b)(a - b)$

Note:
Exercise:

TRY IT : : 6.102

Problem: Factor: $x^4 - 16$.

Solution:

$\left(x^2 + 4\right)(x + 2)(x - 2)$

As always, you should look for a common factor first whenever you have an expression to factor. Sometimes a common factor may “disguise” the difference of squares and you won’t recognize the perfect squares until you factor the GCF.

Example:
Exercise:

Problem: **Factor: $8x^2y - 98y$.

Solution:
Solution

	$8x^2y - 98y$
Is there a GCF? Yes, $2y$—factor it out!	$2y\left(4x^2 - 49\right)$
Is the binomial a difference of squares? Yes.	$2y\left((2x)^2 - (7)^2\right)$
Factor as a product of conjugates.	$2y(2x-7)(2x+7)$

Check by multiplying.

$$2y(2x-7)(2x+7)$$
$$2y[(2x-7)(2x+7)]$$
$$2y\left(4x^2 - 49\right)$$
$$8x^2y - 98y \checkmark$$

Note:
Exercise:

TRY IT : : 6.103
Problem: Factor: $7xy^2 - 175x$.

Solution:

$7x(y-5)(y+5)$

Note:
Exercise:

TRY IT : : 6.104
Problem: Factor: $45a^2b - 80b$.

Solution:

$5b\,(3a-4)\,(3a+4)$

Example:
Exercise:

Problem: **Factor: $6x^2+96$.

Solution:
Solution

$6x^2+96$

Is there a GCF? Yes, 6—factor it out! $6\left(x^2+16\right)$

Is the binomial a difference of squares? No, it is a sum of squares. Sums of squares do not factor!

Check by multiplying.

$6\left(x^2+16\right)$
$6x^2+96$ ✓

Note:
Exercise:

TRY IT : : 6.105
Problem: Factor: $8a^2+200$.

Solution:

$8\left(a^2+25\right)$

Note:
Exercise:

TRY IT : : 6.106
Problem: Factor: $36y^2+81$.

Solution:

$9\left(4y^2+9\right)$

Factor Sums and Differences of Cubes

There is another special pattern for factoring, one that we did not use when we multiplied polynomials. This is the pattern for the sum and difference of cubes. We will write these formulas first and then check them by multiplication.

Equation:

$$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$$
$$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$$

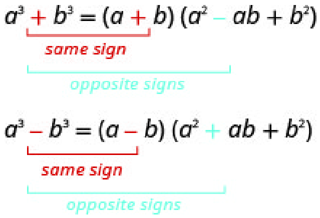
We’ll check the first pattern and leave the second to you.

	$(a + b)\,(a^2 - ab + b^2)$
Distribute.	$a(a^2 - ab + b^2) + b(a^2 - ab + b^2)$
Multiply.	$a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$
Combine like terms.	$a^3 + b^3$

Note:
Sum and Difference of Cubes Pattern
Equation:

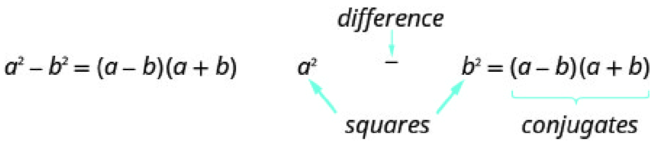
$$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$$
$$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$$

The two patterns look very similar, don’t they? But notice the signs in the factors. The sign of the binomial factor matches the sign in the original binomial. And the sign of the middle term of the trinomial factor is the opposite of the sign in the original binomial. If you recognize the pattern of the signs, it may help you memorize the patterns.



The trinomial factor in the sum and difference of cubes pattern cannot be factored.

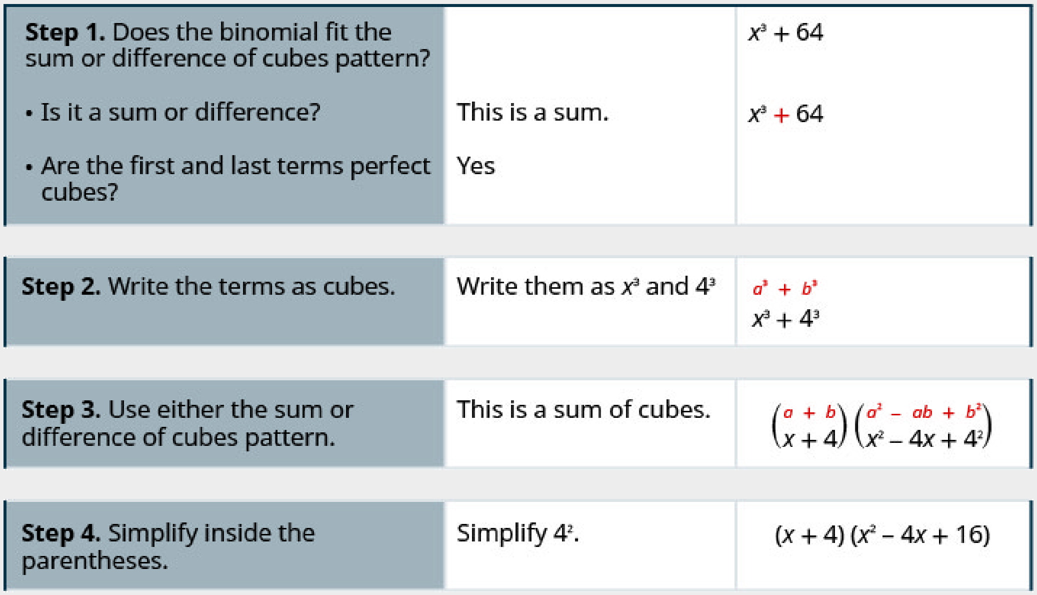
It can be very helpful if you learn to recognize the cubes of the integers from 1 to 10, just like you have learned to recognize squares. We have listed the cubes of the integers from 1 to 10 in [link].



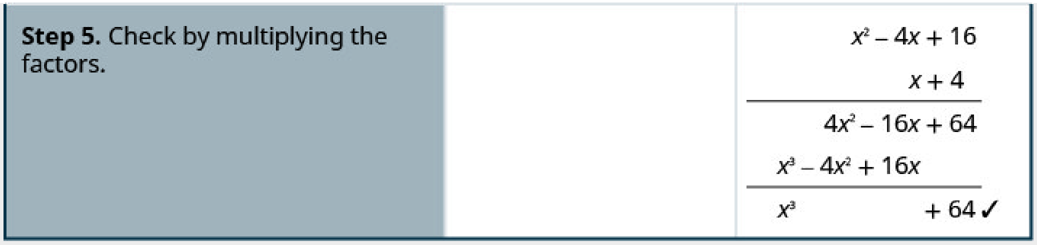
Example:
How to Factor the Sum or Difference of Cubes
Exercise:

Problem: **Factor: x^3+64.

Solution:
Solution



Step 1. Does the binomial fit the sum or difference of cubes pattern?		x^3+64
• Is it a sum or difference?	This is a sum.	x^3+64
• Are the first and last terms perfect cubes?	Yes	
Step 2. Write the terms as cubes.	Write them as x^3 and 4^3	$\overset{a^3}{x^3}+\overset{b^3}{4^3}$
Step 3. Use either the sum or difference of cubes pattern.	This is a sum of cubes.	$\left(\overset{a}{x}+\overset{b}{4}\right)\left(\overset{a^2}{x^2}-\overset{ab}{4x}+\overset{b^2}{4^2}\right)$
Step 4. Simplify inside the parentheses.	Simplify 4^2.	$(x+4)(x^2-4x+16)$



Step 5. Check by multiplying the factors.		$\begin{array}{r} x^2-4x+16 \\ x+4 \\ \hline 4x^2-16x+64 \\ x^3-4x^2+16x \\ \hline x^3 \qquad\quad +64\checkmark \end{array}$

Note:
Exercise:

TRY IT : : 6.107

Problem: Factor: x^3+27.

Solution:

$(x+3)\left(x^2-3x+9\right)$

Note:
Exercise:

TRY IT : : 6.108

Problem: Factor: y^3+8.

Solution:

$(y+2)\left(y^2-2y+4\right)$

Note:
Factor the sum or difference of cubes.
To factor the sum or difference of cubes:

Does the binomial fit the sum or difference of cubes pattern?

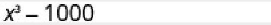
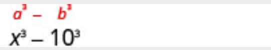
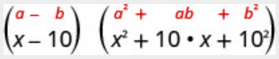
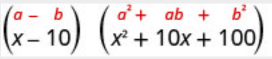
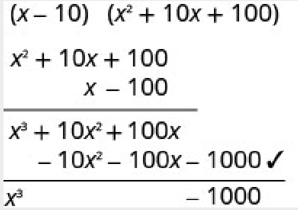
- Is it a sum or difference?
- Are the first and last terms perfect cubes?

Write them as cubes.
Use either the sum or difference of cubes pattern.
Simplify inside the parentheses
Check by multiplying the factors.

Example:
Exercise:

Problem: **Factor: $x^3 - 1000$.

Solution:
Solution

	 $x^3 - 1000$
This binomial is a difference. The first and last terms are perfect cubes.	
Write the terms as cubes.	 $\overset{a^3}{x^3} \overset{-}{-} \overset{b^3}{10^3}$
Use the difference of cubes pattern.	 $\left(\overset{a}{x} - \overset{b}{10}\right)\left(\overset{a^2}{x^2} + \overset{ab}{10 \cdot x} + \overset{b^2}{10^2}\right)$
Simplify.	 $\left(\overset{a}{x} - \overset{b}{10}\right)\left(\overset{a^2}{x^2} + \overset{ab}{10x} + \overset{b^2}{100}\right)$
Check by multiplying.	
 $\begin{array}{l}(x-10)\quad(x^2+10x+100) \\ x^2+10x+100 \\ \qquad x-100 \\ \hline x^3+10x^2+100x \\ \quad -10x^2-100x-1000 \checkmark \\ \hline x^3 \qquad\qquad -1000\end{array}$	

Note:
Exercise:

TRY IT : : 6.109
Problem: Factor: $u^3 - 125$.

Solution:

$\left(u - 5\right)\left(u^2 + 5u + 25\right)$

Note:
Exercise:

TRY IT : : 6.110
Problem: Factor: $v^3 - 343$.

Solution:

$\left(v - 7\right)\left(v^2 + 7v + 49\right)$

Be careful to use the correct signs in the factors of the sum and difference of cubes.

Example:
Exercise:

Problem: **Factor: $512 - 125p^3$.

Solution:
Solution

	$512 - 125p^3$
This binomial is a difference. The first and last terms are perfect cubes.	
Write the terms as cubes.	$\overset{a^3}{8^3} \overset{-}{-} \overset{b^3}{(5p)^3}$
Use the difference of cubes pattern.	

	$\left(\overset{a}{8}\overset{-}{-}\overset{b}{5p}\right)\left(\overset{a^2}{8^2}\overset{+}{+}\overset{ab}{8\cdot 5p}\overset{+}{+}\overset{b^2}{(5p)^2}\right)$
Simplify.	$\left(\overset{a}{8}\overset{-}{-}\overset{b}{5p}\right)\left(\overset{a^2}{64}\overset{+}{+}\overset{ab}{40p}\overset{+}{+}\overset{b^2}{25p^2}\right)$
Check by multiplying.	We'll leave the check to you.

Note:
Exercise:

TRY IT : : 6.111
Problem: Factor: $64-27x^3$.

Solution:

$(4-3x)\left(16+12x+9x^2\right)$

Note:
Exercise:

TRY IT : : 6.112
Problem: Factor: $27-8y^3$.

Solution:

$(3-2y)\left(9+6y+4y^2\right)$

Example:
Exercise:

Problem: **Factor: $27u^3-125v^3$.

Solution:
Solution

	$27u^3 - 125v^3$
This binomial is a difference. The first and last terms are perfect cubes.	
Write the terms as cubes.	$\overset{a^3}{(3u)^3} \overset{-}{-} \overset{b^3}{(5v)^3}$
Use the difference of cubes pattern.	$\left(\overset{a}{3u} \overset{-}{-} \overset{b}{5v}\right)\left(\overset{a^2}{(3u)^2} \overset{+}{+} \overset{ab}{3u \cdot 5v} \overset{+}{+} \overset{b^2}{(5v)^2}\right)$
Simplify.	$\left(\overset{a}{3u} \overset{-}{-} \overset{b}{5v}\right)\left(\overset{a^2}{9u^2} \overset{+}{+} \overset{ab}{15uv} \overset{+}{+} \overset{b^2}{25v^2}\right)$
Check by multiplying.	We'll leave the check to you.

Note:
Exercise:

TRY IT : : 6.113
Problem: Factor: $8x^3 - 27y^3$.

Solution:

$(2x - 3y)\left(4x^2 + 6xy + 9y^2\right)$

Note:
Exercise:

TRY IT : : 6.114
Problem: Factor: $1000m^3 - 125n^3$.

Solution:

$(10m - 5n)\left(100m^2 - 50mn + 25n^2\right)$

In the next example, we first factor out the GCF. Then we can recognize the sum of cubes.

Example:
Exercise:

Problem: **Factor: $5m^3 + 40n^3$.

Solution:
Solution

	$5m^3 + 40n^3$
Factor the common factor.	$5(m^3 + 8n^3)$
This binomial is a sum. The first and last terms are perfect cubes.	
Write the terms as cubes.	$5\left(\overset{a^3}{m^3} \overset{+}{+} \overset{b^3}{(2n)^3}\right)$
Use the sum of cubes pattern.	$5\left(\overset{a}{m} \overset{+}{+} \overset{b}{2n}\right)\left(\overset{a^2}{m^2} \overset{-}{-} \overset{ab}{m \cdot 2n} \overset{+}{+} \overset{b^2}{(2n)^2}\right)$
Simplify.	$5\left(\overset{a}{m} \overset{+}{+} \overset{b}{2n}\right)\left(\overset{a^2}{m^2} \overset{-}{-} \overset{ab}{2m\,n} \overset{+}{+} \overset{b^2}{4n^2}\right)$

Check. To check, you may find it easier to multiply the sum of cubes factors first, then multiply that product by 5. We’ll leave the multiplication for you.

$$5\left(m + 2n\right)\left(m^2 - 2mn + 4n^2\right)$$

Note:
Exercise:

TRY IT : : 6.115

Problem: Factor: $500p^3+4q^3$.

Solution:

$4(5p+q)\left(25p^2-5pq+q^2\right)$

Note:
Exercise:

TRY IT : : 6.116

Problem: Factor: $432c^3+686d^3$.

Solution:

$2\,(6c+7d)\left(36c^2-42cd+49d^2\right)$

Note:
Access these online resources for additional instruction and practice with factoring special products.

- Sum of Difference of Cubes
- Difference of Cubes Factoring

Key Concepts

- **Factor perfect square trinomials** See [link].

Step 1. Does the trinomial fit the pattern?	$a^2+2ab+b^2$	$a^2-2ab+b^2$
Is the first term a perfect square? Write it as a square.	$(a)^2$	$(a)^2$
Is the last term a perfect square? Write it as a square.	$(a)^2 \qquad (b)^2$	$(a)^2 \qquad (b)^2$
Check the middle term. Is it $2ab$?	$(a)^2 \underset{2\cdot a\cdot b}{\searrow\ \swarrow} (b)^2$	$(a)^2 \underset{2\cdot a\cdot b}{\searrow\ \swarrow} (b)^2$
Step 2. Write the square of the binomial.	$(a+b)^2$	$(a-b)^2$
Step 3. Check by multiplying.		

- **Factor differences of squares** See [link].
  Step 1. Does the binomial fit the pattern? $a^2 - b^2$
  Is this a difference? $\_\_\_\_ - \_\_\_\_$
  Are the first and last terms perfect squares?
  Step 2. Write them as squares. $(a)^2 - (b)^2$
  Step 3. Write the product of conjugates. $(a - b)(a + b)$
  Step 4. Check by multiplying.
- **Factor sum and difference of cubes** To factor the sum or difference of cubes: See [link].

  Does the binomial fit the sum or difference of cubes pattern? Is it a sum or difference? Are the first and last terms perfect cubes?
  Write them as cubes.
  Use either the sum or difference of cubes pattern.
  Simplify inside the parentheses
  Check by multiplying the factors.

Practice Makes Perfect

Factor Perfect Square Trinomials

In the following exercises, factor.

Exercise:

Problem: $16y^2 + 24y + 9$

Solution:

$(4y + 3)^2$

Exercise:

Problem: $25v^2 + 20v + 4$

Exercise:

Problem: $36s^2 + 84s + 49$

Solution:

$(6s + 7)^2$

Exercise:

Problem: $49s^2 + 154s + 121$

Exercise:

Problem: $100x^2 - 20x + 1$

Solution:

$(10x-1)^2$

Exercise:

Problem: $64z^2-16z+1$

Exercise:

Problem: $25n^2-120n+144$

Solution:

$(5n-12)^2$

Exercise:

Problem: $4p^2-52p+169$

Exercise:

Problem: $49x^2-28xy+4y^2$

Solution:

$(7x-2y)^2$

Exercise:

Problem: $25r^2-60rs+36s^2$

Exercise:

Problem: $25n^2+25n+4$

Solution:

$(5n+4)(5n+1)$

Exercise:

Problem: $100y^2-20y+1$

Exercise:

Problem: $64m^2-16m+1$

Solution:

$(8m-1)^2$

Exercise:

Problem: $100x^2 - 25x + 1$

Exercise:

Problem: $10k^2 + 80k + 160$

Solution:

$10(k+4)^2$

Exercise:

Problem: $64x^2 - 96x + 36$

Exercise:

Problem: $75u^3 - 30u^2v + 3uv^2$

Solution:

$3u(5u - v)^2$

Exercise:

Problem: $90p^3 + 300p^2q + 250pq^2$

Factor Differences of Squares

In the following exercises, factor.

Exercise:

Problem: $x^2 - 16$

Solution:

$(x-4)\,(x+4)$

Exercise:

Problem: $n^2 - 9$

Exercise:

Problem: $25v^2 - 1$

Solution:

$(5v-1)\,(5v+1)$

Exercise:

Problem: $169q^2 - 1$

Exercise:

Problem: $121x^2 - 144y^2$

Solution:

$\left(11x - 12y\right)\left(11x + 12y\right)$

Exercise:

Problem: $49x^2 - 81y^2$

Exercise:

Problem: $169c^2 - 36d^2$

Solution:

$\left(13c - 6d\right)\left(13c + 6d\right)$

Exercise:

Problem: $36p^2 - 49q^2$

Exercise:

Problem: $4 - 49x^2$

Solution:

$(7x - 2)(7x + 2)\ (2 - 7x)(2 + 7x)$

Exercise:

Problem: $121 - 25s^2$

Exercise:

Problem: $16z^4 - 1$

Solution:

$(2z - 1)(2z + 1)(4z^2 + 1)$

Exercise:

Problem: $m^4 - n^4$

Exercise:

Problem: $5q^2 - 45$

Solution:

$5(q-3)(q+3)$

Exercise:

Problem: $98r^3 - 72r$

Exercise:

Problem: $24p^2 + 54$

Solution:

$6\left(4p^2+9\right)$

Exercise:

Problem: $20b^2 + 140$

Factor Sums and Differences of Cubes

In the following exercises, factor.

Exercise:

Problem: $x^3 + 125$

Solution:

$(x+5)(x^2-5x+25)$

Exercise:

Problem: $n^3 + 512$

Exercise:

Problem: $z^3 - 27$

Solution:

$(z-3)(z^2+3z+9)$

Exercise:

Problem: $v^3 - 216$

Exercise:

Problem: $8 - 343t^3$

Solution:

$(2-7t)(4+14t+49t^2)$

Exercise:

Problem: $125-27w^3$

Exercise:

Problem: $8y^3-125z^3$

Solution:

$(2y-5z)(4y^2+10yz+25z^2)$

Exercise:

Problem: $27x^3-64y^3$

Exercise:

Problem: $7k^3+56$

Solution:

$7(k+2)(k^2-2k+4)$

Exercise:

Problem: $6x^3-48y^3$

Exercise:

Problem: $2-16y^3$

Solution:

$2(1-2y)(1+2y+4y^2)$

Exercise:

Problem: $-2x^3-16y^3$

Mixed Practice

In the following exercises, factor.

Exercise:

Problem: $64a^2-25$

Solution:

$(8a-5)\,(8a+5)$

Exercise:

Problem: $121x^2 - 144$

Exercise:

Problem: $27q^2 - 3$

Solution:

$3\left(3q-1\right)\left(3q+1\right)$

Exercise:

Problem: $4p^2 - 100$

Exercise:

Problem: $16x^2 - 72x + 81$

Solution:

$\left(4x-9\right)^2$

Exercise:

Problem: $36y^2 + 12y + 1$

Exercise:

Problem: $8p^2 + 2$

Solution:

$2\left(4p^2+1\right)$

Exercise:

Problem: $81x^2 + 169$

Exercise:

Problem: $125 - 8y^3$

Solution:

$(5-2y)(25+10y+4y^2)$

Exercise:

Problem: $27u^3 + 1000$

Exercise:

Problem: $45n^2 + 60n + 20$

Solution:

$5(3n + 2)^2$

Exercise:

Problem: $48q^3 - 24q^2 + 3q$

Everyday Math

Exercise:

Problem:

Landscaping Sue and Alan are planning to put a 15 foot square swimming pool in their backyard. They will surround the pool with a tiled deck, the same width on all sides. If the width of the deck is w, the total area of the pool and deck is given by the trinomial $4w^2 + 60w + 225$. Factor the trinomial.

Solution:

$(2w + 15)^2$

Exercise:

Problem:

Home repair The height a twelve foot ladder can reach up the side of a building if the ladder's base is b feet from the building is the square root of the binomial $144 - b^2$. Factor the binomial.

Writing Exercises

Exercise:

Problem:

Why was it important to practice using the binomial squares pattern in the chapter on multiplying polynomials?

Solution:

Answers may vary.

Exercise:

Problem: How do you recognize the binomial squares pattern?

Exercise:

Problem: Explain why $n^2 + 25 \neq (n+5)^2$. Use algebra, words, or pictures.

Solution:

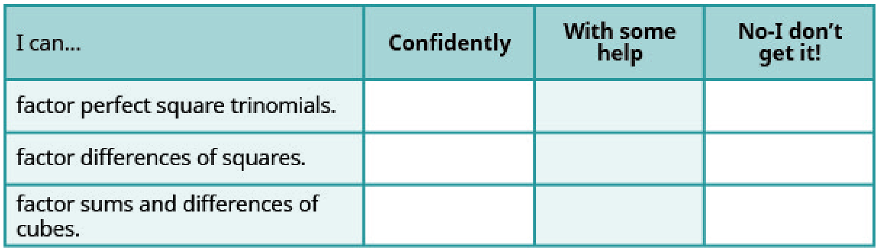
Answers may vary.

Exercise:

Problem: Maribel factored $y^2 - 30y + 81$ as $(y-9)^2$. Was she right or wrong? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
factor perfect square trinomials.			
factor differences of squares.			
factor sums and differences of cubes.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

perfect square trinomials pattern

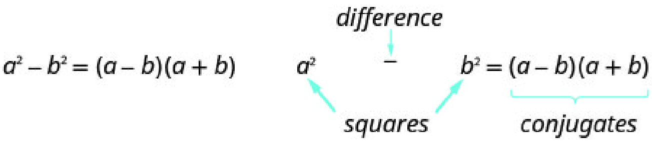
If *a* and *b* are real numbers,

Equation:

$$a^2 + 2ab + b^2 = (a+b)^2 \qquad a^2 - 2ab + b^2 = (a-b)^2$$

difference of squares pattern

If *a* and *b* are real numbers,



$$a^2 - b^2 = (a-b)(a+b) \qquad \underbrace{a^2}_{\text{squares}} \overset{\text{difference}}{-} \underbrace{b^2}_{\text{squares}} = \underbrace{(a-b)(a+b)}_{\text{conjugates}}$$

sum and difference of cubes pattern

Equation:

$$a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$$

$$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$$

General Strategy for Factoring Polynomials
By the end of this section, you will be able to:

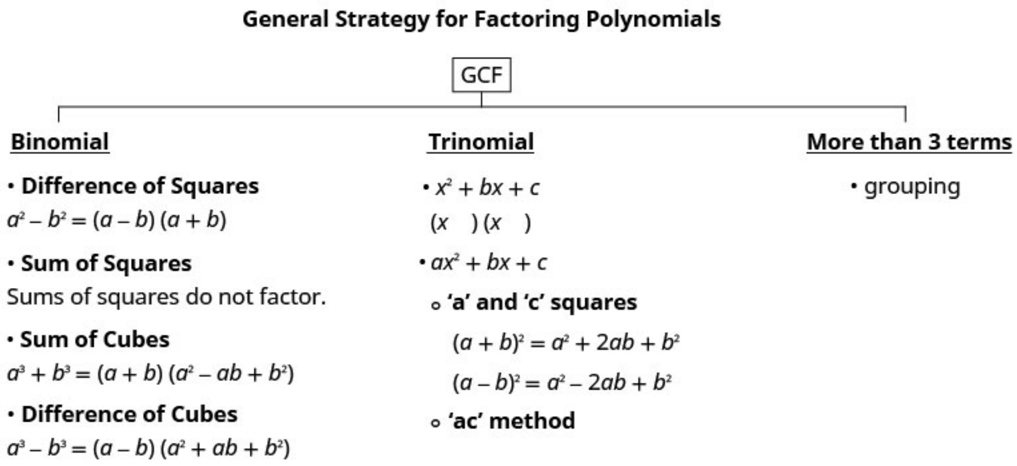
- Recognize and use the appropriate method to factor a polynomial completely

Note:
Before you get started, take this readiness quiz.

1. Factor $y^2 - 2y - 24$.
   If you missed this problem, review [link].
2. Factor $3t^2 + 17t + 10$.
   If you missed this problem, review [link].
3. Factor $36p^2 - 60p + 25$.
   If you missed this problem, review [link].
4. Factor $5x^2 - 80$.
   If you missed this problem, review [link].

Recognize and Use the Appropriate Method to Factor a Polynomial Completely

You have now become acquainted with all the methods of factoring that you will need in this course. (In your next algebra course, more methods will be added to your repertoire.) The figure below summarizes all the factoring methods we have covered. [link] outlines a strategy you should use when factoring polynomials.


General Strategy for Factoring Polynomials
GCF
Binomial
• Difference of Squares
$a^2 - b^2 = (a - b)(a + b)$
• Sum of Squares
Sums of squares do not factor.
• Sum of Cubes
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
• Difference of Cubes
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Trinomial
• $x^2 + bx + c$
$(x\quad)(x\quad)$
• $ax^2 + bx + c$
∘ 'a' and 'c' squares
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
∘ 'ac' method
More than 3 terms
• grouping

Note:
Factor polynomials.

Is there a greatest common factor?
   - Factor it out.

Is the polynomial a binomial, trinomial, or are there more than three terms?
   - If it is a binomial:
     Is it a sum?

    - Of squares? Sums of squares do not factor.
    - Of cubes? Use the sum of cubes pattern.

  Is it a difference?

    - Of squares? Factor as the product of conjugates.
    - Of cubes? Use the difference of cubes pattern.

- If it is a trinomial:
  Is it of the form x^2+bx+c? Undo FOIL.
  Is it of the form ax^2+bx+c?

    - If a and c are squares, check if it fits the trinomial square pattern.
    - Use the trial and error or "ac" method.

- If it has more than three terms:
  Use the grouping method.

Check.

- Is it factored completely?
- Do the factors multiply back to the original polynomial?

Remember, a polynomial is completely factored if, other than monomials, its factors are prime!

Example:
Exercise:

Problem: **Factor completely: $4x^5+12x^4$.

Solution:
Solution

Is there a GCF?	Yes, $4x^4$.	$4x^5+12x^4$
	Factor out the GCF.	$4x^4\left(x+3\right)$
In the parentheses, is it a binomial, a trinomial, or are there more than three terms?	Binomial.	
Is it a sum?		Yes.
Of squares? Of cubes?		No.
Check.		
Is the expression factored completely?		Yes.
Multiply.		

$$4x^4\left(x+3\right)$$
$$4x^4\cdot x+4x^4\cdot 3$$
$$4x^5+12x^4\checkmark$$

Note:
Exercise:

TRY IT : : 6.117
Problem: Factor completely: $3a^4 + 18a^3$.

Solution:

$3a^3\,(a + 6)$

Note:
Exercise:

TRY IT : : 6.118
Problem: Factor completely: $45b^6 + 27b^5$.

Solution:

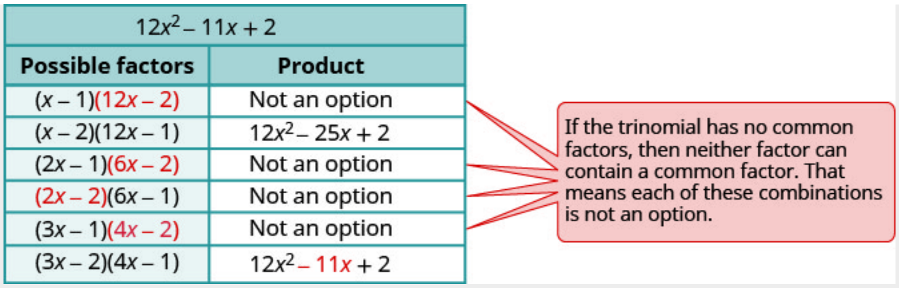
$9b^5\,(5b + 3)$

Example:
Exercise:

Problem: **Factor completely: $12x^2 - 11x + 2$.

Solution:
Solution

		$12x^2 - 11x + 2$
Is there a GCF?	No.	
Is it a binomial, trinomial, or are there more than three terms?	Trinomial.	
Are *a* and *c* perfect squares?	No, *a* = 12, not a perfect square.	
Use trial and error or the “ac” method. We will use trial and error here.		$12x^2 - 11x + 2$ $1x, 12x$ $\quad -1, -2$ $2x, 6x$ $3x, 4x$



$12x^2 - 11x + 2$	
Possible factors	**Product**
$(x-1)(12x-2)$	Not an option
$(x-2)(12x-1)$	$12x^2 - 25x + 2$
$(2x-1)(6x-2)$	Not an option
$(2x-2)(6x-1)$	Not an option
$(3x-1)(4x-2)$	Not an option
$(3x-2)(4x-1)$	$12x^2 - 11x + 2$

If the trinomial has no common factors, then neither factor can contain a common factor. That means each of these combinations is not an option.

Check.

$$(3x-2)(4x-1)$$

$$12x^2 - 3x - 8x + 2$$

$$12x^2 - 11x + 2 \checkmark$$

Note:
Exercise:

TRY IT : : 6.119

Problem: Factor completely: $10a^2 - 17a + 6$.

Solution:

$(5a-6)(2a-1)$

Note:
Exercise:

TRY IT : : 6.120

Problem: Factor completely: $8x^2 - 18x + 9$.

Solution:

$(2x-3)(4x-3)$

Example:
Exercise:

Problem: **Factor completely: $g^3 + 25g$.

Solution:
Solution

Is there a GCF?	Yes, g.	$g^3 + 25g$
Factor out the GCF.		$g\left(g^2 + 25\right)$
In the parentheses, is it a binomial, trinomial, or are there more than three terms?	Binomial.	
Is it a sum ? Of squares?	Yes.	Sums of squares are prime.
Check.		
Is the expression factored completely?	Yes.	

Multiply.

$g\left(g^2 + 25\right)$

$g^3 + 25g$ ✓

Note:
Exercise:

TRY IT : : 6.121
Problem: Factor completely: $x^3 + 36x$.

Solution:

$x\left(x^2 + 36\right)$

Note:
Exercise:

TRY IT : : 6.122
Problem: Factor completely: $27y^2 + 48$.

Solution:

$3\left(9y^2 + 16\right)$

Example:
Exercise:

Problem: **Factor completely: $12y^2 - 75$.

Solution:
Solution

Is there a GCF?	Yes, 3.	$12y^2 - 75$
Factor out the GCF.		$3\left(4y^2 - 25\right)$
In the parentheses, is it a binomial, trinomial, or are there more than three terms?	Binomial.	
Is it a sum?	No.	
Is it a difference? Of squares or cubes?	Yes, squares.	$3\left((2y)^2 - (5)^2\right)$
Write as a product of conjugates.		$3(2y-5)(2y+5)$
Check.		
Is the expression factored completely? Neither binomial is a difference of squares.	Yes.	

Multiply.

$3(2y-5)(2y+5)$

$3\left(4y^2-25\right)$

$12y^2 - 75$ ✓

Note:
Exercise:

TRY IT : : 6.123

Problem: Factor completely: $16x^3 - 36x$.

Solution:

$4x(2x-3)(2x+3)$

Note:
Exercise:

TRY IT : : 6.124

Problem: Factor completely: $27y^2 - 48$.

Solution:

$3(3y-4)(3y+4)$

Example:
Exercise:

Problem: *Factor completely: $4a^2 - 12ab + 9b^2$.

Solution:
Solution

Is there a GCF?	No.	$4a^2 - 12ab + 9b^2$
Is it a binomial, trinomial, or are there more terms?		
Trinomial with $a \neq 1$. But the first term is a perfect square.		
Is the last term a perfect square?	Yes.	$(2a)^2 - 12ab + (3b)^2$
Does it fit the pattern, $a^2 - 2ab + b^2$?	Yes.	$(2a)^2 - 12ab + (3b)^2$ $-2(2a)(3b)$ $12ab$
Write it as a square.		$(2a - 3b)^2$
Check your answer.		
Is the expression factored completely?		
Yes.		
The binomial is not a difference of squares.		
Multiply.		
$(2a - 3b)^2$		
$(2a)^2 - 2 \cdot 2a \cdot 3b + (3b)^2$		
$4a^2 - 12ab + 9b^2$✓		

Note:
Exercise:

TRY IT : : 6.125

Problem: Factor completely: $4x^2 + 20xy + 25y^2$.

Solution:

$(2x + 5y)^2$

Note:
Exercise:

TRY IT : : 6.126
Problem: Factor completely: $9m^2 + 42mn + 49n^2$.

Solution:

$(3m + 7n)^2$

Example:
Exercise:

Problem: **Factor completely: $6y^2 - 18y - 60$.

Solution:
Solution

Is there a GCF?	Yes, 6.	$6y^2 - 1$
Factor out the GCF.	Trinomial with leading coefficient 1.	$6\left(y^2 -\right.$
In the parentheses, is it a binomial, trinomial, or are there more terms?		
"Undo" FOIL.	$6(y\quad)(y\quad)$	$6(y + 2$

Check your answer.
Is the expression factored completely? Ye
Neither binomial is a difference of squares.
Multiply.

$$6(y + 2)(y - 5)$$
$$6\left(y^2 - 5y + 2y - 10\right)$$
$$6\left(y^2 - 3y - 10\right)$$
$$6y^2 - 18y - 60 \checkmark$$

Note:
Exercise:

TRY IT : : 6.127
Problem: Factor completely: $8y^2 + 16y - 24$.

Solution:

$8(y - 1)(y + 3)$

Note:

Exercise:

TRY IT : : 6.128
Factor completely: $5u^2 - 15u - 270$.

Problem:

Solution:

$5\,(u - 9)\,(u + 6)$

Example:
Exercise:

Problem: **Factor completely: $24x^3 + 81$.

Solution:
Solution

Is there a GCF?	Yes, 3.	$24x^3 + 81$
Factor it out.		$3\left(8x^3 + 27\right)$
In the parentheses, is it a binomial, trinomial, or are there more than three terms?	Binomial.	
Is it a sum or difference?	Sum.	
Of squares or cubes?	Sum of cubes.	$3\left(\overset{a^3}{(2x)^3} \overset{+}{+} \overset{b^3}{(3)^3}\right)$
Write it using the sum of cubes pattern.		$3\left(\overset{a}{2x} \overset{+}{+} \overset{b}{3}\right)\left(\overset{a^2}{(2x)^2} \overset{-}{-} \overset{ab}{2x \cdot 3} \overset{+}{+} \overset{b^2}{3^2}\right)$
Is the expression factored completely?	Yes.	$3\left(2x + 3\right)\left(4x^2 - 6x + 9\right)$
Check by multiplying.		We leave the check to you.

Note:
Exercise:

TRY IT : : 6.129

Problem: Factor completely: $250m^3 + 432$.

Solution:

$2\left(5m+6\right)\left(25m^2-30m+36\right)$

Note:
Exercise:

TRY IT : : 6.130

Problem: Factor completely: $81q^3 + 192$.

Solution:

$3\left(3q+4\right)\left(9q^2-12q+16\right)$

Example:
Exercise:

Problem: **Factor completely: $2x^4 - 32$.

Solution:
Solution

Is there a GCF?	Yes, 2.	$2x^4 - 32$
Factor it out.		$2(x^4 - 16$
In the parentheses, is it a binomial, trinomial, or are there more than three terms?	Binomial.	
Is it a sum or difference?	Yes.	
Of squares or cubes?	Difference of squares.	$2\Big((x^2)^2 - ($
Write it as a product of conjugates.		$2(x^2-4)(x^2$
The first binomial is again a difference of squares.		$2\left((x)^2 - (2)^2\right)$
Write it as a product of conjugates.		$2(x-2)(x+2)$
Is the expression factored completely?	Yes.	
None of these binomials is a difference of squares.		
Check your answer.		
Multiply.		

$$2(x-2)(x+2)(x^2+4)$$
$$2(x^2-4)(x^2+4)$$
$$2(x^4-16)$$
$$2x^4-32\checkmark$$

Note:
Exercise:

TRY IT : : 6.131

Problem: Factor completely: $4a^4 - 64$.

Solution:

$4\left(a^2+4\right)(a-2)(a+2)$

Note:
Exercise:

TRY IT : : 6.132

Problem: Factor completely: $7y^4 - 7$.

Solution:

$7\left(y^2+1\right)(y-1)(y+1)$

Example:

Exercise:

Problem: **Factor completely: $3x^2 + 6bx - 3ax - 6ab$.

Solution:
Solution

Is there a GCF?	Yes, 3.	$3x^2 + 6bx - 3ax - 6ab$
Factor out the GCF.		$3\left(x^2 + 2bx - ax - 2ab\right)$
In the parentheses, is it a binomial, trinomial, or are there more terms?	More than 3 terms.	
Use grouping.		$3\left[x\left(x+2b\right) - a\left(x+2b\right)\right]$ $3\left(x+2b\right)\left(x-a\right)$

Check your answer.

Is the expression factored completely? Yes.
Multiply.

$$3\left(x+2b\right)\left(x-a\right)$$
$$3\left(x^2 - ax + 2bx - 2ab\right)$$
$$3x^2 - 3ax + 6bx - 6ab\ \checkmark$$

Note:
Exercise:

TRY IT : : 6.133
Problem: Factor completely: $6x^2 - 12xc + 6bx - 12bc$.

Solution:

$6\left(x+b\right)\left(x-2c\right)$

Note:
Exercise:

TRY IT : : 6.134
Problem: Factor completely: $16x^2 + 24xy - 4x - 6y$.

Solution:

$2(4x-1)(2x+3y)$

Example:
Exercise:

Problem: **Factor completely: $10x^2 - 34x - 24$.

Solution:
Solution

Is there a GCF?	Yes, 2.	$10x^2 - 34x - 24$
Factor out the GCF.		$2\left(5x^2 - 17x - 12\right)$
In the parentheses, is it a binomial, trinomial, or are there more than three terms?	Trinomial with $a \neq 1$.	
Use trial and error or the “ac” method.		$2(5x^2 - 17x{-}12)$
		$2\left(5x + 3\right)\left(x - 4\right)$
Check your answer. Is the expression factored completely? Yes.		

Multiply.

$$2\left(5x + 3\right)\left(x - 4\right)$$
$$2\left(5x^2 - 20x + 3x - 12\right)$$
$$2\left(5x^2 - 17x - 12\right)$$
$$10x^2 - 34x - 24 \checkmark$$

Note:
Exercise:

TRY IT : : 6.135
Problem: Factor completely: $4p^2 - 16p + 12$.

Solution:

$4\left(p - 1\right)\left(p - 3\right)$

Note:
Exercise:

TRY IT : : 6.136
Problem: Factor completely: $6q^2 - 9q - 6$.

Solution:

$3\left(q - 2\right)\left(2q + 1\right)$

Key Concepts

- **General Strategy for Factoring Polynomials See [link].**
- **How to Factor Polynomials**

  Is there a greatest common factor? Factor it out.
  Is the polynomial a binomial, trinomial, or are there more than three terms?

  - If it is a binomial:
    Is it a sum?
    - Of squares? Sums of squares do not factor.
    - Of cubes? Use the sum of cubes pattern.

    Is it a difference?
    - Of squares? Factor as the product of conjugates.
    - Of cubes? Use the difference of cubes pattern.
  - If it is a trinomial:
    Is it of the form $x^2 + bx + c$? Undo FOIL.
    Is it of the form $ax^2 + bx + c$?
    - If 'a' and 'c' are squares, check if it fits the trinomial square pattern.
    - Use the trial and error or 'ac' method.
  - If it has more than three terms:
    Use the grouping method.

  Check. Is it factored completely? Do the factors multiply back to the original polynomial?

Practice Makes Perfect

Recognize and Use the Appropriate Method to Factor a Polynomial Completely

In the following exercises, factor completely.
Exercise:

Problem: $10x^4 + 35x^3$

Solution:

$5x^3(2x + 7)$

Exercise:

Problem: $18p^6 + 24p^3$

Exercise:

Problem: $y^2 + 10y - 39$

Solution:

$(y - 3)(y + 13)$

Exercise:

Problem: $b^2 - 17b + 60$

Exercise:

Problem: $2n^2 + 13n - 7$

Solution:

$(2n - 1)(n + 7)$

Exercise:

Problem: $8x^2 - 9x - 3$

Exercise:

Problem: $a^5 + 9a^3$

Solution:

$a^3(a^2 + 9)$

Exercise:

Problem: $75m^3 + 12m$

Exercise:

Problem: $121r^2 - s^2$

Solution:

$(11r - s)(11r + s)$

Exercise:

Problem: $49b^2 - 36a^2$

Exercise:

Problem: $8m^2 - 32$

Solution:

$8(m - 2)(m + 2)$

Exercise:

Problem: $36q^2 - 100$

Exercise:

Problem: $25w^2 - 60w + 36$

Solution:

$(5w-6)^2$

Exercise:

Problem: $49b^2-112b+64$

Exercise:

Problem: $m^2+14mn+49n^2$

Solution:

$(m+7n)^2$

Exercise:

Problem: $64x^2+16xy+y^2$

Exercise:

Problem: $7b^2+7b-42$

Solution:

$7(b+3)(b-2)$

Exercise:

Problem: $3n^2+30n+72$

Exercise:

Problem: $3x^3-81$

Solution:

$3\left(x-3\right)\left(x^2+3x+9\right)$

Exercise:

Problem: $5t^3-40$

Exercise:

Problem: k^4-16

Solution:

$(k-2)(k+2)(k^2+4)$

Exercise:

Problem: m^4-81

Exercise:

Problem: $15pq-15p+12q-12$

Solution:

$3(5p+4)(q-1)$

Exercise:

Problem: $12ab-6a+10b-5$

Exercise:

Problem: $4x^2+40x+84$

Solution:

$4\left(x+3\right)\left(x+7\right)$

Exercise:

Problem: $5q^2-15q-90$

Exercise:

Problem: u^5+u^2

Solution:

$u^2\left(u+1\right)\left(u^2-u+1\right)$

Exercise:

Problem: $5n^3+320$

Exercise:

Problem: $4c^2+20cd+81d^2$

Solution:

prime

Exercise:

Problem: $25x^2+35xy+49y^2$

Exercise:

Problem: $10m^4-6250$

Solution:

$10\left(m-5\right)\left(m+5\right)\left(m^2+25\right)$

Exercise:

Problem: $3v^4-768$

Everyday Math

Exercise:

Problem:

Watermelon drop A springtime tradition at the University of California San Diego is the Watermelon Drop, where a watermelon is dropped from the seventh story of Urey Hall.

ⓐ The binomial $-16t^2 + 80$ gives the height of the watermelon t seconds after it is dropped. Factor the greatest common factor from this binomial.
ⓑ If the watermelon is thrown down with initial velocity 8 feet per second, its height after t seconds is given by the trinomial $-16t^2 - 8t + 80$. Completely factor this trinomial.

Solution:

ⓐ $-16\left(t^2 - 5\right)$ ⓑ $-8\,(2t + 5)\,(t - 2)$

Exercise:

Problem:

Pumpkin drop A fall tradition at the University of California San Diego is the Pumpkin Drop, where a pumpkin is dropped from the eleventh story of Tioga Hall.

ⓐ The binomial $-16t^2 + 128$ gives the height of the pumpkin t seconds after it is dropped. Factor the greatest common factor from this binomial.
ⓑ If the pumpkin is thrown down with initial velocity 32 feet per second, its height after t seconds is given by the trinomial $-16t^2 - 32t + 128$. Completely factor this trinomial.

Writing Exercises

Exercise:

Problem:

The difference of squares $y^4 - 625$ can be factored as $\left(y^2 - 25\right)\left(y^2 + 25\right)$. But it is not *completely* factored. What more must be done to completely factor it?

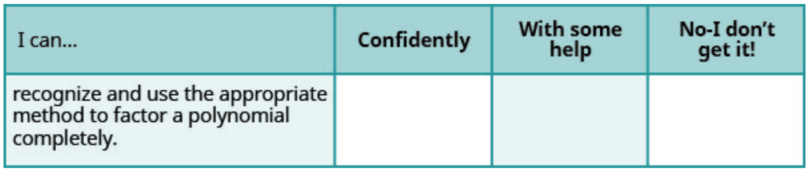
Exercise:

Problem:

Of all the factoring methods covered in this chapter (GCF, grouping, undo FOIL, ‘ac’ method, special products) which is the easiest for you? Which is the hardest? Explain your answers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
recognize and use the appropriate method to factor a polynomial completely.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

Quadratic Equations

By the end of this section, you will be able to:

- Solve quadratic equations by using the Zero Product Property
- Solve quadratic equations factoring
- Solve applications modeled by quadratic equations

Note:

Before you get started, take this readiness quiz.

1. Solve: $5y-3=0$.
   If you missed this problem, review [link].
2. Solve: $10a=0$.
   If you missed this problem, review [link].
3. Combine like terms: $12x^2-6x+4x$.
   If you missed this problem, review [link].
4. Factor n^3-9n^2-22n completely.
   If you missed this problem, review [link].

We have already solved linear equations, equations of the form $ax+by=c$. In linear equations, the variables have no exponents. Quadratic equations are equations in which the variable is squared. Listed below are some examples of quadratic equations:

Equation:

$$x^2+5x+6=0 \qquad 3y^2+4y=10 \qquad 64u^2-81=0 \qquad n\left(n+1\right)=42$$

The last equation doesn't appear to have the variable squared, but when we simplify the expression on the left we will get n^2+n.

The general form of a quadratic equation is $ax^2+bx+c=0$, with $a\ne 0$.

Note:

Quadratic Equation

An equation of the form $ax^2+bx+c=0$ is called a quadratic equation.

Equation:

$$a,b,\text{ and } c \text{ are real numbers and } a\ne 0$$

To solve quadratic equations we need methods different than the ones we used in solving linear equations. We will look at one method here and then several others in a later chapter.

Solve Quadratic Equations Using the Zero Product Property

We will first solve some quadratic equations by using the Zero Product Property. The **Zero Product Property** says that if the product of two quantities is zero, it must be that at least one of the quantities is zero. The only way to get a product equal to zero is to multiply by zero itself.

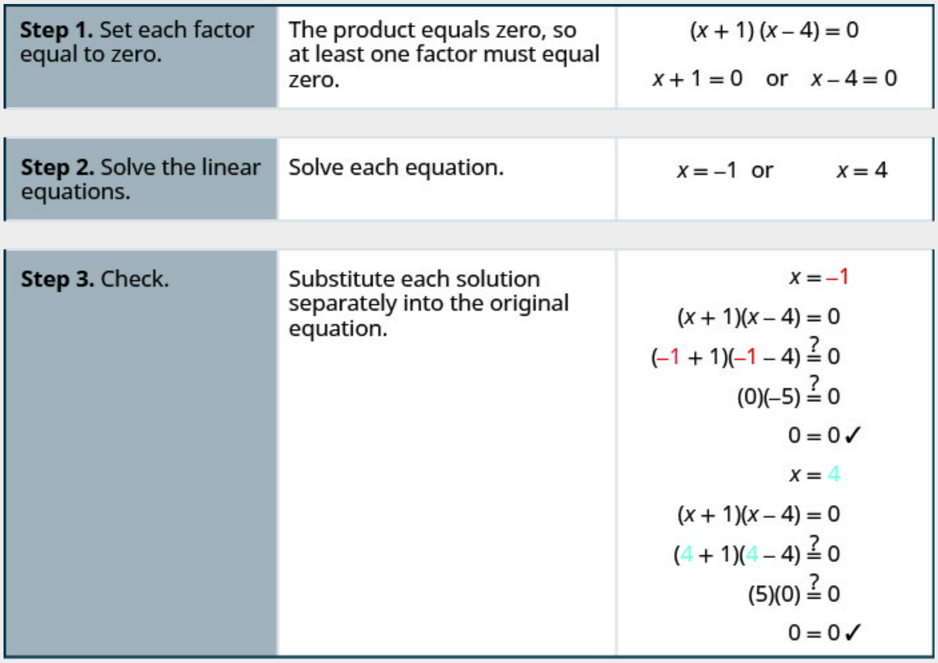
Note:
Zero Product Property
If $a \cdot b = 0$, then either $a = 0$ or $b = 0$ or both.

We will now use the Zero Product Property, to solve a quadratic equation.

Example:
How to Use the Zero Product Property to Solve a Quadratic Equation
Exercise:

Problem: **Solve: $(x + 1)\,(x - 4) = 0$.

Solution:
Solution



Step 1. Set each factor equal to zero.	The product equals zero, so at least one factor must equal zero.	$(x + 1)\,(x - 4) = 0$ $x + 1 = 0 \quad \text{or} \quad x - 4 = 0$
Step 2. Solve the linear equations.	Solve each equation.	$x = -1 \quad \text{or} \quad x = 4$
Step 3. Check.	Substitute each solution separately into the original equation.	$x = -1$ $(x + 1)(x - 4) = 0$ $(-1 + 1)(-1 - 4) \overset{?}{=} 0$ $(0)(-5) \overset{?}{=} 0$ $0 = 0$ ✓ $x = 4$ $(x + 1)(x - 4) = 0$ $(4 + 1)(4 - 4) \overset{?}{=} 0$ $(5)(0) \overset{?}{=} 0$ $0 = 0$ ✓

Note:
Exercise:

TRY IT : : 6.137
Problem: Solve: $(x - 3)\,(x + 5) = 0$.

Solution:

$x = 3, x = -5$

Note:
Exercise:

TRY IT : : 6.138
Problem: Solve: $(y-6)(y+9)=0$.

Solution:

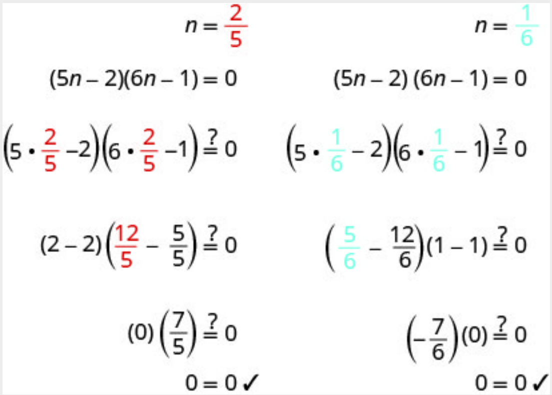
$y=6, y=-9$

We usually will do a little more work than we did in this last example to solve the linear equations that result from using the Zero Product Property.

Example:
Exercise:

Problem: **Solve: $(5n-2)(6n-1)=0$.

Solution:
Solution

	$(5n-2)(6n-1)=0$	
Use the Zero Product Property to set each factor to 0.	$5n-2=0$	$6n-1=0$
Solve the equations.	$n=\frac{2}{5}$	$n=\frac{1}{6}$
Check your answers.		
 $n=\frac{2}{5}$ $(5n-2)(6n-1)=0$ $\left(5\cdot\frac{2}{5}-2\right)\left(6\cdot\frac{2}{5}-1\right)\overset{?}{=}0$ $(2-2)\left(\frac{12}{5}-\frac{5}{5}\right)\overset{?}{=}0$ $(0)\left(\frac{7}{5}\right)\overset{?}{=}0$ $0=0$ ✓ $n=\frac{1}{6}$ $(5n-2)(6n-1)=0$ $\left(5\cdot\frac{1}{6}-2\right)\left(6\cdot\frac{1}{6}-1\right)\overset{?}{=}0$ $\left(\frac{5}{6}-\frac{12}{6}\right)(1-1)\overset{?}{=}0$ $\left(-\frac{7}{6}\right)(0)\overset{?}{=}0$ $0=0$ ✓		

Note:
Exercise:

TRY IT : : 6.139

Problem: Solve: $(3m-2)(2m+1)=0$.

Solution:

$m=\frac{2}{3}, m=-\frac{1}{2}$

Note:
Exercise:

TRY IT : : 6.140

Problem: Solve: $(4p+3)(4p-3)=0$.

Solution:

$p=-\frac{3}{4}, p=\frac{3}{4}$

Notice when we checked the solutions that each of them made just one factor equal to zero. But the product was zero for both solutions.

Example:
Exercise:

Problem: **Solve: $3p(10p+7)=0$.

Solution:
Solution

	$3p(10p+7)=0$	
Use the Zero Product Property to set each factor to 0.	$3p=0$	$10p+7=0$
Solve the equations.	$p=0$	$10p=-7$
		$p=-\frac{7}{10}$
Check your answers.		

$p = 0$	$p = -\frac{7}{10}$
$3p(10p + 7) = 0$	$3p(10p + 7) = 0$
$3 \cdot 0(10 \cdot 0 + 7) \overset{?}{=} 0$	$3\left(-\frac{7}{10}\right)10\left(-\frac{7}{10}\right) + 7 \overset{?}{=} 0$
$0(0 + 7) \overset{?}{=} 0$	$\left(-\frac{21}{10}\right)(-7 + 7) \overset{?}{=} 0$
$0(7) \overset{?}{=} 0$	$\left(-\frac{21}{10}\right)(0) \overset{?}{=} 0$
$0 = 0$ ✓	$0 = 0$ ✓

Note:
Exercise:

TRY IT : : 6.141
Problem: Solve: $2u\left(5u - 1\right) = 0$.

Solution:

$u = 0, u = \frac{1}{5}$

Note:
Exercise:

TRY IT : : 6.142
Problem: Solve: $w\left(2w + 3\right) = 0$.

Solution:

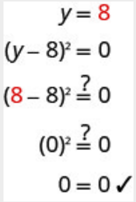
$w = 0, w = -\frac{3}{2}$

It may appear that there is only one factor in the next example. Remember, however, that $\left(y - 8\right)^2$ means $\left(y - 8\right)\left(y - 8\right)$.

Example:
Exercise:

Problem: **Solve: $\left(y - 8\right)^2 = 0$.

Solution:
Solution

	$(y-8)^2=0$	
Rewrite the left side as a product.	$(y-8)(y-8)=0$	
Use the Zero Product Property and set each factor to 0.	$y-8=0$	$y-8=0$
Solve the equations.	$y=8$	$y=8$
When a solution repeats, we call it a double root.		
Check your answer.		
 $y=8$ $(y-8)^2=0$ $(8-8)^2\overset{?}{=}0$ $(0)^2\overset{?}{=}0$ $0=0$ ✓		

Note:
Exercise:

TRY IT : : 6.143
Problem: Solve: $(x+1)^2=0$.

Solution:

$x=1$

Note:
Exercise:

TRY IT : : 6.144
Problem: Solve: $(v-2)^2=0$.

Solution:

$v=2$

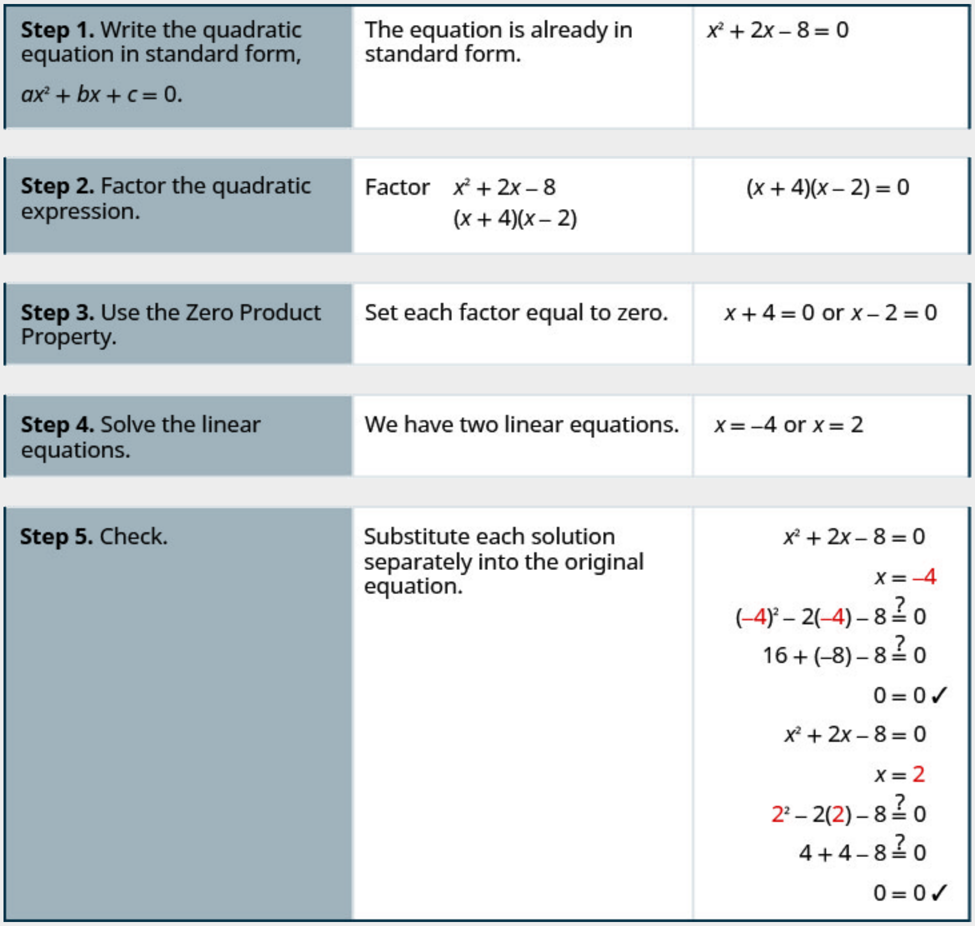
Solve Quadratic Equations by Factoring

Each of the equations we have solved in this section so far had one side in factored form. In order to use the Zero Product Property, the quadratic equation must be factored, with zero on one side. So we be sure to start with the quadratic equation in standard form, $ax^2 + bx + c = 0$. Then we factor the expression on the left.

Example:
How to Solve a Quadratic Equation by Factoring
Exercise:

Problem: **Solve: $x^2 + 2x - 8 = 0$.

Solution:
Solution



Step 1. Write the quadratic equation in standard form, $ax^2 + bx + c = 0$.	The equation is already in standard form.	$x^2 + 2x - 8 = 0$
Step 2. Factor the quadratic expression.	Factor $x^2 + 2x - 8$ $(x + 4)(x - 2)$	$(x + 4)(x - 2) = 0$
Step 3. Use the Zero Product Property.	Set each factor equal to zero.	$x + 4 = 0$ or $x - 2 = 0$
Step 4. Solve the linear equations.	We have two linear equations.	$x = -4$ or $x = 2$
Step 5. Check.	Substitute each solution separately into the original equation.	$x^2 + 2x - 8 = 0$ $x = -4$ $(-4)^2 - 2(-4) - 8 \stackrel{?}{=} 0$ $16 + (-8) - 8 \stackrel{?}{=} 0$ $0 = 0$ ✓ $x^2 + 2x - 8 = 0$ $x = 2$ $2^2 - 2(2) - 8 \stackrel{?}{=} 0$ $4 + 4 - 8 \stackrel{?}{=} 0$ $0 = 0$ ✓

Note:
Exercise:

TRY IT : : 6.145
Problem: Solve: $x^2 - x - 12 = 0$.

Solution:

$x = 4, x = -3$

Note:
Exercise:

TRY IT : : 6.146
Problem: Solve: $b^2 + 9b + 14 = 0$.

Solution:

$b = -2, b = -7$

Note:
Solve a quadratic equation by factoring.

Write the quadratic equation in standard form,$ax^2 + bx + c = 0$.
Factor the quadratic expression.
Use the Zero Product Property.
Solve the linear equations.
Check.

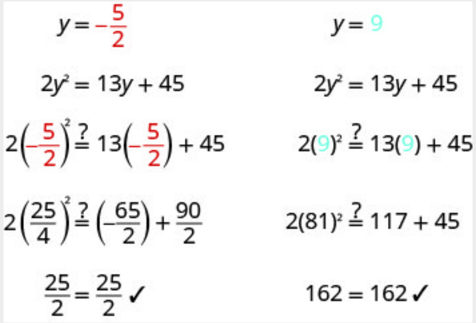
Before we factor, we must make sure the quadratic equation is in standard form.

Example:
Exercise:

Problem: **Solve: $2y^2 = 13y + 45$.

Solution:
Solution

	$2y^2 = 13y + 45$	
Write the quadratic equation in standard form.	$2y^2 - 13y - 45 = 0$	
Factor the quadratic expression.	$(2y + 5)(y - 9) = 0$	
Use the Zero Product Property to set each factor to 0.	$2y + 5 = 0$	$y - 9 = 0$
Solve each equation.	$y = -\frac{5}{2}$	$y = 9$

Check your answers.		
 $y = -\frac{5}{2}$ $\quad$ $y = 9$ $2y^2 = 13y + 45$ $\quad$ $2y^2 = 13y + 45$ $2\left(-\frac{5}{2}\right)^2 \overset{?}{=} 13\left(-\frac{5}{2}\right) + 45$ $\quad$ $2(9)^2 \overset{?}{=} 13(9) + 45$ $2\left(\frac{25}{4}\right)^2 \overset{?}{=} \left(-\frac{65}{2}\right) + \frac{90}{2}$ $\quad$ $2(81)^2 \overset{?}{=} 117 + 45$ $\frac{25}{2} = \frac{25}{2}$ ✓ $\quad$ $162 = 162$ ✓		

Note:
Exercise:

TRY IT : : 6.147
Problem: Solve: $3c^2 = 10c - 8$.

Solution:

$c = 2, c = \frac{4}{3}$

Note:
Exercise:

TRY IT : : 6.148
Problem: Solve: $2d^2 - 5d = 3$.

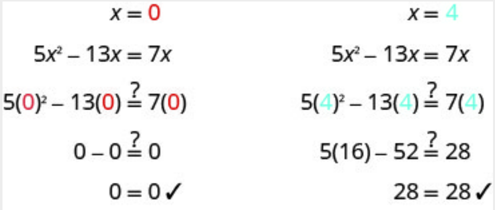
Solution:

$d = 3, d = -\frac{1}{2}$

Example:
Exercise:

Problem: **Solve: $5x^2 - 13x = 7x$.

Solution:
Solution

	$5x^2 - 13x = 7x$	
Write the quadratic equation in standard form.	$5x^2 - 20x = 0$	
Factor the left side of the equation.	$5x(x-4) = 0$	
Use the Zero Product Property to set each factor to 0.	$5x = 0$	$x - 4 = 0$
Solve each equation.	$x = 0$	$x = 4$
Check your answers.		
 $x = 0$ $5x^2 - 13x = 7x$ $5(0)^2 - 13(0) \overset{?}{=} 7(0)$ $0 - 0 \overset{?}{=} 0$ $0 = 0$ ✓ $x = 4$ $5x^2 - 13x = 7x$ $5(4)^2 - 13(4) \overset{?}{=} 7(4)$ $5(16) - 52 \overset{?}{=} 28$ $28 = 28$ ✓		

Note:
Exercise:

TRY IT : : 6.149

Problem: Solve: $6a^2 + 9a = 3a$.

Solution:

$a = 0, a = -1$

Note:
Exercise:

TRY IT : : 6.150

Problem: Solve: $45b^2 - 2b = -17b$.

Solution:

$b = 0, b = -\frac{1}{3}$

Solving quadratic equations by factoring will make use of all the factoring techniques you have learned in this chapter! Do you recognize the special product pattern in the next example?

Example:
Exercise:

Problem: **Solve: $144q^2 = 25$.

Solution:
Solution

	$144q^2 = 25$	
Write the quadratic equation in standard form.	$144q^2 - 25 = 0$	
Factor. It is a difference of squares.	$(12q - 5)(12q + 5) = 0$	
Use the Zero Product Property to set each factor to 0.	$12q - 5 = 0$	$12q + 5 = 0$
Solve each equation.	$12q = 5$ $q = \frac{5}{12}$	$12q = -5$ $q = -\frac{5}{12}$
Check your answers.		

Note:
Exercise:

TRY IT : : 6.151
Problem: Solve: $25p^2 = 49$.

Solution:

$p = \frac{7}{5}, \mathrm{p} = -\frac{7}{5}$

Note:
Exercise:

TRY IT : : 6.152
Problem: Solve: $36x^2 = 121$.

Solution:

$x = \frac{11}{6}, x = -\frac{11}{6}$

The left side in the next example is factored, but the right side is not zero. In order to use the Zero Product Property, one side of the equation must be zero. We'll multiply the factors and then write the equation in standard form.

Example:
Exercise:

Problem: **Solve: $(3x-8)(x-1) = 3x$.

Solution:
Solution

	$(3x-8)(x-1) = 3x$
Multiply the binomials.	$3x^2 - 11x + 8 = 3x$
Write the quadratic equation in standard form.	$3x^2 - 14x + 8 = 0$
Factor the trinomial.	$(3x-2)(x-4) = 0$
Use the Zero Product Property to set each factor to 0.	$3x - 2 = 0 \quad x - 4 = 0$
Solve each equation.	$3x = 2 \quad x = 4$
	$x = \frac{2}{3}$
Check your answers.	The check is left to you!

Note:
Exercise:

TRY IT : : 6.153
Problem: Solve: $(2m+1)(m+3) = 12m$.

Solution:

$m = 1, m = \frac{3}{2}$

Note:
Exercise:

TRY IT : : 6.154
Problem: Solve: $(k+1)(k-1) = 8$.

Solution:

$k=3, k=-3$

The Zero Product Property also applies to the product of three or more factors. If the product is zero, at least one of the factors must be zero. We can solve some equations of degree more than two by using the Zero Product Property, just like we solved quadratic equations.

Example:
Exercise:

Problem: **Solve: $9m^3+100m=60m^2$.

Solution:
Solution

	$9m^3+100m=60m^2$
Bring all the terms to one side so that the other side is zero.	$9m^3-60m^2+100m=0$
Factor the greatest common factor first.	$m(9m^2-60m+100)=0$
Factor the trinomial.	$m(3m-10)(3m-10)=0$
Use the Zero Product Property to set each factor to 0.	$m=0 \quad 3m-10=0 \quad 3m-10=0$
Solve each equation.	$m=0 \quad m=\frac{10}{3} \quad m=\frac{10}{3}$
Check your answers.	The check is left to you.

Note:
Exercise:

TRY IT : : 6.155
Problem: Solve: $8x^3 = 24x^2 - 18x$.

Solution:

$x = 0, x = \frac{3}{2}$

Note:
Exercise:

TRY IT : : 6.156
Problem: Solve: $16y^2 = 32y^3 + 2y$.

Solution:

$y = 0, y = \frac{1}{4}$

When we factor the quadratic equation in the next example we will get three factors. However the first factor is a constant. We know that factor cannot equal 0.

Example:
Exercise:

Problem: **Solve: $4x^2 = 16x + 84$.

Solution:
Solution

	$4x^2 = 16x + 84$
Write the quadratic equation in standard form.	$4x^2 - 16x - 84 = 0$
Factor the greatest common factor first.	$4\left(x^2 - 4x - 21\right) = 0$
Factor the trinomial.	$4\left(x - 7\right)\left(x + 3\right) = 0$
Use the Zero Product Property to set each factor to 0.	$4 \neq 0 \qquad x - 7 = 0 \qquad x + 3 =$
Solve each equation.	$4 \neq 0 \qquad x = 7 \qquad x =$
Check your answers.	The check is left to you.

Note:
Exercise:

TRY IT : : 6.157

Problem: Solve: $18a^2 - 30 = -33a$.

Solution:

$a = -\frac{5}{2}, a = \frac{2}{3}$

Note:
Exercise:

TRY IT : : 6.158

Problem: Solve: $123b = -6 - 60b^2$.

Solution:

$b = -2, b = -\frac{1}{20}$

Solve Applications Modeled by Quadratic Equations

The problem solving strategy we used earlier for applications that translate to linear equations will work just as well for applications that translate to quadratic equations. We will copy the problem solving strategy here so we can use it for reference.

Note:
Use a problem-solving strategy to solve word problems.

Readthe problem. Make sure all the words and ideas are understood.
Identifywhat we are looking for.
Namewhat we are looking for. Choose a variable to represent that quantity.
Translateinto an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
Solvethe equation using good algebra techniques.
Checkthe answer in the problem and make sure it makes sense.
Answerthe question with a complete sentence.

We will start with a number problem to get practice translating words into a quadratic equation.

Example:
Exercise:

Problem: **The product of two consecutive integers is 132. Find the integers.

Solution:

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for. We are looking for two consecutive integers

Step 3. Name what we are looking for.

Let $n =$ the first integer
$n + 1 =$ the next consecutive integer

Step 4. Translate into an equation. Restate the problem in a sentence.

The product of the two consecutive integer

The first integer times the next integer is 1

Translate to an equation. $n\left(n+1\right) = 132$

Step 5. Solve the equation. $n^2 + n = 132$

Bring all the terms to one side. $n^2 + n - 132 = 0$

Factor the trinomial. $\left(n-11\right)\left(n+12\right) = 0$

Use the zero product property. $n - 11 = 0$ $\quad n + 12 =$

Solve the equations. $n = 11$ $\quad n =$

There are two values for n that are solutions to this problem. So there are two sets of consecutive integers that will work.

If the first integer is $n = 11$	If the first integer is $n = -12$
then the next integer is $n + 1$	then the next integer is $n + 1$
$11 + 1$	$-12 + 1$
12	-11

Step 6. Check the answer.

The consecutive integers are $11, 12$ and $-11, -12$. The product $11 \cdot 12 = 132$ and the product $-11\left(-12\right) = 132$. Both pairs of consecutive integers are solutions.

Step 7. Answer the question. The consecutive integers are $11, 12$ and $-11, -12$.

Note:
Exercise:

TRY IT : : 6.159
Problem: The product of two consecutive integers is 240. Find the integers.

Solution:

$-15, -16$ and $15, 16$

Note:
Exercise:

TRY IT : : 6.160

Problem: The product of two consecutive integers is 420. Find the integers.

Solution:

$-21, -20 \text{ and } 20, 21$

Were you surprised by the pair of negative integers that is one of the solutions to the previous example? The product of the two positive integers and the product of the two negative integers both give 132.

In some applications, negative solutions will result from the algebra, but will not be realistic for the situation.

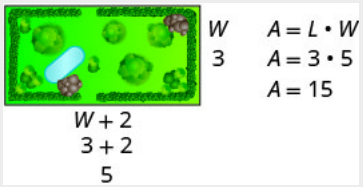
Example:
Exercise:

Problem:

**A rectangular garden has an area 15 square feet. The length of the garden is two feet more than the width. Find the length and width of the garden.

Solution:
Solution

Step 1. Read the problem. In problems involving geometric figures, a sketch can help you visualize the situation.	
Step 2. Identify what you are looking for.	We are looking for the length and width.
Step 3. Name what you are looking for. The length is two feet more than width.	Let W = the width of the garden. $W + 2$ = the length of the garden
Step 4. Translate into an equation. Restate the important information in a sentence.	The area of the rectangular garden is 15 square feet.
Use the formula for the area of a rectangle.	$A = L \cdot W$
Substitute in the variables.	$15 = (W + 2)W$
Step 5. Solve the equation. Distribute first.	$15 = W^2 + 2W$

Get zero on one side.	$0 = W^2 + 2W - 15$	
Factor the trinomial.	$0 = (W + 5)(W - 3)$	
Use the Zero Product Property.	$0 = W + 5$	$0 = W - 3$
Solve each equation.	$-5 = W$	$3 = W$
Since *W* is the width of the garden, it does not make sense for it to be negative. We eliminate that value for *W*.	$\cancel{-5 = W}$ $W = 3$	$3 = W$ Width is 3 feet.
Find the value of the length.	$W + 2 = \text{length}$	
	$3 + 2$	
	5	Length is 5 feet.
Step 6. Check the answer. Does the answer make sense?		
 W 3 $A = L \cdot W$ $A = 3 \cdot 5$ $A = 15$ $W + 2$ $3 + 2$ 5		
	Yes, this makes sense.	
Step 7. Answer the question.	The width of the garden is 3 feet and the length is 5 feet.	

Note:
Exercise:

Problem:

TRY IT : : 6.161
A rectangular sign has an area of 30 square feet. The length of the sign is one foot more than the width. Find the length and width of the sign.

Solution:

5 feet and 6 feet

Note:
Exercise:

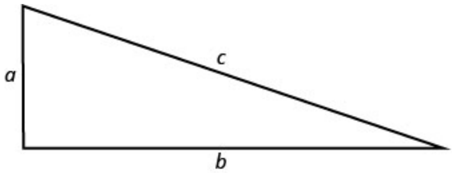
Problem:

TRY IT : : 6.162
A rectangular patio has an area of 180 square feet. The width of the patio is three feet less than the length. Find the length and width of the patio.

Solution:

12 feet and 15 feet

In an earlier chapter, we used the Pythagorean Theorem $\left(a^2 + b^2 = c^2\right)$. It gave the relation between the legs and the hypotenuse of a right triangle.

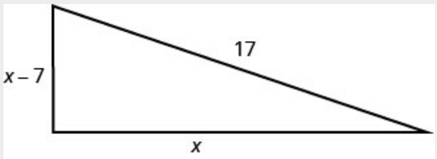


We will use this formula to in the next example.

Example:
Exercise:

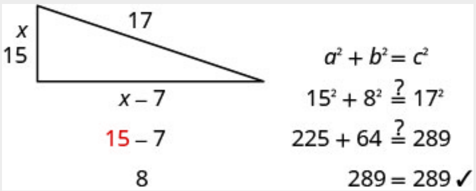
Problem:

**Justine wants to put a deck in the corner of her backyard in the shape of a right triangle, as shown below. The hypotenuse will be 17 feet long. The length of one side will be 7 feet less than the length of the other side. Find the lengths of the sides of the deck.



Solution:
Solution

Step 1. Read the problem.			
Step 2. Identify what you are looking for.	We are looking for the lengths of the sides of the deck.		

Step 3. Name what you are looking for. One side is 7 less than the other.	Let *x* = length of a side of the deck *x* − 7 = length of other side		
Step 4. Translate into an equation. Since this is a right triangle we can use the Pythagorean Theorem.	$a^2+b^2=c^2$		
Substitute in the variables.	$x^2+(x-7)^2=17^2$		
Step 5. Solve the equation.	$x^2+x^2-14x+49=289$		
Simplify.	$2x^2-14x+49=289$		
It is a quadratic equation, so get zero on one side.	$2x^2-14x-240=0$		
Factor the greatest common factor.	$2(x^2-7x-120)=0$		
Factor the trinomial.	$2(x-15)(x+8)=0$		
Use the Zero Product Property.	$2\neq0$	$x-15=0$	$x+8=0$
Solve.	$2\neq0$	$x=15$	$x=-8$
Since *x* is a side of the triangle, $x=-8$ does not make sense.	$2\neq0$	$x=15$	$\cancel{x=-8}$
Find the length of the other side.			
If the length of one side is		$x=15$	
then the length of the other side is		$x-7$	
		$15-7$	
	8 is the length of the other side.		
Step 6. Check the answer. Do these numbers make sense?			
 x 15 17 x – 7 15 – 7 8 a² + b² = c² 15² + 8² ≟ 17² 225 + 64 ≟ 289 289 = 289 ✓			
Step 7. Answer the question.	The sides of the deck are 8, 15, and 17 feet.		

Note:
Exercise:

Problem:

TRY IT : : 6.163
A boat's sail is a right triangle. The length of one side of the sail is 7 feet more than the other side. The hypotenuse is 13. Find the lengths of the two sides of the sail.

Solution:

5 feet and 12 feet

Note:
Exercise:

Problem:

TRY IT : : 6.164
A meditation garden is in the shape of a right triangle, with one leg 7 feet. The length of the hypotenuse is one more than the length of one of the other legs. Find the lengths of the hypotenuse and the other leg.

Solution:

24 feet and 25 feet

Key Concepts

- **Zero Product Property** If $a \cdot b = 0$, then either $a = 0$ or $b = 0$ or both. See [link].
- **Solve a quadratic equation by factoring** To solve a quadratic equation by factoring: See [link].

  Write the quadratic equation in standard form,$ax^2 + bx + c = 0$.
  Factor the quadratic expression.
  Use the Zero Product Property.
  Solve the linear equations.
  Check.

- **Use a problem solving strategy to solve word problems** See [link].

  Readthe problem. Make sure all the words and ideas are understood.
  Identifywhat we are looking for.
  Namewhat we are looking for. Choose a variable to represent that quantity.
  Translateinto an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  Solvethe equation using good algebra techniques.
  Checkthe answer in the problem and make sure it makes sense.
  Answerthe question with a complete sentence.

Section Exercises

Practice Makes Perfect

Use the Zero Product Property

In the following exercises, solve.

Exercise:

Problem: $\left(x-3\right)\left(x+7\right)=0$

Solution:

$x=3,x=-7$

Exercise:

Problem: $\left(y-11\right)\left(y+1\right)=0$

Exercise:

Problem: $\left(3a-10\right)\left(2a-7\right)=0$

Solution:

$a=10/3,a=7/2$

Exercise:

Problem: $\left(5b+1\right)\left(6b+1\right)=0$

Exercise:

Problem: $6m\left(12m-5\right)=0$

Solution:

$m=0,m=5/12$

Exercise:

Problem: $2x\left(6x-3\right)=0$

Exercise:

Problem: $\left(y-3\right)^{2}=0$

Solution:

$y=3$

Exercise:

Problem: $\left(b+10\right)^{2}=0$

Exercise:

Problem: $\left(2x-1\right)^{2}=0$

Solution:

$x = 1/2$

Exercise:

Problem: $(3y + 5)^2 = 0$

Solve Quadratic Equations by Factoring

In the following exercises, solve.

Exercise:

Problem: $x^2 + 7x + 12 = 0$

Solution:

$x = -3, x = -4$

Exercise:

Problem: $y^2 - 8y + 15 = 0$

Exercise:

Problem: $5a^2 - 26a = 24$

Solution:

$a = -4/5, a = 6$

Exercise:

Problem: $4b^2 + 7b = -3$

Exercise:

Problem: $4m^2 = 17m - 15$

Solution:

$m = 5/4, m = 3$

Exercise:

Problem: $n^2 = 5 - 6n$ $n^2 = 5n - 6$

Exercise:

Problem: $7a^2 + 14a = 7a$

Solution:

$a = -1, a = 0$

Exercise:

Problem: $12b^2 - 15b = -9b$

Exercise:

Problem: $49m^2 = 144$

Solution:

$m = 12/7, m = -12/7$

Exercise:

Problem: $625 = x^2$

Exercise:

Problem: $(y - 3)(y + 2) = 4y$

Solution:

$y = -1, y = 6$

Exercise:

Problem: $(p - 5)(p + 3) = -7$

Exercise:

Problem: $(2x + 1)(x - 3) = -4x$

Solution:

$x = 3/2, x = -1$

Exercise:

Problem: $(x + 6)(x - 3) = -8$

Exercise:

Problem: $16p^3 = 24p^2 + 9p$

Solution:

$p = 0, p =$ ¾

Exercise:

Problem: $m^3 - 2m^2 = -m$

Exercise:

Problem: $20x^2 - 60x = -45$

Solution:

$x = 3/2$

Exercise:

Problem: $3y^2 - 18y = -27$

Solve Applications Modeled by Quadratic Equations

In the following exercises, solve.

Exercise:

Problem: The product of two consecutive integers is 56. Find the integers.

Solution:

$7 \text{ and } 8; -8 \text{ and } -7$

Exercise:

Problem: The product of two consecutive integers is 42. Find the integers.

Exercise:

Problem:

The area of a rectangular carpet is 28 square feet. The length is three feet more than the width. Find the length and the width of the carpet.

Solution:

$4 \text{ feet and } 7 \text{ feet}$

Exercise:

Problem:

A rectangular retaining wall has area 15 square feet. The height of the wall is two feet less than its length. Find the height and the length of the wall.

Exercise:

Problem:

A pennant is shaped like a right triangle, with hypotenuse 10 feet. The length of one side of the pennant is two feet longer than the length of the other side. Find the length of the two sides of the pennant.

Solution:

$6 \text{ feet and } 8 \text{ feet}$

Exercise:

Problem:

A reflecting pool is shaped like a right triangle, with one leg along the wall of a building. The hypotenuse is 9 feet longer than the side along the building. The third side is 7 feet longer than the side along the building. Find the lengths of all three sides of the reflecting pool.

Mixed Practice

In the following exercises, solve.

Exercise:

Problem: $(x+8)(x-3)=0$

Solution:

$x=-8, x=3$

Exercise:

Problem: $(3y-5)(y+7)=0$

Exercise:

Problem: $p^2+12p+11=0$

Solution:

$p=-1, p=-11$

Exercise:

Problem: $q^2-12q-13=0$

Exercise:

Problem: $m^2=6m+16$

Solution:

$m=-2, m=8$

Exercise:

Problem: $4n^2+19n=5$

Exercise:

Problem: $a^3-a^2-42a=0$

Solution:

$a=0, a=-6, a=7$

Exercise:

Problem: $4b^2-60b+224=0$

Exercise:

Problem: The product of two consecutive integers is 110. Find the integers.

Solution:

10 and 11; -11 and -10

Exercise:

Problem:

The length of one leg of a right triangle is three more than the other leg. If the hypotenuse is 15, find the lengths of the two legs.

Everyday Math

Exercise:

Problem:

Area of a patio If each side of a square patio is increased by 4 feet, the area of the patio would be 196 square feet. Solve the equation $(s+4)^2 = 196$ for s to find the length of a side of the patio.

Solution:

10 feet

Exercise:

Problem:

Watermelon drop A watermelon is dropped from the tenth story of a building. Solve the equation $-16t^2 + 144 = 0$ for t to find the number of seconds it takes the watermelon to reach the ground.

Writing Exercises

Exercise:

Problem:

Explain how you solve a quadratic equation. How many answers do you expect to get for a quadratic equation?

Solution:

Answers may vary.

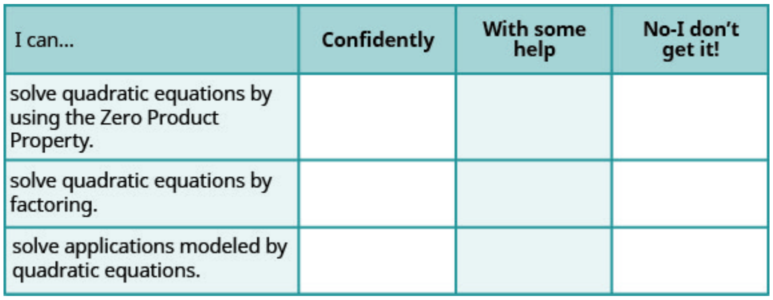
Exercise:

Problem:

Give an example of a quadratic equation that has a GCF and none of the solutions to the equation is zero.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
solve quadratic equations by using the Zero Product Property.			
solve quadratic equations by factoring.			
solve applications modeled by quadratic equations.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

Glossary

quadratic equations	are equations in which the variable is squared.
Zero Product Property	The Zero Product Property states that, if the product of two quantities is zero, at least one of the quantities is zero.

Review & Practice Test
By the end of this section, you will be prepared to take your Chapter 6 Assessment.

Chapter 6 Review Exercises

Greatest Common Factor and Factor by Grouping

Find the Greatest Common Factor of Two or More Expressions

In the following exercises, find the greatest common factor.

Exercise:

Problem: $42, 60$

Solution:

6

Exercise:

Problem: $450, 420$

Exercise:

Problem: $90, 150, 105$

Solution:

15

Exercise:

Problem: $60, 294, 630$

Factor the Greatest Common Factor from a Polynomial

In the following exercises, factor the greatest common factor from each polynomial.

Exercise:

Problem: $24x - 42$

Solution:

$6(4x - 7)$

Exercise:

Problem: $35y + 84$

Exercise:

Problem: $15m^4 + 6m^2n$

Solution:

$3m^2(5m^2 + 2n)$

Exercise:

Problem: $24pt^4 + 16t^7$

Factor by Grouping

In the following exercises, factor by grouping.

Exercise:

Problem: $ax - ay + bx - by$

Solution:

$(a + b)\,(x - y)$

Exercise:

Problem: $x^2y - xy^2 + 2x - 2y$

Exercise:

Problem: $x^2 + 7x - 3x - 21$

Solution:

$(x - 3)\,(x + 7)$

Exercise:

Problem: $4x^2 - 16x + 3x - 12$

Exercise:

Problem: $m^3 + m^2 + m + 1$

Solution:

$(m^2 + 1)(m + 1)$

Exercise:

Problem: $5x - 5y - y + x$

Factor Trinomials of the form $x^2 + bx + c$

Factor Trinomials of the Form $x^2 + bx + c$

In the following exercises, factor each trinomial of the form $x^2 + bx + c$.

Exercise:

Problem: $u^2 + 17u + 72$

Solution:

$(u+8)(u+9)$

Exercise:

Problem: $a^2+14a+33$

Exercise:

Problem: $k^2-16k+60$

Solution:

$(k-6)(k-10)$

Exercise:

Problem: $r^2-11r+28$

Exercise:

Problem: y^2+6y-7

Solution:

$(y+7)(y-1)$

Exercise:

Problem: $m^2+3m-54$

Exercise:

Problem: s^2-2s-8

Solution:

$(s-4)(s+2)$

Exercise:

Problem: $x^2-3x-10$

Factor Trinomials of the Form $x^2+bxy+cy^2$

In the following examples, factor each trinomial of the form $x^2+bxy+cy^2$.

Exercise:

Problem: $x^2+12xy+35y^2$

Solution:

$(x+5y)(x+7y)$

Exercise:

Problem: $u^2+14uv+48v^2$

Exercise:

Problem: $a^2+4ab-21b^2$

Solution:

$(a+7b)(a-3b)$

Exercise:

Problem: $p^2-5pq-36q^2$

Factoring Trinomials of the form ax^2+bx+c

Recognize a Preliminary Strategy to Factor Polynomials Completely

In the following exercises, identify the best method to use to factor each polynomial.

Exercise:

Problem: $y^2 - 17y + 42$

Solution:

Undo FOIL

Exercise:

Problem: $12r^2 + 32r + 5$

Exercise:

Problem: $8a^3 + 72a$

Solution:

Factor the GCF

Exercise:

Problem: $4m - mn - 3n + 12$

Factor Trinomials of the Form $ax^2 + bx + c$ with a GCF

In the following exercises, factor completely.

Exercise:

Problem: $6x^2 + 42x + 60$

Solution:

$6\left(x+2\right)\left(x+5\right)$

Exercise:

Problem: $8a^2+32a+24$

Exercise:

Problem: $3n^4-12n^3-96n^2$

Solution:

$3n^2\left(n-8\right)\left(n+4\right)$

Exercise:

Problem: $5y^4+25y^2-70y$

Factor Trinomials Using the “ac” Method

In the following exercises, factor.

Exercise:

Problem: $2x^2+9x+4$

Solution:

$\left(x+4\right)\left(2x+1\right)$

Exercise:

Problem: $3y^2+17y+10$

Exercise:

Problem: $18a^2-9a+1$

Solution:

$(3a-1)(6a-1)$

Exercise:

Problem: $8u^2-14u+3$

Exercise:

Problem: $15p^2+2p-8$

Solution:

$(5p+4)(3p-2)$

Exercise:

Problem: $15x^2+6x-2$

Exercise:

Problem: $40s^2-s-6$

Solution:

$(5s-2)(8s+3)$

Exercise:

Problem: $20n^2-7n-3$

Factor Trinomials with a GCF Using the “ac” Method

In the following exercises, factor.

Exercise:

Problem: $3x^2+3x-36$

Solution:

$3\left(x+4\right)\left(x-3\right)$

Exercise:

Problem: $4x^2+4x-8$

Exercise:

Problem: $60y^2-85y-25$

Solution:

$5\left(4y+1\right)\left(3y-5\right)$

Exercise:

Problem: $18a^2-57a-21$

Factoring Special Products

Factor Perfect Square Trinomials

In the following exercises, factor.

Exercise:

Problem: $25x^2+30x+9$

Solution:

$\left(5x+3\right)^2$

Exercise:

Problem: $16y^2+72y+81$

Exercise:

Problem: $36a^2 - 84ab + 49b^2$

Solution:

$(6a - 7b)^2$

Exercise:

Problem: $64r^2 - 176rs + 121s^2$

Exercise:

Problem: $40x^2 + 360x + 810$

Solution:

$10(2x + 9)^2$

Exercise:

Problem: $75u^2 + 180u + 108$

Exercise:

Problem: $2y^3 - 16y^2 + 32y$

Solution:

$2y(y - 4)^2$

Exercise:

Problem: $5k^3 - 70k^2 + 245k$

Factor Differences of Squares

In the following exercises, factor.

Exercise:

Problem: $81r^2 - 25$

Solution:

$\left(9r - 5\right)\left(9r + 5\right)$

Exercise:

Problem: $49a^2 - 144$

Exercise:

Problem: $169m^2 - n^2$

Solution:

$\left(13m + n\right)\left(13m - n\right)$

Exercise:

Problem: $64x^2 - y^2$

Exercise:

Problem: $25p^2 - 1$

Solution:

$\left(5p - 1\right)\left(5p + 1\right)$

Exercise:

Problem: $1 - 16s^2$

Exercise:

Problem: $9-121y^2$

Solution:

$(3+11y)(3-11y)$

Exercise:

Problem: $100k^2-81$

Exercise:

Problem: $20x^2-125$

Solution:

$5(2x-5)(2x+5)$

Exercise:

Problem: $18y^2-98$

Exercise:

Problem: $49u^3-9u$

Solution:

$u(7u+3)(7u-3)$

Exercise:

Problem: $169n^3-n$

Factor Sums and Differences of Cubes

In the following exercises, factor.

Exercise:

Problem: $a^3 - 125$

Solution:

$\left(a-5\right)\left(a^2+5a+25\right)$

Exercise:

Problem: $b^3 - 216$

Exercise:

Problem: $2m^3 + 54$

Solution:

$2\left(m+3\right)\left(m^2-3m+9\right)$

Exercise:

Problem: $81x^3 + 3$

General Strategy for Factoring Polynomials

Recognize and Use the Appropriate Method to Factor a Polynomial Completely

In the following exercises, factor completely.

Exercise:

Problem: $24x^3 + 44x^2$

Solution:

$4x^2(6x + 11)$

Exercise:

Problem: $24a^4 - 9a^3$

Exercise:

Problem: $16n^2 - 56mn + 49m^2$

Solution:

$(4n - 7m)^2$

Exercise:

Problem: $6a^2 - 25a - 9$

Exercise:

Problem: $5r^2 + 22r - 48$

Solution:

$(r + 6)(5r - 8)$

Exercise:

Problem: $5u^4 - 45u^2$

Exercise:

Problem: $n^4 - 81$

Solution:

$\left(n^2 + 9\right)(n + 3)(n - 3)$

Exercise:

Problem: $64j^2 + 225$

Exercise:

Problem: $5x^2 + 5x - 60$

Solution:

$5\left(x - 3\right)\left(x + 4\right)$

Exercise:

Problem: $b^3 - 64$

Exercise:

Problem: $m^3 + 125$

Solution:

$(m + 5)(m^2 - 5m + 25)$

Exercise:

Problem: $2b^2 - 2bc + 5cb - 5c^2$

Quadratic Equations

Use the Zero Product Property

In the following exercises, solve.

Exercise:

Problem: $(a - 3)(a + 7) = 0$

Solution:

$a = 3\, a = -7$

Exercise:

Problem: $(b-3)(b+10)=0$

Exercise:

Problem: $3m(2m-5)(m+6)=0$

Solution:

$m = 0\, m = \frac{5}{2}\, m = -6$

Exercise:

Problem: $7n(3n+8)(n-5)=0$

Solve Quadratic Equations by Factoring

In the following exercises, solve.

Exercise:

Problem: $x^2+9x+20=0$

Solution:

$x=-4, x=-5$

Exercise:

Problem: $y^2-y-72=0$

Exercise:

Problem: $2p^2 - 11p = 40$

Solution:

$p = -\frac{5}{2}, p = 8$

Exercise:

Problem: $q^3 + 3q^2 + 2q = 0$

Exercise:

Problem: $144m^2 - 25 = 0$

Solution:

$m = \frac{5}{12}, m = -\frac{5}{12}$

Exercise:

Problem: $4n^2 = 36$

Solve Applications Modeled by Quadratic Equations

In the following exercises, solve.

Exercise:

Problem:

The product of two consecutive numbers is 462. Find the numbers.

Solution:

$-21, -22\ 21, 22$

Exercise:

Problem:

The area of a rectangular shaped patio 400 square feet. The length of the patio is 9 feet more than its width. Find the length and width.

Practice Test

In the following exercises, find the Greatest Common Factor in each expression.

Exercise:

Problem: $14y - 42$

Solution:

$7\left(y - 6\right)$

Exercise:

Problem: $-6x^2 - 30x$

Exercise:

Problem: $80a^2 + 120a^3$

Solution:

$40a^2(2 + 3a)$

Exercise:

Problem: $5m\left(m - 1\right) + 3\left(m - 1\right)$

In the following exercises, factor completely.

Exercise:

Problem: $x^2 + 13x + 36$

Solution:

$(x + 7)(x + 6)$

Exercise:

Problem: $p^2 + pq - 12q^2$

Exercise:

Problem: $3a^3 - 6a^2 - 72a$

Solution:

$3a(a + 4)(a - 6)$

Exercise:

Problem: $s^2 - 25s + 84$

Exercise:

Problem: $5n^2 + 30n + 45$

Solution:

$5(n + 3)^2$

Exercise:

Problem: $64y^2 - 49$

Exercise:

Problem: $xy - 8y + 7x - 56$

Solution:

$(x-8)\,(y+7)$

Exercise:

Problem: $40r^2+810$

Exercise:

Problem: $9s^2-12s+4$

Solution:

$(3s-2)^2$

Exercise:

Problem: $n^2+12n+36$

Exercise:

Problem: $100-a^2$

Solution:

$(10-a)\,(10+a)$

Exercise:

Problem: $6x^2-11x-10$

Exercise:

Problem: $3x^2-75y^2$

Solution:

$3\,(x+5y)\,(x-5y)$

Exercise:

Problem: $c^3 - 1000d^3$

Exercise:

Problem: $ab - 3b - 2a + 6$

Solution:

$(a - 3)(b - 2)$

Exercise:

Problem: $6u^2 + 3u - 18$

Exercise:

Problem: $8m^2 + 22m + 5$

Solution:

$(4m + 1)(2m + 5)$

In the following exercises, solve.

Exercise:

Problem: $x^2 + 9x + 20 = 0$

Exercise:

Problem: $y^2 = y + 132$

Solution:

$\text{y} = -11, \text{y} = 12$

Exercise:

Problem: $5a^2 + 26a = 24$

Exercise:

Problem: $9b^2 - 9 = 0$

Solution:

$b = 1, b = -1$

Exercise:

Problem: $16 - m^2 = 0$

Exercise:

Problem: $4n^2 + 19n + 21 = 0$

Solution:

$n = -\frac{7}{4}, n = -3$

Exercise:

Problem: $(x - 3)(x + 2) = 6$

Exercise:

Problem:

The product of two consecutive integers is 156. Find the integers.

Solution:

12 and 13; -13 and -12

Exercise:

Problem:

The area of a rectangular place mat is 168 square inches. Its length is two inches longer than the width. Find the length and width of the placemat.

Introduction class="introduction"

Rowing a boat downstream can be very relaxing, but it takes much more effort to row the boat upstream.

Like rowing a boat, riding a bicycle is a situation in which going in one direction, downhill, is easy, but going in the opposite direction, uphill, can be more work. The trip to reach a destination may be quick, but the return trip whether upstream or uphill will take longer.

Rational equations are used to model situations like these. In this chapter, we will work with simplifying rational expressions.

Simplify Rational Expressions
By the end of this section, you will be able to:

- Determine the values for which a rational expression is undefined
- Evaluate rational expressions
- Simplify rational expressions
- Simplify rational expressions with opposite factors

Note:
Before you get started, take this readiness quiz.
If you miss a problem, go back to the section listed and review the material.

1. Simplify: $\frac{90y}{15y^2}$.
   If you missed this problem, review [link].
2. Factor: $6x^2 - 7x + 2$.
   If you missed this problem, review [link].
3. Factor: $n^3 + 8$.
   If you missed this problem, review [link].

In Chapter 1, we reviewed the properties of fractions and their operations. We introduced rational numbers, which are just fractions where the numerators and denominators are integers, and the denominator is not zero.

In this chapter, we will work with fractions whose numerators and denominators are polynomials. We call these rational expressions.

Note:
Rational Expression
A **rational expression** is an expression of the form $\frac{p(x)}{q(x)}$, where p and q are polynomials and $q \neq 0$.

Remember, division by 0 is undefined.

Here are some examples of rational expressions:
Equation:

$$-\frac{13}{42} \qquad \frac{7y}{8z} \qquad \frac{5x+2}{x^2-7} \qquad \frac{4x^2+3x-1}{2x-8}$$

Notice that the first rational expression listed above, $-\frac{13}{42}$, is just a fraction. Since a constant is a polynomial with degree zero, the ratio of two constants is a rational expression, provided the denominator is not zero.

We will perform same operations with rational expressions that we do with fractions. We will simplify, add, subtract, multiply, divide, and use them in applications.

Determine the Values for Which a Rational Expression is Undefined

When we work with a numerical fraction, it is easy to avoid dividing by zero, because we can see the number in the denominator. In order to avoid dividing by zero in a rational expression, we must not allow values of the variable that will make the denominator be zero.

If the denominator is zero, the rational expression is undefined. The numerator of a rational expression may be 0—but not the denominator.

So before we begin any operation with a rational expression, we examine it first to find the values that would make the denominator zero. That way, when we solve a rational equation for example, we will know whether the algebraic solutions we find are allowed or not.

Note:
Determine the Values for Which a Rational Expression is Undefined.

Set the denominator equal to zero.
Solve the equation in the set of reals, if possible.

Example:
Exercise:

Problem: **Determine the values for which the rational expression is undefined:

ⓐ $\frac{9y}{x}$ ⓑ $\frac{4b-3}{2b+5}$ ⓒ $\frac{x+4}{x^2+5x+6}$

Solution:
Solution

The expression will be undefined when the denominator is zero.

ⓐ

Set the denominator equal to zero. Solve for the variable.

$$\frac{9y}{x}$$

$$x = 0$$

$\frac{9y}{x}$ is undefined for $x = 0$.

ⓑ

Set the denominator equal to zero. Solve for the variable.

$$\frac{4b-3}{2b+5}$$

$$\begin{aligned} 2b+5 &= 0 \\ 2b &= -5 \\ b &= -\frac{5}{2} \end{aligned}$$

$\frac{4b-3}{2b+5}$ is undefined for $b = -\frac{5}{2}$.

ⓒ

$\frac{x+4}{x^2+5x+6}$

Set the denominator equal to zero. Solve for the variable.

$$\begin{aligned} x^2+5x+6 &= 0 \\ (x+2)(x+3) &= 0 \\ x+2=0 \text{ or } x+3 &= 0 \\ x=-2 \text{ or } x &= -3 \end{aligned}$$

$\frac{x+4}{x^2+5x+6}$ is undefined for $x=-2$ or $x=-3$.

Saying that the rational expression $\frac{x+4}{x^2+5x+6}$ is undefined for $x=-2$ or $x=-3$ is similar to writing the phrase “void where prohibited” in contest rules.

Note:
Exercise:

TRY IT : : 7.1

Problem: Determine the values for which the rational expression is undefined:

ⓐ $\frac{3y}{x}$ ⓑ $\frac{8n-5}{3n+1}$ ⓒ $\frac{a+10}{a^2+4a+3}$

Solution:

ⓐ $x=0$ ⓑ $n=-\frac{1}{3}$ ⓒ $a=-1, a=-3$

Note:
Exercise:

TRY IT : : 7.2

Problem: Determine the values for which the rational expression is undefined:

ⓐ $\frac{4p}{5q}$ ⓑ $\frac{y-1}{3y+2}$ ⓒ $\frac{m-5}{m^2+m-6}$

Solution:

ⓐ $q=0$ ⓑ $y=-\frac{2}{3}$ ⓒ $m=2, m=-3$

Evaluate Rational Expressions

To evaluate a rational expression, we substitute values of the variables into the expression and simplify, just as we have for many other expressions in this book.

Example:
Exercise:

Problem: **Evaluate $\frac{2x+3}{3x-5}$ for each value:

ⓐ $x=0$ ⓑ $x=2$ ⓒ $x=-3$

Solution:
Solution

ⓐ

	$\frac{2x+3}{3x-5}$
Substitute 0 for *x*.	$\frac{2(0)+3}{3(0)-5}$
Simplify.	$-\frac{3}{5}$

ⓑ

	$\frac{2x+3}{3x-5}$
Substitute 2 for *x*.	$\frac{2(2)+3}{3(2)-5}$
Simplify.	$\frac{4+3}{6-5}$
	$\frac{7}{1}$
	7

ⓒ

	$\frac{2x+3}{3x-5}$
Substitute -3 for x.	$\frac{2(-3)+3}{3(-3)-5}$
Simplify.	$\frac{-6+3}{-9-5}$
	$\frac{-3}{-14}$
	$\frac{3}{14}$

Note:
Exercise:

TRY IT : : 7.3

Problem: Evaluate $\frac{y+1}{2y-3}$ for each value:

ⓐ $y=1$ ⓑ $y=-3$ ⓒ $y=0$

Solution:

ⓐ -2 ⓑ $\frac{2}{9}$ ⓒ $-\frac{1}{3}$

Note:
Exercise:

TRY IT : : 7.4

Problem: Evaluate $\frac{5x-1}{2x+1}$ for each value:

ⓐ $x=1$ ⓑ $x=-1$ ⓒ $x=0$

Solution:

ⓐ $\frac{4}{3}$ ⓑ 6 ⓒ -1

Example:
Exercise:

Problem: **Evaluate $\frac{x^2+8x+7}{x^2-4}$ for each value:

ⓐ $x=0$ ⓑ $x=2$ ⓒ $x=-1$

Solution:
Solution

ⓐ

	$\frac{x^2+8x+7}{x^2-4}$
Substitute 0 for *x*.	$\frac{(0)^2+8(0)+7}{(0)^2-4}$
Simplify.	$\frac{7}{-4}$
	$-\frac{7}{4}$

ⓑ

	$\frac{x^2+8x+7}{x^2-4}$
Substitute 2 for *x*.	$\frac{(2)^2+8(2)+7}{(2)^2-4}$

Simplify.	$\frac{4+16+7}{4-4}$
	$\frac{27}{0}$
This rational expression is undefined for $x = 2$.	

ⓒ

	$\frac{x^2+8x+7}{x^2-4}$
Substitute -1 for x.	$\frac{(-1)^2+8(-1)+7}{(-1)^2-4}$
Simplify.	$\frac{1-8+7}{1-4}$
	$\frac{-7+7}{-3}$
	$\frac{0}{-3}$
	0

Note:
Exercise:

TRY IT : : 7.5

Problem: Evaluate $\frac{x^2+1}{x^2-3x+2}$ for each value:

ⓐ $x = 0$ ⓑ $x = -1$ ⓒ $x = 3$

Solution:

ⓐ $\frac{1}{2}$ ⓑ $\frac{1}{3}$ ⓒ 5

Note:
Exercise:

TRY IT : : 7.6

Problem: Evaluate $\frac{x^2+x-6}{x^2-9}$ for each value:

ⓐ $x = 0$ ⓑ $x = -2$ ⓒ $x = 1$

Solution:

ⓐ $\frac{2}{3}$ ⓑ $\frac{4}{5}$ ⓒ $\frac{1}{2}$

Remember that a fraction is simplified when it has no common factors, other than 1, in its numerator and denominator. When we evaluate a rational expression, we make sure to simplify the resulting fraction.

Example:
Exercise:

Problem: **Evaluate $\frac{a^2+2ab+b^2}{3ab^3}$ for each value:

ⓐ $a = 1, b = 2$ ⓑ $a = -2, b = -1$ ⓒ $a = \frac{1}{3}, b = 0$

Solution:
Solution

ⓐ

	$\frac{a^2+2ab+b^2}{3ab^2}$ when $a = 1, b = 2.$
Substitute 1 for *a* and 2 for *b*.	$\frac{(1)^2+2(1)(2)+(2)^2}{3(1)(2)^2}$
Simplify.	$\frac{1+4+4}{3(4)}$

	$\frac{9}{12}$
	$\frac{3}{4}$

ⓑ

	$\frac{a^2+2ab+b^2}{3ab^2}$ when $a=-2, b=-1.$
Substitute -2 for *a* and -1 for *b*.	$\frac{(-2)^2+2(-2)(-1)+(-1)^2}{3(-2)(-1)^2}$
Simplify.	$\frac{4+4+1}{-6}$
	$-\frac{9}{6}$
	$-\frac{3}{2}$

ⓒ

	$\frac{a^2+2ab+b^2}{3ab^2}$ when $a=\frac{1}{3}, b=0.$
Substitute $\frac{1}{3}$ for *a* and 0 for *b*.	$\frac{\left(\frac{1}{3}\right)^2+2\left(\frac{1}{3}\right)(0)+(0)^2}{3\left(\frac{1}{3}\right)(0)^2}$
Simplify.	

	$\frac{\frac{1}{9}+0+0}{0}$
	$\frac{\frac{1}{9}}{0}$
	The expression is undefined.

Note:
Exercise:

TRY IT : : 7.7

Problem: Evaluate $\frac{2a^3b}{a^2+2ab+b^2}$ for each value:

ⓐ $a=-1, b=2$ ⓑ $a=0, b=-1$ ⓒ $a=1, b=\frac{1}{2}$

Solution:

ⓐ -4 ⓑ 0 ⓒ $\frac{4}{9}$

Note:
Exercise:

TRY IT : : 7.8

Problem: Evaluate $\frac{a^2-b^2}{8ab^3}$ for each value:

ⓐ $a=1, b=-1$ ⓑ $a=\frac{1}{2}, b=-1$ ⓒ $a=-2, b=1$

Solution:

ⓐ 0 ⓑ $\frac{3}{16}$ ⓒ $\frac{3}{16}$

Simplify Rational Expressions

Just like a fraction is considered simplified if there are no common factors, other than 1, in its numerator and denominator, a rational expression is *simplified* if it has no common factors, other than 1, in its numerator and denominator.

Note:
Simplified Rational Expression

A rational expression is considered simplified if there are no common factors in its numerator and denominator.

For example:

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{2x}{3x}$ is not simplified because x is a common factor of $2x$ and $3x$.

We use the Equivalent Fractions Property to simplify numerical fractions. We restate it here as we will also use it to simplify rational expressions.

Note:
Equivalent Fractions Property
If a, b, and c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.

Notice that in the Equivalent Fractions Property, the values that would make the denominators zero are specifically disallowed. We see $b \neq 0, c \neq 0$ clearly stated. Every time we write a rational expression, we should make a similar statement disallowing values that would make a denominator zero. However, to let us focus on the work at hand, we will omit writing it in the examples.

Let’s start by reviewing how we simplify numerical fractions.

Example:
Exercise:

Problem: **Simplify: $-\frac{36}{63}$.

Solution:
Solution

	$-\frac{36}{63}$
Rewrite the numerator and denominator showing the common factors.	$-\frac{4 \cdot 9}{7 \cdot 9}$
Simplify using the Equivalent Fractions Property.	$-\frac{4}{7}$

Notice that the fraction $-\frac{4}{7}$ is simplified because there are no more common factors.

Note:
Exercise:

TRY IT : : 7.9
Problem: Simplify: $-\frac{45}{81}$.

Solution:

$-\frac{5}{9}$.

Note:
Exercise:

TRY IT : : 7.10
Problem: Simplify: $-\frac{42}{54}$.

Solution:

$-\frac{7}{9}$

Throughout this chapter, we will assume that all numerical values that would make the denominator be zero are excluded. We will not write the restrictions for each rational expression, but keep in mind that the denominator can never be zero. So in this next example, $x \neq 0$ and $y \neq 0$.

Example:
Exercise:

Problem: **Simplify: $\frac{3xy}{18x^2y^2}$.

Solution:
Solution

	$\frac{3xy}{18x^2y^2}$
Rewrite the numerator and denominator showing the common factors.	$\frac{1 \cdot 3xy}{6xy \cdot 3xy}$

Simplify using the Equivalent Fractions Property.	$\frac{1}{6xy}$

Did you notice that these are the same steps we took when we divided monomials in Polynomials?

Note:
Exercise:

TRY IT : : 7.11

Problem: Simplify: $\frac{4x^2y}{12xy^2}$.

Solution:

$\frac{x}{3y}$

Note:
Exercise:

TRY IT : : 7.12

Problem: Simplify: $\frac{16x^2y}{2xy^2}$.

Solution:

$\frac{8x}{y}$

To simplify rational expressions we first write the numerator and denominator in factored form. Then we remove the common factors using the Equivalent Fractions Property.

Be very careful as you remove common factors. Factors are multiplied to make a product. You can remove a factor from a product. You cannot remove a term from a sum.

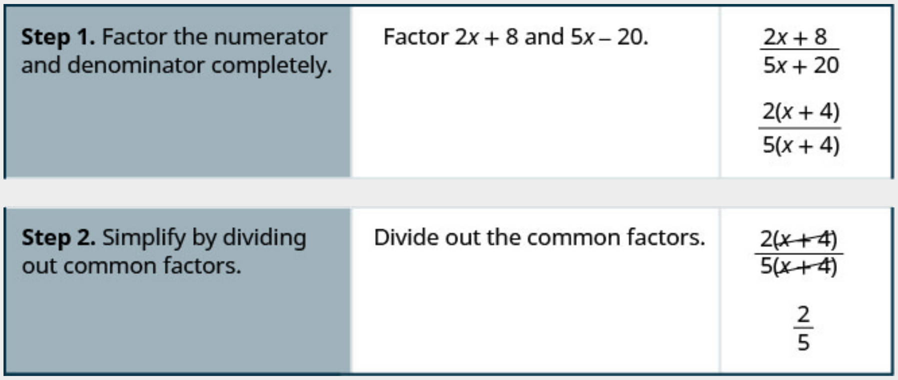
$\frac{2\cdot\cancel{3}\cdot\cancel{7}}{\cancel{3}\cdot5\cdot\cancel{7}}$	$\frac{3x(x-9)}{5(x-9)}$	$\frac{x+5}{x}$
$\frac{2}{5}$	$\frac{3x}{5}$	NO COMMON FACTORS
We removed the common factors 3 and 7. They are factors of the product.	We removed the common factor $(x-9)$. It is a factor of the product.	While there is an x in both the numerator and denominator, the x in the numerator is a term of a sum!

Note that removing the x's from $\frac{x+5}{x}$ would be like cancelling the 2's in the fraction $\frac{2+5}{2}$!

Example:
How to Simplify Rational Binomials
Exercise:

Problem: **Simplify: $\frac{2x+8}{5x+20}$.

Solution:
Solution



Step 1. Factor the numerator and denominator completely.	Factor 2*x* + 8 and 5*x* – 20.	$\frac{2x+8}{5x+20}$ $\frac{2(x+4)}{5(x+4)}$
Step 2. Simplify by dividing out common factors.	Divide out the common factors.	$\frac{2\cancel{(x+4)}}{5\cancel{(x+4)}}$ $\frac{2}{5}$

Note:
Exercise:

TRY IT : : 7.13
Problem: Simplify: $\frac{3x-6}{2x-4}$.

Solution:

$\frac{3}{2}$

Note:
Exercise:

TRY IT : : 7.14
Problem: Simplify: $\frac{7y+35}{5y+25}$.

Solution:

$\frac{7}{5}$

We now summarize the steps you should follow to simplify rational expressions.

Note:
Simplify a Rational Expression.

Factor the numerator and denominator completely.
Simplify by dividing out common factors.

Usually, we leave the simplified rational expression in factored form. This way it is easy to check that we have removed all the common factors!

We'll use the methods we covered in Factoring to factor the polynomials in the numerators and denominators in the following examples.

Example:
Exercise:

Problem: **Simplify: $\frac{x^2+5x+6}{x^2+8x+12}$.

Solution:
Solution

$$\frac{x^2+5x+6}{x^2+8x+12}$$

Factor the numerator and denominator.

$$\frac{(x+2)(x+3)}{(x+2)(x+6)}$$

Remove the common factor $x+2$ from the numerator and the denominator.

$$\frac{\cancel{(x+2)}\,(x+3)}{\cancel{(x+2)}\,(x+6)}$$

$$\frac{x+3}{x+6}$$

Can you tell which values of *x* must be excluded in this example?

Note:
Exercise:

TRY IT : : 7.15

Problem: Simplify: $\frac{x^2-x-2}{x^2-3x+2}$.

Solution:

$\frac{x+1}{x-1}$

Note:
Exercise:

TRY IT : : 7.16

Problem: Simplify: $\frac{x^2-3x-10}{x^2+x-2}$.

Solution:

$\frac{x-5}{x-1}$

Example:
Exercise:

Problem: **Simplify: $\frac{y^2+y-42}{y^2-36}$.

Solution:
Solution

$$\frac{y^2+y-42}{y^2-36}$$

Factor the numerator and denominator.

$$\frac{(y+7)(y-6)}{(y+6)(y-6)}$$

Remove the common factor $y-6$ from the numerator and the denominator.

$$\frac{(y+7)\cancel{(y-6)}}{(y+6)\cancel{(y-6)}}$$

$$\frac{y+7}{y+6}$$

Note:
Exercise:

TRY IT : : 7.17

Problem: Simplify: $\frac{x^2+x-6}{x^2-4}$.

Solution:

$\frac{x+3}{x+2}$

Note:
Exercise:

TRY IT : : 7.18

Problem: Simplify: $\frac{x^2+8x+7}{x^2-49}$.

Solution:

$\frac{x+1}{x-7}$

Example:
Exercise:

Problem: **Simplify: $\frac{p^3-2p^2+2p-4}{p^2-7p+10}$.

Solution:
Solution

$$\frac{p^3-2p^2+2p-4}{p^2-7p+10}$$

Factor the numerator and denominator, using grouping to factor the numerator.

$$\frac{p^2(p-2)+2(p-2)}{(p-5)(p-2)}$$

$$\frac{\left(p^2+2\right)(p-2)}{(p-5)(p-2)}$$

Remove the common factor of $p-2$ from the numerator and the denominator.

$$\frac{\left(p^2+2\right)\cancel{(p-2)}}{(p-5)\cancel{(p-2)}}$$

$$\frac{p^2+2}{p-5}$$

Note:
Exercise:

TRY IT : : 7.19

Problem: Simplify: $\frac{y^3-3y^2+y-3}{y^2-y-6}$.

Solution:

$\frac{y^2+1}{y+2}$

Note:
Exercise:

TRY IT : : 7.20

Problem: Simplify: $\frac{p^3-p^2+2p-2}{p^2+4p-5}$.

Solution:

$\frac{p^2+2}{p+5}$

Example:
Exercise:

Problem: **Simplify: $\frac{2n^2-14n}{4n^2-16n-48}$.

Solution:
Solution

	$\frac{2n^2-14n}{4n^2-16n-48}$
Factor the numerator and denominator, first factoring out the GCF.	$\frac{2n(n-7)}{4(n^2-4n-12)}$ $\frac{2n(n-7)}{4(n-6)(n+2)}$
Remove the common factor, 2.	$\frac{\cancel{2}\,n(n-7)}{\cancel{2}\cdot 2(n-6)(n+2)}$ $\frac{n(n-7)}{2(n-6)(n+2)}$

Note:
Exercise:

TRY IT : : 7.21

Problem: Simplify: $\frac{2n^2-10n}{4n^2-16n-20}$.

Solution:

$\frac{n}{2(n+1)}$

Note:
Exercise:

TRY IT : : 7.22

Problem: Simplify: $\frac{4x^2-16x}{8x^2-16x-64}$.

Solution:

$\frac{x}{2(x+2)}$

Example:
Exercise:

Problem: **Simplify: $\frac{3b^2-12b+12}{6b^2-24}$.

Solution:
Solution

	$\frac{3b^2-12b+12}{6b^2-24}$
Factor the numerator and denominator, first factoring out the GCF.	$\frac{3\left(b^2-4b+4\right)}{6(b^2-4)}$
	$\frac{3(b-2)(b-2)}{6(b+2)(b-2)}$
Remove the common factors of $b-2$ and 3.	$\frac{\cancel{3}(b-2)\,\cancel{(b-2)}}{\cancel{3}\cdot 2(b+2)\,\cancel{(b-2)}}$
	$\frac{b-2}{2(b+2)}$

Note:
Exercise:

TRY IT : : 7.23

Problem: Simplify: $\frac{2x^2-12x+18}{3x^2-27}$.

Solution:

$\frac{2(x-3)}{3(x+3)}$

Note:
Exercise:

TRY IT : : 7.24

Problem: Simplify: $\frac{5y^2-30y+25}{2y^2-50}$.

Solution:

$\frac{5(y-1)}{2(y+5)}$

Example:
Exercise:

Problem: **Simplify: $\frac{m^3+8}{m^2-4}$.

Solution:
Solution

$$\frac{m^3+8}{m^2-4}$$

Factor the numerator and denominator, using the formulas for sum of cubes and difference of squares.

$$\frac{(m+2)\left(m^2-2m+4\right)}{(m+2)(m-2)}$$

Remove the common factor of $m+2$.

$$\frac{\cancel{(m+2)}\left(m^2-2m+4\right)}{\cancel{(m+2)}(m-2)}$$

$$\frac{m^2-2m+4}{m-2}$$

Note:
Exercise:

TRY IT : : 7.25

Problem: Simplify: $\frac{p^3-64}{p^2-16}$.

Solution:

$\frac{p^2+4p+16}{p+4}$

Note:
Exercise:

TRY IT : : 7.26

Problem: Simplify: $\frac{x^3+8}{x^2-4}$.

Solution:

$\frac{x^2-2x+4}{x-2}$

Simplify Rational Expressions with Opposite Factors

Now we will see how to simplify a rational expression whose numerator and denominator have opposite factors. Let’s start with a numerical fraction, say $\frac{7}{-7}$. We know this fraction simplifies to -1. We also recognize that the numerator and denominator are opposites.

In Foundations, we introduced opposite notation: the opposite of a is $-a$. We remember, too, that $-a=-1\cdot a$.

We simplify the fraction $\frac{a}{-a}$, whose numerator and denominator are opposites, in this way:

$$\frac{a}{-a}$$

We could rewrite this. $\frac{1\cdot a}{-1\cdot a}$

Remove the common factors. $\frac{1}{-1}$

Simplify. -1

So, in the same way, we can simplify the fraction $\frac{x-3}{-(x-3)}$:

We could rewrite this. $\frac{1\cdot(x-3)}{-1\cdot(x-3)}$

Remove the common factors. $\frac{1}{-1}$

Simplify. -1

But the opposite of $x-3$ could be written differently:

$-(x-3)$

Distribute. $-x+3$

Rewrite. $3-x$

This means the fraction $\frac{x-3}{3-x}$ simplifies to -1.

In general, we could write the opposite of $a-b$ as $b-a$. So the rational expression $\frac{a-b}{b-a}$ simplifies to -1.

Note:
Opposites in a Rational Expression
The opposite of $a-b$ is $b-a$.
Equation:

$$\frac{a-b}{b-a}=-1 \qquad a\neq b$$

An expression and its opposite divide to -1.

We will use this property to simplify rational expressions that contain opposites in their numerators and denominators.

Example:
Exercise:

Problem: **Simplify: $\frac{x-8}{8-x}$.

Solution:
Solution

$\frac{x-8}{8-x}$

Recognize that $x-8$ and $8-x$ are opposites. -1

Note:
Exercise:

TRY IT : : 7.27

Problem: Simplify: $\frac{y-2}{2-y}$.

Solution:

-1

Note:
Exercise:

TRY IT : : 7.28

Problem: Simplify: $\frac{n-9}{9-n}$.

Solution:

-1

Remember, the first step in simplifying a rational expression is to factor the numerator and denominator completely.

Example:
Exercise:

Problem: **Simplify: $\frac{14-2x}{x^2-49}$.

Solution:
Solution

	$\frac{14-2x}{x^2-49}$
Factor the numerator and denominator.	$\frac{2(7-x)}{(x+7)(x-7)}$
Recognize that $7-x$ and $x-7$ are opposites.	$\frac{2\cancel{(7-x)}}{(x+7)\cancel{(x-7)}}(-1)$
Simplify.	

	$-\frac{2}{x+7}$

Note:
Exercise:

TRY IT : : 7.29

Problem: Simplify: $\frac{10-2y}{y^2-25}$.

Solution:

$-\frac{2}{y+5}$.

Note:
Exercise:

TRY IT : : 7.30

Problem: Simplify: $\frac{3y-27}{81-y^2}$.

Solution:

$-\frac{3}{9+y}$

Example:
Exercise:

Problem: **Simplify: $\frac{x^2-4x-32}{64-x^2}$.

Solution:
Solution

	$\frac{x^2-4x-32}{64-x^2}$
Factor the numerator and denominator.	$\frac{(x-8)(x+4)}{(8-x)(8+x)}$

Recognize the factors that are opposites.	$(-1)\,\frac{\cancel{(x-8)}(x+4)}{\cancel{(8-x)}(8+x)}$
Simplify.	$-\frac{x+4}{x+8}$

Note:
Exercise:

TRY IT : : 7.31

Problem: Simplify: $\frac{x^2-4x-5}{25-x^2}$.

Solution:

$-\frac{x+1}{x+5}$

Note:
Exercise:

TRY IT : : 7.32

Problem: Simplify: $\frac{x^2+x-2}{1-x^2}$.

Solution:

$-\frac{x+2}{x+1}$

Key Concepts

- **Determine the Values for Which a Rational Expression is Undefined**

  Set the denominator equal to zero.
  Solve the equation, if possible.

- **Simplified Rational Expression**
  - A rational expression is considered simplified if there are no common factors in its numerator and denominator.
- **Simplify a Rational Expression**

  Factor the numerator and denominator completely.
  Simplify by dividing out common factors.

- **Opposites in a Rational Expression**

- The opposite of $a-b$ is $b-a$.
  $\frac{a-b}{b-a}=-1 \qquad a\neq 0, b\neq 0, \text{a}\neq \text{b}$

Practice Makes Perfect

In the following exercises, determine the values for which the rational expression is undefined.

Exercise:

Problem: ⓐ $\frac{2x}{z}$
ⓑ $\frac{4p-1}{6p-5}$
ⓒ $\frac{n-3}{n^2+2n-8}$

Solution:

ⓐ $z=0$ ⓑ $p=\frac{5}{6}$
ⓒ $n=-4, n=2$

Exercise:

Problem: ⓐ $\frac{10m}{11n}$
ⓑ $\frac{6y+13}{4y-9}$
ⓒ $\frac{b-8}{b^2-36}$

Exercise:

Problem: ⓐ $\frac{4x^2y}{3y}$
ⓑ $\frac{3x-2}{2x+1}$
ⓒ $\frac{u-1}{u^2-3u-28}$

Solution:

ⓐ $y=0$ ⓑ $x=-\frac{1}{2}$
ⓒ $u=-4, u=7$

Exercise:

Problem: ⓐ $\frac{5pq^2}{9q}$
ⓑ $\frac{7a-4}{3a+5}$
ⓒ $\frac{1}{x^2-4}$

Evaluate Rational Expressions

In the following exercises, evaluate the rational expression for the given values.

Exercise:

Problem: $\frac{2x}{x-1}$

ⓐ $x=0$
ⓑ $x=2$
ⓒ $x=-1$

Solution:

ⓐ 0 ⓑ 4 ⓒ 1

Exercise:

Problem: $\frac{4y-1}{5y-3}$

ⓐ $y=0$
ⓑ $y=2$
ⓒ $y=-1$

Exercise:

Problem: $\frac{2p+3}{p^2+1}$

ⓐ $p=0$
ⓑ $p=1$
ⓒ $p=-2$

Solution:

ⓐ 3 ⓑ $\frac{5}{2}$ ⓒ $-\frac{1}{5}$

Exercise:

Problem: $\frac{x+3}{2-3x}$

ⓐ $x=0$
ⓑ $x=1$
ⓒ $x=-2$

Exercise:

Problem: $\frac{y^2+5y+6}{y^2-1}$

ⓐ $y=0$
ⓑ $y=2$
ⓒ $y=-2$

Solution:

ⓐ -6 ⓑ $\frac{20}{3}$ ⓒ 0

Exercise:

Problem: $\frac{z^2+3z-10}{z^2-1}$

ⓐ $z = 0$
ⓑ $z = 2$
ⓒ $z = -2$

Exercise:

Problem: $\frac{a^2-4}{a^2+5a+4}$

ⓐ $a = 0$
ⓑ $a = 1$
ⓒ $a = -2$

Solution:

ⓐ -1 ⓑ $-\frac{3}{10}$ ⓒ 0

Exercise:

Problem: $\frac{b^2+2}{b^2-3b-4}$

ⓐ $b = 0$
ⓑ $b = 2$
ⓒ $b = -2$

Exercise:

Problem: $\frac{x^2+3xy+2y^2}{2x^3y}$

ⓐ $x = 1, y = -1$
ⓑ $x = 2, y = 1$
ⓒ $x = -1, y = -2$

Solution:

ⓐ 0 ⓑ $\frac{3}{4}$ ⓒ $\frac{15}{4}$

Exercise:

Problem: $\frac{c^2+cd-2d^2}{cd^3}$

ⓐ $c = 2, d = -1$
ⓑ $c = 1, d = -1$
ⓒ $c = -1, d = 2$

Exercise:

Problem: $\frac{m^2-4n^2}{5mn^3}$

ⓐ $m = 2, n = 1$
ⓑ $m = -1, n = -1$
ⓒ $m = 3, n = 2$

Solution:

ⓐ 0 ⓑ $-\frac{3}{5}$ ⓒ $-\frac{7}{20}$

Exercise:

Problem: $\frac{2s^2t}{s^2-9t^2}$

ⓐ $s=4, t=1$
ⓑ $s=-1, t=-1$
ⓒ $s=0, t=2$

Simplify Rational Expressions

In the following exercises, simplify.

Exercise:

Problem: $-\frac{4}{52}$

Solution:

$-\frac{1}{13}$

Exercise:

Problem: $-\frac{44}{55}$

Exercise:

Problem: $\frac{56}{63}$

Solution:

$\frac{8}{9}$

Exercise:

Problem: $\frac{65}{104}$

Exercise:

Problem: $\frac{6ab^2}{12a^2b}$

Solution:

$\frac{b}{2a}$

Exercise:

Problem: $\frac{15xy}{3x^3y^3}$

Exercise:

Problem: $\frac{8m^3n}{12mn^2}$

Solution:

$\frac{2m^2}{3n}$

Exercise:

Problem: $\frac{36v^3w^2}{27vw^3}$

Exercise:

Problem: $\frac{3a+6}{4a+8}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{5b+5}{6b+6}$

Exercise:

Problem: $\frac{3c-9}{5c-15}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{4d+8}{9d+18}$

Exercise:

Problem: $\frac{7m+63}{5m+45}$

Solution:

$\frac{7}{5}$

Exercise:

Problem: $\frac{8n-96}{3n-36}$

Exercise:

Problem: $\frac{12p-240}{5p-100}$

Solution:

$\frac{12}{5}$

Exercise:

Problem: $\frac{6q+210}{5q+175}$

Exercise:

Problem: $\frac{a^2-a-12}{a^2-8a+16}$

Solution:

$\frac{a+3}{a-4}$

Exercise:

Problem: $\frac{x^2+4x-5}{x^2-2x+1}$

Exercise:

Problem: $\frac{y^2+3y-4}{y^2-6y+5}$

Solution:

$\frac{y+4}{y-5}$

Exercise:

Problem: $\frac{v^2+8v+15}{v^2-v-12}$

Exercise:

Problem: $\frac{x^2-25}{x^2+2x-15}$

Solution:

$\frac{x-5}{x-3}$

Exercise:

Problem: $\frac{a^2-4}{a^2+6a-16}$

Exercise:

Problem: $\frac{y^2-2y-3}{y^2-9}$

Solution:

$\frac{y+1}{y+3}$

Exercise:

Problem: $\frac{b^2+9b+18}{b^2-36}$

Exercise:

Problem: $\frac{y^3+y^2+y+1}{y^2+2y+1}$

Solution:

$\frac{y^2+1}{y+1}$

Exercise:

Problem: $\frac{p^3+3p^2+4p+12}{p^2+p-6}$

Exercise:

Problem: $\frac{x^3-2x^2-25x+50}{x^2-25}$

Solution:

$x-2$

Exercise:

Problem: $\frac{q^3+3q^2-4q-12}{q^2-4}$

Exercise:

Problem: $\frac{3a^2+15a}{6a^2+6a-36}$

Solution:

$\frac{a(a+5)}{2(a+3)(a-2)}$

Exercise:

Problem: $\frac{8b^2-32b}{2b^2-6b-80}$

Exercise:

Problem: $\frac{-5c^2-10c}{-10c^2+30c+100}$

Solution:

$\frac{c}{2(c-5)}$

Exercise:

Problem: $\frac{4d^2-24d}{2d^2-4d-48}$

Exercise:

Problem: $\frac{3m^2+30m+75}{4m^2-100}$

Solution:

$\frac{3(m+5)}{4(m-5)}$

Exercise:

Problem: $\frac{5n^2+30n+45}{2n^2-18}$

Exercise:

Problem: $\frac{5r^2+30r-35}{r^2-49}$

Solution:

$\frac{5(r-1)}{r+7}$

Exercise:

Problem: $\frac{3s^2+30s+24}{3s^2-48}$

Exercise:

Problem: $\frac{t^3-27}{t^2-9}$

Solution:

$\frac{t^2+3t+9}{t+3}$

Exercise:

Problem: $\frac{v^3-1}{v^2-1}$

Exercise:

Problem: $\frac{w^3+216}{w^2-36}$

Solution:

$\frac{w^2-6w+36}{w-6}$

Exercise:

Problem: $\frac{v^3+125}{v^2-25}$

Simplify Rational Expressions with Opposite Factors

In the following exercises, simplify each rational expression.

Exercise:

Problem: $\frac{a-5}{5-a}$

Solution:

-1

Exercise:

Problem: $\frac{b-12}{12-b}$

Exercise:

Problem: $\frac{11-c}{c-11}$

Solution:

-1

Exercise:

Problem: $\frac{5-d}{d-5}$

Exercise:

Problem: $\frac{12-2x}{x^2-36}$

Solution:

$-\frac{2}{x+6}$

Exercise:

Problem: $\frac{20-5y}{y^2-16}$

Exercise:

Problem: $\frac{4v-32}{64-v^2}$

Solution:

$-\frac{4}{8+v}$

Exercise:

Problem: $\frac{7w-21}{9-w^2}$

Exercise:

Problem: $\frac{y^2-11y+24}{9-y^2}$

Solution:

$\left(-\frac{y-8)}{3+y}\right.$

Exercise:

Problem: $\frac{z^2-9z+20}{16-z^2}$

Exercise:

Problem: $\frac{a^2-5a-36}{81-a^2}$

Solution:

$-\frac{a+4}{9+a}$

Exercise:

Problem: $\frac{b^2+b-42}{36-b^2}$

Everyday Math

Exercise:

Problem:

Tax Rates For the tax year 2015, the amount of tax owed by a single person earning between $37,450 and $90,750, can be found by evaluating the formula $0.25x-4206.25$, where *x* is income. The average tax rate for this income can be found by evaluating the formula $\frac{0.25x-4206.25}{x}$. What would be the average tax rate for a single person earning $50,000?

Solution:

16.5%

Exercise:

Problem:

Work The length of time it takes for two people for perform the same task if they work together can be found by evaluating the formula $\frac{xy}{x+y}$. If Tom can paint the den in $x=45$ minutes and his brother Bobby can paint it in $y=60$ minutes, how many minutes will it take them if they work together?

Writing Exercises

Exercise:

Problem: Explain how you find the values of *x* for which the rational expression $\frac{x^2-x-20}{x^2-4}$ is undefined.

Solution:

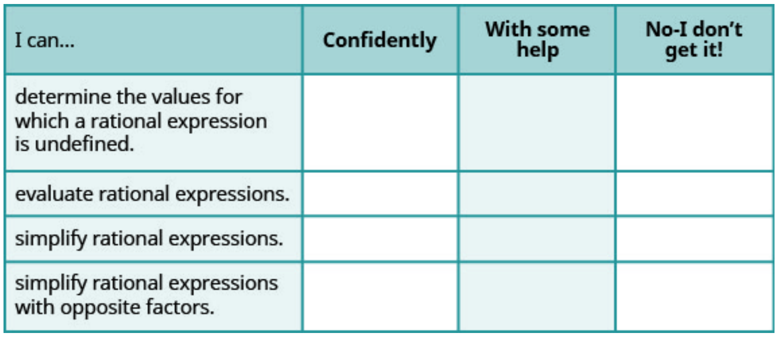
Answers will vary, but all should reference setting the denominator function to zero.

Exercise:

Problem: Explain all the steps you take to simplify the rational expression $\frac{p^2+4p-21}{9-p^2}$.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
determine the values for which a rational expression is undefined.			
evaluate rational expressions.			
simplify rational expressions.			
simplify rational expressions with opposite factors.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved your goals in this section! Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific!

…with some help. This must be addressed quickly as topics you do not master become potholes in your road to success. Math is sequential - every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is critical and you must not ignore it. You need to get help immediately or you will quickly be overwhelmed. See your instructor as soon as possible to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

rational expression

A rational expression is an expression of the form $\frac{p}{q}$, where p and q are polynomials and $q \neq 0$.

Multiply and Divide Rational Expressions
By the end of this section, you will be able to:

- Multiply rational expressions
- Divide rational expressions

Note:
Before you get started, take this readiness quiz.
If you miss a problem, go back to the section listed and review the material.

1. Multiply: $\frac{14}{15}\cdot\frac{6}{35}$.
   If you missed this problem, review [link].
2. Divide: $\frac{14}{15}\div\frac{6}{35}$.
   If you missed this problem, review [link].
3. Factor completely: $2x^2-98$.
   If you missed this problem, review [link].
4. Factor completely: $10n^3+10$.
   If you missed this problem, review [link].
5. Factor completely: $10p^2-25pq-15q^2$.
   If you missed this problem, review [link].

Multiply Rational Expressions

To multiply rational expressions, we do just what we did with numerical fractions. We multiply the numerators and multiply the denominators. Then, if there are any common factors, we remove them to simplify the result.

Note:
Multiplication of Rational Expressions
If p,q,r,s are polynomials where $q\neq0$ and $s\neq0$, then
Equation:

$$\frac{p}{q}\cdot\frac{r}{s}=\frac{pr}{qs}$$

To multiply rational expressions, multiply the numerators and multiply the denominators.

We’ll do the first example with numerical fractions to remind us of how we multiplied fractions without variables.

Example:
Exercise:

Problem: **Multiply: $\frac{10}{28} \cdot \frac{8}{15}$.

Solution:
Solution

	$\frac{10}{28} \cdot \frac{8}{15}$
Multiply the numerators and denominators.	$\frac{10 \cdot 8}{28 \cdot 15}$
Look for common factors, and then remove them.	$\frac{2 \cdot \cancel{5} \cdot 2 \cdot \cancel{4}}{7 \cdot \cancel{4} \cdot 3 \cdot \cancel{5}}$
Simplify.	$\frac{4}{21}$

Note:
Exercise:

TRY IT : : 7.33

Problem: Mulitply: $\frac{6}{10} \cdot \frac{15}{12}$.

Solution:

$\frac{3}{4}$

Note:
Exercise:

TRY IT : : 7.34
Problem: Mulitply: $\frac{20}{15} \cdot \frac{6}{8}$.

Solution:

1

Remember, throughout this chapter, we will assume that all numerical values that would make the denominator be zero are excluded. We will not write the restrictions for each rational expression, but keep in mind that the denominator can never be zero. So in this next example, $x \neq 0$ and $y \neq 0$.

Example:
Exercise:

Problem: **Mulitply: $\frac{2x}{3y^2} \cdot \frac{6xy^3}{x^2y}$.

Solution:
Solution

	$\frac{2x}{3y^2} \cdot \frac{6xy^3}{x^2y}$
Multiply.	$\frac{2x \cdot 6xy^3}{3y^2 \cdot x^2y}$

Factor the numerator and denominator completely, and then remove common factors.	$\frac{2\cdot\cancel{x}\cdot 2\cdot\cancel{3}\cdot\cancel{x}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}}{\cancel{3}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{y}}$
Simplify.	4

Note:
Exercise:

TRY IT : : 7.35

Problem: Mulitply: $\frac{3pq}{q^2}\cdot\frac{5p^2q}{6pq}$.

Solution:

$\frac{5p^2}{2q}$

Note:
Exercise:

TRY IT : : 7.36

Problem: Mulitply: $\frac{6x^3y}{7x^2}\cdot\frac{2xy^3}{x^2y}$.

Solution:

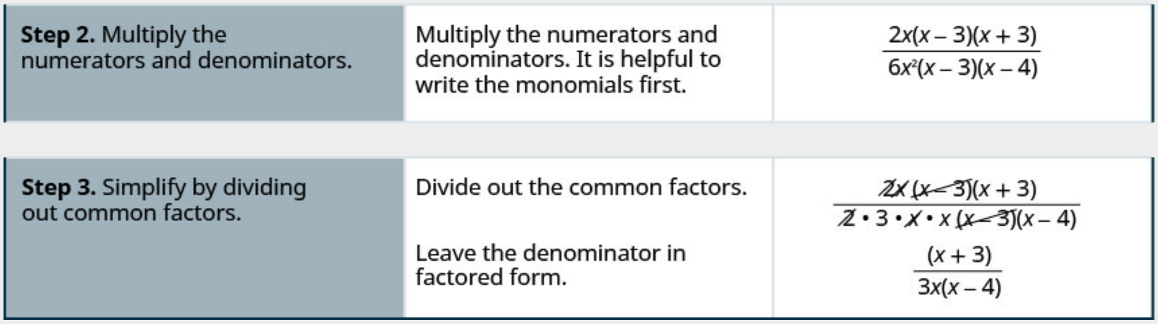
$\frac{12y^3}{7}$

Example:
How to Multiply Rational Expressions
Exercise:

Problem: **Mulitply: $\frac{2x}{x^2-7x+12}\cdot\frac{x^2-9}{6x^2}$.

Solution:
Solution

[missing_resource: CNX_ElemAlg_Figure_08_02_003a_img.jpg]



Step 2. Multiply the numerators and denominators.	Multiply the numerators and denominators. It is helpful to write the monomials first.	$\frac{2x(x-3)(x+3)}{6x^2(x-3)(x-4)}$
Step 3. Simplify by dividing out common factors.	Divide out the common factors. Leave the denominator in factored form.	$\frac{\cancel{2}\cancel{x}\cancel{(x-3)}(x+3)}{\cancel{2}\cdot 3\cdot\cancel{x}\cdot x\cancel{(x-3)}(x-4)}$ $\frac{(x+3)}{3x(x-4)}$

Note:
Exercise:

TRY IT : : 7.37

Problem: Mulitply: $\frac{5x}{x^2+5x+6}\cdot\frac{x^2-4}{10x}$.

Solution:

$\frac{x-2}{2(x+3)}$

Note:
Exercise:

TRY IT : : 7.38

Problem: Mulitply: $\frac{9x^2}{x^2+11x+30}\cdot\frac{x^2-36}{3x^2}$.

Solution:

$\frac{3(x-6)}{x+5}$

Note:
Multiply a rational expression.

Factor each numerator and denominator completely.
Multiply the numerators and denominators.

Simplify by dividing out common factors.

Example:
Exercise:

Problem: **Multiply: $\frac{n^2-7n}{n^2+2n+1} \cdot \frac{n+1}{2n}$.

Solution:
Solution

	$\frac{n^2-7n}{n^2+2n+1} \cdot \frac{n+1}{2n}$
Factor each numerator and denominator.	$\frac{n(n-7)}{(n+1)(n+1)} \cdot \frac{n+1}{2n}$
Multiply the numerators and the denominators.	$\frac{n(n-7)(n+1)}{(n+1)(n+1)2n}$
Remove common factors.	$\frac{\cancel{n}\,(n-7)\,\cancel{(n+1)}}{(n+1)\,\cancel{(n+1)}\,2\,\cancel{n}}$
Simplify.	$\frac{n-7}{2(n+1)}$

Note:
Exercise:

TRY IT : : 7.39

Problem: Multiply: $\frac{x^2-25}{x^2-3x-10} \cdot \frac{x+2}{x}$.

Solution:

$\frac{x+5}{x}$

Note:
Exercise:

TRY IT : : 7.40

Problem: Multiply: $\frac{x^2-4x}{x^2+5x+6} \cdot \frac{x+2}{x}$.

Solution:

$\frac{x-4}{x+3}$

Example:
Exercise:

Problem: **Multiply: $\frac{16-4x}{2x-12} \cdot \frac{x^2-5x-6}{x^2-16}$.

Solution:
Solution

	$\frac{16-4x}{2x-12} \cdot \frac{x^2-5x-6}{x^2-16}$
Factor each numerator and denominator.	$\frac{4(4-x)}{2(x-6)} \cdot \frac{(x-6)(x+1)}{(x-4)(x+4)}$
Multiply the numerators and the denominators.	$\frac{4(4-x)(x-6)(x+1)}{2(x-6)(x-4)(x+4)}$
Remove common factors.	$(-1)\frac{\cancel{2}\cdot 2\,\cancel{(4-x)}\,\cancel{(x-6)}\,(x+1)}{\cancel{2}\,\cancel{(x-6)}\,\cancel{(x-4)}\,(x+4)}$
Simplify.	$-\frac{2(x+1)}{(x+4)}$

Note:
Exercise:

TRY IT : : 7.41

Problem: Multiply: $\frac{12x-6x^2}{x^2+8x} \cdot \frac{x^2+11x+24}{x^2-4}$.

Solution:

$-\frac{6(x+3)}{x+2}$

Note:
Exercise:

TRY IT : : 7.42

Problem: Multiply: $\frac{9v-3v^2}{9v+36} \cdot \frac{v^2+7v+12}{v^2-9}$.

Solution:

$-\frac{v}{3}$

Example:
Exercise:

Problem: **Multiply: $\frac{2x-6}{x^2-8x+15} \cdot \frac{x^2-25}{2x+10}$.

Solution:
Solution

	$\frac{2x-6}{x^2-8x+15} \cdot \frac{x^2-25}{2x+10}$
Factor each numerator and denominator.	$\frac{2(x-3)}{(x-3)(x-5)} \cdot \frac{(x-5)(x+5)}{2(x+5)}$
Multiply the numerators and denominators.	$\frac{2(x-3)(x-5)(x+5)}{2(x-3)(x-5)(x+5)}$
Remove common factors.	$\frac{\cancel{2}\cancel{(x-3)}\cancel{(x-5)}\cancel{(x+5)}}{\cancel{2}\cancel{(x-3)}\cancel{(x-5)}\cancel{(x+5)}}$

Simplify.	1

Note:
Exercise:

TRY IT : : 7.43

Problem: Multiply: $\frac{3a-21}{a^2-9a+14} \cdot \frac{a^2-4}{3a+6}$.

Solution:

1

Note:
Exercise:

TRY IT : : 7.44

Problem: Multiply: $\frac{b^2-b}{b^2+9b-10} \cdot \frac{b^2-100}{b^2-10b}$.

Solution:

1

Divide Rational Expressions

To divide rational expressions we multiply the first fraction by the reciprocal of the second, just like we did for numerical fractions.

Remember, the **reciprocal** of $\frac{a}{b}$ is $\frac{b}{a}$. To find the reciprocal we simply put the numerator in the denominator and the denominator in the numerator. We “flip” the fraction.

Note:
Division of Rational Expressions

If p, q, r, s are polynomials where $q \neq 0, r \neq 0, s \neq 0$, then
Equation:

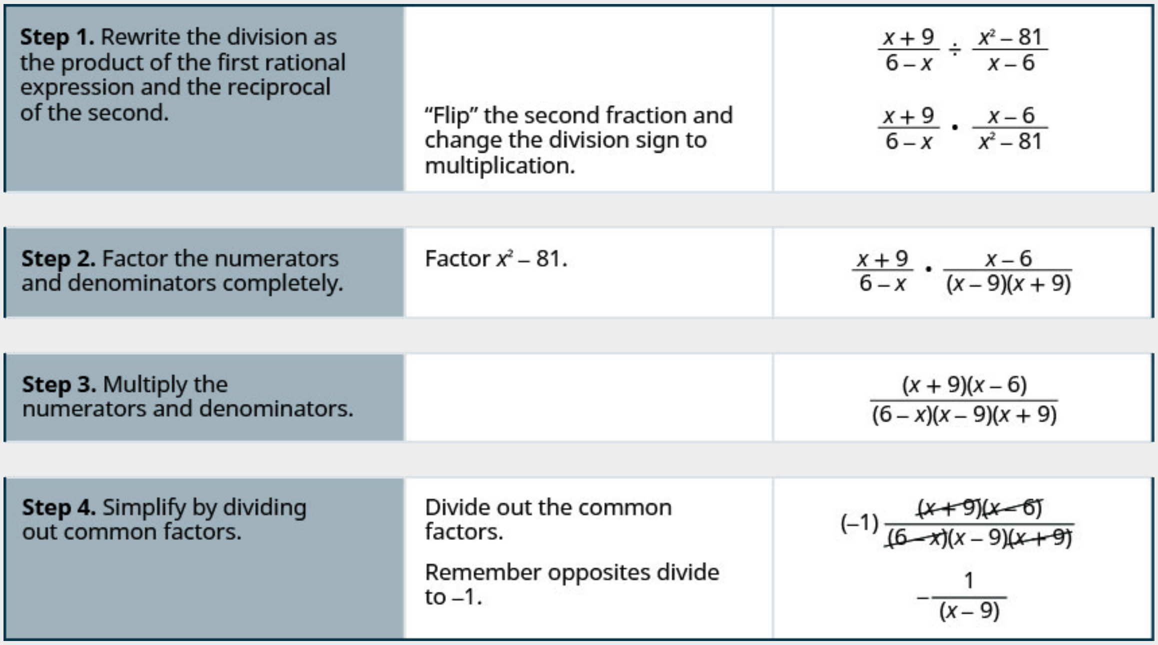
$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \cdot \frac{s}{r}$$

To divide rational expressions multiply the first fraction by the reciprocal of the second.

Example:
How to Divide Rational Expressions
Exercise:

Problem: **Divide: $\frac{x+9}{6-x} \div \frac{x^2-81}{x-6}$.

Solution:
Solution



Step 1. Rewrite the division as the product of the first rational expression and the reciprocal of the second.	"Flip" the second fraction and change the division sign to multiplication.	$\frac{x+9}{6-x} \div \frac{x^2-81}{x-6}$ $\frac{x+9}{6-x} \cdot \frac{x-6}{x^2-81}$
Step 2. Factor the numerators and denominators completely.	Factor $x^2 - 81$.	$\frac{x+9}{6-x} \cdot \frac{x-6}{(x-9)(x+9)}$
Step 3. Multiply the numerators and denominators.		$\frac{(x+9)(x-6)}{(6-x)(x-9)(x+9)}$
Step 4. Simplify by dividing out common factors.	Divide out the common factors. Remember opposites divide to –1.	$(-1)\frac{\cancel{(x+9)}\cancel{(x-6)}}{\cancel{(6-x)}(x-9)\cancel{(x+9)}}$ $-\frac{1}{(x-9)}$

Note:
Exercise:

TRY IT : : 7.45
Problem: Divide: $\frac{c+3}{5-c} \div \frac{c^2-9}{c-5}$.

Solution:

$-\frac{1}{c-3}$

Note:
Exercise:

TRY IT : : 7.46

Problem: Divide: $\frac{2-d}{d-4} \div \frac{4-d^2}{4-d}$.

Solution:

$-\frac{1}{2+d}$

Note:
Divide rational expressions.

Rewrite the division as the product of the first rational expression and the reciprocal of the second.
Factor the numerators and denominators completely.
Multiply the numerators and denominators together.
Simplify by dividing out common factors.

Example:
Exercise:

Problem: **Divide: $\frac{3n^2}{n^2-4n} \div \frac{9n^2-45n}{n^2-7n+10}$.

Solution:
Solution

	$\frac{3n^2}{n^2-4n} \div \frac{9n^2-45n}{n^2-7n+10}$
Rewrite the division as the product of the first rational expression and the reciprocal of the second.	$\frac{3n^2}{n^2-4n} \cdot \frac{n^2-7n+10}{9n^2-45n}$
Factor the numerators and denominators and then multiply.	$\frac{3 \cdot n \cdot n \cdot (n-5)(n-2)}{n(n-4) \cdot 3 \cdot 3 \cdot n \cdot (n-5)}$
Simplify by dividing out common factors.	$\frac{\cancel{3} \cdot \cancel{n} \cdot \cancel{n}\cancel{(n-5)}(n-2)}{\cancel{n}(n-4)\cancel{3} \cdot 3 \cdot \cancel{n}\cancel{(n-5)}}$
	$\frac{n-2}{3(n-4)}$

Note:
Exercise:

TRY IT : : 7.47

Problem: Divide: $\frac{2m^2}{m^2-8m} \div \frac{8m^2+24m}{m^2+m-6}$.

Solution:

$\frac{(m-2)}{4(m-8)}$

Note:
Exercise:

TRY IT : : 7.48

Problem: Divide: $\frac{15n^2}{3n^2+33n} \div \frac{5n-5}{n^2+9n-22}$.

Solution:

$\frac{n(n-2)}{n-1}$

Remember, first rewrite the division as multiplication of the first expression by the reciprocal of the second. Then factor everything and look for common factors.

Example:
Exercise:

Problem: **Divide: $\frac{2x^2+5x-12}{x^2-16} \div \frac{2x^2-13x+15}{x^2-8x+16}$.

Solution:
Solution

$$\frac{2x^2+5x-12}{x^2-16} \div \frac{2x^2-13x+15}{x^2-8x+16}$$

Rewrite the division as multiplication of the first expression by the reciprocal of the second.

$$\frac{2x^2+5x-12}{x^2-16} \cdot \frac{x^2-8x+16}{2x^2-13x+15}$$

Factor the numerators and denominators and then multiply.

$$\frac{(2x-3)(x+4)(x-4)(x-4)}{(x-4)(x+4)(2x-3)(x-5)}$$

Simplify by dividing out common factors.

$$\frac{\cancel{(2x-3)}\,\cancel{(x+4)}\,\cancel{(x-4)}\,(x-4)}{\cancel{(x-4)}\,\cancel{(x+4)}\,\cancel{(2x-3)}\,(x-5)}$$

Simplify.

$$\frac{x-4}{x-5}$$

Note:
Exercise:

TRY IT : : 7.49

Problem: Divide: $\frac{3a^2-8a-3}{a^2-25} \div \frac{3a^2-14a-5}{a^2+10a+25}$.

Solution:

$$\frac{(a-3)(a+5)}{(a-5)(a-5)}$$

Note:

Exercise:

TRY IT : : 7.50

Problem: Divide: $\frac{4b^2+7b-2}{1-b^2} \div \frac{4b^2+15b-4}{b^2-2b+1}$.

Solution:

$-\frac{(b+2)(b-1)}{(1+b)(b+4)}$

Example:
Exercise:

Problem: **Divide: $\frac{p^3+q^3}{2p^2+2pq+2q^2} \div \frac{p^2-q^2}{6}$.

Solution:
Solution

$$\frac{p^3+q^3}{2p^2+2pq+2q^2} \div \frac{p^2-q^2}{6}$$

Rewrite the division as a multiplication of the first expression times the reciprocal of the second.

$$\frac{p^3+q^3}{2p^2+2pq+2q^2} \cdot \frac{6}{p^2-q^2}$$

Factor the numerators and denominators and then multiply.

$$\frac{(p+q)\left(p^2-pq+q^2\right)6}{2(p^2+pq+q^2)(p-q)(p+q)}$$

Simplify by dividing out common factors.

$$\frac{\cancel{(p+q)}\left(p^2-pq+q^2\right)\cancel{6}^{3}}{\cancel{2}(p^2+pq+q^2)(p-q)\cancel{(p+q)}}$$

Simplify.

$$\frac{3\left(p^2-pq+q^2\right)}{(p-q)(p^2+pq+q^2)}$$

Note:
Exercise:

TRY IT : : 7.51

Problem: Divide: $\frac{x^3-8}{3x^2-6x+12} \div \frac{x^2-4}{6}$.

Solution:

$\frac{2\left(x^2+2x+4\right)}{(x+2)(x^2-2x+4)}$

Note:
Exercise:

TRY IT : : 7.52

Problem: Divide: $\frac{2z^2}{z^2-1} \div \frac{z^3-z^2+z}{z^3-1}$.

Solution:

$\frac{2z(z^2+z+1)}{(z+1)(z^2-z+1)}$

Before doing the next example, let’s look at how we divide a fraction by a whole number. When we divide $\frac{3}{5} \div 4$, we first write 4 as a fraction so that we can find its reciprocal.

Equation:

$$\begin{gathered}\frac{3}{5} \div 4 \\ \frac{3}{5} \div \frac{4}{1} \\ \frac{3}{5} \cdot \frac{1}{4}\end{gathered}$$

We do the same thing when we divide rational expressions.

Example:
Exercise:

Problem: **Divide: $\frac{a^2-b^2}{3ab} \div \left(a^2+2ab+b^2\right)$.

Solution:
Solution

	$\frac{a^2-b^2}{3ab} \div \left(a^2+2ab+b^2\right)$
Write the second expression as a fraction.	$\frac{a^2-b^2}{3ab} \div \frac{a^2+2ab+b^2}{1}$
Rewrite the division as the first expression times the reciprocal of the second expression.	$\frac{a^2-b^2}{3ab} \cdot \frac{1}{a^2+2ab+b^2}$
Factor the numerators and the denominators, and then multiply.	$\frac{(a-b)(a+b)\cdot 1}{3ab\cdot(a+b)(a+b)}$
Simplify by dividing out common factors.	$\frac{(a-b)\cancel{(a+b)}}{3ab\cdot\cancel{(a+b)}(a+b)}$
Simplify.	$\frac{(a-b)}{3ab(a+b)}$

Note:
Exercise:

TRY IT : : 7.53

Problem: Divide: $\frac{2x^2-14x-16}{4} \div \left(x^2+2x+1\right)$.

Solution:

$\frac{x-8}{2(x+1)}$

Note:
Exercise:

TRY IT : : 7.54

Problem: Divide: $\frac{y^2-6y+8}{y^2-4y} \div \left(3y^2-12y\right)$.

Solution:

$\frac{y-2}{3y^2(y-4)}$

Remember a fraction bar means division. A complex fraction is another way of writing division of two fractions.

Example:
Exercise:

Problem: **Divide: $\frac{\frac{6x^2-7x+2}{4x-8}}{\frac{2x^2-7x+3}{x^2-5x+6}}$.

Solution:
Solution

	$\frac{\frac{6x^2-7x+2}{4x-8}}{\frac{2x^2-7x+3}{x^2-5x+6}}$
Rewrite with a division sign.	$\frac{6x^2-7x+2}{4x-8} \div \frac{2x^2-7x+3}{x^2-5x+6}$
Rewrite as product of first times reciprocal of second.	$\frac{6x^2-7x+2}{4x-8} \cdot \frac{x^2-5x+6}{2x^2-7x+3}$
Factor the numerators and the denominators, and then multiply.	$\frac{(2x-1)(3x-2)(x-2)(x-3)}{4(x-2)(2x-1)(x-3)}$
Simplify by dividing out common factors.	$\frac{\cancel{(2x-1)}(3x-2)\cancel{(x-2)}\cancel{(x-3)}}{4\cancel{(x-2)}\cancel{(2x-1)}\cancel{(x-3)}}$
Simplify.	$\frac{3x-2}{4}$

Note:
Exercise:

TRY IT : : 7.55

Problem: Divide: $\frac{\frac{3x^2+7x+2}{4x+24}}{\frac{3x^2-14x-5}{x^2+x-30}}$.

Solution:

$\frac{x+2}{4}$

Note:
Exercise:

TRY IT : : 7.56

Problem: Divide: $\frac{\frac{y^2-36}{2y^2+11y-6}}{\frac{2y^2-2y-60}{8y-4}}$.

Solution:

$\frac{2}{y+5}$

If we have more than two rational expressions to work with, we still follow the same procedure. The first step will be to rewrite any division as multiplication by the reciprocal. Then we factor and multiply.

Example:
Exercise:

Problem: **Divide: $\frac{3x-6}{4x-4}\cdot\frac{x^2+2x-3}{x^2-3x-10}\div\frac{2x+12}{8x+16}$.

Solution:
Solution

	$\frac{3x-6}{4x-4}\cdot\frac{x^2+2x-3}{x^2-3x-10}\div\frac{2x+12}{8x+16}$
Rewrite the division as multiplication by the reciprocal.	$\frac{3x-6}{4x-4}\cdot\frac{x^2+2x-3}{x^2-3x-10}\cdot\frac{8x+16}{2x+12}$
Factor the numerators and the denominators, and then multiply.	$\frac{3\cdot 8(x-2)(x+3)(x-1)(x+2)}{4\cdot 2(x-1)(x+2)(x-5)(x+6)}$

Simplify by dividing out common factors.	$\frac{3\cdot\cancel{8}(x-2)(x+3)\cancel{(x-1)}\cancel{(x+2)}}{\cancel{4}\cdot\cancel{2}\cancel{(x-1)}\cancel{(x+2)}(x-5)(x+6)}$
Simplify.	$\frac{3(x-2)(x+3)}{(x-5)(x+6)}$

Note:
Exercise:

TRY IT : : 7.57

Problem: Divide: $\frac{4m+4}{3m-15}\cdot\frac{m^2-3m-10}{m^2-4m-32}\div\frac{12m-36}{6m-48}$.

Solution:

$\frac{2(m+1)(m+2)}{3(m+4)(m-3)}$

Note:
Exercise:

TRY IT : : 7.58

Problem: Divide: $\frac{2n^2+10n}{n-1}\div\frac{n^2+10n+24}{n^2+8n-9}\cdot\frac{n+4}{8n^2+12n}$.

Solution:

$\frac{(n+5)(n+9)}{2(n+6)(2n+3)}$

Key Concepts

- **Multiplication of Rational Expressions**
  - If p, q, r, s are polynomials where $q \neq 0, s \neq 0$, then $\frac{p}{q}\cdot\frac{r}{s}=\frac{pr}{qs}$.
  - To multiply rational expressions, multiply the numerators and multiply the denominators

- **Multiply a Rational Expression**

  Factor each numerator and denominator completely.
  Multiply the numerators and denominators.
  Simplify by dividing out common factors.

- **Division of Rational Expressions**
  - If p, q, r, s are polynomials where $q \neq 0, r \neq 0, s \neq 0$, then $\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \cdot \frac{s}{r}$.
  - To divide rational expressions multiply the first fraction by the reciprocal of the second.

- **Divide Rational Expressions**

  Rewrite the division as the product of the first rational expression and the reciprocal of the second.
  Factor the numerators and denominators completely.
  Multiply the numerators and denominators together.
  Simplify by dividing out common factors.

Practice Makes Perfect

Multiply Rational Expressions

In the following exercises, multiply.

Exercise:

Problem: $\frac{12}{16} \cdot \frac{4}{10}$

Solution:

$\frac{3}{10}$

Exercise:

Problem: $\frac{32}{5} \cdot \frac{16}{24}$

Exercise:

Problem: $\frac{18}{10} \cdot \frac{4}{30}$

Solution:

$\frac{6}{25}$

Exercise:

Problem: $\frac{21}{36} \cdot \frac{45}{24}$

Exercise:

Problem: $\frac{5x^2y^4}{12xy^3} \cdot \frac{6x^2}{20y^2}$

Solution:

$\frac{x^3}{8y}$

Exercise:

Problem: $\frac{8w^3y}{9y^2} \cdot \frac{3y}{4w^4}$

Exercise:

Problem: $\frac{12a^3b}{b^2} \cdot \frac{2ab^2}{9b^3}$

Solution:

$\frac{8a^3}{3b^2}$

Exercise:

Problem: $\frac{4mn^2}{5n^3} \cdot \frac{mn^3}{8m^2n^2}$

Exercise:

Problem: $\frac{5p^2}{p^2-5p-36} \cdot \frac{p^2-16}{10p}$

Solution:

$\frac{p(p-4)}{2(p-9)}$

Exercise:

Problem: $\frac{3q^2}{q^2+q-6} \cdot \frac{q^2-9}{9q}$

Exercise:

Problem: $\frac{4r}{r^2-3r-10} \cdot \frac{r^2-25}{8r^2}$

Solution:

$\frac{r+5}{2r(r+2)}$

Exercise:

Problem: $\frac{s}{s^2-9s+14} \cdot \frac{s^2-49}{7s^2}$

Exercise:

Problem: $\frac{x^2-7x}{x^2+6x+9} \cdot \frac{x+3}{4x}$

Solution:

$\frac{x-7}{4(x+3)}$

Exercise:

Problem: $\frac{2y^2-10y}{y^2+10y+25} \cdot \frac{y+5}{6y}$

Exercise:

Problem: $\frac{z^2+3z}{z^2-3z-4} \cdot \frac{z-4}{z^2}$

Solution:

$\frac{z+3}{z(z+1)}$

Exercise:

Problem: $\frac{2a^2+8a}{a^2-9a+20} \cdot \frac{a-5}{a^2}$

Exercise:

Problem: $\frac{28-4b}{3b-3} \cdot \frac{b^2+8b-9}{b^2-49}$

Solution:

$-\frac{4(b+9)}{3(b+7)}$

Exercise:

Problem: $\frac{18c-2c^2}{6c+30} \cdot \frac{c^2+7c+10}{c^2-81}$

Exercise:

Problem: $\frac{35d-7d^2}{d^2+7d} \cdot \frac{d^2+12d+35}{d^2-25}$

Solution:

-7

Exercise:

Problem: $\frac{72m-12m^2}{8m+32} \cdot \frac{m^2+10m+24}{m^2-36}$

Exercise:

Problem: $\frac{4n+20}{n^2+n-20} \cdot \frac{n^2-16}{4n+16}$

Solution:

1

Exercise:

Problem: $\frac{6p^2-6p}{p^2+7p-18} \cdot \frac{p^2-81}{3p^2-27p}$

Exercise:

Problem: $\frac{q^2-2q}{q^2+6q-16} \cdot \frac{q^2-64}{q^2-8q}$

Solution:

1

Exercise:

Problem: $\frac{2r^2-2r}{r^2+4r-5} \cdot \frac{r^2-25}{2r^2-10r}$

Divide Rational Expressions

In the following exercises, divide.

Exercise:

Problem: $\frac{t-6}{3-t} \div \frac{t^2-9}{t-5}$

Solution:

$-\frac{2t}{t^3-5t-9}$

Exercise:

Problem: $\frac{v-5}{11-v} \div \frac{v^2-25}{v-11}$

Exercise:

Problem: $\frac{10+w}{w-8} \div \frac{100-w^2}{8-w}$

Solution:

$-\frac{1}{10-w}$

Exercise:

Problem: $\frac{7+x}{x-6} \div \frac{49-x}{x+6}^{2}$

Exercise:

Problem: $\frac{27y^2}{3y-21} \div \frac{3y^2+18}{y^2+13y+42}$

Solution:

$\frac{3y^2(y+6)(y+7)}{(y-7)(y^2+6)}$

Exercise:

Problem: $\frac{24z^2}{2z-8} \div \frac{4z-28}{z^2-11z+28}$

Exercise:

Problem: $\frac{16a^2}{4a+36} \div \frac{4a^2-24a}{a^2+4a-45}$

Solution:

$\frac{a(a-5)}{a-6}$

Exercise:

Problem: $\frac{24b^2}{2b-4} \div \frac{12b^2+36b}{b^2-11b+18}$

Exercise:

Problem: $\frac{5c^2+9c+2}{c^2-25} \div \frac{3c^2-14c-5}{c^2+10c+25}$

Solution:

$\frac{(c+2)(c+2)}{(c-2)(c-3)}$

Exercise:

Problem: $\frac{2d^2+d-3}{d^2-16} \div \frac{2d^2-9d-18}{d^2-8d+16}$

Exercise:

Problem: $\frac{6m^2-2m-10}{9-m^2} \div \frac{6m^2+29m-20}{m^2-6m+9}$

Solution:

$-\frac{(m-2)(m-3)}{(3+m)(m+4)}$

Exercise:

Problem: $\frac{2n^2-3n-14}{25-n^2} \div \frac{2n^2-13n+21}{n^2-10n+25}$

Exercise:

Problem: $\frac{3s^2}{s^2-16} \div \frac{s^3+4s^2+16s}{s^3-64}$

Solution:

$\frac{3s}{s+4}$

Exercise:

Problem: $\frac{r^2-9}{15} \div \frac{r^3-27}{5r^2+15r+45}$

Exercise:

Problem: $\frac{p^3+q^3}{3p^2+3pq+3q^2} \div \frac{p^2-q^2}{12}$

Solution:

$\frac{4(p^2-pq+q^2)}{(p-q)(p^2+pq+q^2)}$

Exercise:

Problem: $\frac{v^3-8w^3}{2v^2+4vw+8w^2} \div \frac{v^2-4w^2}{4}$

Exercise:

Problem: $\frac{t^2-9}{2t} \div (t^2-6t+9)$

Solution:

$\frac{t+3}{2t(t-3)}$

Exercise:

Problem: $\frac{x^2+3x-10}{4x} \div (2x^2+20x+50)$

Exercise:

Problem: $\frac{2y^2-10yz-48z^2}{2y-1} \div (4y^2-32yz)$

Solution:

$\frac{y+3z}{2y(2y-1)}$

Exercise:

Problem: $\frac{2m^2-98n^2}{2m+6} \div (m^2-7mn)$

Exercise:

Problem: $\frac{\frac{2a^2-a-21}{5a+20}}{\frac{a^2+7a+12}{a^2+8a+16}}$

Solution:

$\frac{2a-7}{5}$

Exercise:

Problem: $\frac{\frac{3b^2+2b-8}{12b+18}}{\frac{3b^2+2b-8}{2b^2-7b-15}}$

Exercise:

Problem: $\frac{\frac{12c^2-12}{2c^2-3c+1}}{\frac{4c+4}{6c^2-13c+5}}$

Solution:

$$3\left(3c-5\right)$$

Exercise:

Problem: $\frac{\frac{4d^2+7d-2}{35d+10}}{\frac{d^2-4}{7d^2-12d-4}}$

Exercise:

Problem: $\frac{10m^2+80m}{3m-9}\cdot\frac{m^2+4m-21}{m^2-9m+20}\div\frac{5m^2+10m}{2m-10}$

Solution:

$$\frac{4(m+8)(m+7)}{3(m-4)(m+2)}$$

Exercise:

Problem: $\frac{4n^2+32n}{3n+2}\cdot\frac{3n^2-n-2}{n^2+n-30}\div\frac{108n^2-24n}{n+6}$

Exercise:

Problem: $\frac{12p^2+3p}{p+3}\div\frac{p^2+2p-63}{p^2-p-12}\cdot\frac{p-7}{9p^3-9p^2}$

Solution:

$$\frac{(4p+1)(p-7)}{3p(p+9)(p-1)}$$

Exercise:

Problem: $\frac{6q+3}{9q^2-9q}\div\frac{q^2+14q+33}{q^2+4q-5}\cdot\frac{4q^2+12q}{12q+6}$

Everyday Math

Exercise:

Problem:

Probability The director of large company is interviewing applicants for two identical jobs. If $w =$ the number of women applicants and $m =$ the number of men applicants, then the probability that two women are selected for the jobs is $\frac{w}{w+m} \cdot \frac{w-1}{w+m-1}$.

ⓐ Simplify the probability by multiplying the two rational expressions.
ⓑ Find the probability that two women are selected when $w = 5$ and $m = 10$.

Solution:

ⓐ $\frac{w(w-1)}{(w+m)(w+m-1)}$
ⓑ $\frac{2}{21}$

Exercise:

Problem:

Area of a triangle The area of a triangle with base b and height h is $\frac{bh}{2}$. If the triangle is stretched to make a new triangle with base and height three times as much as in the original triangle, the area is $\frac{9bh}{2}$. Calculate how the area of the new triangle compares to the area of the original triangle by dividing $\frac{9bh}{2}$ by $\frac{bh}{2}$.

Writing Exercises

Exercise:

Problem:

ⓐ Multiply $\frac{7}{4} \cdot \frac{9}{10}$ and explain all your steps.
ⓑ Multiply $\frac{n}{n-3} \cdot \frac{9}{n+3}$ and explain all your steps.
ⓒ Evaluate your answer to part (b) when $n = 7$. Did you get the same answer you got in part (a)? Why or why not?

Solution:

Answers will vary.

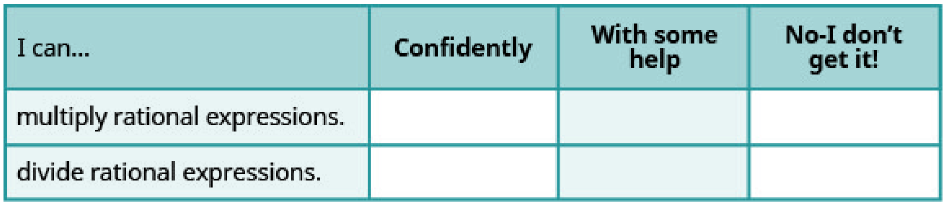
Exercise:

Problem:

ⓐ Divide $\frac{24}{5} \div 6$ and explain all your steps.
ⓑ Divide $\frac{x^2-1}{x} \div (x+1)$ and explain all your steps.
ⓒ Evaluate your answer to part (b) when $x = 5$. Did you get the same answer you got in part (a)? Why or why not?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
multiply rational expressions.			
divide rational expressions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Add and Subtract Rational Expressions with a Common Denominator
By the end of this section, you will be able to:

- Add rational expressions with a common denominator
- Subtract rational expressions with a common denominator
- Add and subtract rational expressions whose denominators are opposites

Note:
Before you get started, take this readiness quiz.
If you miss a problem, go back to the section listed and review the material.

1. Add: $\frac{y}{3}+\frac{9}{3}$.
   If you missed this problem, review [link].
2. Subtract: $\frac{10}{x}-\frac{2}{x}$.
   If you missed this problem, review [link].
3. Factor completely: $8n^5-20n^3$.
   If you missed this problem, review [link].
4. Factor completely: $45a^3-5ab^2$.
   If you missed this problem, review [link].

Add Rational Expressions with a Common Denominator

What is the first step you take when you add numerical fractions? You check if they have a common denominator. If they do, you add the numerators and place the sum over the common denominator. If they do not have a common denominator, you find one before you add.

It is the same with rational expressions. To add rational expressions, they must have a common denominator. When the denominators are the same, you add the numerators and place the sum over the common denominator.

Note:

Rational Expression Addition
If $p, q,$ and r are polynomials where $r \neq 0$, then
Equation:

$$\frac{p}{r} + \frac{q}{r} = \frac{p+q}{r}$$

To add rational expressions with a common denominator, add the numerators and place the sum over the common denominator.

We will add two numerical fractions first, to remind us of how this is done.

Example:
Exercise:

Problem: **Add: $\frac{5}{18} + \frac{7}{18}$.

Solution:
Solution

	$\frac{5}{18} + \frac{7}{18}$
The fractions have a common denominator, so add the numerators and place the sum over the common denominator.	$\frac{5+7}{18}$
Add in the numerator.	$\frac{12}{18}$
Factor the numerator and denominator to show the common factors.	$\frac{6 \cdot 2}{6 \cdot 3}$
Remove common factors.	$\frac{\cancel{6} \cdot 2}{\cancel{6} \cdot 3}$
Simplify.	$\frac{2}{3}$

Note:
Exercise:

TRY IT : : 7.59
Problem: Add: $\frac{7}{16} + \frac{5}{16}$.

Solution:

$\frac{3}{4}$

Note:
Exercise:

TRY IT : : 7.60
Problem: Add: $\frac{3}{10} + \frac{1}{10}$.

Solution:

$\frac{2}{5}$

Remember, we do not allow values that would make the denominator zero. What value of y should be excluded in the next example?

Example:
Exercise:

Problem: **Add: $\frac{3y}{4y-3} + \frac{7}{4y-3}$.

Solution:
Solution

$\frac{3y}{4y-3} + \frac{7}{4y-3}$

The fractions have a common denominator, so add the numerators and place the sum over the common denominator.

$\frac{3y+7}{4y-3}$

The numerator and denominator cannot be factored. The fraction is simplified.

Note:
Exercise:

TRY IT : : 7.61

Problem: Add: $\frac{5x}{2x+3} + \frac{2}{2x+3}$.

Solution:

$\frac{5x+2}{2x+3}$.

Note:
Exercise:

TRY IT : : 7.62

Problem: Add: $\frac{x}{x-2} + \frac{1}{x-2}$.

Solution:

$\frac{x+1}{x-2}$

Example:
Exercise:

Problem: **Add: $\frac{7x+12}{x+3} + \frac{x^2}{x+3}$.

Solution:
Solution

	$\frac{7x+12}{x+3}+\frac{x^2}{x+3}$
The fractions have a common denominator, so add the numerators and place the sum over the common denominator.	$\frac{7x+12+x^2}{x+3}$
Write the degrees in descending order.	$\frac{x^2+7x+12}{x+3}$
Factor the numerator.	$\frac{(x+3)(x+4)}{x+3}$
Simplify by removing common factors.	$\frac{\cancel{(x+3)}\,(x+4)}{\cancel{x+3}}$
Simplify.	$x+4$

Note:
Exercise:

TRY IT : : 7.63

Problem: Add: $\frac{9x+14}{x+7}+\frac{x^2}{x+7}$.

Solution:

$x+2$

Note:
Exercise:

TRY IT : : 7.64

Problem: Add: $\frac{x^2+8x}{x+5}+\frac{15}{x+5}$.

Solution:

$x+3$

Subtract Rational Expressions with a Common Denominator

To subtract rational expressions, they must also have a common denominator. When the denominators are the same, you subtract the numerators and place the difference over the common denominator.

Note:
Rational Expression Subtraction
If $p, q,$ and r are polynomials where $r \neq 0$, then
Equation:

$$\frac{p}{r} - \frac{q}{r} = \frac{p-q}{r}$$

To subtract rational expressions, subtract the numerators and place the difference over the common denominator.

We always simplify rational expressions. Be sure to factor, if possible, after you subtract the numerators so you can identify any common factors.

Example:
Exercise:

Problem: **Subtract: $\frac{n^2}{n-10} - \frac{100}{n-10}$.

Solution:
Solution

	$\frac{n^2}{n-10} - \frac{100}{n-10}$
The fractions have a common denominator, so subtract the numerators and place the difference over the common denominator.	$\frac{n^2-100}{n-10}$
Factor the numerator.	$\frac{(n-10)(n+10)}{n-10}$
Simplify by removing common factors.	$\frac{\cancel{(n-10)}\,(n+10)}{\cancel{n-10}}$
Simplify.	$n+10$

Note:
Exercise:

TRY IT : : 7.65

Problem: Subtract: $\frac{x^2}{x+3} - \frac{9}{x+3}$.

Solution:

$x-3$

Note:
Exercise:

TRY IT : : 7.66

Problem: Subtract: $\frac{4x^2}{2x-5} - \frac{25}{2x-5}$.

Solution:

$2x+5$

Be careful of the signs when you subtract a binomial!

Example:
Exercise:

Problem: **Subtract: $\frac{y^2}{y-6} - \frac{2y+24}{y-6}$.

Solution:
Solution

	$\frac{y^2}{y-6} - \frac{2y+24}{y-6}$
The fractions have a common denominator, so subtract the numerators and place the difference over the common denominator.	$\frac{y^2-(2y+24)}{y-6}$
Distribute the sign in the numerator.	$\frac{y^2-2y-24}{y-6}$
Factor the numerator.	$\frac{(y-6)(y+4)}{y-6}$
Remove common factors.	$\frac{\cancel{(y-6)}\,(y+4)}{\cancel{y-6}}$
Simplify.	$y+4$

Note:
Exercise:

TRY IT : : 7.67

Problem: Subtract: $\frac{n^2}{n-4} - \frac{n+12}{n-4}$.

Solution:

$n+3$

Note:
Exercise:

TRY IT : : 7.68

Problem: Subtract: $\frac{y^2}{y-1} - \frac{9y-8}{y-1}$.

Solution:

$y - 8$

Example:
Exercise:

Problem: **Subtract: $\frac{5x^2-7x+3}{x^2-3x-18} - \frac{4x^2+x-9}{x^2-3x-18}$.

Solution:
Solution

	$\frac{5x^2-7x+3}{x^2-3x-18} - \frac{4x^2+x-9}{x^2-3x-18}$
Subtract the numerators and place the difference over the common denominator.	$\frac{5x^2-7x+3-\left(4x^2+x-9\right)}{x^2-3x-18}$
Distribute the sign in the numerator.	$\frac{5x^2-7x+3-4x^2-x+9}{x^2-3x-18}$
Combine like terms.	$\frac{x^2-8x+12}{x^2-3x-18}$
Factor the numerator and the denominator.	$\frac{(x-2)(x-6)}{(x+3)(x-6)}$
Simplify by removing common factors.	$\frac{(x-2)\,\cancel{(x-6)}}{(x+3)\,\cancel{(x-6)}}$
Simplify.	$\frac{(x-2)}{(x+3)}$

Note:
Exercise:

TRY IT : : 7.69

Problem: Subtract: $\frac{4x^2-11x+8}{x^2-3x+2} - \frac{3x^2+x-3}{x^2-3x+2}$.

Solution:

$\frac{x-11}{x-2}$

Note:
Exercise:

TRY IT : : 7.70

Problem: Subtract: $\frac{6x^2-x+20}{x^2-81} - \frac{5x^2+11x-7}{x^2-81}$.

Solution:

$\frac{x-3}{x+9}$

Add and Subtract Rational Expressions whose Denominators are Opposites

When the denominators of two rational expressions are opposites, it is easy to get a common denominator. We just have to multiply one of the fractions by $\frac{-1}{-1}$.

Let’s see how this works.

	$\frac{7}{d}+\frac{5}{-d}$
Multiply the second fraction by $\frac{-1}{-1}$.	$\frac{7}{d}+\frac{(-1)5}{(-1)(-d)}$
The denominators are the same.	$\frac{7}{d}+\frac{-5}{d}$
Simplify.	$\frac{2}{d}$

Example:
Exercise:

Problem: **Add: $\frac{4u-1}{3u-1}+\frac{u}{1-3u}$.

Solution:
Solution

	$\frac{4u-1}{3u-1}+\frac{u}{1-3u}$
The denominators are opposites, so multiply the second fraction by $\frac{-1}{-1}$.	$\frac{4u-1}{3u-1}+\frac{(-1)u}{(-1)(1-3u)}$
Simplify the second fraction.	$\frac{4u-1}{3u-1}+\frac{-u}{3u-1}$
The denominators are the same. Add the numerators.	$\frac{4u-1-u}{3u-1}$
Simplify.	$\frac{3u-1}{3u-1}$
Simplify.	1

Note:
Exercise:

TRY IT : : 7.71

Problem: Add: $\frac{8x-15}{2x-5}+\frac{2x}{5-2x}$.

Solution:

3

Note:
Exercise:

TRY IT : : 7.72

Problem: Add: $\frac{6y^2+7y-10}{4y-7}+\frac{2y^2+2y+11}{7-4y}$.

Solution:

$y+3$

Example:
Exercise:

Problem: **Subtract: $\frac{m^2-6m}{m^2-1}-\frac{3m+2}{1-m^2}$.

Solution:
Solution

	$\frac{m^2-6m}{m^2-1}-\frac{3m+2}{1-m^2}$
The denominators are opposites, so multiply the	

second fraction by $\frac{-1}{-1}$.	$\frac{m^2-6m}{m^2-1} - \frac{-1(3m+2)}{-1(1-m^2)}$
Simplify the second fraction.	$\frac{m^2-6m}{m^2-1} - \frac{-3m-2}{m^2-1}$
The denominators are the same. Subtract the numerators.	$\frac{m^2-6m-(-3m-2)}{m^2-1}$
Distribute.	$\frac{m^2-6m+3m+2}{m^2-1}$
Combine like terms.	$\frac{m^2-3m+2}{m^2-1}$
Factor the numerator and denominator.	$\frac{(m-1)(m-2)}{(m-1)(m+1)}$
Simplify by removing common factors.	$\frac{\cancel{(m-1)}(m-2)}{\cancel{(m-1)}(m+1)}$
Simplify.	$\frac{m-2}{m+1}$

Note:
Exercise:

TRY IT : : 7.73

Problem: Subtract: $\frac{y^2-5y}{y^2-4} - \frac{6y-6}{4-y^2}$.

Solution:

$\frac{y+3}{y+2}$

Note:
Exercise:

TRY IT : : 7.74

Problem: Subtract: $\frac{2n^2+8n-1}{n^2-1} - \frac{n^2-7n-1}{1-n^2}$.

Solution:

$\frac{3n-2}{n-1}$

Key Concepts

- **Rational Expression Addition**
  - If $p, q,$ and r are polynomials where $r \neq 0$, then
    Equation:

$$\frac{p}{r} + \frac{q}{r} = \frac{p+q}{r}$$

  - To add rational expressions with a common denominator, add the numerators and place the sum over the common denominator.
- **Rational Expression Subtraction**
  - If $p, q,$ and r are polynomials where $r \neq 0$, then
    Equation:

$$\frac{p}{r} - \frac{q}{r} = \frac{p-q}{r}$$

  - To subtract rational expressions, subtract the numerators and place the difference over the common denominator.

Practice Makes Perfect

Add Rational Expressions with a Common Denominator

In the following exercises, add.

Exercise:

Problem: $\frac{2}{15}+\frac{7}{15}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{4}{21}+\frac{3}{21}$

Exercise:

Problem: $\frac{7}{24}+\frac{11}{24}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{7}{36}+\frac{13}{36}$

Exercise:

Problem: $\frac{3a}{a-b}+\frac{1}{a-b}$

Solution:

$\frac{3a+1}{a-b}$

Exercise:

Problem: $\frac{3c}{4c-5}+\frac{5}{4c-5}$

Exercise:

Problem: $\frac{d}{d+8}+\frac{5}{d+8}$

Solution:

$\frac{d+5}{d+8}$

Exercise:

Problem: $\frac{7m}{2m+n}+\frac{4}{2m+n}$

Exercise:

Problem: $\frac{p^2+10p}{p+2}+\frac{16}{p+2}$

Solution:

$p+8$

Exercise:

Problem: $\frac{q^2+12q}{q+3}+\frac{27}{q+3}$

Exercise:

Problem: $\frac{2r^2}{2r-1}+\frac{15r-8}{2r-1}$

Solution:

$r+8$

Exercise:

Problem: $\frac{3s^2}{3s-2}+\frac{13s-10}{3s-2}$

Exercise:

Problem: $\frac{8t^2}{t+4}+\frac{32t}{t+4}$

Solution:

$8t$

Exercise:

Problem: $\frac{6v^2}{v+5} + \frac{30v}{v+5}$

Exercise:

Problem: $\frac{2w^2}{w^2-16} + \frac{8w}{w^2-16}$

Solution:

$\frac{2w}{w-4}$

Exercise:

Problem: $\frac{7x^2}{x^2-9} + \frac{21x}{x^2-9}$

Subtract Rational Expressions with a Common Denominator

In the following exercises, subtract.

Exercise:

Problem: $\frac{y^2}{y+8} - \frac{64}{y+8}$

Solution:

$y - 8$

Exercise:

Problem: $\frac{z^2}{z+2} - \frac{4}{z+2}$

Exercise:

Problem: $\frac{9a^2}{3a-7} - \frac{49}{3a-7}$

Solution:

$3a + 7$

Exercise:

Problem: $\frac{25b^2}{5b-6} - \frac{36}{5b-6}$

Exercise:

Problem: $\frac{c^2}{c-8} - \frac{6c+16}{c-8}$

Solution:

$c+2$

Exercise:

Problem: $\frac{d^2}{d-9} - \frac{6d+27}{d-9}$

Exercise:

Problem: $\frac{3m^2}{6m-30} - \frac{21m-30}{6m-30}$

Solution:

$\frac{m-2}{3}$

Exercise:

Problem: $\frac{2n^2}{4n-32} - \frac{30n-16}{4n-32}$

Exercise:

Problem: $\frac{6p^2+3p+4}{p^2+4p-5} - \frac{5p^2+p+7}{p^2+4p-5}$

Solution:

$\frac{p+3}{p+5}$

Exercise:

Problem: $\frac{5q^2+3q-9}{q^2+6q+8} - \frac{4q^2+9q+7}{q^2+6q+8}$

Exercise:

Problem: $\frac{5r^2+7r-33}{r^2-49} - \frac{4r^2-5r-30}{r^2-49}$

Solution:

$\frac{r+9}{r+7}$

Exercise:

Problem: $\frac{7t^2-t-4}{t^2-25} - \frac{6t^2+2t-1}{t^2-25}$

Add and Subtract Rational Expressions whose Denominators are Opposites

In the following exercises, add.

Exercise:

Problem: $\frac{10v}{2v-1} + \frac{2v+4}{1-2v}$

Solution:

4

Exercise:

Problem: $\frac{20w}{5w-2} + \frac{5w+6}{2-5w}$

Exercise:

Problem: $\frac{10x^2+16x-7}{8x-3} + \frac{2x^2+3x-1}{3-8x}$

Solution:

$x + 2$

Exercise:

Problem: $\frac{6y^2+2y-11}{3y-7} + \frac{3y^2-3y+17}{7-3y}$

In the following exercises, subtract.

Exercise:

Problem: $\frac{z^2+6z}{z^2-25} - \frac{3z+20}{25-z^2}$

Solution:

$\frac{z+4}{z-5}$

Exercise:

Problem: $\frac{a^2+3a}{a^2-9} - \frac{3a-27}{9-a^2}$

Exercise:

Problem: $\frac{2b^2+30b-13}{b^2-49} - \frac{2b^2-5b-8}{49-b^2}$

Solution:

$\frac{4b-3}{b-7}$

Exercise:

Problem: $\frac{c^2+5c-10}{c^2-16} - \frac{c^2-8c-10}{16-c^2}$

Everyday Math

Exercise:

Problem:

Sarah ran 8 miles and then biked 24 miles. Her biking speed is 4 mph faster than her running speed. If r represents Sarah’s speed when she ran, then her running time is modeled by the expression $\frac{8}{r}$ and her biking time is modeled by the expression $\frac{24}{r+4}$. Add the rational expressions $\frac{8}{r} + \frac{24}{r+4}$ to get an expression for the total amount of time Sarah ran and biked.

Solution:

$\frac{32r+32}{r(r+4)}$

Exercise:

Problem:

If Pete can paint a wall in p hours, then in one hour he can paint $\frac{1}{p}$ of the wall. It would take Penelope 3 hours longer than Pete to paint the wall, so in one hour she can paint $\frac{1}{p+3}$ of the wall. Add the rational expressions $\frac{1}{p}+\frac{1}{p+3}$ to get an expression for the part of the wall Pete and Penelope would paint in one hour if they worked together.

Writing Exercises

Exercise:

Problem: Donald thinks that $\frac{3}{x}+\frac{4}{x}$ is $\frac{7}{2x}$. Is Donald correct? Explain.

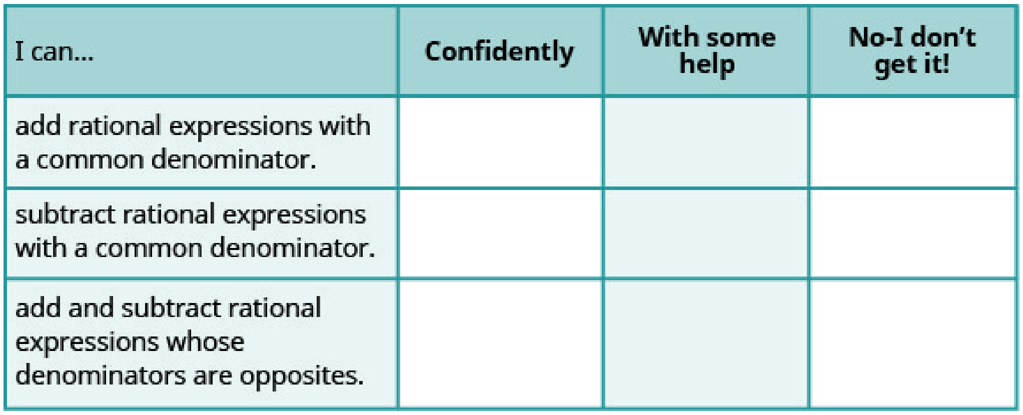
Exercise:

Problem:

Explain how you find the Least Common Denominator of x^2+5x+4 and x^2-16.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
add rational expressions with a common denominator.			
subtract rational expressions with a common denominator.			
add and subtract rational expressions whose denominators are opposites.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Add and Subtract Rational Expressions with Unlike Denominators
By the end of this section, you will be able to:

- Find the least common denominator of rational expressions
- Find equivalent rational expressions
- Add rational expressions with different denominators
- Subtract rational expressions with different denominators

Note:
Before you get started, take this readiness quiz.
If you miss a problem, go back to the section listed and review the material.

1. Add: $\frac{7}{10}+\frac{8}{15}$.
   If you missed this problem, review [link].
2. Subtract: $6\left(2x+1\right)-4\left(x-5\right)$.
   If you missed this problem, review [link].
3. Find the Greatest Common Factor of $9x^2y^3$ and $12xy^5$.
   If you missed this problem, review [link].
4. Factor completely $-48n-12$.
   If you missed this problem, review [link].

Find the Least Common Denominator of Rational Expressions

When we add or subtract rational expressions with unlike denominators we will need to get common denominators. If we review the procedure we used with numerical fractions, we will know what to do with rational expressions.

Let’s look at the example $\frac{7}{12}+\frac{5}{18}$ from Arithmetic Review. Since the denominators are not the same, the first step was to find the least common denominator (LCD). Remember, the LCD is the least common multiple of the denominators. It is the smallest number we can use as a common denominator.

To find the LCD of 12 and 18, we factored each number into primes, lining up any common primes in columns. Then we “brought down” one prime from each column. Finally, we multiplied the factors to find the LCD.
Equation:

$$\begin{array}{r}12 = 2 \cdot 2 \cdot 3 \\ 18 = 2 \cdot 3 \cdot 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3 \\ \text{LCD} = 36\end{array}$$

We do the same thing for rational expressions. However, we leave the LCD in factored form.

Note:
Find the least common denominator of rational expressions.

Factor each expression completely.
List the factors of each expression. Match factors vertically when possible.
Bring down the columns.
Multiply the factors.

Remember, we always exclude values that would make the denominator zero. What values of *x* should we exclude in this next example?

Example:
Exercise:

Problem: **Find the LCD for $\frac{8}{15a^2b}, \frac{3x}{27ab^2}$.

Solution:
Solution

	Find the LCD for $\frac{8}{15a^2b}, \frac{3x}{27ab^2}$.
Factor each expression completely, lining up common factors. Bring down the columns.	$$\begin{array}{l}15a^2b = 3* \quad\quad 5*a*a*b \\ 27ab^2 = 3*3*3* \quad a* \quad b*b \\ \hline \text{LCD} = 3*3*3*5*a*a*b*b\end{array}$$
Multiply the factors.	The LCD is $135a^2b^2$.

Note:
Exercise:

TRY IT :: 7.75

Problem: Find the LCD for $\frac{2}{12xy^2}, \frac{1}{6x^2}$.

Solution:

$12x^2y^2$

Note:
Exercise:

TRY IT :: 7.76

Problem: Find the LCD for $\frac{r}{9s^3t}, \frac{5}{6rs}$.

Solution:

$18rs^3t$

Find Equivalent Rational Expressions

When we add numerical fractions, once we find the LCD, we rewrite each fraction as an equivalent fraction with the LCD.

$$\frac{7}{12} + \frac{5}{18}$$

$$\frac{7 \cdot 3}{12 \cdot 3} + \frac{5 \cdot 2}{18 \cdot 2}$$

$$\frac{21}{36} + \frac{10}{36}$$

$$\begin{array}{rl} 12 = & 2 \cdot 2 \cdot 3 \\ 18 = & 2 \cdot \phantom{2 \cdot {}} 3 \cdot 3 \\ \hline \text{LCD} = & 2 \cdot 2 \cdot 3 \cdot 3 \\ \text{LCD} = & 36 \end{array}$$

We will do the same thing for rational expressions.

Example:
Exercise:

Problem:

**Rewrite as equivalent rational expressions with denominator $21x^2y$: $\frac{2}{3x^2}, \frac{5}{7y}$.

Solution:
Solution

	$\frac{2}{3x^2}\ ,\ \frac{5}{7y}$
Factor each denominator.	$\frac{2}{3\cdot x\cdot x}\ ,\ \frac{5}{7y}$
Find the LCD. $\begin{array}{rl} 3x^2 = & 3\cdot\bigcirc\ x\cdot x\ \bigcirc \\ 7y = & \bigcirc\ 7\cdot\bigcirc\bigcirc\ y \\ \hline 21x^2y = & 3\cdot 7\cdot x\cdot x\cdot y \end{array}$	
Multiply each denominator by the 'missing' factor and multiply each numerator by the same factor.	$\frac{2\cdot 7\cdot y}{3x^2\cdot 7\cdot y}, \frac{5\cdot 3\cdot x\cdot x}{7y\cdot 3\cdot x\cdot x}$
Simplify the numerators.	$\frac{14y}{21x^{2y}}, \frac{15x^2}{21x^{2y}}$

Note:
Exercise:

Problem:

TRY IT :: 7.77
Rewrite as equivalent rational expressions with denominator $15a^2b$: $\frac{2}{3a^2}, \frac{5}{5ab}$.

Solution:

$\frac{10b}{15a^2b}, \frac{15a}{15a^2b}$.

Note:
Exercise:

Problem:

TRY IT :: 7.78
Rewrite as equivalent rational expressions with denominator $24y$: $\frac{y}{3}, \frac{5}{8y}$.

Solution:

$\frac{8y^2}{24y}, \frac{15}{24y}$.

Add Rational Expressions with Different Denominators

Now we have all the steps we need to add rational expressions with different denominators. As we have done previously, we will do one example of adding numerical fractions first.

Example:
Exercise:

Problem: **Add: $\frac{7}{12}+\frac{5}{18}$.

Solution:
Solution

	$\frac{7}{12}+\frac{5}{18}$
Find the LCD of 12 and 18. $12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3$ $\text{LCD} = 36$	
Rewrite each fraction as an equivalent fraction with the LCD.	$\frac{7 \cdot 3}{12 \cdot 3}+\frac{5 \cdot 2}{18 \cdot 2}$
Add the fractions.	$\frac{21}{36}+\frac{10}{36}$
The fraction cannot be simplified.	$\frac{31}{36}$

Note:
Exercise:

TRY IT : : 7.79

Problem: Add: $\frac{11}{30}+\frac{7}{12}$.

Solution:

$\frac{57}{60}$

Note:
Exercise:

TRY IT : : 7.80

Problem: Add: $\frac{3}{8}+\frac{9}{20}$.

Solution:

$\frac{33}{40}$

Now we will add rational expressions whose denominators are monomials.

Example:
Exercise:

Problem: **Add: $\frac{5}{12x^2y}+\frac{4}{21xy^2}$.

Solution:
Solution

		$\frac{5}{12x^2y}+\frac{4}{21xy^2}$

Find the LCD of $12x^2y$ and $21xy^2$. $\begin{array}{ll} 12x^2y & = 2\cdot 2\cdot 3\cdot \quad x\cdot x\cdot y \\ 21xy^2 & = \qquad 3\cdot 7\cdot x\cdot \quad y\cdot y \\ \hline \text{LCD} & = 2\cdot 2\cdot 3\cdot 7\cdot x\cdot x\cdot y\cdot y \\ \text{LCD} & = 84x^2y^2 \end{array}$		
		$\frac{5}{12x^2y}+\frac{4}{21xy^2}$
Rewrite each rational expression as an equivalent fraction with the LCD.		$\frac{5\cdot 7y}{12x^2y\cdot 7y}+\frac{4\cdot 4x}{21xy^2\cdot 4x}$
Simplify.		$\frac{35y}{84x^2y^2}+\frac{16x}{84x^2y^2}$
Add the rational expressions.		$\frac{16x+35y}{84x^2y^2}$
There are no factors common to the numerator and denominator. The fraction cannot be simplified.		

Note:
Exercise:

TRY IT : : 7.81

Problem: Add: $\frac{2}{15a^2b}+\frac{5}{6ab^2}$.

Solution:

$\frac{4b+25a}{30a^2b^2}$

Note:

Exercise:

TRY IT : : 7.82

Problem: Add: $\frac{5}{16c}+\frac{3}{8cd^2}$.

Solution:

$\frac{5d^2+6}{16cd^2}$

Now we are ready to tackle polynomial denominators.

Example:

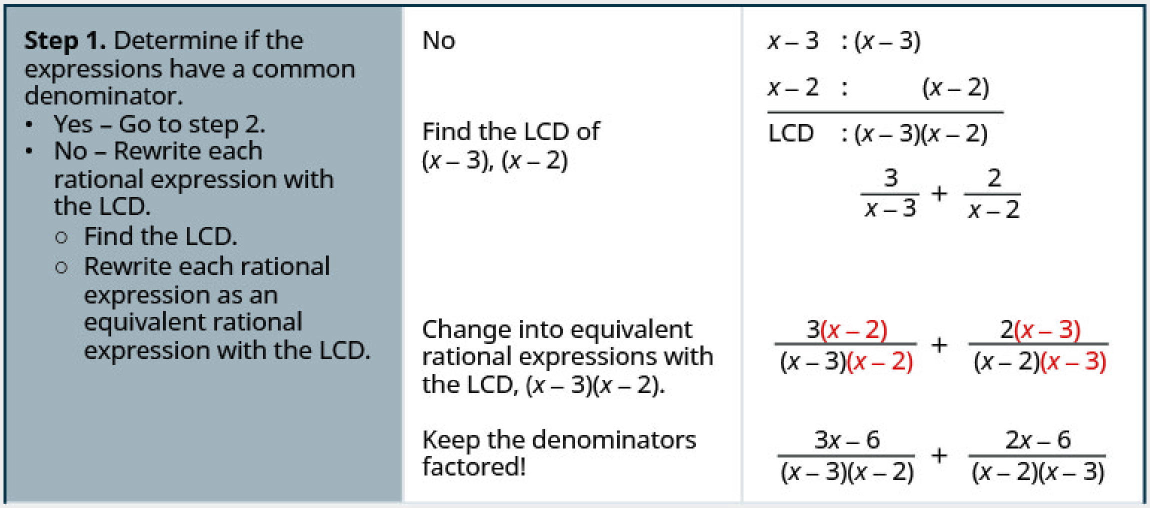
How to Add Rational Expressions with Different Denominators

Exercise:

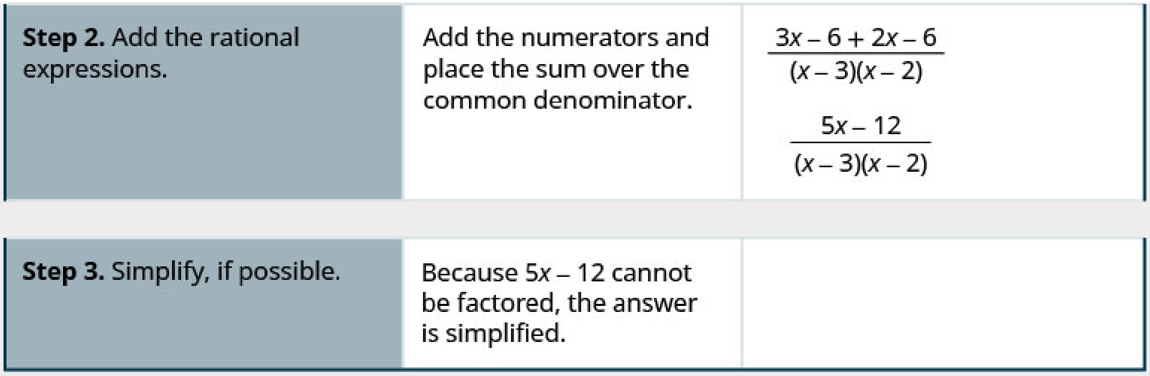
Problem: **Add: $\frac{3}{x-3}+\frac{2}{x-2}$.

Solution:

Solution



Step 1. Determine if the expressions have a common denominator. • Yes – Go to step 2. • No – Rewrite each rational expression with the LCD. ○ Find the LCD. ○ Rewrite each rational expression as an equivalent rational expression with the LCD.	No Find the LCD of $(x-3)$, $(x-2)$	$x-3 \;:\; (x-3)$ $x-2 \;:\; (x-2)$ LCD $:\; (x-3)(x-2)$ $\frac{3}{x-3}+\frac{2}{x-2}$
	Change into equivalent rational expressions with the LCD, $(x-3)(x-2)$.	$\frac{3(x-2)}{(x-3)(x-2)}+\frac{2(x-3)}{(x-2)(x-3)}$
	Keep the denominators factored!	$\frac{3x-6}{(x-3)(x-2)}+\frac{2x-6}{(x-2)(x-3)}$



Step 2. Add the rational expressions.	Add the numerators and place the sum over the common denominator.	$\frac{3x-6+2x-6}{(x-3)(x-2)}$ $\frac{5x-12}{(x-3)(x-2)}$
Step 3. Simplify, if possible.	Because 5*x* – 12 cannot be factored, the answer is simplified.	

Note:
Exercise:

TRY IT :: 7.83

Problem: Add: $\frac{2}{x-2}+\frac{5}{x+3}$.

Solution:

$\frac{7x-4}{(x+3)(x-2)}$

Note:
Exercise:

TRY IT :: 7.84

Problem: Add: $\frac{4}{m+3}+\frac{3}{m+4}$.

Solution:

$\frac{7m+25}{(m+3)(m+4)}$

The steps to use to add rational expressions are summarized in the following procedure box.

Note:
Add rational expressions.

Determine if the expressions have a common denominator.	**Yes**– go to step 2.	**No**– Rewrite each rational expression with the LCD.	Find the LCD.	Rewrite each rational expression as an equivalent rational expression with the LCD.

Add the rational expressions.
Simplify, if possible.

Example:
Exercise:

Problem: **Add: $\frac{2a}{2ab+b^2}+\frac{3a}{4a^2-b^2}$.

Solution:
Solution

	$\frac{2a}{2ab+b^2}+\frac{3a}{4a^2-b^2}$
Do the expressions have a common denominator? No. Rewrite each expression with the LCD.	
Find the LCD. $\begin{array}{ll} 2ab+b^2 & = b(2a+b) \\ 4a^2-b^2 & = (2a+b)(2a-b) \\ \hline \text{LCD} & = b(2a+b)(2a-b) \end{array}$	
Rewrite each rational expression as an equivalent rational expression with the LCD.	$\frac{2a(2a-b)}{b(2a+b)(2a-b)}+\frac{3a\cdot b}{(2a+b)(2a-b)\cdot b}$

Simplify the numerators.	$\frac{4a^2-2ab}{b(2a+b)(2a-b)}+\frac{3ab}{b(2a+b)(2a-b)}$
Add the rational expressions.	$\frac{4a^2-2ab+3ab}{b(2a+b)(2a-b)}$
Simplify the numerator.	$\frac{4a^2+ab}{b(2a+b)(2a-b)}$
Factor the numerator.	$\frac{a(4a+b)}{b(2a+b)(2a-b)}$
There are no factors common to the numerator and denominator. The fraction cannot be simplified.	

Note:
Exercise:

TRY IT :: 7.85

Problem: Add: $\frac{5x}{xy-y^2}+\frac{2x}{x^2+y^2}$.

Solution:

$\frac{x(5x+7y)}{y(x-y)(x+y)}$

Note:
Exercise:

TRY IT :: 7.86

Problem: Add: $\frac{7}{2m+6}+\frac{4}{m^2+4m+3}$.

Solution:

$\frac{7m+15}{2(m+3)(m+1)}$

Avoid the temptation to simplify too soon! In the example above, we must leave the first rational expression as $\frac{2a(2a-b)}{b(2a+b)(2a-b)}$ to be able to add it to $\frac{3a\cdot b}{(2a+b)(2a-b)\cdot b}$. Simplify only after you have combined the numerators.

Example:
Exercise:

Problem: **Add: $\frac{8}{x^2-2x-3}+\frac{3x}{x^2+4x+3}$.

Solution:
Solution

	$\frac{8}{x^2-2x-3}+\frac{3x}{x^2+4x+3}$
Do the expressions have a common denominator? No. Rewrite each expression with the LCD.	
Find the LCD. $x^2-2x-3=(x+1)(x-3)$ $x^2+4x+3=(x+1)\quad(x+3)$ $\text{LCD}\quad=(x+1)(x-3)(x+3)$	
Rewrite each rational expression as an equivalent fraction with the LCD.	$\frac{8(x+3)}{(x+1)(x-3)(x+3)}+\frac{3x(x-3)}{(x+1)(x+3)(x-3)}$
Simplify the numerators.	

	$\frac{8x+24}{(x+1)(x-3)(x+3)} + \frac{3x^2-9x}{(x+1)(x+3)(x-3)}$
Add the rational expressions.	$\frac{8x+24+3x^2+9x}{(x+1)(x-3)(x+3)}$
Simplify the numerator.	$\frac{3x^2-x^2+24}{(x+1)(x-3)(x+3)}$
The numerator is prime, so there are no common factors.	

Note:
Exercise:

TRY IT :: 7.87

Problem: Add: $\frac{1}{m^2-m-2} + \frac{5m}{m^2+3m+2}$.

Solution:

$\frac{5m^2-9m+2}{(m-2)(m+1)(m+2)}$

Note:
Exercise:

TRY IT :: 7.88

Problem: Add: $\frac{2n}{n^2-3n-10} + \frac{6}{n^2+5n+6}$.

Solution:

$\frac{2(n^2+6n-15)}{(n+2)(n-5)(n+3)}$

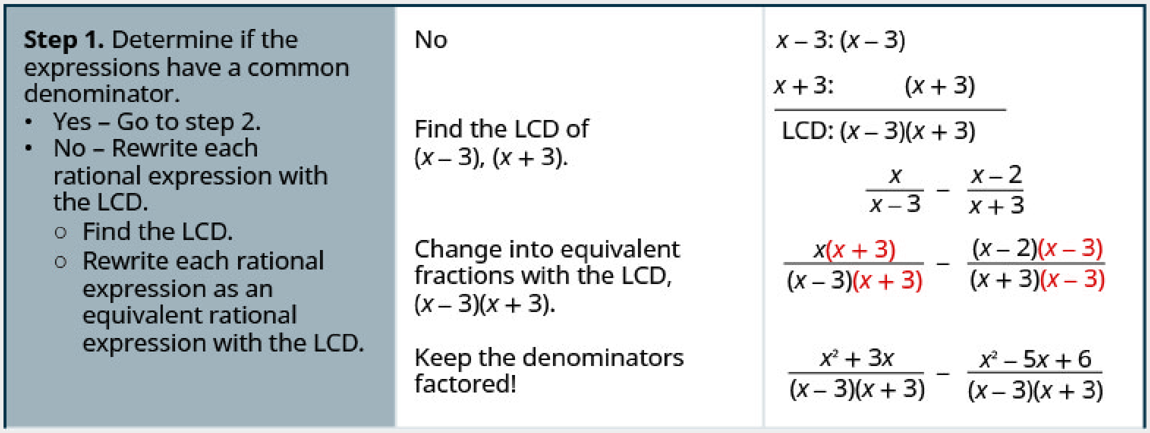
Subtract Rational Expressions with Different Denominators

The process we use to subtract rational expressions with different denominators is the same as for addition. We just have to be very careful of the signs when subtracting the numerators.

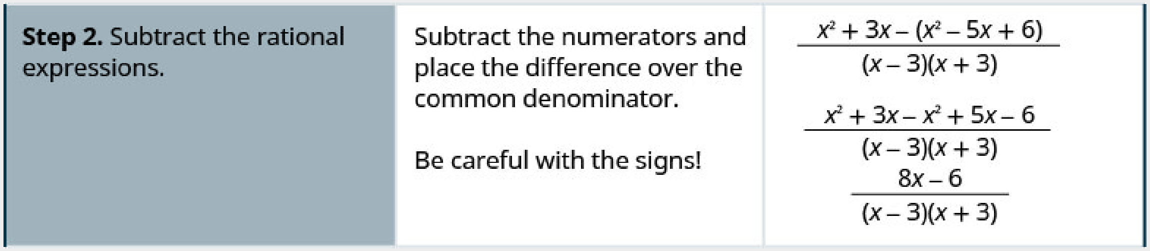
Example:
How to Subtract Rational Expressions with Different Denominators
Exercise:

Problem: **Subtract: $\frac{x}{x-3}-\frac{x-2}{x+3}$.

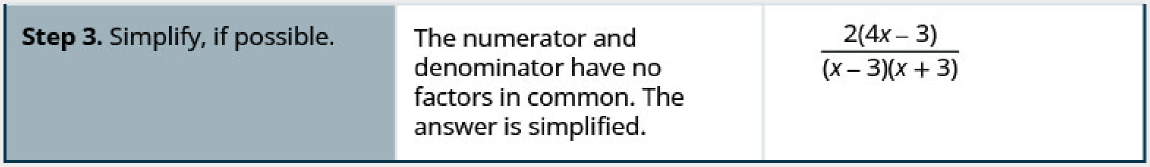
Solution:
Solution



Step 1. Determine if the expressions have a common denominator. • Yes – Go to step 2. • No – Rewrite each rational expression with the LCD. ○ Find the LCD. ○ Rewrite each rational expression as an equivalent rational expression with the LCD.	No	$x-3$: $(x-3)$ $x+3$: $\quad(x+3)$
	Find the LCD of $(x-3)$, $(x+3)$.	LCD: $(x-3)(x+3)$
		$\frac{x}{x-3}-\frac{x-2}{x+3}$
	Change into equivalent fractions with the LCD, $(x-3)(x+3)$.	$\frac{x(x+3)}{(x-3)(x+3)}-\frac{(x-2)(x-3)}{(x+3)(x-3)}$
	Keep the denominators factored!	$\frac{x^2+3x}{(x-3)(x+3)}-\frac{x^2-5x+6}{(x-3)(x+3)}$



Step 2. Subtract the rational expressions.	Subtract the numerators and place the difference over the common denominator.	$\frac{x^2+3x-(x^2-5x+6)}{(x-3)(x+3)}$
	Be careful with the signs!	$\frac{x^2+3x-x^2+5x-6}{(x-3)(x+3)}$ $\frac{8x-6}{(x-3)(x+3)}$



Step 3. Simplify, if possible.	The numerator and denominator have no factors in common. The answer is simplified.	$\frac{2(4x-3)}{(x-3)(x+3)}$

Note:
Exercise:

TRY IT :: 7.89

Problem: Subtract: $\frac{y}{y+4} - \frac{y-2}{y-5}$.

Solution:

$\frac{-7y+8}{(y+4)(y-5)}$

Note:
Exercise:

TRY IT :: 7.90

Problem: Subtract: $\frac{z+3}{z+2} - \frac{z}{z+3}$.

Solution:

$\frac{4z+9}{(z+2)(z+3)}$

The steps to take to subtract rational expressions are listed below.

Note:
Subtract rational expressions.

Determine if they have a common denominator.	**Yes**– go to step 2.	**No**– Rewrite each rational expression with the LCD.	Find the LCD.	Rewrite each rational expression as an equivalent rational expression with the LCD.

Subtract the rational expressions.
Simplify, if possible.

Example:
Exercise:

Problem: **Subtract: $\frac{8y}{y^2-16} - \frac{4}{y-4}$.

Solution:
Solution

	$\frac{8y}{y^2-16} - \frac{4}{y-4}$
Do the expressions have a common denominator? No. Rewrite each expression with the LCD.	
Find the LCD. $\begin{aligned} y^2-16 &= (y-4)(y+4) \\ y-4 &= y-4 \\ \hline \text{LCD} &= (y-4)(y+4) \end{aligned}$	
Rewrite each rational expression as an equivalent rational expression with the LCD.	$\frac{8y}{(y-4)(y+4)} - \frac{4(y+4)}{(y-4)(y+4)}$
Simplify the numerators.	$\frac{8y}{(y-4)(y+4)} - \frac{4y+16}{(y-4)(y+4)}$
Subtract the rational expressions.	$\frac{8y-4y-16}{(y-4)(y+4)}$
Simplify the numerators.	$\frac{4y-16}{(y-4)(y+4)}$

Factor the numerator to look for common factors.	$\frac{4(y-4)}{(y-4)(y+4)}$
Remove common factors.	$\frac{4\cancel{(y-4)}}{\cancel{(y-4)}(y+4)}$
Simplify.	$\frac{4}{(y+4)}$

Note:
Exercise:

TRY IT :: 7.91

Problem: Subtract: $\frac{2x}{x^2-4} - \frac{1}{x+2}$.

Solution:

$\frac{1}{x-2}$

Note:
Exercise:

TRY IT :: 7.92

Problem: Subtract: $\frac{3}{z+3} - \frac{6z}{z^2-9}$.

Solution:

$\frac{-3}{z-3}$

There are lots of negative signs in the next example. Be extra careful!

Example:
Exercise:

Problem: **Subtract: $\frac{-3n-9}{n^2+n-6} - \frac{n+3}{2-n}$.

Solution:
Solution

	$\frac{-3n-9}{n^2+n-6} - \frac{n+3}{2-n}$
Factor the denominator.	$\frac{-3n-9}{(n-2)(n+3)} - \frac{n+3}{2-n}$
Since $n-2$ and $2-n$ are opposites, we will mutliply the second rational expression by $\frac{-1}{-1}$.	$\frac{-3n-9}{(n-2)(n+3)} - \frac{(-1)(n+3)}{(-1)(2-n)}$
Simplify.	$\frac{-3n-9}{(n-2)(n+3)} + \frac{(n+3)}{(n-2)}$
Do the expressions have a common denominator? No.	
Find the LCD. $\begin{array}{r} n^2+n-6=(n-2)(n+3) \\ n-2=(n-2) \\ \hline \text{LCD}=(n-2)(n+3) \end{array}$	
Rewrite each rational expression as an equivalent rational expression with the LCD.	$\frac{-3n-9}{(n-2)(n+3)} + \frac{(n+3)(n+3)}{(n-2)(n+3)}$
Simplify the numerators.	$\frac{-3n-9}{(n-2)(n+3)} + \frac{n^2+6n+9}{(n-2)(n+3)}$

Simplify the rational expressions.	$\frac{-3n-9+n^2+6n+9}{(n-2)(n+3)}$
Somplify the numerator.	$\frac{n^2+3n}{(n-2)(n+3)}$
Factor the numerator to look for common factors.	$\frac{n\cancel{(n+3)}}{(n-2)\cancel{(n+3)}}$
Simplify.	$\frac{n}{(n-2)}$

Note:
Exercise:

TRY IT :: 7.93

Problem: Subtract: $\frac{3x-1}{x^2-5x-6} - \frac{2}{6-x}$.

Solution:

$\frac{1}{x-6}$

Note:
Exercise:

TRY IT :: 7.94

Problem: Subtract: $\frac{-2y-2}{y^2+2y-8} - \frac{y-1}{2-y}$.

Solution:

$\frac{y+3}{y+4}$

When one expression is not in fraction form, we can write it as a fraction with denominator 1.

Example:
Exercise:

Problem: **Subtract: $\frac{5c+4}{c-2} - 3$.

Solution:
Solution

	$\frac{5c+4}{c-2} - 3$
Write 3 as $\frac{3}{1}$ to have 2 rational expressions.	$\frac{5c+4}{c-2} - \frac{3}{1}$
Do the rational expressions have a common denominator? No.	
Find the LCD of $c-2$ and 1. LCD = $c-2$.	
Rewrite $\frac{3}{1}$ as an equivalent rational expression with the LCD.	$\frac{5c+4}{c-2} - \frac{3(c-2)}{1(c-2)}$
Simplify.	$\frac{5c+4}{c-2} - \frac{3c-6}{(c-2)}$
Subtract the rational expressions.	$\frac{5c+4-(3c-6)}{c-2}$

Simplify.	$\frac{2c+10}{c-2}$
Factor to check for common factors.	$\frac{2(c+5)}{c-2}$
There are no common factors; the rational expression is simplified.	

Note:
Exercise:

TRY IT :: 7.95

Problem: Subtract: $\frac{2x+1}{x-7} - 3$.

Solution:

$\frac{-x+22}{x-7}$

Note:
Exercise:

TRY IT :: 7.96

Problem: Subtract: $\frac{4y+3}{2y-1} - 5$.

Solution:

$\frac{-2(3y-4)}{2y-1}$

Note:

Add or subtract rational expressions.

Determine if the expressions have a common denominator. **Yes**– go to step 2. **No**– Rewrite each rational expression with the LCD. Find the LCD. Rewrite each rational expression as an equivalent rational expression with the LCD.
Add or subtract the rational expressions.
Simplify, if possible.

We follow the same steps as before to find the LCD when we have more than two rational expressions. In the next example we will start by factoring all three denominators to find their LCD.

Example:
Exercise:

Problem: **Simplify: $\frac{2u}{u-1}+\frac{1}{u}-\frac{2u-1}{u^2-u}$.

Solution:
Solution

	$\frac{2u}{u-1}+\frac{1}{u}-\frac{2u-1}{u^2-u}$
Do the rational expressions have a common denominator? No.	
Find the LCD. $\begin{array}{rl} u-1 = & u-1 \\ u = & u \\ u^2-u = & u(u-1) \\ \hline \text{LCD} = & u(u-1) \end{array}$	

Rewrite each rational expression as an equivalent rational expression with the LCD.	$\frac{2u \cdot u}{(u-1)u} + \frac{1 \cdot (u-1)}{u \cdot (u-1)} - \frac{2u-1}{u(u-1)}$
	$\frac{2u^2}{(u-1)u} + \frac{u-1}{u \cdot (u-1)} - \frac{2u-1}{u(u-1)}$
Write as one rational expression.	$\frac{2u^2 + u - 1 - 2u + 1}{u(u-1)}$
Simplify.	$\frac{2u^2 - u}{u(u-1)}$
Factor the numerator, and remove common factors.	$\frac{\cancel{u}(2u-1)}{\cancel{u}(u-1)}$
Simplify.	$\frac{2u-1}{u-1}$

Note:
Exercise:

TRY IT :: 7.97

Problem: Simplify: $\frac{v}{v+1} + \frac{3}{v-1} - \frac{6}{v^2-1}$.

Solution:

$\frac{v+3}{v+1}$

Note:
Exercise:

TRY IT :: 7.98

Problem: Simplify: $\frac{3w}{w+2}+\frac{2}{w+7}-\frac{17w+4}{w^2+9w+14}$.

Solution:

$\frac{3w}{w+7}$

Key Concepts

- **Find the Least Common Denominator of Rational Expressions**

  Factor each expression completely.
  List the factors of each expression. Match factors vertically when possible.
  Bring down the columns.
  Multiply the factors.

- **Add or Subtract Rational Expressions**

Determine if the expressions have a common denominator.	**Yes**– go to step 2.	**No**– Rewrite each rational expression with the LCD.	▪ Find the LCD. ▪ Rewrite each rational expression as an equivalent rational expression with the LCD.

  Add or subtract the rational expressions.
  Simplify, if possible.

Practice Makes Perfect

In the following exercises, find the LCD.

Exercise:

Problem: $\frac{5}{x^2-2x-8}$, $\frac{2x}{x^2-x-12}$

Solution:

$(x-4)(x+2)(x+3)$

Exercise:

Problem: $\frac{8}{y^2+12y+35}, \frac{3y}{y^2+y-42}$

Exercise:

Problem: $\frac{9}{z^2+2z-8}, \frac{4z}{z^2-4}$

Solution:

$(z-2)(z+4)(z+2)$

Exercise:

Problem: $\frac{6}{a^2+14a+45}, \frac{5a}{a^2-81}$

Exercise:

Problem: $\frac{4}{b^2+6b+9}, \frac{2b}{b^2-2b-15}$

Solution:

$(b+3)(b+3)(b-5)$

Exercise:

Problem: $\frac{5}{c^2-4c+4}, \frac{3c}{c^2-10c+16}$

Exercise:

Problem: $\frac{2}{3d^2+14d-5}, \frac{5d}{3d^2-19d+6}$

Solution:

$(3d-1)(d+5)(d-6)$

Exercise:

Problem: $\frac{3}{5m^2-3m-2}, \frac{6m}{5m^2+17m+6}$

In the following exercises, write as equivalent rational expressions with the given LCD.

Exercise:

Problem: $\frac{5}{x^2-2x-8}, \frac{2x}{x^2-x-12}$ LCD $(x-4)(x+2)(x+3)$

Solution:

$\frac{5x+15}{(x-4)(x+2)(x+3)}$, $\frac{2x^2+4x}{(x-4)(x+2)(x+3)}$

Exercise:

Problem: $\frac{8}{y^2+12y+35}, \frac{3y}{y^2+y-42}$ LCD $(y+7)(y+5)(y-6)$

Exercise:

Problem: $\frac{9}{z^2+2z-8}, \frac{4z}{z^2-4}$ LCD $(z-2)(z+4)(z+2)$

Solution:

$\frac{9z+18}{(z-2)(z+4)(z+2)}$, $\frac{4z^2+16}{(z-2)(z+4)(z+2)}$

Exercise:

Problem: $\frac{6}{a^2+14a+45}, \frac{5a}{a^2-81}$ LCD $(a+9)(a+5)(a-9)$

Exercise:

Problem: $\frac{4}{b^2+6b+9}, \frac{2b}{b^2-2b-15}$ LCD $(b+3)(b+3)(b-5)$

Solution:

$\frac{4b-20}{(b+3)(b+3)(b-5)}$,
$\frac{2b^2+6b}{(b+3)(b+3)(b-5)}$

Exercise:

$\frac{5}{c^2-4c+4}$, $\frac{3c}{c^2-10c+10}$

Problem: LCD $(c-2)(c-2)(c-8)$

Exercise:

$\frac{2}{3d^2+14d-5}$, $\frac{5d}{3d^2-19d+6}$

Problem: LCD $(3d-1)(d+5)(d-6)$

Solution:

$\frac{2d-12}{(3d-1)(d+5)(d-6)}$,
$\frac{5d^2+25d}{(3d-1)(d+5)(d-6)}$

Exercise:

$\frac{3}{5m^2-3m-2}$, $\frac{6m}{5m^2+17m+6}$

Problem: LCD $(5m+2)(m-1)(m+3)$

In the following exercises, add.

Exercise:

Problem: $\frac{5}{24}+\frac{11}{36}$

Solution:

$\frac{37}{72}$

Exercise:

Problem: $\frac{7}{30}+\frac{13}{45}$

Exercise:

Problem: $\frac{9}{20}+\frac{11}{30}$

Solution:

$\frac{49}{60}$

Exercise:

Problem: $\frac{8}{27} + \frac{7}{18}$

Exercise:

Problem: $\frac{7}{10x^2y} + \frac{4}{15xy^2}$

Solution:

$\frac{21y+8x}{30x^2y^2}$

Exercise:

Problem: $\frac{1}{12a^3b^2} + \frac{5}{9a^2b^3}$

Exercise:

Problem: $\frac{1}{2m} + \frac{7}{8m^2n}$

Solution:

$\frac{mn+14}{16m^2n}$

Exercise:

Problem: $\frac{5}{6p^2q} + \frac{1}{4p}$

Exercise:

Problem: $\frac{3}{r+4} + \frac{2}{r-5}$

Solution:

$\frac{5r-7}{(r+4)(r-5)}$

Exercise:

Problem: $\frac{4}{s-7}+\frac{5}{s+3}$

Exercise:

Problem: $\frac{8}{t+5}+\frac{6}{t-5}$

Solution:

$\frac{14t-10}{(t+5)(t-5)}$

Exercise:

Problem: $\frac{7}{v+5}+\frac{9}{v-5}$

Exercise:

Problem: $\frac{5}{3w-2}+\frac{2}{w+1}$

Solution:

$\frac{11w+1}{(3w-2)(w+1)}$

Exercise:

Problem: $\frac{4}{2x+5}+\frac{2}{x-1}$

Exercise:

Problem: $\frac{2y}{y+3}+\frac{3}{y-1}$

Solution:

$\frac{2y^2+y+9}{(y+3)(y-1)}$

Exercise:

Problem: $\frac{3z}{z-2}+\frac{1}{z+5}$

Exercise:

Problem: $\frac{5b}{a^2b-2a^2}+\frac{2b}{b^2-4}$

Solution:

$\frac{b(5b+10+2a^2)}{a^2(b-2)(b+2)}$

Exercise:

Problem: $\frac{4}{cd+3c}+\frac{1}{d^2-9}$

Exercise:

Problem: $\frac{2m}{3m-3}+\frac{5m}{m^2+3m-4}$

Solution:

$\frac{2m^2+23m}{3(m-1)(m+4)}$

Exercise:

Problem: $\frac{3}{4n+4}+\frac{6}{n^2-n-2}$

Exercise:

Problem: $\frac{3}{n^2+3n-18}+\frac{4n}{n^2+8n+12}$

Solution:

$\frac{4n^2-9n+6}{(p-3)(n+6)(n+2)}$

Exercise:

Problem: $\frac{6}{q^2-3q-10}+\frac{5q}{q^2-8q+15}$

Exercise:

Problem: $\frac{3r}{r^2+7r+6}+\frac{9}{r^2+4r+3}$

Solution:

$\frac{3(r^2+6r+18)}{(r+1)(r+6)(r+3)}$

Exercise:

Problem: $\frac{2s}{s^2+2s-8} + \frac{4}{s^2+3s-10}$

In the following exercises, subtract.

Exercise:

Problem: $\frac{t}{t-6} - \frac{t-2}{t+6}$

Solution:

$\frac{2(7t-6)}{(t-6)(t+6)}$

Exercise:

Problem: $\frac{v}{v-3} - \frac{v-6}{v+1}$

Exercise:

Problem: $\frac{w+2}{w+4} - \frac{w}{w-2}$

Solution:

$\frac{-4(1+w)}{(w+4)(w-2)}$

Exercise:

Problem: $\frac{x-3}{x+6} - \frac{x}{x+3}$

Exercise:

Problem: $\frac{y-4}{y+1} - \frac{1}{y+7}$

Solution:

$\frac{y^2+2y-29}{(y+1)(y+7)}$

Exercise:

Problem: $\frac{z+8}{z-3} - \frac{z}{z-2}$

Exercise:

Problem: $\frac{5a}{a+3} - \frac{a+2}{a+6}$

Solution:

$\frac{4a^2+25a-6}{(a+3)(a+6)}$

Exercise:

Problem: $\frac{3b}{b-2} - \frac{b-6}{b-8}$

Exercise:

Problem: $\frac{6c}{c^2-25} - \frac{3}{c+5}$

Solution:

$\frac{3}{c-5}$

Exercise:

Problem: $\frac{4d}{d^2-81} - \frac{2}{d+9}$

Exercise:

Problem: $\frac{6}{m+6} - \frac{12m}{m^2-36}$

Solution:

$\frac{-6}{m-6}$

Exercise:

Problem: $\frac{4}{n+4} - \frac{8n}{n^2-16}$

Exercise:

Problem: $\frac{-9p-17}{p^2-4p-21}-\frac{p+1}{7-p}$

Solution:

$\frac{p+2}{p+3}$

Exercise:

Problem: $\frac{7q+8}{q^2-2q-24}-\frac{q+2}{4-q}$

Exercise:

Problem: $\frac{-2r-16}{r^2+6r-16}-\frac{5}{2-r}$

Solution:

$\frac{3}{r-2}$

Exercise:

Problem: $\frac{2t-30}{t^2+6t-27}-\frac{2}{3-t}$

Exercise:

Problem: $\frac{5v-2}{v+3}-4$

Solution:

$\frac{-v-14}{v+3}$

Exercise:

Problem: $\frac{6w+5}{w-1}+2$

Exercise:

Problem: $\frac{2x+7}{10x-1}+3$

Solution:

$\frac{4(8x+1)}{10x-1}$

Exercise:

Problem: $\frac{8y-4}{5y+2} - 6$

In the following exercises, add and subtract.

Exercise:

Problem: $\frac{5a}{a-2} + \frac{9}{a} - \frac{2a+18}{a^2-2a}$

Solution:

$\frac{5a^2+7a-36}{a(a-2)}$

Exercise:

Problem: $\frac{2b}{b-5} + \frac{3}{2b} - \frac{2b-15}{2b^2-10b}$

Exercise:

Problem: $\frac{c}{c+2} + \frac{5}{c-2} - \frac{11c}{c^2-4}$

Solution:

$\frac{c-5}{c+2}$

Exercise:

Problem: $\frac{6d}{d-5} + \frac{1}{d+4} - \frac{7d-5}{d^2-d-20}$

In the following exercises, simplify.

Exercise:

Problem: $\frac{6a}{3ab+b^2} + \frac{3a}{9a^2-b^2}$

Solution:

$\frac{3a(6a-b)}{b(3a+b)(3a-b)}$

Exercise:

Problem: $\frac{2c}{2c+10}+\frac{7c}{c^2+9c+20}$

Exercise:

Problem: $\frac{6d}{d^2-64}-\frac{3}{d-8}$

Solution:

$\frac{3}{d+8}$

Exercise:

Problem: $\frac{5}{n+7}-\frac{10n}{n^2-49}$

Exercise:

Problem: $\frac{4m}{m^2+6m-7}+\frac{2}{m^2+10m+21}$

Solution:

$\frac{2(2m+7m-1)}{(m+7)(m-1)(m+3)}$

Exercise:

Problem: $\frac{3p}{p^2+4p-12}+\frac{1}{p^2+p-30}$

Exercise:

Problem: $\frac{-5n-5}{n^2+n-6}+\frac{n+1}{2-n}$

Solution:

$\frac{n+1}{n+3}$

Exercise:

Problem: $\frac{-4b-24}{b^2+b-30}+\frac{b+7}{5-b}$

Exercise:

Problem: $\frac{7}{15p}+\frac{5}{18pq}$

Solution:

$\frac{42q+25}{90pq}$

Exercise:

Problem: $\frac{3}{20a^2}+\frac{11}{12ab^2}$

Exercise:

Problem: $\frac{4}{x-2}+\frac{3}{x+5}$

Solution:

$\frac{7(x+2)}{(x-2)(x+5)}$

Exercise:

Problem: $\frac{6}{m+4}+\frac{9}{m-8}$

Exercise:

Problem: $\frac{2q+7}{y+4}-2$

Solution:

$\frac{17q+2}{3q-1}$

Exercise:

Problem: $\frac{3y-1}{y+4}-2$

Exercise:

Problem: $\frac{z+2}{z-5}-\frac{z}{z+1}$

Solution:

$\frac{8z+2}{(z-5)(z+1)}$

Exercise:

Problem: $\frac{t}{t-5} - \frac{t-1}{t+5}$

Exercise:

Problem: $\frac{3d}{d+2} + \frac{4}{d} - \frac{d+8}{d^2+2d}$

Solution:

$\frac{3(d+1)}{d+2}$

Exercise:

Problem: $\frac{2q}{q+5} + \frac{3}{q-3} - \frac{13q+15}{q^2+2q-15}$

Everyday Math

Exercise:

Problem:

Decorating cupcakes Victoria can decorate an order of cupcakes for a wedding in t hours, so in 1 hour she can decorate $\frac{1}{t}$ of the cupcakes. It would take her sister 3 hours longer to decorate the same order of cupcakes, so in 1 hour she can decorate $\frac{1}{t+3}$ of the cupcakes.

ⓐ Find the fraction of the decorating job that Victoria and her sister, working together, would complete in one hour by adding the rational expressions $\frac{1}{t} + \frac{1}{t+3}$.
ⓑ Evaluate your answer to part (a) when $t = 5$.

Solution:

$\frac{2t+3}{t(t+3)}$
$\frac{13}{40}$

Exercise:

Problem:

Kayaking When Trina kayaks upriver, it takes her $\frac{5}{3-c}$ hours to go 5 miles, where c is the speed of the river current. It takes her $\frac{5}{3+c}$ hours to kayak 5 miles down the river.

ⓐ Find an expression for the number of hours it would take Trina to kayak 5 miles up the river and then return by adding $\frac{5}{3-c}+\frac{5}{3+c}$.
ⓑ Evaluate your answer to part (a) when $c=1$ to find the number of hours it would take Trina if the speed of the river current is 1 mile per hour.

Writing Exercises

Exercise:

Problem: Felipe thinks $\frac{1}{x}+\frac{1}{y}$ is $\frac{2}{x+y}$.

ⓐ Choose numerical values for x and y and evaluate $\frac{1}{x}+\frac{1}{y}$.
ⓑ Evaluate $\frac{2}{x+y}$ for the same values of x and y you used in part (a).
ⓒ Explain why Felipe is wrong.
ⓓ Find the correct expression for $\frac{1}{x}+\frac{1}{y}$.

Solution:

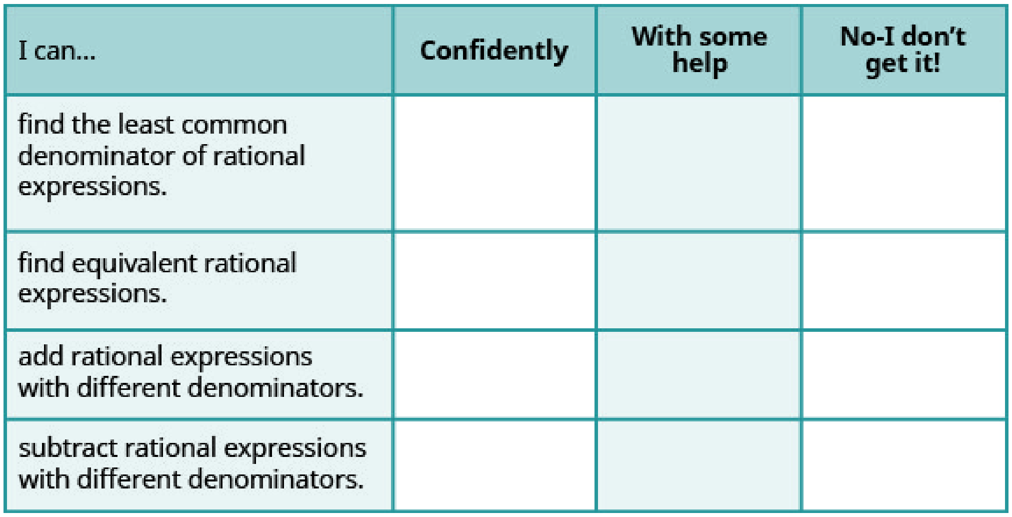
Answers may vary.

Exercise:

Problem: Simplify the expression $\frac{4}{n^2+6n+9}-\frac{1}{n^2-9}$ and explain all your steps.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
find the least common denominator of rational expressions.			
find equivalent rational expressions.			
add rational expressions with different denominators.			
subtract rational expressions with different denominators.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Review & Practice Test
By the end of this section, you will be prepared to take the Chapter 7 Assessment.

Chapter 7 Review Exercises

Simplify Rational Expressions

Determine the Values for Which a Rational Expression is Undefined

In the following exercises, determine the values for which the rational expression is undefined.

Exercise:

Problem: $\frac{2a+1}{3a-2}$

Solution:

$a \neq \frac{2}{3}$

Exercise:

Problem: $\frac{b-3}{b^2-16}$

Exercise:

Problem: $\frac{3xy^2}{5y}$

Solution:

$y \neq 0$

Exercise:

Problem: $\frac{u-3}{u^2-u-30}$

Evaluate Rational Expressions

In the following exercises, evaluate the rational expressions for the given values.

Exercise:

Problem: $\frac{4p-1}{p^2+5}$ when $p=-1$

Solution:

$-\frac{5}{6}$

Exercise:

Problem: $\frac{q^2-5}{q+3}$ when $\text{q}=7$

Exercise:

Problem: $\frac{y^2-8}{y^2-y-2}$ when $y=1$

Solution:

$\frac{7}{2}$

Exercise:

Problem: $\frac{z^2+2}{4z-z^2}$ when $\text{z}=3$

Simplify Rational Expressions

In the following exercises, simplify.

Exercise:

Problem: $\frac{10}{24}$

Solution:

$\frac{5}{12}$

Exercise:

Problem: $\frac{8m^4}{16mn^3}$

Exercise:

Problem: $\frac{14a-14}{a-1}$

Solution:

14

Exercise:

Problem: $\frac{b^2+7b+12}{b^2+8b+16}$

Simplify Rational Expressions with Opposite Factors

In the following exercises, simplify.

Exercise:

Problem: $\frac{c^2-c-2}{4-c^2}$

Solution:

$-\frac{c+1}{c+2}$

Exercise:

Problem: $\frac{d-16}{16-d}$

Exercise:

Problem: $\frac{7v-35}{25-v^2}$

Solution:

$-\frac{7}{5+v}$

Exercise:

Problem: $\frac{w^2-3w-28}{49-w^2}$

Multiply and Divide Rational Expressions

Multiply Rational Expressions

In the following exercises, multiply.

Exercise:

Problem: $\frac{3}{8} \cdot \frac{2}{15}$

Solution:

$\frac{1}{20}$

Exercise:

Problem: $\frac{2xy^2}{8y^3} \cdot \frac{16y}{24x}$

Exercise:

Problem: $\frac{3a^2+21a}{a^2+6a-7} \cdot \frac{a-1}{ab}$

Solution:

$\frac{3}{b}$

Exercise:

Problem: $\frac{5z^2}{5z^2+40z+35} \cdot \frac{z^2-1}{3z}$

Divide Rational Expressions

In the following exercises, divide.

Exercise:

Problem: $\frac{t^2-4t-12}{t^2+8t+12} \div \frac{t^2-36}{6t}$

Solution:

$\frac{6t}{(t+6)^2}$

Exercise:

Problem: $\frac{r^2-16}{4} \div \frac{r^3-64}{2r^2-8r+32}$

Exercise:

Problem: $\frac{11+w}{w-9} \div \frac{121-w^2}{9-w}$

Solution:

$\frac{1}{11+w}$

Exercise:

Problem: $\frac{3y^2-12y-63}{4y+3} \div (6y^2 - 42y)$

Exercise:

Problem: $\frac{\frac{c^2-64}{3c^2+26c+16}}{\frac{c^2-4c-32}{15c+10}}$

Solution:

$\frac{5}{c+4}$

Exercise:

Problem: $\frac{8m^2-8m}{m-4} \cdot \frac{m^2+2m-24}{m^2+7m+10} \div \frac{2m^2-6m}{m+5}$

Add and Subtract Rational Expressions with a Common Denominator

Add Rational Expressions with a Common Denominator

In the following exercises, add.

Exercise:

Problem: $\frac{3}{5} + \frac{2}{5}$

Solution:

1

Exercise:

Problem: $\frac{4a^2}{2a-1} - \frac{1}{2a-1}$

Exercise:

Problem: $\frac{p^2+10p}{p+5} + \frac{25}{p+5}$

Solution:

$p+5$

Exercise:

Problem: $\frac{3x}{x-1}+\frac{2}{x-1}$

Subtract Rational Expressions with a Common Denominator

In the following exercises, subtract.

Exercise:

Problem: $\frac{d^2}{d+4}-\frac{3d+28}{d+4}$

Solution:

$d–7$

Exercise:

Problem: $\frac{z^2}{z+10}-\frac{100}{z+10}$

Exercise:

Problem: $\frac{4q^2-q+3}{q^2+6q+5}-\frac{3q^2+q+6}{q^2+6q+5}$

Solution:

$\frac{q-3}{q+5}$

Exercise:

Problem: $\frac{5t+4t+3}{t^2-25}-\frac{4t^2-8t-32}{t^2-25}$

Add and Subtract Rational Expressions whose Denominators are Opposites

In the following exercises, add and subtract.

Exercise:

Problem: $\frac{18w}{6w-1}+\frac{3w-2}{1-6w}$

Solution:

$\frac{15w+2}{6w-1}$

Exercise:

Problem: $\frac{a^2+3a}{a^2-4}-\frac{3a-8}{4-a^2}$

Exercise:

Problem: $\frac{2b^2+3b-15}{b^2-49}-\frac{b^2+16b-1}{49-b^2}$

Solution:

$\frac{3b-2}{b+7}$

Exercise:

Problem: $\frac{8y^2-10y+7}{2y-5}+\frac{2y^2+7y+2}{5-2y}$

Add and Subtract Rational Expressions With Unlike Denominators

Find the Least Common Denominator of Rational Expressions

In the following exercises, find the LCD.

Exercise:

Problem: $\frac{4}{m^2-3m-10},\frac{2m}{m^2-m-20}$

Solution:

$(m+2)(m-5)(m+4)$

Exercise:

Problem: $\frac{6}{n^2-4}$, $\frac{2n}{n^2-4n+4}$

Exercise:

Problem: $\frac{5}{3p^2+17p-6}$, $\frac{2m}{3p^2-23p-8}$

Solution:

$(3p+1)(p+6)(p+8)$

Find Equivalent Rational Expressions

In the following exercises, rewrite as equivalent rational expressions with the given denominator.

Exercise:

Problem:

Rewrite as equivalent rational expressions with denominator $(m+2)(m-5)(m+4)$:

Equation:

$$\frac{4}{m^2-3m-10}, \frac{2m}{m^2-m-20}.$$

Exercise:

Problem:

Rewrite as equivalent rational expressions with denominator $(n-2)(n-2)(n+2)$:

Equation:

$$\frac{6}{n^2-4n+4}, \frac{2n}{n^2-4}.$$

Solution:

$\frac{6n+12}{(n-2)(n-2)(n+2)}$, $\frac{2n^2-4n}{(n-2)(n-2)(n+2)}$

Exercise:

Problem:

Rewrite as equivalent rational expressions with denominator $(3p+1)(p+6)(p+8)$:

Equation:

$$\frac{5}{3p^2+19p+6}, \frac{7p}{3p^2+25p+8}$$

Add Rational Expressions with Different Denominators

In the following exercises, add.

Exercise:

Problem: $\frac{2}{3}+\frac{3}{5}$

Solution:

$\frac{19}{15}$

Exercise:

Problem: $\frac{7}{5a}+\frac{3}{2b}$

Exercise:

Problem: $\frac{2}{c-2}+\frac{9}{c+3}$

Solution:

$\frac{11c-12}{(c-2)(c+3)}$

Exercise:

Problem: $\frac{3d}{d^2-9}+\frac{5}{d^2+6d+9}$

Exercise:

Problem: $\frac{2x}{x^2+10x+24}+\frac{3x}{x^2+8x+16}$

Solution:

$\frac{5x^2+26x}{(x+4)(x+4)(x+6)}$

Exercise:

Problem: $\frac{5q}{p^2q-p^2}+\frac{4q}{q^2-1}$

Subtract Rational Expressions with Different Denominators

In the following exercises, subtract and add.

Exercise:

Problem: $\frac{3v}{v+2}-\frac{v+2}{v+8}$

Solution:

$\frac{2(v^2+10v-2)}{(v+2)(v+8)}$

Exercise:

Problem: $\frac{-3w-15}{w^2+w-20} - \frac{w+2}{4-w}$

Exercise:

Problem: $\frac{7m+3}{m+2} - 5$

Solution:

$\frac{2m-7}{m+2}$

Exercise:

Problem: $\frac{n}{n+3} + \frac{2}{n-3} - \frac{n-9}{n^2-9}$

Exercise:

Problem: $\frac{8d}{d^2-64} - \frac{4}{d+8}$

Solution:

$\frac{4}{d-8}$

Exercise:

Problem: $\frac{5}{12x^2y} + \frac{7}{20xy^3}$

Practice Test

In the following exercises, simplify.

Exercise:

Problem: $\frac{3a^2b}{6ab^2}$

Solution:

$\frac{a}{2b}$

Exercise:

Problem: $\frac{5b-25}{b^2-25}$

In the following exercises, perform the indicated operation and simplify.

Exercise:

Problem: $\frac{4x}{x+2} \cdot \frac{x^2+5x+6}{12x^2}$

Solution:

$\frac{x+3}{3x}$

Exercise:

Problem: $\frac{5y}{4y-8} \cdot \frac{y^2-4}{10}$

Exercise:

Problem: $\frac{4}{pq} + \frac{5}{p}$

Solution:

$\frac{4+5q}{pq}$

Introduction
class="introduction"

Square roots are used to determine the time it would take for a stone falling from the edge of this cliff to hit the land below.

Suppose a stone falls from the edge of a cliff. The number of feet the stone has dropped after seconds can be found by multiplying 16 times the square of . But to calculate the number of seconds it would take the stone to hit the land below, we need to use a square root. In this chapter, we will introduce and apply the properties of square roots.

Simplify and Use Square Roots
By the end of this section, you will be able to:

- Simplify expressions with square roots
- Estimate square roots
- Approximate square roots
- Simplify variable expressions with square roots

Note:
Before you get started, take this readiness quiz.

1. Simplify: ⓐ 9^2 ⓑ $(-9)^2$ ⓒ -9^2.
   If you missed this problem, review [link].
2. Round 3.846 to the nearest hundredth.
   If you missed this problem, review [link].
3. For each number, identify whether it is a real number or not a real number:
   ⓐ $-\sqrt{100}$ ⓑ $\sqrt{-100}$.
   If you missed this problem, review [link].

Simplify Expressions with Square Roots

Remember that when a number n is multiplied by itself, we write n^2 and read it “n squared.” For example, 15^2 reads as “15 squared,” and 225 is called the square of 15, since $15^2 = 225$.

Note:
Square of a Number
If $n^2 = m$, then m is the square of n.

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because 225 is the square of 15, we can also say that 15 is a square root of 225. A number whose square is m is called a *square root* of m.

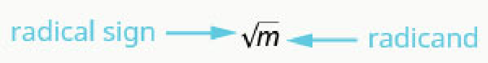
Note:
Square Root of a Number
If $n^2 = m$, then n is a square root of m.

Notice $(-15)^2 = 225$ also, so -15 is also a square root of 225. Therefore, both 15 and -15 are square roots of 225.

So, every positive number has two square roots—one positive and one negative. What if we only wanted the positive square root of a positive number? The *radical sign*, $\sqrt{m}$, denotes the positive square root. The positive square root is also called the principal square root.

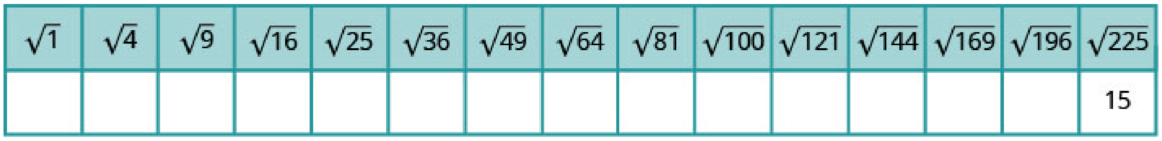
We also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

Note:
Square Root Notation



$\sqrt{m}$ is read as "the square root of m."
If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.
The square root of m, $\sqrt{m}$, is the positive number whose square is m.

Since 15 is the positive square root of 225, we write $\sqrt{225} = 15$. Fill in [link] to make a table of square roots you can refer to as you work this chapter.



$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
														15

We know that every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{225} = 15$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{225} = -15$.

Example:
Exercise:

Problem: **Simplify: ⓐ $\sqrt{36}$ ⓑ $\sqrt{196}$ ⓒ $-\sqrt{81}$ ⓓ $-\sqrt{289}$.

Solution:
Solution

ⓐ	$\sqrt{36}$
Since $6^2 = 36$	6
ⓑ	$\sqrt{196}$
Since $14^2 = 196$	14
ⓒ	$-\sqrt{81}$
The negative is in front of the radical sign.	-9
ⓓ	$-\sqrt{289}$
The negative is in front of the radical sign.	-17

Note:
Exercise:

TRY IT : : 8.1

Problem: Simplify: ⓐ $-\sqrt{49}$ ⓑ $\sqrt{225}$.

Solution:

ⓐ -7 ⓑ 15

Note:
Exercise:

TRY IT : : 8.2

Problem: Simplify: ⓐ $\sqrt{64}$ ⓑ $-\sqrt{121}$.

Solution:

ⓐ 8 ⓑ -11

Example:
Exercise:

Problem: **Simplify: ⓐ $\sqrt{-169}$ ⓑ $-\sqrt{64}$.

Solution:

Solution

ⓐ

$\sqrt{-169}$

There is no real number whose square is -169.	$\sqrt{-169}$ is not a real number.

ⓑ

$-\sqrt{64}$

The negative is in front of the radical.	-8

Note:
Exercise:

TRY IT : : 8.3

Problem: Simplify: ⓐ $\sqrt{-196}$ ⓑ $-\sqrt{81}$.

Solution:

ⓐ not a real number ⓑ -9

Note:
Exercise:

TRY IT : : 8.4

Problem: Simplify: ⓐ $-\sqrt{49}$ ⓑ $\sqrt{-121}$.

Solution:

ⓐ -7 ⓑ not a real number

When using the order of operations to simplify an expression that has square roots, we treat the radical as a grouping symbol.

Example:
Exercise:

Problem: **Simplify: ⓐ $\sqrt{25}+\sqrt{144}$ ⓑ $\sqrt{25+144}$.

Solution:
Solution

ⓐ

	$\sqrt{25}+\sqrt{144}$
Use the order of operations.	$5+12$
Simplify.	17

ⓑ

	$\sqrt{25+144}$
Simplify under the radical sign.	$\sqrt{169}$
Simplify.	13

Notice the different answers in parts ⓐ and ⓑ!

Note:
Exercise:

TRY IT : : 8.5
Problem: Simplify: ⓐ $\sqrt{9}+\sqrt{16}$ ⓑ $\sqrt{9+16}$.

Solution:

ⓐ 7 ⓑ 5

Note:
Exercise:

TRY IT : : 8.6
Problem: Simplify: ⓐ $\sqrt{64+225}$ ⓑ $\sqrt{64}+\sqrt{225}$.

Solution:

ⓐ 17 ⓑ 23

Estimate Square Roots

So far we have only considered square roots of perfect square numbers. The square roots of other numbers are not whole numbers. Look at [link] below.

Number	**Square Root**
4	$\sqrt{4}$ = 2
5	$\sqrt{5}$
6	$\sqrt{6}$
7	$\sqrt{7}$
8	$\sqrt{8}$
9	$\sqrt{9}$ = 3

The square roots of numbers between 4 and 9 must be between the two consecutive whole numbers 2 and 3, and they are not whole numbers. Based on the pattern in the table above, we could say that $\sqrt{5}$ must be between 2 and 3. Using inequality symbols, we write:

Equation:

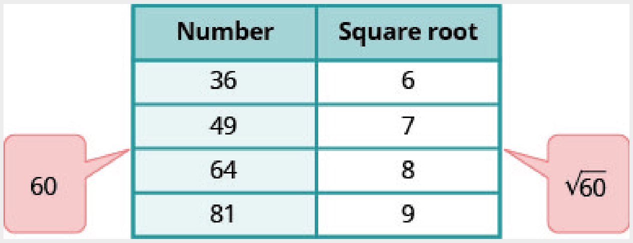
$$2 < \sqrt{5} < 3$$

Example:
Exercise:

Problem: **Estimate $\sqrt{60}$ between two consecutive whole numbers.

Solution:
Solution

Think of the perfect square numbers closest to 60. Make a small table of these perfect squares and their squares roots.



Number	Square root
36	6
49	7
64	8
81	9

Locate 60 between two consecutive perfect squares.	$49 < 60 < 64$
$\sqrt{60}$ is between their square roots.	$7 < \sqrt{60} < 8$

Note:
Exercise:

TRY IT : : 8.7

Problem: Estimate the square root $\sqrt{38}$ between two consecutive whole numbers.

Solution:

$6 < \sqrt{38} < 7$

Note:
Exercise:

TRY IT : : 8.8

Problem: Estimate the square root $\sqrt{84}$ between two consecutive whole numbers.

Solution:

$9 < \sqrt{84} < 10$

Approximate Square Roots

There are mathematical methods to approximate square roots, but nowadays most people use a calculator to find them. Find the $\sqrt{x}$ key on your calculator. You will use this key to approximate square roots.

When you use your calculator to find the square root of a number that is not a perfect square, the answer that you see is not the exact square root. It is an approximation, accurate to the number of digits shown on your calculator's display. The symbol for an approximation is $\approx$ and it is read 'approximately.'

Suppose your calculator has a 10-digit display. You would see that
Equation:

$$\sqrt{5} \approx 2.236067978$$

If we wanted to round $\sqrt{5}$ to two decimal places, we would say
Equation:

$$\sqrt{5} \approx 2.24$$

How do we know these values are approximations and not the exact values? Look at what happens when we square them:
Equation:

$$\begin{aligned} (2.236067978)^2 &= 5.000000002 \\ (2.24)^2 &= 5.0176 \end{aligned}$$

Their squares are close to 5, but are not exactly equal to 5.

Using the square root key on a calculator and then rounding to two decimal places, we can find:
Equation:

$$\begin{aligned}\sqrt{4} &= 2\\ \sqrt{5} &\approx 2.24\\ \sqrt{6} &\approx 2.45\\ \sqrt{7} &\approx 2.65\\ \sqrt{8} &\approx 2.83\\ \sqrt{9} &= 3\end{aligned}$$

Example:
Exercise:

Problem: **Round $\sqrt{17}$ to two decimal places.

Solution:
Solution

	$\sqrt{17}$
Use the calculator square root key.	$4.123105626...$
Round to two decimal places.	4.12
	$\sqrt{17} \approx 4.12$

Note:
Exercise:

TRY IT : : 8.9
Problem: Round $\sqrt{11}$ to two decimal places.

Solution:

≈ 3.32

Note:
Exercise:

TRY IT : : 8.10

Problem: Round $\sqrt{13}$ to two decimal places.

Solution:

≈ 3.61

Simplify Variable Expressions with Square Roots

What if we have to find a square root of an expression with a variable? Consider $\sqrt{9x^2}$. Can you think of an expression whose square is $9x^2$?

Equation:

$$\begin{aligned} (?)^2 &= 9x^2 \\ (3x)^2 &= 9x^2, \qquad \text{so } \sqrt{9x^2} = 3x \end{aligned}$$

When we use the radical sign to take the square root of a variable expression, we should specify that $x \geq 0$ to make sure we get the *principal square root*.

However, in this chapter we will assume that each variable in a square-root expression represents a non-negative number and so we will not write $x \geq 0$ next to every radical.

What about square roots of higher powers of variables? Think about the Power Property of Exponents we used in Chapter 6.

Equation:

$$(a^m)^n = a^{m \cdot n}$$

If we square a^m, the exponent will become $2m$.

Equation:

$$(a^m)^2 = a^{2m}$$

How does this help us take square roots? Let’s look at a few:

Equation:

$$\begin{aligned} \sqrt{25u^8} &= 5u^4 \quad \text{because } \left(5u^4\right)^2 = 25u^8 \\ \sqrt{16r^{20}} &= 4r^{10} \quad \text{because } \left(4r^{10}\right)^2 = 16r^{20} \\ \sqrt{196q^{36}} &= 14q^{18} \quad \text{because } \left(14q^{18}\right)^2 = 196q^{36} \end{aligned}$$

Example:
Exercise:

Problem: **Simplify: ⓐ $\sqrt{x^6}$ ⓑ $\sqrt{y^{16}}$.

Solution:
Solution

ⓐ

	$\sqrt{x^6}$
Since $\left(x^3\right)^2 = x^6$.	x^3

ⓑ

	$\sqrt{y^{16}}$
Since $\left(y^8\right)^2 = y^{16}$.	y^8

Note:
Exercise:

TRY IT : : 8.11

Problem: Simplify: ⓐ $\sqrt{y^8}$ ⓑ $\sqrt{z^{12}}$.

Solution:

ⓐ y^4 ⓑ z^6

Note:
Exercise:

TRY IT : : 8.12

Problem: Simplify: ⓐ $\sqrt{m^4}$ ⓑ $\sqrt{b^{10}}$.

Solution:

ⓐ m^2 ⓑ b^5

Example:
Exercise:

Problem: **Simplify: $\sqrt{16n^2}$.

Solution:
Solution

	$\sqrt{16n^2}$
Since $(4n)^2 = 16n^2$.	$4n$

Note:
Exercise:

TRY IT : : 8.13

Problem: Simplify: $\sqrt{64x^2}$.

Solution:

$8x$

Note:
Exercise:

TRY IT : : 8.14

Problem: Simplify: $\sqrt{169y^2}$.

Solution:

$13y$

Example:
Exercise:

Problem: **Simplify: $-\sqrt{81c^2}$.

Solution:
Solution

	$-\sqrt{81c^2}$
Since $(9c)^2 = 81c^2$.	$-9c$

Note:
Exercise:

TRY IT : : 8.15
Problem: Simplify: $-\sqrt{121y^2}$.

Solution:

$-11y$

Note:
Exercise:

TRY IT : : 8.16
Problem: Simplify: $-\sqrt{100p^2}$.

Solution:

$-10p$

Example:
Exercise:

Problem: **Simplify: $\sqrt{36x^2y^2}$.

Solution:
Solution

	$\sqrt{36x^2y^2}$
Since $(6xy)^2 = 36x^2y^2$.	$6xy$

Note:
Exercise:

TRY IT : : 8.17

Problem: Simplify: $\sqrt{100a^2b^2}$.

Solution:

$10ab$

Note:
Exercise:

TRY IT : : 8.18

Problem: Simplify: $\sqrt{225m^2n^2}$.

Solution:

$15mn$

Example:
Exercise:

Problem: **Simplify: $\sqrt{64p^{64}}$.

Solution:
Solution

	$\sqrt{64p^{64}}$
Since $\left(8p^{32}\right)^2 = 64p^{64}$.	$8p^{32}$

Note:
Exercise:

TRY IT : : 8.19

Problem: Simplify: $\sqrt{49x^{30}}$.

Solution:

$7x^{15}$

Note:
Exercise:

TRY IT : : 8.20

Problem: Simplify: $\sqrt{81w^{36}}$.

Solution:

$9w^{18}$

Example:
Exercise:

Problem: **Simplify: $\sqrt{121a^6b^8}$

Solution:
Solution

	$\sqrt{121a^6b^8}$
Since $\left(11a^3b^4\right)^2 = 121a^6b^8$.	$11a^3b^4$

Note:
Exercise:

TRY IT : : 8.21

Problem: Simplify: $\sqrt{169x^{10}y^{14}}$.

Solution:

$13x^5y^7$

Note:
Exercise:

TRY IT : : 8.22

Problem: Simplify: $\sqrt{144p^{12}q^{20}}$.

Solution:

$12p^6q^{10}$

Note:
Access this online resource for additional instruction and practice with square roots.

- Square Roots

Key Concepts

- Note that the square root of a negative number is not a real number.
- Every positive number has two square roots, one positive and one negative. The positive square root of a positive number is the principal square root.
- We can estimate square roots using nearby perfect squares.
- We can approximate square roots using a calculator.
- When we use the radical sign to take the square root of a variable expression, we should specify that $x \geq 0$ to make sure we get the principal square root.

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{36}$

Solution:

6

Exercise:

Problem: $\sqrt{4}$

Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{169}$

Exercise:

Problem: $\sqrt{9}$

Solution:

3

Exercise:

Problem: $\sqrt{16}$

Exercise:

Problem: $\sqrt{100}$

Solution:

10

Exercise:

Problem: $\sqrt{144}$

Exercise:

Problem: $-\sqrt{4}$

Solution:

-2

Exercise:

Problem: $-\sqrt{100}$

Exercise:

Problem: $-\sqrt{1}$

Solution:

-1

Exercise:

Problem: $-\sqrt{121}$

Exercise:

Problem: $\sqrt{-121}$

Solution:

not a real number

Exercise:

Problem: $\sqrt{-36}$

Exercise:

Problem: $\sqrt{-9}$

Solution:

not a real number

Exercise:

Problem: $\sqrt{-49}$

Exercise:

Problem: $\sqrt{9+16}$

Solution:

5

Exercise:

Problem: $\sqrt{25+144}$

Exercise:

Problem: $\sqrt{9}+\sqrt{16}$

Solution:

7

Exercise:

Problem: $\sqrt{25}+\sqrt{144}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

Exercise:

Problem: $\sqrt{70}$

Solution:

$8<\sqrt{70}<9$

Exercise:

Problem: $\sqrt{55}$

Exercise:

Problem: $\sqrt{200}$

Solution:

$14 < \sqrt{200} < 15$

Exercise:

Problem: $\sqrt{172}$

Approximate Square Roots

In the following exercises, approximate each square root and round to two decimal places.

Exercise:

Problem: $\sqrt{19}$

Solution:

4.36

Exercise:

Problem: $\sqrt{21}$

Exercise:

Problem: $\sqrt{53}$

Solution:

7.28

Exercise:

Problem: $\sqrt{47}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{y^2}$

Solution:

y

Exercise:

Problem: $\sqrt{b^2}$

Exercise:

Problem: $\sqrt{a^{14}}$

Solution:

a^7

Exercise:

Problem: $\sqrt{w^{24}}$

Exercise:

Problem: $\sqrt{49x^2}$

Solution:

$7x$

Exercise:

Problem: $\sqrt{100y^2}$

Exercise:

Problem: $\sqrt{121m^{20}}$

Solution:

$11m^{10}$

Exercise:

Problem: $\sqrt{25h^{44}}$

Exercise:

Problem: $\sqrt{81x^{36}}$

Solution:

$9x^{18}$

Exercise:

Problem: $\sqrt{144z^{84}}$

Exercise:

Problem: $-\sqrt{81x^{18}}$

Solution:

$-9x^{9}$

Exercise:

Problem: $-\sqrt{100m^{32}}$

Exercise:

Problem: $-\sqrt{64a^{2}}$

Solution:

$-8a$

Exercise:

Problem: $-\sqrt{25x^{2}}$

Exercise:

Problem: $\sqrt{144x^{2}y^{2}}$

Solution:

$12xy$

Exercise:

Problem: $\sqrt{196a^{2}b^{2}}$

Exercise:

Problem: $\sqrt{169w^8y^{10}}$

Solution:

$13w^4y^5$

Exercise:

Problem: $\sqrt{81p^{24}q^6}$

Exercise:

Problem: $\sqrt{9c^8d^{12}}$

Solution:

$3c^4d^6$

Exercise:

Problem: $\sqrt{36r^6s^{20}}$

Everyday Math

Exercise:

Problem:

Decorating Denise wants to have a square accent of designer tiles in her new shower. She can afford to buy 625 square centimeters of the designer tiles. How long can a side of the accent be?

Solution:

25 centimeters

Exercise:

Problem:

Decorating Morris wants to have a square mosaic inlaid in his new patio. His budget allows for 2025 square inch tiles. How long can a side of the mosaic be?

Writing Exercises

Exercise:

Problem: Why is there no real number equal to $\sqrt{-64}$?

Solution:

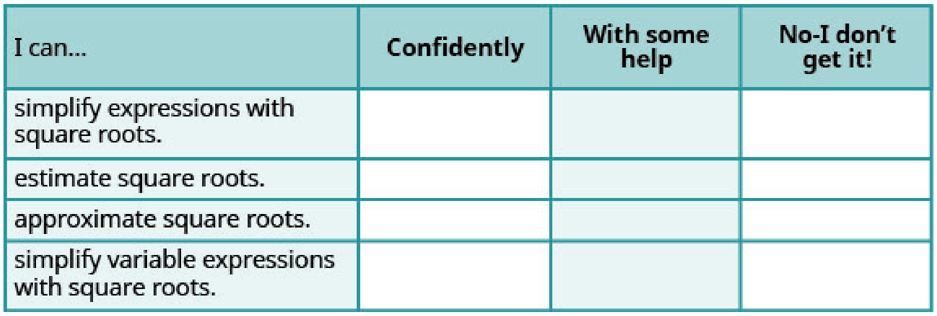
Answers will vary.

Exercise:

Problem: What is the difference between 9^2 and $\sqrt{9}$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
simplify expressions with square roots.			
estimate square roots.			
approximate square roots.			
simplify variable expressions with square roots.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

square of a number

- If $n^2 = m$, then m is the square of n

square root of a number

- If $n^2 = m$, then n is a square root of m

square root notation

- If $m = n^2$, then $\sqrt{m} = n$. We read $\sqrt{m}$ as ‘the square root of m.’

Simplify Square Root

By the end of this section, you will be able to:

- Use the Product Property to simplify square roots
- Use the Quotient Property to simplify square roots

Note:
Before you get started take this readiness quiz.

1. Simplify: $\frac{80}{176}$.
   If you missed this problem, review [link].
2. Simplify: $\frac{n^9}{n^3}$.
   If you missed this problem, review [link].
3. Simplify: $\frac{q^4}{q^{12}}$.
   If you missed this problem, review [link].

In the last section, we estimated the square root of a number between two consecutive whole numbers. We can say that $\sqrt{50}$ is between 7 and 8. This is fairly easy to do when the numbers are small enough that we can use [link].

But what if we want to estimate $\sqrt{500}$? If we simplify the square root first, we'll be able to estimate it easily. There are other reasons, too, to simplify square roots as you'll see later in this chapter.

A square root is considered *simplified* if its radicand contains no perfect square factors.

Note:
Simplified Square Root
$\sqrt{a}$ is considered simplified if a has no perfect square factors.

So $\sqrt{31}$ is simplified. But $\sqrt{32}$ is not simplified, because 16 is a perfect square factor of 32.

Use the Product Property to Simplify Square Roots

The properties we will use to simplify expressions with square roots are similar to the properties of exponents. We know that $(ab)^m = a^m b^m$. The corresponding property of square roots says that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

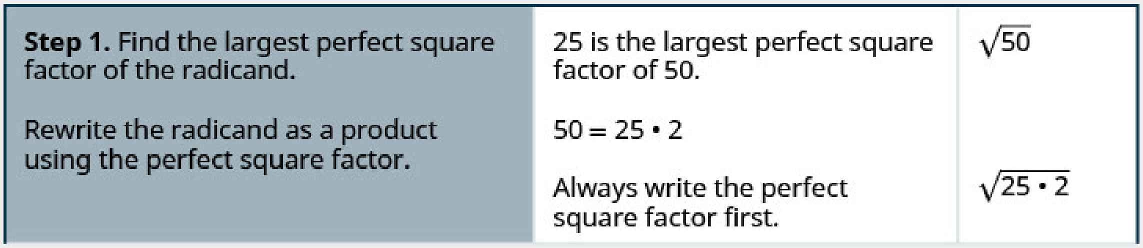
Note:
Product Property of Square Roots
If a, b are non-negative real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

We use the Product Property of Square Roots to remove all perfect square factors from a radical. We will show how to do this in [link].

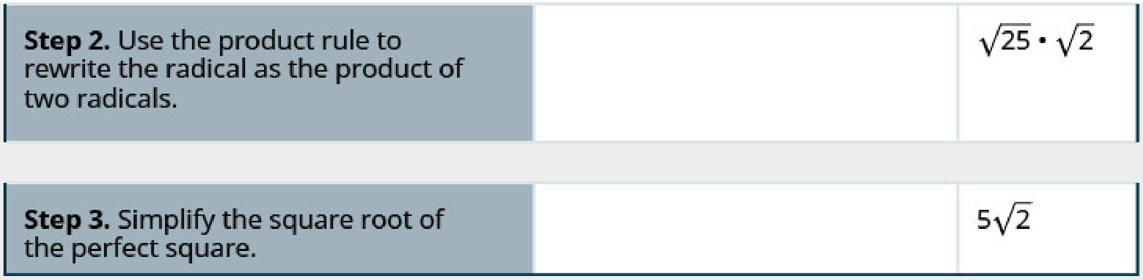
Example:
How To Use the Product Property to Simplify a Square Root
Exercise:

Problem: **Simplify: $\sqrt{50}$.

Solution:
Solution



Step 1. Find the largest perfect square factor of the radicand.	25 is the largest perfect square factor of 50.	$\sqrt{50}$
Rewrite the radicand as a product using the perfect square factor.	50 = 25 • 2 Always write the perfect square factor first.	$\sqrt{25 \cdot 2}$



Step 2. Use the product rule to rewrite the radical as the product of two radicals.		$\sqrt{25} \cdot \sqrt{2}$
Step 3. Simplify the square root of the perfect square.		$5\sqrt{2}$

Note:
Exercise:

TRY IT : : 8.23

Problem: Simplify: $\sqrt{48}$.

Solution:

$4\sqrt{3}$

Note:
Exercise:

TRY IT : : 8.24

Problem: Simplify: $\sqrt{45}$.

Solution:

$3\sqrt{5}$

Notice in the previous example that the simplified form of $\sqrt{50}$ is $5\sqrt{2}$, which is the product of an integer and a square root. We always write the integer in front of the square root.

Note:
Simplify a square root using the product property.

Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect-square factor.
Use the product rule to rewrite the radical as the product of two radicals.
Simplify the square root of the perfect square.

Example:
Exercise:

Problem: **Simplify: $\sqrt{500}$.

Solution:
Solution

	$\sqrt{500}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{100 \cdot 5}$
Rewrite the radical as the product of two radicals.	$\sqrt{100} \cdot \sqrt{5}$
Simplify.	$10\sqrt{5}$

Note:
Exercise:

TRY IT : : 8.25
Problem: Simplify: $\sqrt{288}$.

Solution:

$12\sqrt{2}$

Note:
Exercise:

TRY IT : : 8.26
Problem: Simplify: $\sqrt{432}$.

Solution:

$12\sqrt{3}$

We could use the simplified form $10\sqrt{5}$ to estimate $\sqrt{500}$. We know 5 is between 2 and 3, and $\sqrt{500}$ is $10\sqrt{5}$. So $\sqrt{500}$ is between 20 and 30.

The next example is much like the previous examples, but with variables.

Example:
Exercise:

Problem: **Simplify: $\sqrt{x^3}$.

Solution:
Solution

	$\sqrt{x^3}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{x^2 \cdot x}$
Rewrite the radical as the product of two radicals.	$\sqrt{x^2} \cdot \sqrt{x}$
Simplify.	$x\sqrt{x}$

Note:
Exercise:

TRY IT : : 8.27

Problem: Simplify: $\sqrt{b^5}$.

Solution:

$b^2\sqrt{b}$

Note:
Exercise:

TRY IT : : 8.28

Problem: Simplify: $\sqrt{p^9}$.

Solution:

$p^4\sqrt{p}$

We follow the same procedure when there is a coefficient in the radical, too.

Example:
Exercise:

Problem: **Simplify: $\sqrt{25y^5}$.

Solution:
Solution

	$\sqrt{25y^5}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{25y^4 \cdot y}$
Rewrite the radical as the product of two radicals.	$\sqrt{25y^4} \cdot \sqrt{y}$
Simplify.	$5y^2\sqrt{y}$

Note:
Exercise:

TRY IT : : 8.29

Problem: Simplify: $\sqrt{16x^7}$.

Solution:

$4x^3\sqrt{x}$

Note:
Exercise:

TRY IT : : 8.30

Problem: Simplify: $\sqrt{49v^9}$.

Solution:

$7v^4\sqrt{v}$

In the next example both the constant and the variable have perfect square factors.

Example:
Exercise:

Problem: **Simplify: $\sqrt{72n^7}$.

Solution:
Solution

	$\sqrt{72n^7}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{36n^6 \cdot 2n}$
Rewrite the radical as the product of two radicals.	$\sqrt{36n^6} \cdot \sqrt{2n}$
Simplify.	$6n^3\sqrt{2n}$

Note:
Exercise:

TRY IT : : 8.31
Problem: Simplify: $\sqrt{32y^5}$.

Solution:

$4y^2\sqrt{2y}$

Note:
Exercise:

TRY IT : : 8.32
Problem: Simplify: $\sqrt{75a^9}$.

Solution:

$5a^4\sqrt{3a}$

Example:
Exercise:

Problem: **Simplify: $\sqrt{63u^3v^5}$.

Solution:
Solution

	$\sqrt{63u^3v^5}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{9u^2v^4 \cdot 7uv}$
Rewrite the radical as the product of two radicals.	$\sqrt{9u^2v^4} \cdot \sqrt{7uv}$
Simplify.	$3uv^2\sqrt{7uv}$

Note:
Exercise:

TRY IT : : 8.33

Problem: Simplify: $\sqrt{98a^7b^5}$.

Solution:

$7a^3b^2\sqrt{2ab}$

Note:
Exercise:

TRY IT : : 8.34

Problem: Simplify: $\sqrt{180m^9n^{11}}$.

Solution:

$6m^4n^5\sqrt{5mn}$

We have seen how to use the Order of Operations to simplify some expressions with radicals. To simplify $\sqrt{25}+\sqrt{144}$ we must simplify each square root separately first, then add to get the sum of 17.

The expression $\sqrt{17}+\sqrt{7}$ cannot be simplified—to begin we'd need to simplify each square root, but neither 17 nor 7 contains a perfect square factor.

In the next example, we have the sum of an integer and a square root. We simplify the square root but cannot add the resulting expression to the integer.

Example:
Exercise:

Problem: **Simplify: $3+\sqrt{32}$.

Solution:
Solution

	$3+\sqrt{32}$
Rewrite the radicand as a product using the largest perfect square factor.	$3+\sqrt{16\cdot 2}$
Rewrite the radical as the product of two radicals.	$3+\sqrt{16}\cdot\sqrt{2}$
Simplify.	$3+4\sqrt{2}$

The terms are not like and so we cannot add them. Trying to add an integer and a radical is like trying to add an integer and a variable—they are not like terms!

Note:
Exercise:

TRY IT : : 8.35
Problem: Simplify: $5+\sqrt{75}$.

Solution:

$5+5\sqrt{3}$

Note:
Exercise:

TRY IT : : 8.36
Problem: Simplify: $2+\sqrt{98}$.

Solution:

$2+7\sqrt{2}$

The next example includes a fraction with a radical in the numerator. Remember that in order to simplify a fraction you need a common factor in the numerator and denominator.

Example:
Exercise:

Problem: **Simplify: $\frac{4-\sqrt{48}}{2}$.

Solution:
Solution

	$\frac{4-\sqrt{48}}{2}$
Rewrite the radicand as a product using the largest perfect square factor.	$\frac{4-\sqrt{16\cdot 3}}{2}$
Rewrite the radical as the product of two radicals.	$\frac{4-\sqrt{16}\cdot\sqrt{3}}{2}$
Simplify.	$\frac{4-4\sqrt{3}}{2}$
Factor the common factor from the numerator.	$\frac{4\left(1-\sqrt{3}\right)}{2}$
Remove the common factor, 2, from the numerator and denominator.	$\frac{\cancel{2}\cdot 2\left(1-\sqrt{3}\right)}{\cancel{2}}$
Simplify.	$2\left(1-\sqrt{3}\right)\text{or}2-2\sqrt{3}$

Note:
Exercise:

TRY IT : : 8.37

Problem: Simplify: $\frac{10-\sqrt{75}}{5}$.

Solution:

$2-\sqrt{3}$

Note:
Exercise:

TRY IT : : 8.38

Problem: Simplify: $\frac{6-\sqrt{45}}{3}$.

Solution:

$2-\sqrt{5}$

Use the Quotient Property to Simplify Square Roots

Whenever you have to simplify a square root, the first step you should take is to determine whether the radicand is a perfect square. A *perfect square fraction* is a fraction in which both the numerator and the denominator are perfect squares.

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{9}{64}}$.

Solution:
Solution

	$\sqrt{\frac{9}{64}}$
Since $\left(\frac{3}{8}\right)^2=\frac{9}{64}$	$\frac{3}{8}$

Note:
Exercise:

TRY IT : : 8.39

Problem: Simplify: $\sqrt{\frac{25}{16}}$.

Solution:

$\frac{5}{4}$

Note:
Exercise:

TRY IT : : 8.40

Problem: Simplify: $\sqrt{\frac{49}{81}}$.

Solution:

$\frac{7}{9}$

If the numerator and denominator have any common factors, remove them. You may find a perfect square fraction!

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{45}{80}}$.

Solution:
Solution

	$\sqrt{\frac{45}{80}}$
Simplify inside the radical first. Rewrite showing the common factors of the numerator and denominator.	$\sqrt{\frac{5\cdot 9}{5\cdot 16}}$
Simplify the fraction by removing common factors.	$\sqrt{\frac{9}{16}}$
Simplify. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$	$\frac{3}{4}$

Note:
Exercise:

TRY IT : : 8.41

Problem: Simplify: $\sqrt{\frac{75}{48}}$.

Solution:

$\frac{5}{4}$

Note:
Exercise:

TRY IT : : 8.42

Problem: Simplify: $\sqrt{\frac{98}{162}}$.

Solution:

$\frac{7}{9}$

In the last example, our first step was to simplify the fraction under the radical by removing common factors. In the next example we will use the Quotient Property to simplify under the radical. We divide the like bases by subtracting their exponents, $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$.

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{m^6}{m^4}}$.

Solution:
Solution

	$\sqrt{\frac{m^6}{m^4}}$
Simplify the fraction inside the radical first. Divide the like bases by subtracting the exponents.	$\sqrt{m^2}$
Simplify.	m

Note:
Exercise:

TRY IT : : 8.43

Problem: Simplify: $\sqrt{\frac{a^8}{a^6}}$.

Solution:

a

Note:
Exercise:

TRY IT : : 8.44

Problem: Simplify: $\sqrt{\frac{x^{14}}{x^{10}}}$.

Solution:

x^2

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{48p^7}{3p^3}}$.

Solution:
Solution

	$\sqrt{\frac{48p^7}{3p^3}}$
Simplify the fraction inside the radical first.	$\sqrt{16p^4}$
Simplify.	$4p^2$

Note:
Exercise:

TRY IT : : 8.45

Problem: Simplify: $\sqrt{\frac{75x^5}{3x}}$.

Solution:

$5x^2$

Note:
Exercise:

TRY IT : : 8.46

Problem: Simplify: $\sqrt{\frac{72z^{12}}{2z^{10}}}$.

Solution:

$6z$

Remember the Quotient to a Power Property? It said we could raise a fraction to a power by raising the numerator and denominator to the power separately.
Equation:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$$

We can use a similar property to simplify a square root of a fraction. After removing all common factors from the numerator and denominator, if the fraction is not a perfect square we simplify the numerator and denominator separately.

Note:
Quotient Property of Square Roots
If a, b are non-negative real numbers and $b \neq 0$, then
Equation:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{21}{64}}$.

Solution:
Solution

$$\sqrt{\frac{21}{64}}$$

We cannot simplify the fraction inside the radical. Rewrite using the quotient property.

$$\frac{\sqrt{21}}{\sqrt{64}}$$

Simplify the square root of 64. The numerator cannot be simplified.

$$\frac{\sqrt{21}}{8}$$

Note:
Exercise:

TRY IT : : 8.47

Problem: Simplify: $\sqrt{\frac{19}{49}}$.

Solution:

$\frac{\sqrt{19}}{7}$

Note:
Exercise:

TRY IT : : 8.48

Problem: Simplify: $\sqrt{\frac{28}{81}}$.

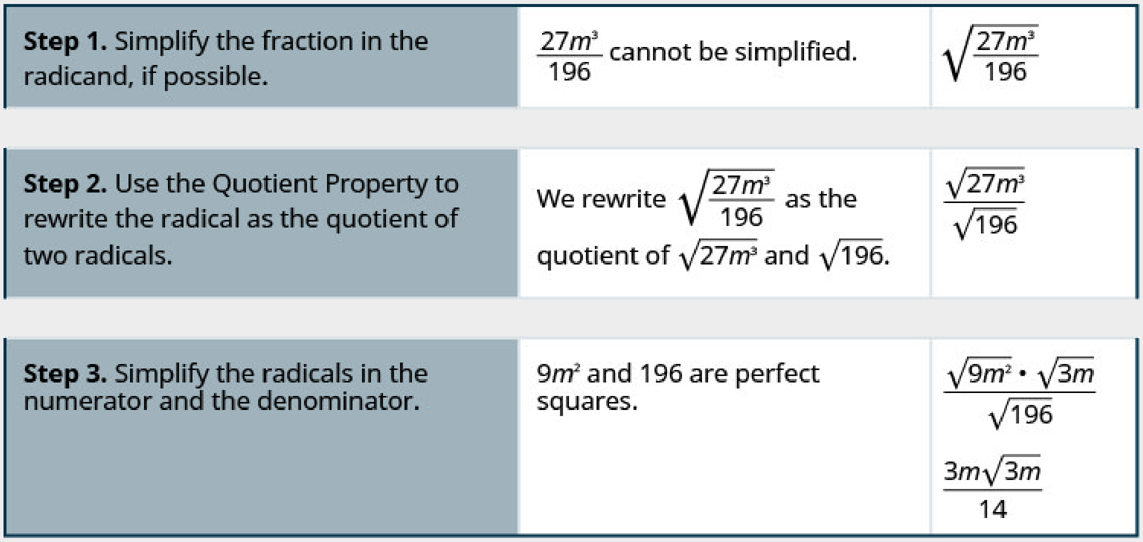
Solution:

$\frac{2\sqrt{7}}{9}$

Example:
How to Use the Quotient Property to Simplify a Square Root
Exercise:

Problem: **Simplify: $\sqrt{\frac{27m^3}{196}}$.

Solution:
Solution



Step 1. Simplify the fraction in the radicand, if possible.	$\frac{27m^3}{196}$ cannot be simplified.	$\sqrt{\frac{27m^3}{196}}$
Step 2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.	We rewrite $\sqrt{\frac{27m^3}{196}}$ as the quotient of $\sqrt{27m^3}$ and $\sqrt{196}$.	$\frac{\sqrt{27m^3}}{\sqrt{196}}$
Step 3. Simplify the radicals in the numerator and the denominator.	$9m^2$ and 196 are perfect squares.	$\frac{\sqrt{9m^2}\cdot\sqrt{3m}}{\sqrt{196}}$ $\frac{3m\sqrt{3m}}{14}$

Note:

Exercise:

TRY IT : : 8.49

Problem: Simplify: $\sqrt{\frac{24p^3}{49}}$.

Solution:

$\frac{2p\sqrt{6p}}{7}$

Note:
Exercise:

TRY IT : : 8.50

Problem: Simplify: $\sqrt{\frac{48x^5}{100}}$.

Solution:

$\frac{2x^2\sqrt{3x}}{5}$

Note:
Simplify a square root using the quotient property.

Simplify the fraction in the radicand, if possible.
Use the Quotient Property to rewrite the radical as the quotient of two radicals.
Simplify the radicals in the numerator and the denominator.

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{45x^5}{y^4}}$.

Solution:
Solution

	$\sqrt{\frac{45x^5}{y^4}}$
We cannot simplify the fraction in the radicand. Rewrite using the Quotient Property.	$\frac{\sqrt{45x^5}}{\sqrt{y^4}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{9x^4}\cdot\sqrt{5x}}{y^2}$
Simplify.	$\frac{3x^2\sqrt{5x}}{y^2}$

Note:
Exercise:

TRY IT : : 8.51

Problem: Simplify: $\sqrt{\frac{80m^3}{n^6}}$.

Solution:

$\frac{4m\sqrt{5m}}{n^3}$

Note:
Exercise:

TRY IT : : 8.52

Problem: Simplify: $\sqrt{\frac{54u^7}{v^8}}$.

Solution:

$\frac{3u^3\sqrt{6u}}{v^4}$

Be sure to simplify the fraction in the radicand first, if possible.

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{81d^9}{25d^4}}$.

Solution:
Solution

	$\sqrt{\frac{81d^9}{25d^4}}$
Simplify the fraction in the radicand.	$\sqrt{\frac{81d^5}{25}}$
Rewrite using the Quotient Property.	$\frac{\sqrt{81d^5}}{\sqrt{25}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{81d^4}\cdot\sqrt{d}}{5}$
Simplify.	$\frac{9d^2\sqrt{d}}{5}$

Note:
Exercise:

TRY IT : : 8.53

Problem: Simplify: $\sqrt{\frac{64x^7}{9x^3}}$.

Solution:

$\frac{8x^2}{3}$

Note:
Exercise:

TRY IT : : 8.54

Problem: Simplify: $\sqrt{\frac{16a^9}{100a^5}}$.

Solution:

$\frac{2a^2}{5}$

Example:
Exercise:

Problem: **Simplify: $\sqrt{\frac{18p^5q^7}{32pq^2}}$.

Solution:
Solution

	$\sqrt{\frac{18p^5q^7}{32pq^2}}$
Simplify the fraction in the radicand, if possible.	$\sqrt{\frac{9p^4q^5}{16}}$
Rewrite using the Quotient Property.	$\frac{\sqrt{9p^4q^5}}{\sqrt{16}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{9p^4q^4}\cdot\sqrt{q}}{4}$
Simplify.	$\frac{3p^2q^2\sqrt{q}}{4}$

Note:
Exercise:

TRY IT : : 8.55

Problem: Simplify: $\sqrt{\frac{50x^5y^3}{72x^4y}}$.

Solution:

$\frac{5y\sqrt{x}}{6}$

Note:
Exercise:

TRY IT : : 8.56

Problem: Simplify: $\sqrt{\frac{48m^7n^2}{125m^5n^9}}$.

Solution:

$\frac{4m\sqrt{3}}{5n^3\sqrt{5n}}$

Key Concepts

- **Simplified Square Root** $\sqrt{a}$ is considered simplified if a has no perfect-square factors.
- **Product Property of Square Roots** If a, b are non-negative real numbers, then
  Equation:

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

- **Simplify a Square Root Using the Product Property** To simplify a square root using the Product Property:

  Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect square factor.
  Use the product rule to rewrite the radical as the product of two radicals.
  Simplify the square root of the perfect square.

- **Quotient Property of Square Roots** If a, b are non-negative real numbers and $b \neq 0$, then
  Equation:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

- **Simplify a Square Root Using the Quotient Property** To simplify a square root using the Quotient Property:

  Simplify the fraction in the radicand, if possible.
  Use the Quotient Rule to rewrite the radical as the quotient of two radicals.
  Simplify the radicals in the numerator and the denominator.

Practice Makes Perfect

Use the Product Property to Simplify Square Roots

In the following exercises, simplify.
Exercise:

Problem: $\sqrt{27}$

Solution:

$3\sqrt{3}$

Exercise:

Problem: $\sqrt{80}$

Exercise:

Problem: $\sqrt{125}$

Solution:

$5\sqrt{5}$

Exercise:

Problem: $\sqrt{96}$

Exercise:

Problem: $\sqrt{200}$

Solution:

$10\sqrt{2}$

Exercise:

Problem: $\sqrt{147}$

Exercise:

Problem: $\sqrt{450}$

Solution:

$15\sqrt{2}$

Exercise:

Problem: $\sqrt{252}$

Exercise:

Problem: $\sqrt{800}$

Solution:

$20\sqrt{2}$

Exercise:

Problem: $\sqrt{288}$

Exercise:

Problem: $\sqrt{675}$

Solution:

$15\sqrt{3}$

Exercise:

Problem: $\sqrt{1250}$

Exercise:

Problem: $\sqrt{x^7}$

Solution:

$x^3\sqrt{x}$

Exercise:

Problem: $\sqrt{y^{11}}$

Exercise:

Problem: $\sqrt{p^3}$

Solution:

$p\sqrt{p}$

Exercise:

Problem: $\sqrt{q^5}$

Exercise:

Problem: $\sqrt{m^{13}}$

Solution:

$m^6\sqrt{m}$

Exercise:

Problem: $\sqrt{n^{21}}$

Exercise:

Problem: $\sqrt{r^{25}}$

Solution:

$r^{12}\sqrt{r}$

Exercise:

Problem: $\sqrt{s^{33}}$

Exercise:

Problem: $\sqrt{49n^{17}}$

Solution:

$7n^8\sqrt{n}$

Exercise:

Problem: $\sqrt{25m^9}$

Exercise:

Problem: $\sqrt{81r^{15}}$

Solution:

$9r^7\sqrt{r}$

Exercise:

Problem: $\sqrt{100s^{19}}$

Exercise:

Problem: $\sqrt{98m^5}$

Solution:

$7m^2\sqrt{2m}$

Exercise:

Problem: $\sqrt{32n^{11}}$

Exercise:

Problem: $\sqrt{125r^{13}}$

Solution:

$5r^6\sqrt{5r}$

Exercise:

Problem: $\sqrt{80s^{15}}$

Exercise:

Problem: $\sqrt{200p^{13}}$

Solution:

$10p^{6}\sqrt{2p}$

Exercise:

Problem: $\sqrt{128q^{3}}$

Exercise:

Problem: $\sqrt{242m^{23}}$

Solution:

$11m^{11}\sqrt{2m}$

Exercise:

Problem: $\sqrt{175n^{13}}$

Exercise:

Problem: $\sqrt{147m^{7}n^{11}}$

Solution:

$7m^{3}n^{5}\sqrt{3mn}$

Exercise:

Problem: $\sqrt{48m^{7}n^{5}}$

Exercise:

Problem: $\sqrt{75r^{13}s^{9}}$

Solution:

$5r^6s^4\sqrt{3rs}$ 70)

Exercise:

Problem: $\sqrt{96r^3s^3}$

Exercise:

Problem: $\sqrt{300p^9q^{11}}$

Solution:

$10p^4q^5\sqrt{3pq}$

Exercise:

Problem: $\sqrt{192q^3r^7}$

Exercise:

Problem: $\sqrt{242m^{13}n^{21}}$

Solution:

$11m^6n^{10}\sqrt{2mn}$

Exercise:

Problem: $\sqrt{150m^9n^3}$

Exercise:

Problem: $5+\sqrt{12}$

Solution:

$5+2\sqrt{3}$

Exercise:

Problem: $8+\sqrt{96}$

Exercise:

Problem: $1+\sqrt{45}$

Solution:

$1+3\sqrt{5}$

Exercise:

Problem: $3+\sqrt{125}$

Exercise:

Problem: $\frac{10-\sqrt{24}}{2}$

Solution:

$5-2\sqrt{6}$

Exercise:

Problem: $\frac{8-\sqrt{80}}{4}$

Exercise:

Problem: $\frac{3+\sqrt{90}}{3}$

Solution:

$1+\sqrt{10}$

Exercise:

Problem: $\frac{15+\sqrt{75}}{5}$

Use the Quotient Property to Simplify Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{\frac{49}{64}}$

Solution:

$\frac{7}{8}$

Exercise:

Problem: $\sqrt{\frac{100}{36}}$

Exercise:

Problem: $\sqrt{\frac{121}{16}}$

Solution:

$\frac{11}{4}$

Exercise:

Problem: $\sqrt{\frac{144}{169}}$

Exercise:

Problem: $\sqrt{\frac{72}{98}}$

Solution:

$\frac{6}{7}$

Exercise:

Problem: $\sqrt{\frac{75}{12}}$

Exercise:

Problem: $\sqrt{\frac{45}{125}}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\sqrt{\frac{300}{243}}$

Exercise:

Problem: $\sqrt{\frac{x^{10}}{x^6}}$

Solution:

x^2

Exercise:

Problem: $\sqrt{\frac{p^{20}}{p^{10}}}$

Exercise:

Problem: $\sqrt{\frac{y^4}{y^8}}$

Solution:

$\frac{1}{y^2}$

Exercise:

Problem: $\sqrt{\frac{q^8}{q^{14}}}$

Exercise:

Problem: $\sqrt{\frac{200x^7}{2x^3}}$

Solution:

$10x^2$

Exercise:

Problem: $\sqrt{\frac{98y^{11}}{2y^5}}$

Exercise:

Problem: $\sqrt{\frac{96p^9}{6p}}$

Solution:

$4p^4$

Exercise:

Problem: $\sqrt{\frac{108q^{10}}{3q^2}}$

Exercise:

Problem: $\sqrt{\frac{36}{35}}$

Solution:

$\frac{6}{\sqrt{35}}$

Exercise:

Problem: $\sqrt{\frac{144}{65}}$

Exercise:

Problem: $\sqrt{\frac{20}{81}}$

Solution:

$\frac{2\sqrt{5}}{9}$

Exercise:

Problem: $\sqrt{\frac{21}{196}}$

Exercise:

Problem: $\sqrt{\frac{96x^7}{121}}$

Solution:

$\frac{4x^3\sqrt{6x}}{11}$

Exercise:

Problem: $\sqrt{\frac{108y^4}{49}}$

Exercise:

Problem: $\sqrt{\frac{300m^5}{64}}$

Solution:

$\frac{10m^2\sqrt{3m}}{8}$

Exercise:

Problem: $\sqrt{\frac{125n^7}{169}}$

Exercise:

Problem: $\sqrt{\frac{98r^5}{100}}$

Solution:

$\frac{7r^2\sqrt{2r}}{10}$

Exercise:

Problem: $\sqrt{\frac{180s^{10}}{144}}$

Exercise:

Problem: $\sqrt{\frac{28q^6}{225}}$

Solution:

$\frac{2q^3\sqrt{7}}{15}$

Exercise:

Problem: $\sqrt{\frac{150r^3}{256}}$

Exercise:

Problem: $\sqrt{\frac{75r^9}{s^8}}$

Solution:

$\frac{5r^4\sqrt{3r}}{s^4}$

Exercise:

Problem: $\sqrt{\frac{72x^5}{y^6}}$

Exercise:

Problem: $\sqrt{\frac{28p^7}{q^2}}$

Solution:

$\frac{4p^3\sqrt{7p}}{q}$

Exercise:

Problem: $\sqrt{\frac{45r^3}{s^{10}}}$

Exercise:

Problem: $\sqrt{\frac{100x^5}{36x^3}}$

Solution:

$\frac{5x}{3}$

Exercise:

Problem: $\sqrt{\frac{49r^{12}}{16r^6}}$

Exercise:

Problem: $\sqrt{\frac{121p^5}{81p^2}}$

Solution:

$\frac{11p\sqrt{p}}{9}$

Exercise:

Problem: $\sqrt{\frac{25r^8}{64r}}$

Exercise:

Problem: $\sqrt{\frac{32x^5y^3}{18x^3y}}$

Solution:

$\frac{4xy}{3}$

Exercise:

Problem: $\sqrt{\frac{75r^6s^8}{48rs^4}}$

Exercise:

Problem: $\sqrt{\frac{27p^2q}{108p^5q^3}}$

Solution:

$\frac{1}{2pq\sqrt{p}}$

Exercise:

Problem: $\sqrt{\frac{50r^5s^2}{128r^2s^5}}$

Everyday Math

Exercise:

Problem:

ⓐ Elliott decides to construct a square garden that will take up 288 square feet of his yard. Simplify $\sqrt{288}$ to determine the length and the

width of his garden. Round to the nearest tenth of a foot.
ⓑ Suppose Elliott decides to reduce the size of his square garden so that he can create a 5-foot-wide walking path on the north and east sides of the garden. Simplify $\sqrt{288} - 5$ to determine the length and width of the new garden. Round to the nearest tenth of a foot.

Solution:

ⓐ $17.0\,\text{feet}$ ⓑ $15.0\,\text{feet}$

Exercise:

Problem:

ⓐ Melissa accidentally drops a pair of sunglasses from the top of a roller coaster, 64 feet above the ground. Simplify $\sqrt{\frac{64}{16}}$ to determine the number of seconds it takes for the sunglasses to reach the ground.
ⓑ Suppose the sunglasses in the previous example were dropped from a height of 144 feet. Simplify $\sqrt{\frac{144}{16}}$ to determine the number of seconds it takes for the sunglasses to reach the ground.

Writing Exercises

Exercise:

Problem: Explain why $\sqrt{x^4} = x^2$. Then explain why $\sqrt{x^{16}} = x^8$.

Solution:

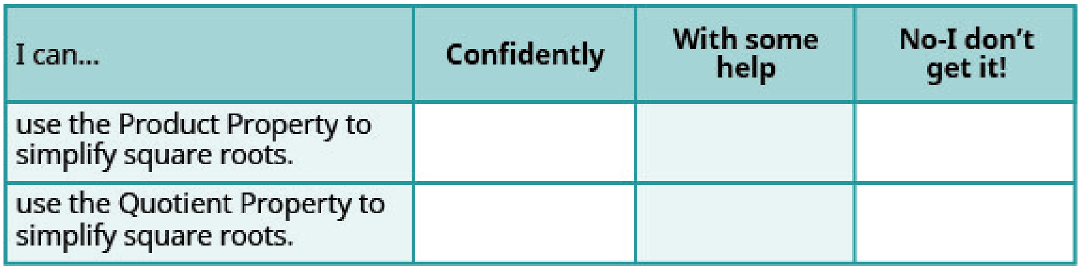
Answers will vary.

Exercise:

Problem: Explain why $7 + \sqrt{9}$ is not equal to $\sqrt{7 + 9}$.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
use the Product Property to simplify square roots.			
use the Quotient Property to simplify square roots.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Review & Practice Test
By the end of this section, you will be prepared to take the Chapter 8 Assessment.

Chapter 8 Review Exercises

Simplify and Use Square Roots

Simplify Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{64}$

Exercise:

Problem: $\sqrt{144}$

Solution:

12

Exercise:

Problem: $-\sqrt{25}$

Exercise:

Problem: $-\sqrt{81}$

Solution:

-9

Exercise:

Problem: $\sqrt{-9}$

Exercise:

Problem: $\sqrt{-36}$

Solution:

not a real number

Exercise:

Problem: $\sqrt{64}+\sqrt{225}$

Exercise:

Problem: $\sqrt{64+225}$

Solution:

17

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

Exercise:

Problem: $\sqrt{28}$

Exercise:

Problem: $\sqrt{155}$

Solution:

$12 < \sqrt{155} < 13$

Approximate Square Roots

In the following exercises, approximate each square root and round to two decimal places.

Exercise:

Problem: $\sqrt{15}$

Exercise:

Problem: $\sqrt{57}$

Solution:

7.55

Simplify Variable Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{q^2}$

Exercise:

Problem: $\sqrt{64b^2}$

Solution:

$8b$

Exercise:

Problem: $-\sqrt{121a^2}$

Exercise:

Problem: $\sqrt{225m^2n^2}$

Solution:

$15mn$

Exercise:

Problem: $-\sqrt{100q^2}$

Exercise:

Problem: $\sqrt{49y^2}$

Solution:

$7y$

Exercise:

Problem: $\sqrt{4a^2b^2}$

Exercise:

Problem: $\sqrt{121c^2d^2}$

Solution:

$11cd$

Simplify Square Roots

Use the Product Property to Simplify Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{300}$

Exercise:

Problem: $\sqrt{98}$

Solution:

$7\sqrt{2}$

Exercise:

Problem: $\sqrt{x^{13}}$

Exercise:

Problem: $\sqrt{y^{19}}$

Solution:

$y^9\sqrt{y}$

Exercise:

Problem: $\sqrt{16m^4}$

Exercise:

Problem: $\sqrt{36n^{13}}$

Solution:

$6n^6\sqrt{n}$

Exercise:

Problem: $\sqrt{288m^{21}}$

Exercise:

Problem: $\sqrt{150n^7}$

Solution:

$5n^3\sqrt{6n}$

Exercise:

Problem: $\sqrt{48r^5s^4}$

Exercise:

Problem: $\sqrt{108r^5s^3}$

Solution:

$6r^2s\sqrt{3rs}$

Exercise:

Problem: $\frac{10-\sqrt{50}}{5}$

Exercise:

Problem: $\frac{6+\sqrt{72}}{6}$

Solution:

$1+\sqrt{2}$

Use the Quotient Property to Simplify Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{\frac{16}{25}}$

Exercise:

Problem: $\sqrt{\frac{81}{36}}$

Solution:

$\frac{3}{2}$

Exercise:

Problem: $\sqrt{\frac{x^8}{x^4}}$

Exercise:

Problem: $\sqrt{\frac{y^6}{y^2}}$

Solution:

y^2

Exercise:

Problem: $\sqrt{\frac{98p^6}{2p^2}}$

Exercise:

Problem: $\sqrt{\frac{72q^8}{2q^4}}$

Solution:

$6q^2$

Exercise:

Problem: $\sqrt{\frac{65}{121}}$

Exercise:

Problem: $\sqrt{\frac{26}{169}}$

Solution:

$\frac{\sqrt{26}}{13}$

Exercise:

Problem: $\sqrt{\frac{64x^4}{25x^2}}$

Exercise:

Problem: $\sqrt{\frac{36r^{10}}{16r^5}}$

Solution:

$\frac{3r^2\sqrt{r}}{2}$

Exercise:

Problem: $\sqrt{\frac{48p^3q^5}{27pq}}$

Exercise:

Problem: $\sqrt{\frac{12r^5s^7}{75r^2s}}$

Solution:

$\frac{2rs^3\sqrt{r}}{5}$

Practice Test

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{81+144}$

Exercise:

Problem: $\sqrt{169m^4n^2}$

Solution:

$13m^2\left|n\right|$

Exercise:

Problem: $\sqrt{36n^{13}}$

Exercise:

Problem: $\left(3\sqrt{6y}\right)\left(2\sqrt{50y^3}\right)$

Solution:

$180y^2\sqrt{3}$

Exercise:

Problem: $\sqrt{\frac{64r^{12}}{25r^6}}$

Exercise:

Problem: $\sqrt{\frac{14y^3}{7y}}$

Solution:

$y\sqrt{2}$